

$n$ : number of clusters

$x_i = \begin{cases} 1, & \text{station at cluster } i \\ 0, & \text{otherwise} \end{cases}$

$y_i = \# \text{ bikes in station of cluster } i$

~~$z_i = \# \text{ docks in station } i$~~

$z_i = \# \text{ docks in station } i$

$x_i \leq z_i \leq M_i x_i$  ① where  $M_i$  is the maximum number of docks for station  $i$

$x_i \leq y_i \leq z_i$  ②

$m_{ij} = \overset{\text{average}}{\#} \text{ times a bike started from somewhere in cluster } i \text{ and ended-up in cluster } j$

$m_{ii} = 0 \quad \forall i$

$\sum_j m_{ij} = \# \text{ times a bike started from } i$

$\sum_i m_{ij} = \# \text{ times a bike ended up in } j$

$m = \sum_i \sum_j m_{ij} = \text{total number of trips}$

$\frac{1}{2m} (\sum_j m_{ij} + \sum_j m_{ji}) = \text{proportion of demand for a station at cluster } i$

$p_i \in \{0, 1, 2\}$ : population density weight

$\varepsilon_i = \begin{cases} 1, & \text{cluster } i \text{ point of interest} \\ 0, & \text{otherwise} \end{cases}$



$$- \text{social\_value} = \sum_i \frac{(p_i + \varepsilon_i)}{\sum m} \sum_j (m_{ij} + m_{ji}) y_i$$

!!! Constraint addition = extra docks for more bikes based on previous stats (confidence)!!

$h_{ij}$  = average time to go from  $i$  to  $j$  with bike

$C_1$  = cost of making a dock  
 $C_2$  = cost of making a bike

$C_{\text{bike}}$  = user cost for bike / time

$C_{\text{car}}$  = user cost for car fuel / time

$p_{\text{car}}$  = proportion of population initially using car

$$- \text{total\_cost} = -C_1 \sum_i z_i - C_2 \sum_i y_i + C_{\text{bike}} \sum_i \sum_j m_{ij} h_{ij} x_i$$

$$- \text{environmental\_value} = p_{\text{car}} C_{\text{car}} \sum_i \sum_j m_{ij} h_{ij} x_i$$

$$\{ \text{if } p_i = \varepsilon_i = 0 \Rightarrow x_i = 0 \} \Rightarrow x_i \leq p_i + \varepsilon_i \quad \forall i \quad (3)$$

if  $x_i = 1$ :

$$y_i \geq \sum_j (m_{ij} - m_{ji}) \quad (4)$$