Code

September 11, 2019

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import math
        import pandas as pd
```

1 Linear regression

1.1 Task 1

```
In [2]: def load_csv(filepath):
            df = pd.read_csv(filepath)
            df.insert(loc=0, column='x0', value=np.ones(len(df)))
            return df.loc[:, df.columns != 'y'].values, df['y'].values

        def mse(y, y_pred):
            return np.mean((y_pred - y)**2)

        def linear_regression(X, y):
            return np.linalg.pinv(X.T.dot(X)).dot(X.T).dot(y)
```

1.2 Task 2

```
In [3]: X_train, y_train = load_csv('dataset/regression/train_2d_reg_data.csv')
        X_test, y_test = load_csv('dataset/regression/test_2d_reg_data.csv')
        w_train = linear_regression(X_train, y_train)
        training_error = mse(X_train.dot(w_train), y_train)
        test_error = mse(X_test.dot(w_train), y_test)

        print("w0: " + str(w_train[0]) + "\tw1: " + str(w_train[1])
              + "\tw2: " + str(w_train[2]))
        print("\nMean Squared Error (MSE)")
        print("MSE_training: " + str(training_error) + "\tMSE_test: " + str(test_error))
```

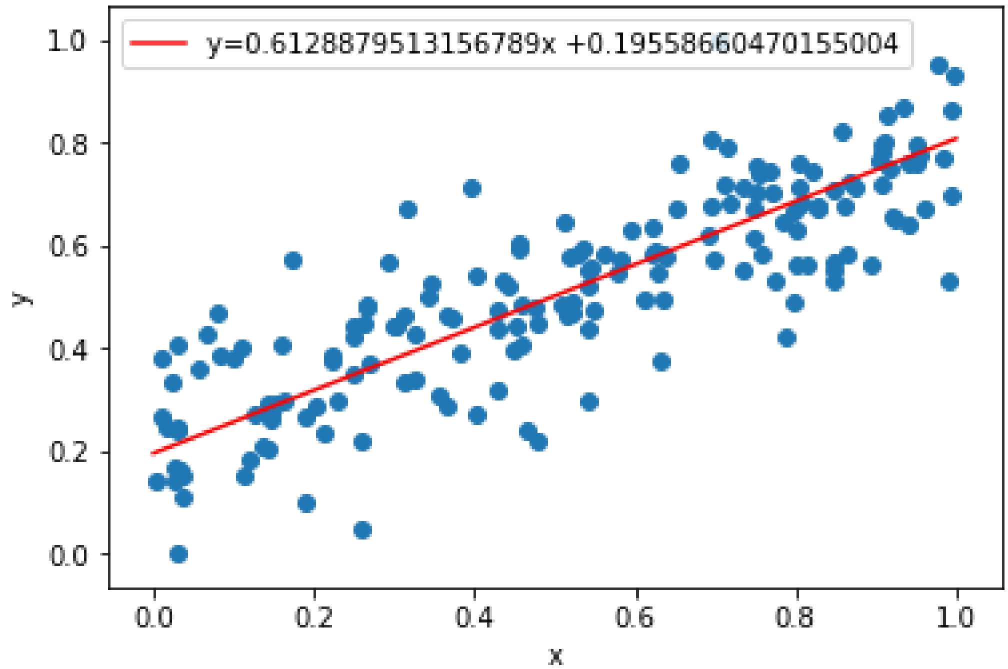
```
w0: 0.24079270870924657        w1: 0.4815568550719211        w2: 0.05864390258297314

Mean Squared Error (MSE)
MSE_training: 0.010386850731462317        MSE_test: 0.009529764450619004
```

As we can see, the model generalizes quite well. The MSE for the training- and the test data are relatively equal. Hence, there is no clear signs of neither over- nor underfitting.

1.3 Task 3

```
In [4]: X_train, y_train = load_csv('dataset/regression/train_1d_reg_data.csv')
        X_test, y_test = load_csv('dataset/regression/test_1d_reg_data.csv')
        w_train = linear_regression(X_train, y_train)
        w_test = linear_regression(X_test, y_test)
        training_error = mse(X_train.dot(w_train), y_train)
        test_error = mse(X_test.dot(w_test), y_test)

        print("w0: " + str(w_train[0]) + "\tw1: " + str(w_train[1]))
        print("\nMean Squared Error (MSE)")
        print("MSE_training: " + str(training_error) + "\tMSE_test: " + str(test_error))

        x_point = X_test[:, -1]
        y_point = y_test
        x_axes = np.linspace(0,1,100)
        line = w_train[1]*x_axes + w_train[0]
        plt.scatter(x_point, y_point)
        plt.plot(x_axes, line, '-r', label= 'y=' + str(w_train[1]) + "x +" + str(w_train[0]))
        plt.legend(loc='upper left')
        plt.xlabel("x")
        plt.ylabel("y")
        plt.show()
```

```
w0: 0.19558660470155004        w1: 0.6128879513156789

Mean Squared Error (MSE)
MSE_training: 0.013758791126537117        MSE_test: 0.012143754475727526
```



The line seem to fit the datapoints relatively well. As we can see from the scatterplot, the data contains a fair bit of variance. Some error could have been remedied by adding some dimensions to the training data, but this might result in overfitting.

2 Logistic regression

```
In [5]: def load_csv(filepath):
            df = pd.read_csv(filepath)
            df.columns = ['x1', 'x2', 'y']
            df['x0'] = df['x1'].pow(0)
            df['x1^2'] = df['x1'].pow(2)
            df['x2^2'] = df['x2'].pow(2)
            return df

        def sigmoid(z):
            return float(1.0 / float((1.0 + math.exp(-1.0 * z))))

        def split_data(data, feature_names):
            return data[feature_names].values, data['y'].values

        def hypothesis(weights, x):
            z = 0
            for i in range(len(weights)):
                z += x[i] * weights[i]
            return sigmoid(z)
```

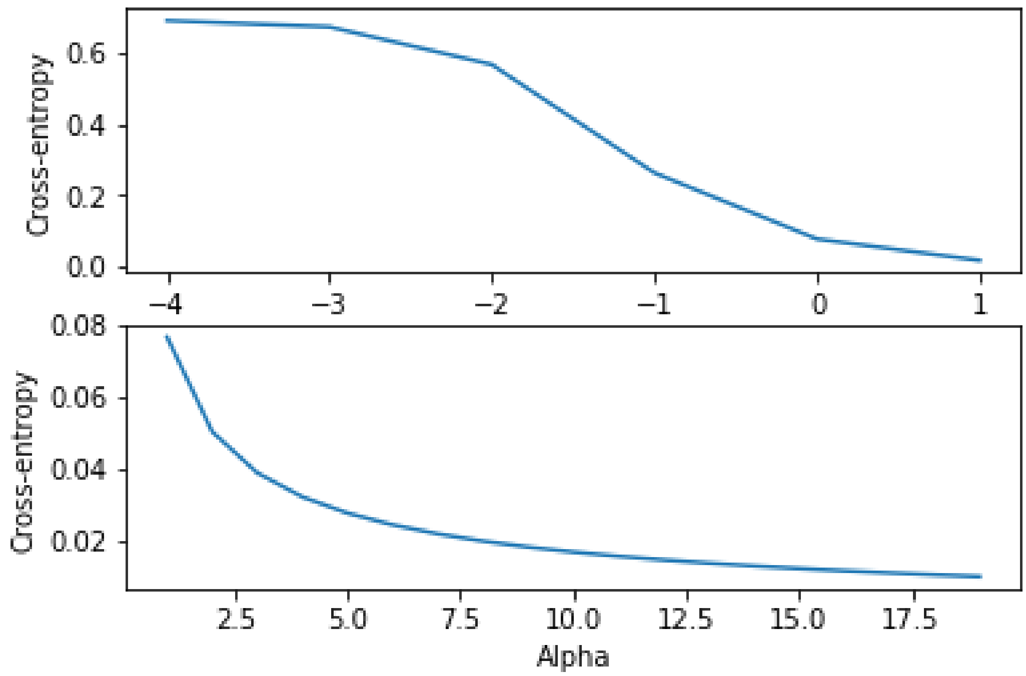
```python
def cross_entropy_error(X, Y, weights):
    sum_errors = 0
    for i in range(len(Y)):
        if Y[i] == 1:
            error = Y[i] * math.log(hypothesis(weights, X[i]))
        else:
            error = (1 - Y[i]) * math.log(1 - hypothesis(weights, X[i]))
        sum_errors += error
    return - 1 / len(Y) * sum_errors

def cross_entropy_derivative(X, Y, weights, j):
    sum_errors = 0
    for i in range(len(Y)):
        sum_errors += (hypothesis(weights, X[i]) - Y[i]) * X[i][j]
    return sum_errors

def update_rule(X, Y, weights, alpha):
    new_weights = []
    error = cross_entropy_error(X, Y, weights)
    for j in range(len(weights)):
        new_weights_value = (weights[j] - float(alpha) /
                             float(len(Y)) * cross_entropy_derivative(X, Y, weights, j))
        new_weights.append(new_weights_value)
    return new_weights, error

def logistic_regression(X, Y, alpha, weights, iter):
    cross_entropy_series = []
    for x in range(iter):
        new_weights, error = update_rule(X, Y, weights, alpha)
        weights = new_weights
        cross_entropy_series.append(error)
    return weights, cross_entropy_series

def train(path_train_data, initial_weights, feature_name, alpha, iterations):
    df_train = load_csv(path_train_data)
    X_train, y_train = split_data(df_train, feature_name)
    return logistic_regression(X_train, y_train, alpha, initial_weights, iterations)

def test(path_test_data, trained_weights, feature_name):
    df = load_csv(path_test_data)
    X, Y = split_data(df, feature_name)
    correct_pred = 0
    for i in range(len(X)):
        if int(round(hypothesis(trained_weights, X[i]))) == Y[i]:
            correct_pred += 1
    accuracy = correct_pred / len(df)
    return accuracy
```

```python
def plot_cross_entropy(cross_entropy_series_traindata,
                       cross_entropy_series_testdata, iterations):
    x_values = np.linspace(0, iterations, iterations)
    plt.plot(x_values, cross_entropy_series_traindata,
             label='Cross-entropy train data')
    plt.plot(x_values, cross_entropy_series_testdata,
             label='Cross-entropy test data')
    plt.xlabel('Iteration number')
    plt.ylabel('Cross-entropy')
    plt.legend()
    plt.show()

def plot_linear_decision_boundary(filepath_data, trained_weights, iterations):
    df = load_csv(filepath_data)
    admitted = df.loc[df['y'] == 1]
    not_admitted = df.loc[df['y'] == 0]
    x_axes = np.linspace(0, 1, iterations)
    line = - ((trained_weights[0] + np.dot(trained_weights[1], x_axes))
              / trained_weights[2])
    plt.scatter(admitted.iloc[:, 0], admitted.iloc[:, 1], s=10, label='y=1')
    plt.scatter(not_admitted.iloc[:, 0], not_admitted.iloc[:, 1], s=10, label='y=0')
    plt.plot(x_axes, line, '-r', label='Decision boundary')
    plt.xlabel('x1')
    plt.ylabel('x2')
    plt.legend()
    plt.show()

def plot_radial_decision_boundary(filepath_data, trained_weights, iterations):
    df = load_csv(filepath_data)
    admitted = df.loc[df['y'] == 1]
    not_admitted = df.loc[df['y'] == 0]
    x = np.linspace(0, 1, 1000)
    y = np.linspace(0, 1, 1000)
    xx, yy = np.meshgrid(x, y)
    z = np.zeros((1000, 1000))

    for i in range(xx.shape[0]):
        for j in range(xx.shape[1]):
            val = np.dot([1, xx[i, j], yy[i, j], np.square(xx[i, j]),
                          np.square(yy[i, j])], trained_weights)
            if val < 0:
                z[i, j] = 1

    plt.contour(xx, yy, z, [1])
    plt.scatter(admitted.iloc[:, 0], admitted.iloc[:, 1], s=10, label='y=1')
    plt.scatter(not_admitted.iloc[:, 0], not_admitted.iloc[:, 1], s=10, label='y=0')
    plt.xlabel('x1')
```

```
    plt.ylabel('x2')
    plt.legend()
    plt.show()
```

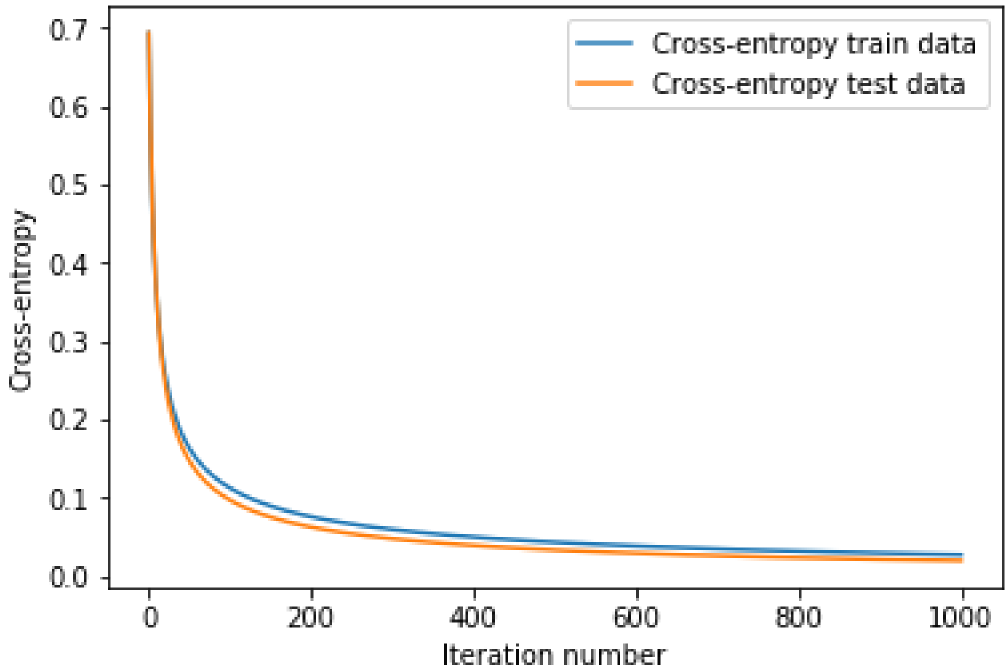
2.1 Task 1

```
In [6]: # Parameters
        path_train_data = 'dataset/classification/cl_train_1.csv'
        path_test_data = 'dataset/classification/cl_test_1.csv'
        initial_weights = [0, 0, 0]
        iterations = 1000
        feature_name = ['x0', 'x1', 'x2']

        # Plot performance with different learning rates
        train_score_range1 = []
        train_score_range2 = []

        # check the model performance for alpha on training data (using powers of 10)
        for a in range(-4, 2):
            w_train, e_train = train(path_train_data, initial_weights,
                                     feature_name, math.pow(10,a), iterations)
            train_score_range1.append(e_train[-1])

        # using a different range
        for a in range(1, 20):
            w_train, e_train = train(path_train_data, initial_weights,
                                     feature_name, a, iterations)
            train_score_range2.append(e_train[-1])

        fig, (ax1, ax2) = plt.subplots(2,1)
        ax1.plot(range(-4,2), train_score_range1)
        ax2.plot(range(1,20), train_score_range2)
        ax1.set(xlabel='Alpha', ylabel='Cross-entropy')
        ax2.set(xlabel='Alpha', ylabel='Cross-entropy')
        plt.show()
```



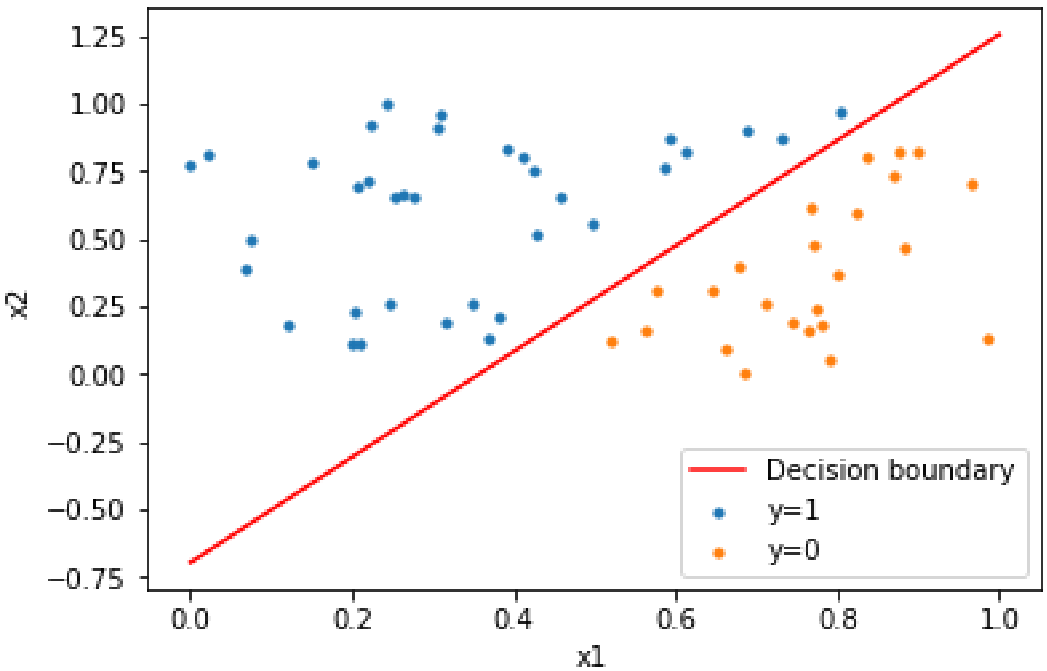
We can see that the performance is best with rather high learning rates. I will use alpha = 5 here.

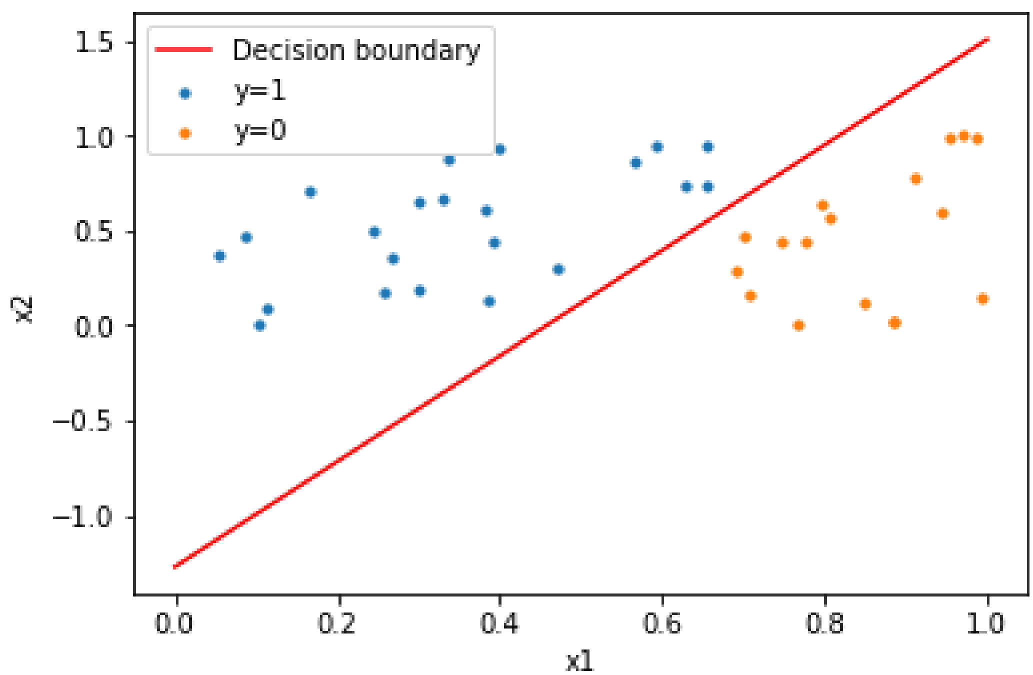
```
In [7]: alpha = 5

        # Train weights
        w_train, ce_train = train(path_train_data, initial_weights,
                                  feature_name, alpha, iterations)
        w_test, ce_test = train(path_test_data, initial_weights,
                                feature_name, alpha, iterations)

        # Plot cross-entropy
        plot_cross_entropy(ce_train, ce_test, iterations)
        print("Cross-entropy error training data:",ce_train[-1])
        print("Cross-entropy error test data:",ce_test[-1])
        print('\n')

        # Test precision
        accuracy = test(path_test_data, w_train, feature_name)
        print("Prediction accuracy:", accuracy)

        # Plot decision boundary
        plot_linear_decision_boundary(path_train_data, w_train, iterations)
        plot_linear_decision_boundary(path_test_data, w_test, iterations)
```




```
Cross-entropy error training data: 0.02780647804921424
Cross-entropy error test data: 0.020269393422011946
```

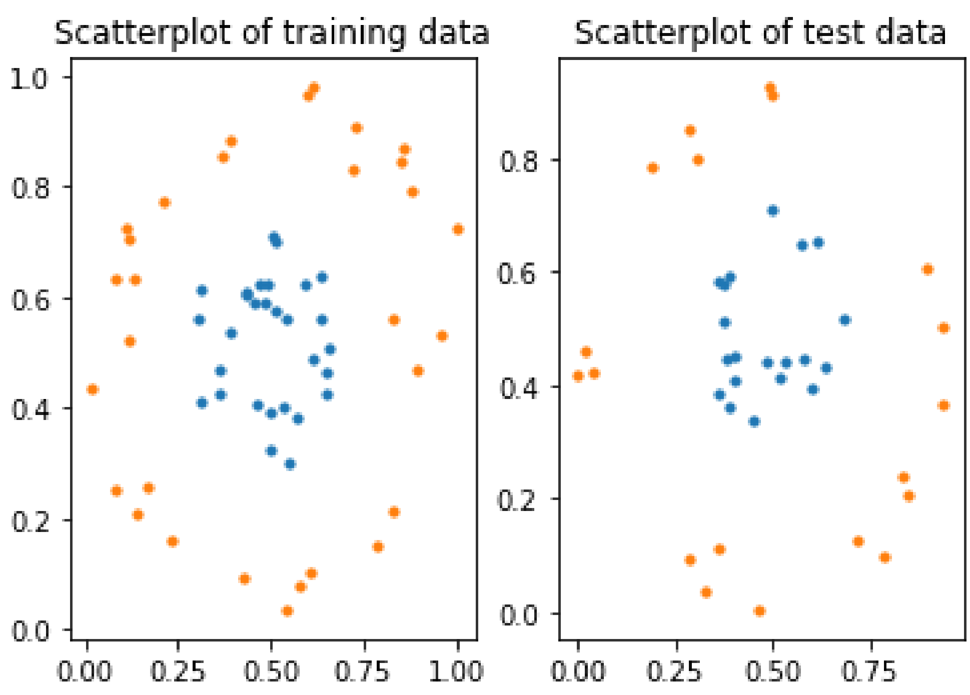
```
Prediction accuracy: 1.0
```


1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
x2
0.0
0.2
0.4
0.6
0.8
1.0
x1
Decision boundary
y=1
y=0


1.5
1.0
0.5
0.0
−0.5
−1.0
x2
0.0
0.2
0.4
0.6
0.8
1.0
x1
Decision boundary
y=1
y=0

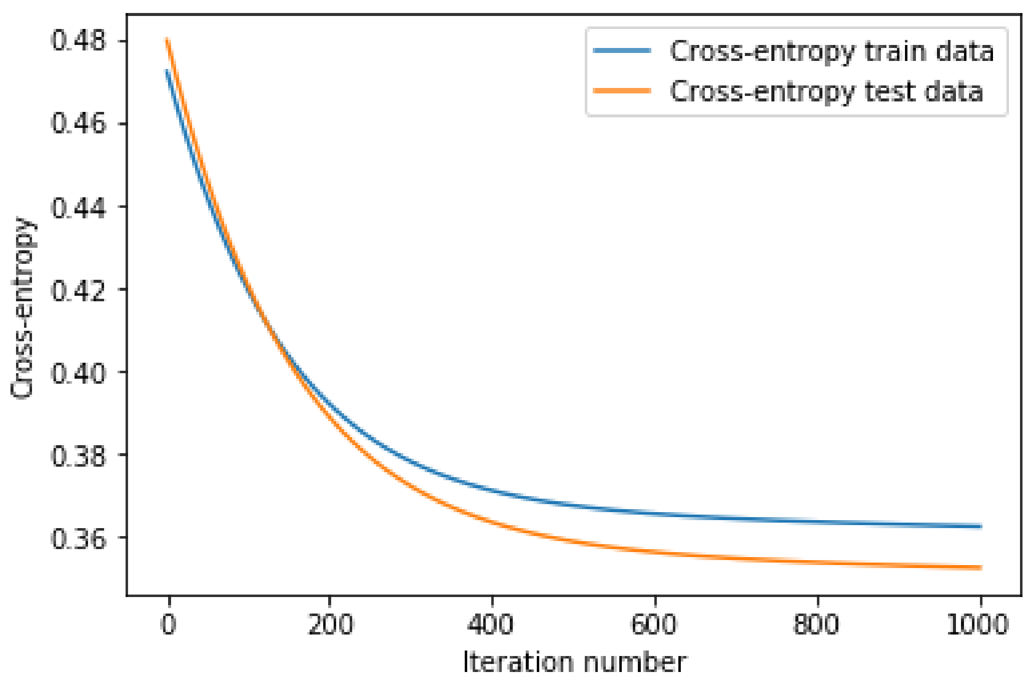
2.2 Task 2

In [8]:
```
path_train_data = 'dataset/classification/cl_train_2.csv'
path_test_data = 'dataset/classification/cl_test_2.csv'

df = load_csv(path_train_data)
admitted = df.loc[df['y'] == 1]
not_admitted = df.loc[df['y'] == 0]

fig, (ax1, ax2) = plt.subplots(1,2)
ax1.scatter(admitted.iloc[:, 0], admitted.iloc[:, 1], s=10, label='y=1')
ax1.scatter(not_admitted.iloc[:, 0], not_admitted.iloc[:, 1], s=10, label='y=0')
ax1.set_title('Scatterplot of training data')

df = load_csv(path_test_data)
admitted = df.loc[df['y'] == 1]
not_admitted = df.loc[df['y'] == 0]

ax2.scatter(admitted.iloc[:, 0], admitted.iloc[:, 1], s=10, label='y=1')
ax2.scatter(not_admitted.iloc[:, 0], not_admitted.iloc[:, 1], s=10, label='y=0')
ax2.set_title('Scatterplot of test data')
plt.show()
```

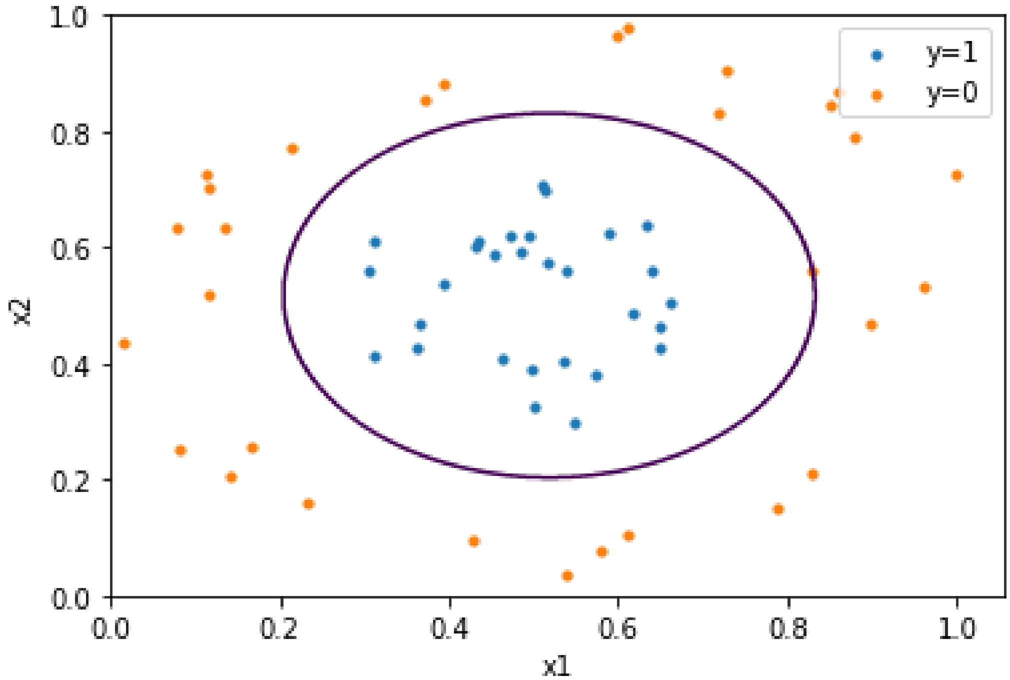


As we can see, the data is not linearly separable. To be able to correctly classify the dataset with logistic regression, we need to add quadratic features to be able to separate the data.

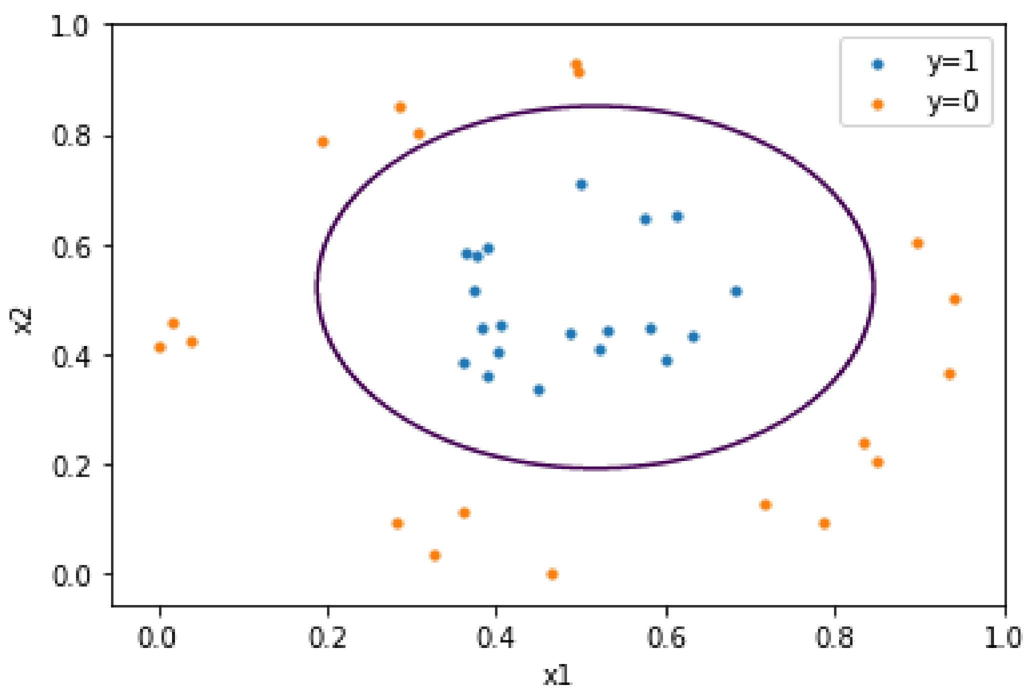
```
In [10]: # Parameters
         initial_weights = [-5, 10, 10, -10, -10]
         alpha = 0.01
         iterations = 1000
         feature_name = ['x0', 'x1', 'x2', 'x1^2', 'x2^2']

         # Train weights
         w_train, ce_train = train(path_train_data, initial_weights,
                                   feature_name, alpha, iterations)
         w_test, ce_test = train(path_test_data, initial_weights,
                                 feature_name, alpha, iterations)

         # Plot cross-entropy
         plot_cross_entropy(ce_train, ce_test, iterations)

         # Test precision
         accuracy = test(path_test_data, w_train, feature_name)
         print("Accuracy:", accuracy)

         # Plot decision boundary
         plot_radial_decision_boundary(path_train_data, w_train, iterations)
         plot_radial_decision_boundary(path_test_data, w_test, iterations)
         print(w_test)
```




```
Accuracy: 1.0
```



y=1
y=0
x1
x2



y=1
y=0
x1
x2

[-4.284107098364902, 10.30209926663444, 10.35045906916575, -9.961908346961593, -9.92154542742388