

The Scala Programming Language

Troy Hut and Benjamin Killeen

Introduction

The Scala
Programming
Language

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Introduction

Class Hierarchy

Nat

Typing

NatOrdering

Variance

OrdList

Scala is:

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Scala is:

- Object oriented

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Scala is:

- Object oriented
- Functional

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Scala is:

- Object oriented
- Functional
- Frustratingly well-typed

The Scala Class Hierarchy

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Class Hierarchy

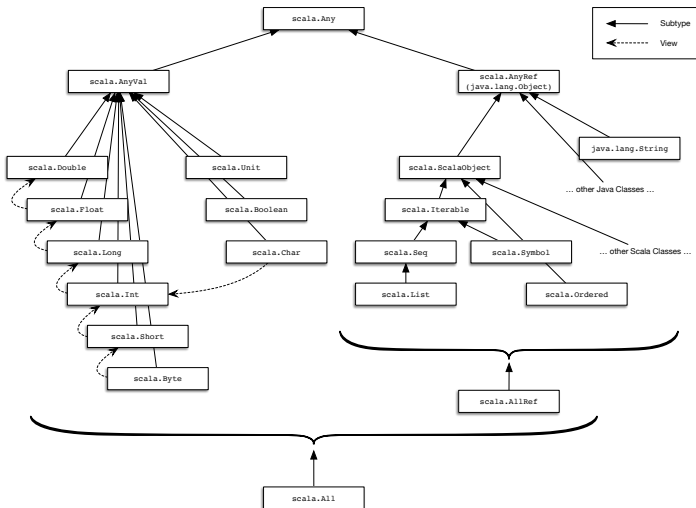
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Traits, Objects, and Classes

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```
trait Nat {  
  def isZero: Boolean;  
  def pred: Nat;  
  def succ: Nat;  
}
```

Traits, Objects, and Classes

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```
trait Nat {  
  def isZero: Boolean;  
  def pred: Nat;  
  def succ: Nat;  
}  
  
object Zero extends Nat {  
  def isZero = true  
  def succ = Succ(this)  
  def pred = this  
}
```


Traits, Objects, and Classes

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```
trait Nat {  
  def isZero: Boolean;  
  def pred: Nat;  
  def succ: Nat;  
}  
  
object Zero extends Nat {  
  def isZero = true  
  def succ = Succ(this)  
  def pred = this  
}  
  
case class Succ(v: Nat) extends Nat {  
  def isZero = false  
  def succ = Succ(this)  
  def pred = v  
}
```

Inferred Methods

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```
package nat

trait Nat {
  def isZero: Boolean;
  def pred: Nat;
  def succ: Nat;
  def + (that: Nat): Nat = if (that.isZero) this else this.succ + that.pred
  def - (that: Nat): Nat = if (that.isZero) this else this.pred - that.pred
  def * (that: Nat): Nat = if (that.isZero) that else this + this * that.pred
  def ^ (that: Nat): Nat = if (that.isZero) that.succ else this * (this ^ that.pred)
}
```

Inferred Methods

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```
package nat

trait Nat {
  def isZero: Boolean;
  def pred: Nat;
  def succ: Nat;
  def + (that: Nat): Nat = if (that.isZero) this else this.succ + that.pred
  def - (that: Nat): Nat = if (that.isZero) this else this.pred - that.pred
  def * (that: Nat): Nat = if (that.isZero) that else this + this * that.pred
  def ^ (that: Nat): Nat = if (that.isZero) that.succ else this * (this ^ that.pred)
}

scala> import nat._
import nat._

scala> val zero = Z; val one = S(Z); val two = S(S(Z))
zero: nat.Z.type = Z
one: nat.S = S(Z)
two: nat.S = S(S(Z))

scala> ((one + two) * two) - (two^two)
res0: nat.Nat = S(S(Z))
```

Type Parameters

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Problem:

What if a trait or class can contain arbitrary types?

Type Parameters

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Problem:

What if a trait or class can contain arbitrary types?

```
package ord
```

```
trait Ordering[T] {  
  def leq (x: T, y: T): Boolean;  
  def geq (x: T, y: T): Boolean = leq(y, x)  
  def gt  (x: T, y: T): Boolean = !leq(x, y)  
  def lt  (x: T, y: T): Boolean = !leq(y, x)  
  def eq  (x: T, y: T): Boolean = leq(x, y) && leq(y, x)  
  def neq (x: T, y: T): Boolean = !leq(x, y) || !leq(y, x)  
}
```

NatOrdering

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An Ordering on Naturals

`NatOrdering` only has to implement `leq` with the proper type, and it gets `geq`, `lt`, etc. for free.

```
object NatOrdering extends Ordering[Nat] {  
  def leq (x: Nat, y: Nat): Boolean =  
    (x.isZero, y.isZero) match {  
      case (true, _) => true  
      case (false, true) => false  
      case (false, false) => leq(x.pred, y.pred)  
    }  
}
```

Nat in Full

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```
trait Nat {  
  def isZero: Boolean;  
  def pred: Nat;  
  def succ: Nat;  
  def + (that: Nat): Nat = if (that.isZero) this else this.succ + that.pred  
  def - (that: Nat): Nat = if (that.isZero) this else this.pred - that.pred  
  def * (that: Nat): Nat = if (that.isZero) that else this + this * that.pred  
  def ^ (that: Nat): Nat = if (that.isZero) that.succ else this * (this ^ that.pred)  
  def <= (that: Nat): Boolean = NatOrdering.leq(this, that)  
  def >= (that: Nat): Boolean = NatOrdering.geq(this, that)  
  def < (that: Nat): Boolean = NatOrdering.lt(this, that)  
  def > (that: Nat): Boolean = NatOrdering.gt(this, that)  
  def == (that: Nat): Boolean = NatOrdering.eq(this, that)  
  def != (that: Nat): Boolean = NatOrdering.neq(this, that)  
}  
  
object NatOrdering extends Ordering[Nat] {  
  def leq (x: Nat, y: Nat): Boolean =  
    (x.isZero, y.isZero) match {  
      case (true, _) => true  
      case (false, true) => false  
      case (false, false) => leq(x.pred, y.pred)  
    }  
}
```

```
scala> one < two  
res5: Boolean = true
```

```
scala> one == two  
res6: Boolean = false
```

```
scala> (one + two) == S(S(S(Z)))  
res7: Boolean = true
```

Variance

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Type parameters immediately raise the question of subtypes.

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Type parameters immediately raise the question of subtypes.

Covariance

If C is a *covariant* type constructor and $S <: T$, then $C[S] <: C[T]$.

Variance

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Type parameters immediately raise the question of subtypes.

Covariance

If C is a *covariant* type constructor and $S <: T$, then $C[S] <: C[T]$.

Contravariance

If C is a *contravariant* type constructor and $S <: T$, then $C[S] >: C[T]$.

Ordered List

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```
package ordlist
import ord._

trait OrdList[+T] {
  def isEmpty: Boolean;
  def insert[U >: T](x: U, o: Ordering[U]): OrdList[U];
}
```

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OrdList

```
package ordlist
import ord._

trait OrdList[+T] {
  def isEmpty: Boolean;
  def insert[U >: T](x: U, o: Ordering[U]): OrdList[U];
}

object Emp extends OrdList[Nothing] {
  override def toString = "Emp"
  def isEmpty = true
  def insert[U >: Nothing](x: U, o: Ordering[U]) =
    Cons(x, this.asInstanceOf[OrdList[U]])
}
```

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```
package ordlist
import ord._

trait OrdList[+T] {
  def isEmpty: Boolean;
  def insert[U >: T](x: U, o: Ordering[U]): OrdList[U];
}

object Emp extends OrdList[Nothing] {
  override def toString = "Emp"
  def isEmpty = true
  def insert[U >: Nothing](x: U, o: Ordering[U]) =
    Cons(x, this.asInstanceOf[OrdList[U]])
}

case class Cons[+T] (head: T, tail: OrdList[T]) extends OrdList[T] {
  override def toString = s"$head :: $tail"
  def isEmpty = false
  def insert[U >: T](x: U, o: Ordering[U]) =
    if (o.leq(x, head)) Cons(x, this) else Cons(head, tail.insert(x, o))
}
```

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```
scala> import ordlist._  
import ordlist._
```

```
scala> val o = NatOrdering  
o: nat.NatOrdering.type = nat.NatOrdering$@236f3885
```

```
scala> Emp  
res12: ordlist.Emp.type = Emp
```

```
scala> Emp.insert(one, o).insert(zero, o).insert(two, o).insert(one, o)  
res13: ordlist.Ordinal[nat.Nat] = Z :: S(Z) :: S(Z) :: S(S(Z)) :: Emp
```

Ordered List

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```
scala> import ordlist._  
import ordlist._
```

```
scala> val o = NatOrdering  
o: nat.NatOrdering.type = nat.NatOrdering$@236f3885
```

```
scala> Emp  
res12: ordlist.Emp.type = Emp
```

```
scala> Emp.insert(one, o).insert(zero, o).insert(two, o).insert(one, o)  
res13: ordlist.OrdList[nat.Nat] = Z :: S(Z) :: S(Z) :: S(S(Z)) :: Emp
```

Note:

Even though `zero: Zero` and `one: Succ` have different types, an `OrdList` can contain both because they have a common **supertype**.