

Practical Session 4: Scheme (Functional programming)

1 Functional Programming

Scheme follows the functional programming paradigm. To put it simply, functional programming is built around the concept of *functions*, viewed as mathematical objects: a function takes some inputs and returns an output, with no other side-effects. Functional programming avoids mutation of data: an argument passed to a function is never modified itself, it's a new object which is returned as the result of applying a function to an argument. In functional programming, a program is viewed as a series of functions applied to input data.

2 First-class functions

Like many other functional programming languages, Scheme has first-class functions. This means that functions can be treated like any other entities and the program, and more specifically, a function can be passed as argument to, can be returned from a function and can be stored in a variable.

Write a `(filter p l)` function. This function takes two arguments: the first is a predicate p and the second is a list l . It returns the list l , in which all the elements for which p is false have been removed. Consider this example:

```
(define mylist '(1 2 -3 4 5 -6 7 8 -9))  
(filter positive? mylist)  
  
-> '(1 2 4 5 7 8)
```

`positive?` is of course a built-in Scheme predicate which returns `#t` if its argument is a positive number.

In the previous session, you wrote a `sum-cubes` function which may have looked like the following:

```
(define (sum-cubes a b)
  (if (> a b)
      0
      (+ (cube a) (sum-cubes (+ a 1) b))))
```

Rewrite a more generic `sum-func` which takes three arguments f , a and b and which returns the sum of f applied to all integers in $\{a, a + 1, \dots b\}$. Uses this function to perform a `sum-cubes`, for instance:

```
(sum-func cube 0 3)
-> 36
```

Finally, define `sum-cubes` in terms of this new `sum-func`.

2.1 Anonymous functions

Sometimes it can be useful to write a small function as argument to another directly in the function call, without `define`'ing it entirely. This is where `lambda`'s become handy. For instance, to use `filter` with a predicate that keeps all items smaller than 5:

```
(filter (lambda (x) (< x 5)) mylist)
-> (1 2 -3 4 -6 -9)
```

The `lambda` expression creates an anonymous function which, to x associates the boolean $x \neq 5$.

2.2 Map and apply

Some functions defined in the Scheme standard take a functional parameter: for instance `apply` or `map`. They are important functional programming tools which are also found in many modern languages (sometimes under a different name)

1. Try to guess / understand what each does based on the following examples:

```
(define mylist '(1 2 -3 4 5 -6 7 8 -9))

(map (lambda (x) (+ x 1)) mylist)
(apply + mylist)
(apply max mylist)
```

2. Write a `(range a b)` utility function which produces a list of all integers from a to b , included. Then use this function to re-implement `sum-cubes` in terms of `map` and `apply`.
3. Write a function which takes a list of lists and returns a list containing the maximum of each list.

```
(define listoflists '((1 2 3 1) (45 1 3 4 5) (4 5 64)
                     (4 6) (144) (0 4 4) (14 464 4 7 6)))

(max-list listoflists)
-> '(3 45 64 6 144 4 464)
```

Another common operation we might want to do on list is to *accumulate* the result of a function over all the arguments of this list. In many languages, this function is called `fold`. However, the Scheme standard does not define this function.

Write an `accumulate` function. This function will have the following interface :

```
(accumulate init-value function list)
```

And its value is the application of the two-parameter `function` successively on all elements of `list`, with the first parameter being the accumulated value so far, starting with `init-value`. In other words, `(accumulate x0 f '(l1 l2 ... ln))` should return

$$f(l_n, f(l_{n-1}, \dots f(l_2, f(l_1, x_0))))$$

For instance :

```
(accumulate 1 * '(2 3 4))  
> 24
```

```
(accumulate 14 + '(2 3 4))  
> 23
```

Take care of the parameter order ($f(a, b)$ may not be equal to $f(b, a)$! Respect the order given above).

Can you use this function to reverse a list?

2.3 Higher-order functions

As said above, it is possible for a function to return a function (think for example about what the `lambda` builtin returns).

Write a function `mean` which takes as argument a numeric function $f : \mathbb{R} \rightarrow \mathbb{R}$ and returns the function:

$$f' : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \quad x, y \mapsto \frac{f(x) + f(y)}{2}$$

Use this function with the `cube` function to generate a `mean-cube` function. Then compute the mean-cube of 4 and 16.

```
(mean-cube 4 16)  
-> 2080
```

3 Special forms and parameter evaluation

From the syntax, it looks like `if` in Scheme is just another function of three arguments: `(if condition true-expr false-expr)`. However, we will see that there is a slight caveat to this approach and that, under the hood, things are a bit different.

Implement a (`my-if condition true-expr false-expr`) function in terms of `cond`. Try it with the following expressions:

```
(define x 4)
(my-if (positive? x) x (- x))

(my-if (positive? x)
      (display "positive")
      (display "negative"))
```

What goes on in your `my-if` function is that the interpreter first evaluates *all* the parameters passed to the function. In the first example, this does not matter, because `x` and `(- x)` simply evaluate to 4 and -4, without any side-effects. But in the second example, both arguments have the side-effect of displaying some text on the output, and their evaluation produces `#<void>` (this is an implementation choice in racket. The standard simply specifies that `display`, returns “an unspecified value”). In other words, both the true and false blocks are evaluated upon parameter passing, and the function call actually becomes `(my-if #t #<void> #<void>)` and the `display` have already printed out their arguments.

3.1 Hygienic macros

Because of this, some constructs in Scheme are actually *special forms* and not functions. In a special form, the arguments are only evaluated when called within the special form, and not when passed as arguments. It is possible to define your own special forms using `define-syntax` and `syntax-rules`. You can refer to the R5RS¹ for a reference, or some tutorials available online².

Rewrite `my-if` using a hygienic macro. Do you observe the problem discovered above?

¹<http://www.schemers.org/Documents/Standards/R5RS/>

²See for instance <http://www.willdonnelly.net/blog/scheme-syntax-rules/>