

$n$ : number of clusters  $T$ : # periods

$x_{i,t} = \begin{cases} 1, & \text{station at cluster } i \text{ during } t \\ 0, & \text{otherwise} \end{cases}$   $t \in 1, \dots, T$

$y_{i,t}$  = # bikes in station of cluster  $i$  during  $t$   $t \in 1, \dots, T$

~~where  $z_{i,t}$  is the number of docks in station  $i$  during  $t$~~

$z_{i,t}$  = # docks in station  $i$  during  $t \in 1, \dots, T$

$x_{i,t} \leq z_{i,t} \leq M_i x_{i,t}$  ① where  $M_i$  is the maximum number of docks for station  $i \forall i \forall t$

$x_{i,t} \leq y_{i,t} \leq z_{i,t}$  ②  $\forall i \forall t$

$m_{ij}$  = <sup>average</sup> # times a bike started from somewhere in cluster  $i$  and ended up in cluster  $j$

$\sum_j m_{ij}$  = # times a bike started from  $i$

$\sum_i m_{ij}$  = # times a bike ended up in  $j$

$m = \sum_i \sum_j m_{ij}$  = total number of trips

$\frac{1}{2m} (\sum_j m_{ij} + \sum_j m_{ji})$  = proportion of demand for a station at cluster  $i$



$a_{ik}$  = number of  $k$  buildings in cluster  $i$

where  $k$  = [residential, commercial, school, university, hospital, library]

$p_k$  = weight for  $k$  building

~~Social value =  $\sum_k \sum_i a_{ik} d_{ik} (m_i + w_k) / i$~~   
(see pg. 4 for reworked version)

objective function: = social value

$C_1$  = cost of making a dock +  $\left( \frac{\text{hangar cost} + \text{docks}}{\text{hangar}} \right)$   
 $C_2$  = cost of making a bike

$C_{car}$  = user cost for car fuel/time

$C_{max}$  = maximum amount of money we can expend

$p_{car}$  = proportion of population initially using car

$$\text{total\_cost} = C_1 \sum_i \bar{z}_i + C_2 \sum_i \bar{y}_i$$

environmental\\_value =

$$= p_{car} C_{car} \sum_i \sum_j m_{ij} h_{ij} \bar{x}_i$$

where  $h_{ij}$  = average time to go from  $i$  to  $j$

$$\text{total\_cost} \leq C_{max} \quad (3)$$

environmental\\_value  $\geq$  (proportion)  $\times$

$\times$  (total cost for car fuel for all residents in a e.g. month) (4)

⑤ forced underperforming stations open regardless of viability  
if  $m_{ij} > m_{ji}$

~~$m_{ij} - m_{ji}$~~

$$\text{total\_cost} \leq l_1 \cdot \text{social\_value} + l_2 \cdot \text{environmental\_value}$$

[6 maybe?]

!!! Constraint 7: extra docks for more bikes  
based on previous stats (confidence interval)!!!

$$\bar{x}_i = \sum_{t=1}^T x_{it} \quad \forall i \in 1, \dots, n : \begin{cases} 1 & \text{if station } i \text{ ever opened} \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{\varphi}_i = \sum_{t=1}^T \varphi_{it} \quad \forall i \in 1, \dots, n : \# \text{ of bikes ever put @ } i$$

$$\bar{z}_i = \sum_{t=1}^T z_{it} \quad \forall i \in 1, \dots, n : \# \text{ docks put @ } i \text{ over all periods}$$

⑧

$$\bar{x}_i \leq 1 : (\text{can only open } i \text{ once})$$

$$d_t = \% \text{ reduction of social value associated w/ opening stations in period } t \quad (d_t \in [0, 1] \forall t)$$

(obj. f(x))

(max)

$$\text{social value} : \sum_t \sum_k \sum_i d_t p_k a_{ik} \sum_j (m_{ij} + m_{ji}) \varphi_{it}$$

$$q_t = \% \text{ of total budget } (C_{\max}) \text{ available during period } t \quad (q_t \in [0, 1] \forall t \wedge \sum_t q_t = 1)$$

⑨

$$\text{periodic costs} : \sum_i c_1 z_{it} + c_2 \varphi_{it} \leq q_t C_{\max} \quad \forall t \in 1, \dots, T$$