Introduction to performance modeling

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Introduction to performance modeling in HPC



Performance modeling

is the process of building analytical or empirical models to predict and quantify the behavior of applications on HPC systems.

- $\boldsymbol{\cdot}$ Aims to identify performance bottlenecks and optimize resource utilization.
- · Enables understanding of the impact of system configuration and parameters on application performance.
- Facilitates design decisions for system upgrades or new architecture choices.



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Approaches for performance modeling

- Analytical modeling uses mathematical equations to represent the performance characteristics.
- · Empirical modeling uses historical data or benchmarking to build models based on observed behavior.



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Challenges in performance modeling

- Heterogeneous architectures and complex interactions between hardware and software components.
- Scalability issues when modeling large-scale systems with millions of concurrent processes.
- · Accuracy and validation of models against empirical performance measurements.
- · Model portability across different platforms and architectures.









Sports car

Family van

Transport truck





Sports car

• Power: 400 hp.

 \cdot Passengers: $2\,\mathrm{pax}$.



Family van

· Power: 150 hp.

· Passengers: 7 pax.



Transport truck

• Power: 800 hp.

· Passengers: 3 pax.





Sports car

• Power: 400 hp.

· Passengers: 2 pax.

• Capacity: $0.5 \,\mathrm{m}^3$.



Family van

· Power: 150 hp.

· Passengers: 7 pax.

• Capacity: $4 \, \mathrm{m}^3$.



Transport truck

- Power: $800\,\mathrm{hp}.$

 \cdot Passengers: $3\,\mathrm{pax}$.

 \cdot Capacity: $45\,\mathrm{m}^3$.





Sports car

• Power: 400 hp.

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· Speed: 350 m/s.



Family van

· Power: 150 hp.

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• Capacity: $4 \, \mathrm{m}^3$.

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Application A

To deliver the maximum power possible.





Sports car

- · Power: 400 hp.
- Passengers: 2 pax.
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Transport truck

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Application B

To win a speed race on a racing circuit.





Sports car

• Power: $400\,\mathrm{hp}$.

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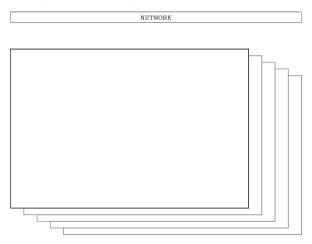
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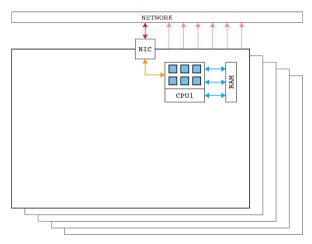
Application C

To travel from point A to B with 5 passengers.

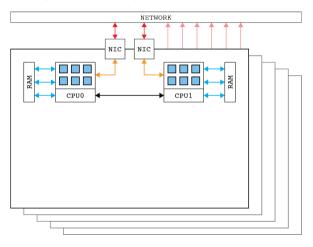




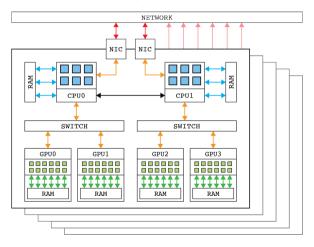






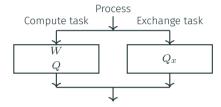






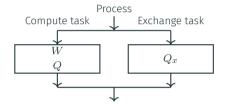
Abstract machine model

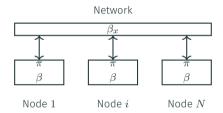




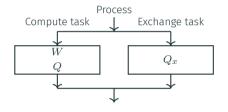
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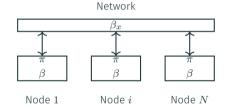






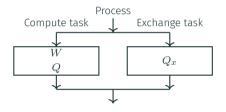




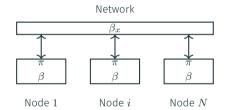


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Consider computing kernels that perform a vast number of independent operations and memory requests which can be pipelined and parallelized.





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$$y \leftarrow \alpha x + y$$
,



$$\boldsymbol{y} \leftarrow \alpha \boldsymbol{x} + \boldsymbol{y},$$

where α is a constant, and \boldsymbol{x} and \boldsymbol{y} are two vectors of size n. In such an operation, the processor performs:

- Work (W): n additions and n multiplications, that is, 2n floating-point operations.
- Memory traffic (Q): 2n elements read and n written, that is, $3n\cdot 8$ bytes in double precision.

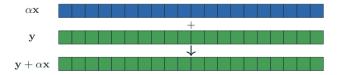


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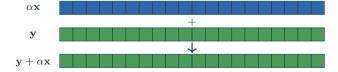


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The time required to perform the calculations is:

$$t^W = \frac{W}{\pi} = \frac{2n}{\pi}.$$



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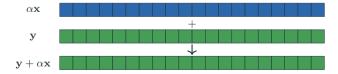
$$t^Q = \frac{Q}{\beta} = \frac{3n \cdot 8}{\beta}.$$

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The time required to execute the kernel:

$$t = \max(t^W, t^Q) = \max(\frac{2n}{\pi}, \frac{3n \cdot 8}{\beta}).$$



$$C \leftarrow \alpha AB + \beta C$$
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where α and β are constants, and $\mathbf{A} \in \mathbb{R}^{m \times k}$, $\mathbf{B} \in \mathbb{R}^{k \times n}$, and $\mathbf{C} \in \mathbb{R}^{m \times n}$ are matrices. In such an operation, the processor performs:

- Work (W): mn(k+3) additions and multiplications, that is, approximately, 2mnk floating-point operations.
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$$t = \max(\frac{2mnk}{\pi}, \frac{8(mk + kn + 2mn)}{\beta}).$$

The roofline model



The elapsed time of a kernel running on a single computing unit is estimated as follows:

$$t^{RL} = \max\left(\frac{W}{\pi}, \frac{Q}{\beta}\right),$$

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Application performance is often considered more relevant than elapsed time itself. Let's divide the work W by $t^{RL}\colon$

$$R^{RL} = \frac{W}{t^{RL}} = \min\left(\pi, \frac{W}{Q}\beta\right) = \min(\pi, I\beta).$$

The model introduces operational intensity, *I*, as the ratio of floating-point operations, in flops, to the memory traffic, in bytes.



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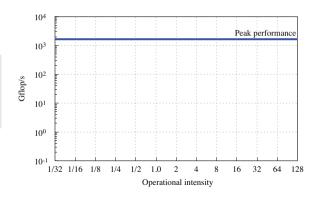
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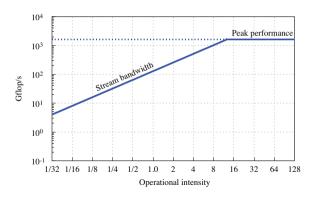
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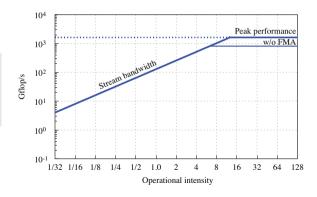
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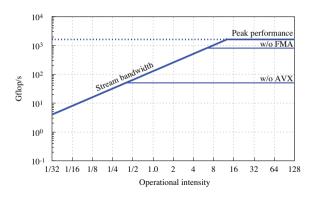
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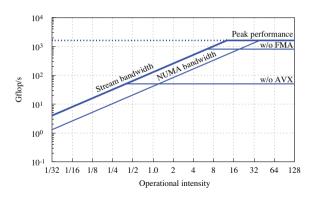
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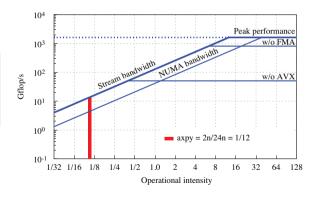
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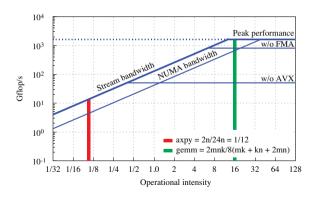
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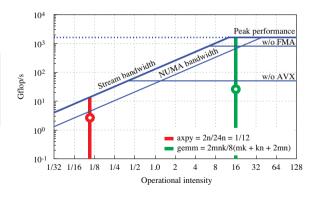
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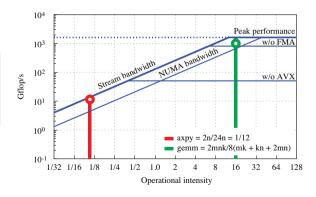
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HPCG benchmark, November 2022

Rank	Site	Computer	Cores	HPL Rmax (Pflop/s)	TOP500 Rank	HPCG (Pflop/s)	Fraction of Peak
1	RIKEN Center for Computational Science Japan	Supercomputer Fugaku — A64FX 48C 2.2GHz, Tofu interconnect D	7,630,848	442.01	2	16.00	3.0%
2	DOE/SC/Oak Ridge National Laboratory United States	Frontier — AMD Optimized 3rd Generation EPYC 64C 2GHz, Slingshot-11, AMD Instinct MI250X	8,730,112	1102.00	1	14.05	0.8%
3	EuroHPC/CSC Finland	LUMI — AMD Optimized 3rd Generation EPYC 64C 2GHz, Slingshot-11, AMD Instinct MI250X	2,220,288	309.10	3	3.408	0.8%
4	DOE/SC/Oak Ridge National Laboratory United States	Summit — IBM POWER9 22C 3.07GHz, Dual-rail Mellanox EDR Infiniband, NVIDIA Volta GV100	2,414,592	148.60	5	2.926	1.5%
5	EuroHPC/CINECA Italy	Leonardo — Xeon Platinum 8358 32C 2.6GHz, Quad-rail NVIDIA HDR100 Infiniband, NVIDIA A100 SXM4 64 GB	1,463,616	174.70	4	2.567	1.0%



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- · Choose specific models for each application based on its characteristics.
- · Analytical models, like the roofline model, identify performance bottlenecks and guide optimization efforts.
- · Identify parallelism, optimize resource utilization, and improve system efficiency through performance modeling.

Thank you for your attention