Torque on the Helix Magnet Due to the Earth's Magnetic Field

Benjamin Killeen

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[1] According to the Biot-Savart law, the general field along a single coil's central axis is

$$B_z = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$$

where R is the radius of the coil, I the current, and z the distance along the central axis.

This implies that for two coils, each a distance z away from the origin, with equivalent I, the magnetic field at the origin is:

$$B = \frac{\mu_0 R^2 I}{(z^2 + R^2)^{3/2}}$$

which implies

$$I = \frac{B(z^2 + R^2)^{3/2}}{\mu_0 R^2}.$$

[2] We then assume a coordinate system with z along the direction of the magnetic moment of the coils, which lie in the xy plane. If $\hat{x}$ points directly upward, then we measure the angle θ from $\hat{x}$ and ϕ from $\hat{y}$. From this, the direction of the Earth's magnetic field B_e is:

$$\hat{B}_e = \sin\theta \cos\phi \hat{y} + \sin\theta \sin\phi \hat{z} + \cos\theta \hat{x}.$$

[3] The torque on a given current-carrying loop is given by $\tau = \vec{\mu} \times \vec{B}$. For each of our loops, $\mu = \pi R^2 I \hat{z}$. Therefore the torque on just one coil due to B_e is:

$$\begin{aligned}\tau_1 &= B_e \pi R^2 I(\hat{z} \times (\sin\theta \cos\phi \hat{y} + \sin\theta \sin\phi \hat{z} + \cos\theta \hat{x}.)) \\ &= B_e \pi R^2 I(\cos\theta \hat{y} - \sin\theta \cos\phi \hat{x})\end{aligned}$$

Therefore the net torque τ on the system is:

$$\tau = 2B_e\pi R^2 I(\cos\theta\hat{y} - \sin\theta\cos\phi\hat{x})$$

[4] Evaluating the worst case scenario, we assume a B value from the coils of 1T, R = 9" = 0.2286 m, z = 15.5" = 0.3937 m, we obtain a value of $I = 1.437 \times 10^6$ A.

Assuming angles which yield the maximal torque, (reducing the unit vector to 1), and a B_e of 0.5 Gauss, we can expect torques of up to 23.59 Nm.