

# Startup meeting master project

Characterization of cardiac cellular dynamics using Physics-Informed  
Neural Networks

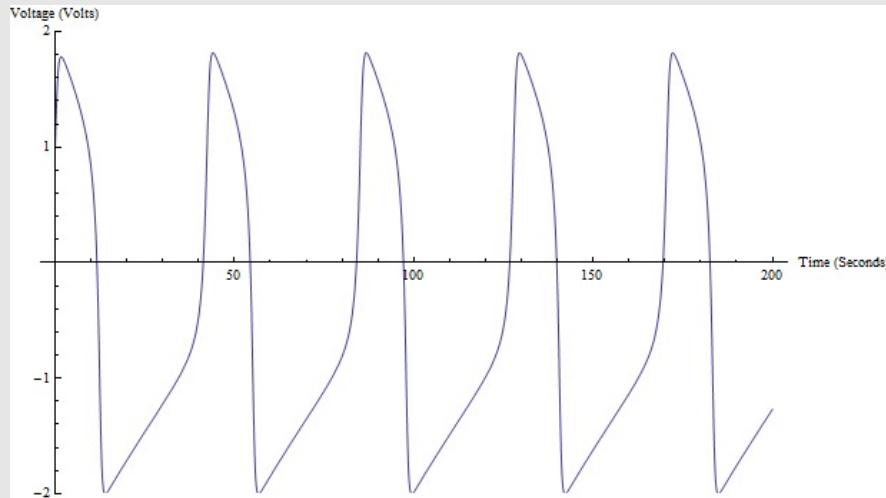
03.09.21

simula

# There exist numerous models for heart cells

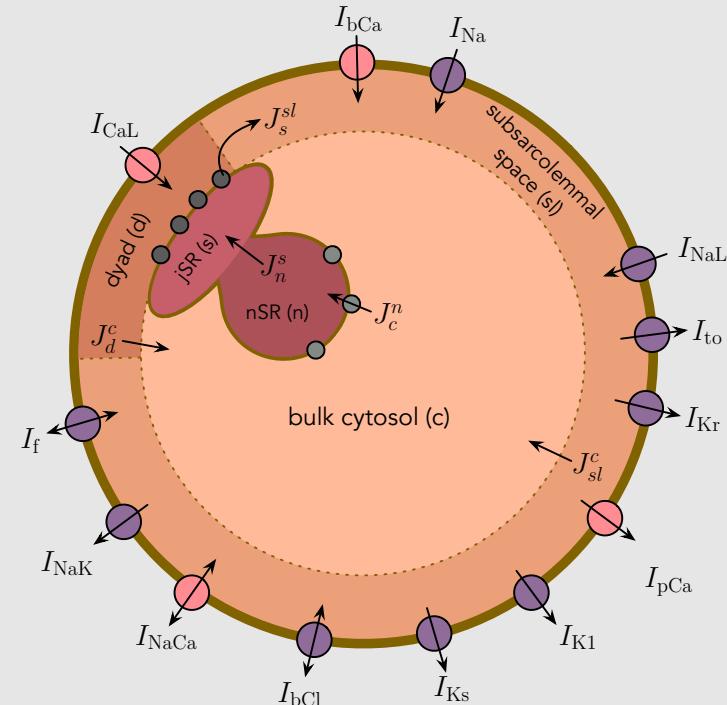
A model developed at Simula (complex)

Fitzhugh-Nagumo (simple)



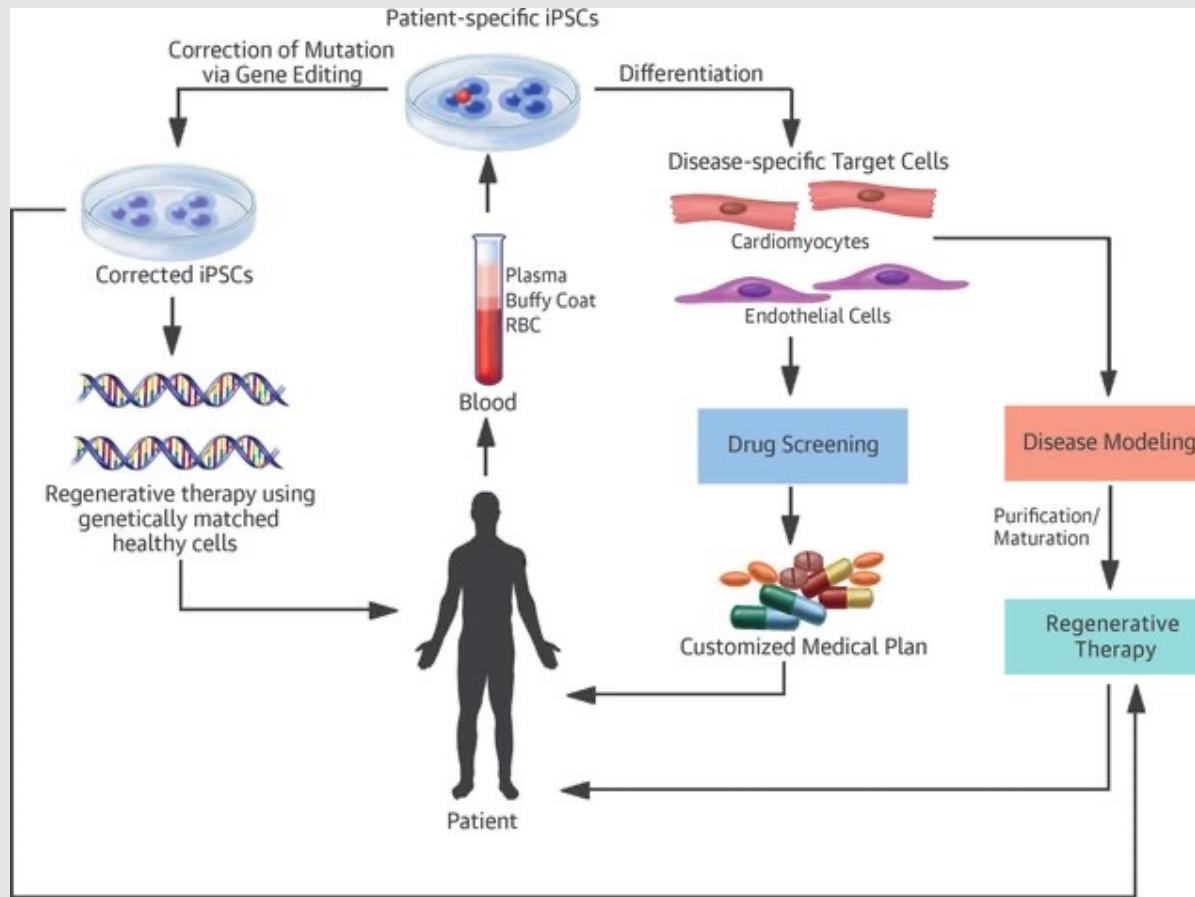
$$\begin{cases} \frac{dv}{dt} = v - v^3 - w + I_{\text{ext}} \\ \tau \frac{dw}{dt} = v - a - bw \end{cases}$$

[https://www.normalesup.org/~doulcier/teaching/modeling/excitatory\\_systems.html](https://www.normalesup.org/~doulcier/teaching/modeling/excitatory_systems.html)

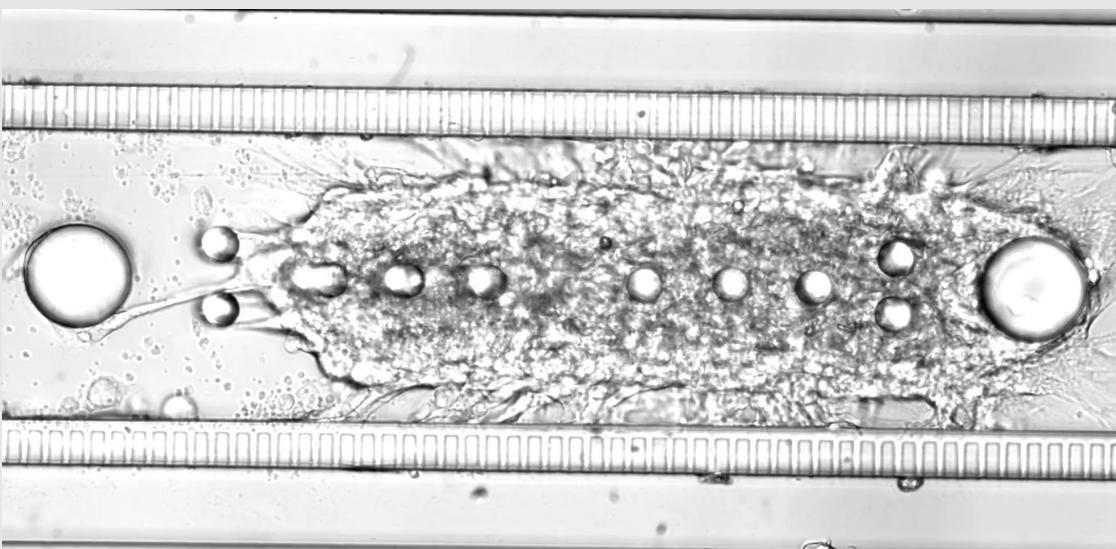
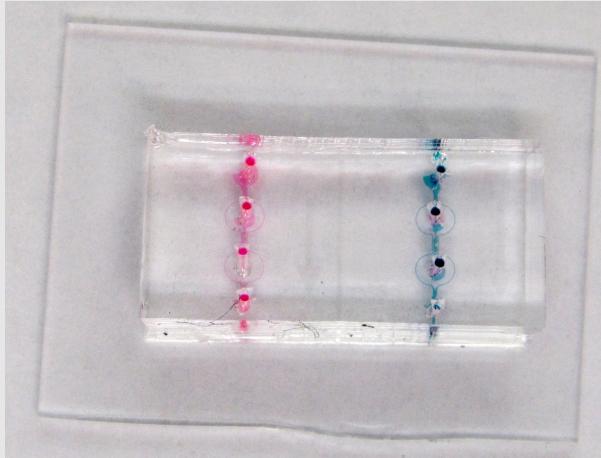
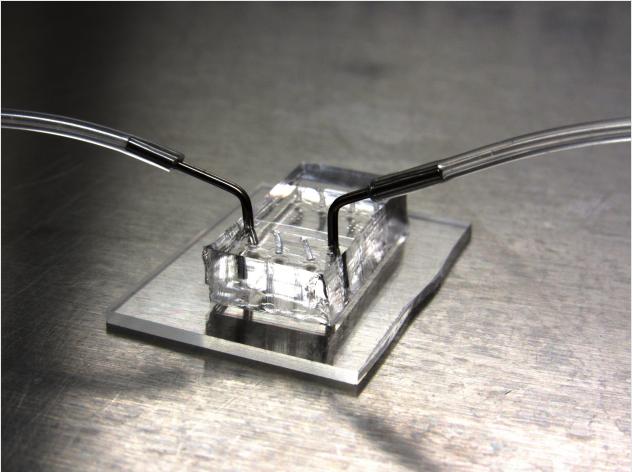


Jæger, K. H., Charwat, V., Charrez, B., Finsberg, H., Maleckar, M. M., Wall, S., ... & Tveito, A. (2020). Improved computational identification of drug response using optical measurements of human stem cell derived cardiomyocytes in microphysiological systems. *Frontiers in pharmacology*, 10, 1648.

# human induced Pluripotent Stem Cells (hiPSC) can be used in personalized drug screening



# Microphysiological systems mimics the in vitro conditions

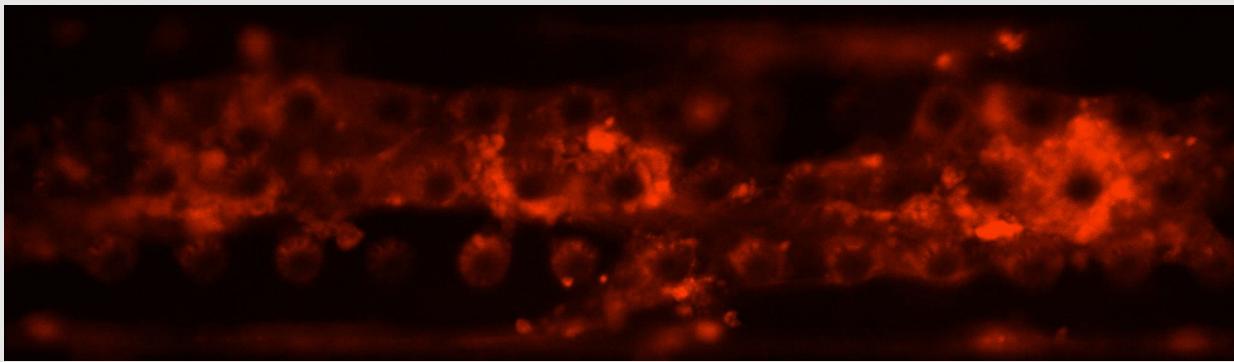


Organ-on-chip

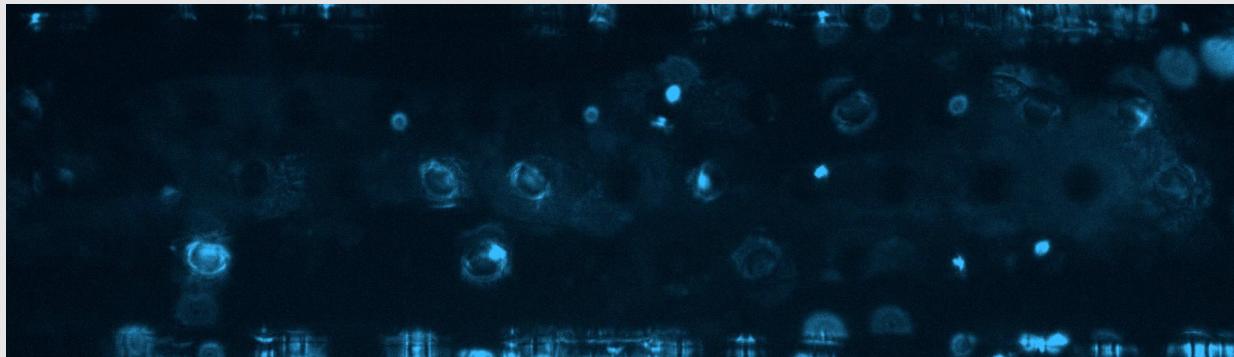
In vitro model for  
(personalized) human  
drug response

# We want to be able to fit models to experimental data

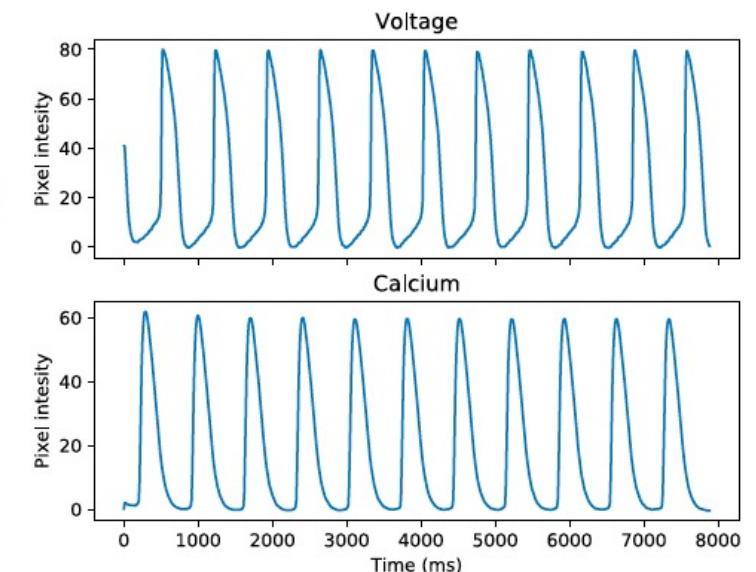
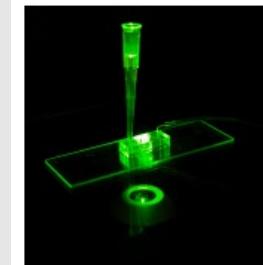
Voltage



Calcium

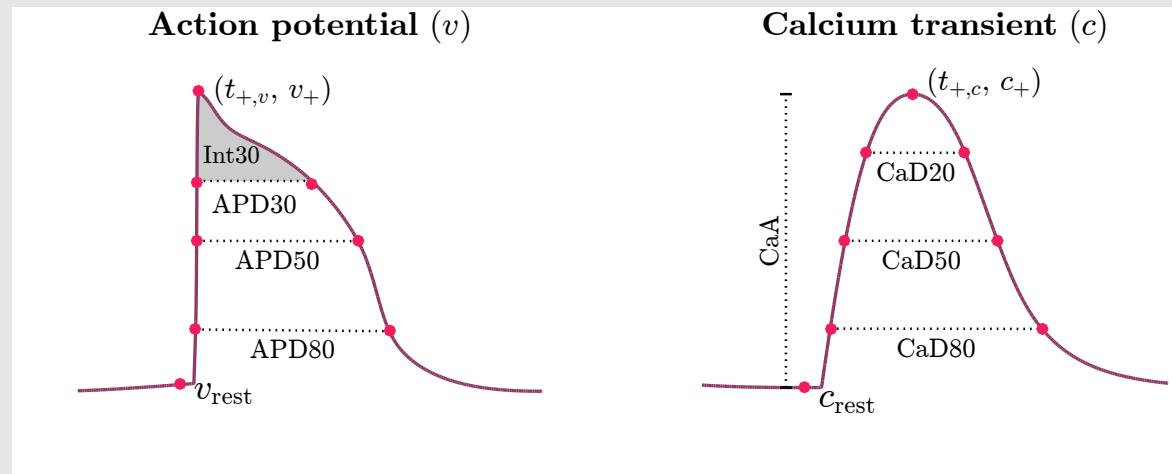


Optical Action  
Potential  
Measurements



# The traditional approach is to form a cost function that represents the model-data mismatch

Compute features of the data and model



Choose control parameters (parameters that we want estimate to fit)

$$\lambda = (\lambda_{CaL}, \lambda_{NaL}, \lambda_{NaK}, \lambda_B, \lambda_{SERCA}, \lambda_{Kr}, \lambda_{NaCa}, \lambda_{diff}, \lambda_{K1}, \lambda_{RyR}),$$

Cost function:

$$J(\lambda) = \sum_i w_{i,v} (f_{i,v}^{\text{model}}(\lambda) - f_{i,v}^{\text{data}}(\lambda))^2 + w_{i,c} (f_{i,c}^{\text{model}}(\lambda) - f_{i,c}^{\text{data}}(\lambda))^2$$

# Challenges with the traditional approach

- Cost function is highly non-convex – multiple local minima
- Gradient decent method does not work – gets stuck in local minima and is very dependent on initial starting point
- Currently we use a Brute Force method
  - Sample the parameter space
  - Run 1 billion simulations and store the resulting features in a database
  - Search through the database for best fit

# Aim of thesis – compare deep learning approach with standard approaches

PLOS COMPUTATIONAL BIOLOGY

RESEARCH ARTICLE

## Systems biology informed deep learning for inferring parameters and hidden dynamics

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# Loss function consist of two supervised losses (data) and one unsupervised loss (ode)

$$\mathcal{L}(\boldsymbol{\theta}, \mathbf{p}) = \mathcal{L}^{data}(\boldsymbol{\theta}) + \mathcal{L}^{ode}(\boldsymbol{\theta}, \mathbf{p}) + \mathcal{L}^{aux}(\boldsymbol{\theta}), \quad (3)$$

where

$$\mathcal{L}^{data}(\boldsymbol{\theta}) = \sum_{m=1}^M w_m^{data} \mathcal{L}_m^{data} = \sum_{m=1}^M w_m^{data} \left[ \frac{1}{N^{data}} \sum_{n=1}^{N^{data}} (y_m(t_n) - \hat{x}_{s_m}(t_n; \boldsymbol{\theta}))^2 \right], \quad (4)$$

$$\mathcal{L}^{ode}(\boldsymbol{\theta}, \mathbf{p}) = \sum_{s=1}^S w_s^{ode} \mathcal{L}_s^{ode} = \sum_{s=1}^S w_s^{ode} \left[ \frac{1}{N^{ode}} \sum_{n=1}^{N^{ode}} \left( \frac{d\hat{x}_s}{dt} \Big|_{\tau_n} - f_s(\hat{x}_s(\tau_n; \boldsymbol{\theta}), \tau_n; \mathbf{p}) \right)^2 \right], \quad (5)$$

$$\mathcal{L}^{aux}(\boldsymbol{\theta}) = \sum_{s=1}^S w_s^{aux} \mathcal{L}_s^{aux} = \sum_{s=1}^S w_s^{aux} \frac{(x_s(T_0) - \hat{x}_s(T_0; \boldsymbol{\theta}))^2 + (x_s(T_1) - \hat{x}_s(T_1; \boldsymbol{\theta}))^2}{2}. \quad (6)$$

# Cardiac cell models have many similarities with the Yeast glycosis model

**Table 1.** Hyperparameters for the problems in this study. The fist and second number in the number of iterations correspond to the first and second training stage.

Model		NN depth	NN width	#Iterations
Yeast glycolysis	Noiseless	4	128	1000, $9 \times 10^4$
	Noisy	4	128	1000, $2 \times 10^6$
Cell apoptosis	Survival	5	256	$0, 1.5 \times 10^6$
	Death	5	256	$0, 1.5 \times 10^6$
Ultradian endocrine	Parameters only	4	128	2000, $6 \times 10^5$
	Hidden nutrition	4	128	2000, $1.5 \times 10^6$

**Table 2.** The feature layer used in the network for each problem.

Model	Features
Yeast glycolysis	$\tilde{t}, \sin(\tilde{t}), \sin(2\tilde{t}), \sin(3\tilde{t}), \sin(4\tilde{t}), \sin(5\tilde{t}), \sin(6\tilde{t})$
Cell apoptosis	$\tilde{t}, e^{-\tilde{t}}$
Ultradian endocrine	$\tilde{t}, \sin(\tilde{t}), \sin(2\tilde{t}), \sin(3\tilde{t}), \sin(4\tilde{t}), \sin(5\tilde{t})$

<https://doi.org/10.1371/journal.pcbi.1007575.t002>

# Use the Yeast glycosis model as a starting point

- <https://github.com/alirezayazdani1/SBINNs/blob/master/glycolysis.py>
- Use a cardiac cell model instead. You can find a few implementations here:  
[https://github.com/ComputationalPhysiology/ap\\_features/tree/master/demo](https://github.com/ComputationalPhysiology/ap_features/tree/master/demo)
- Start with generating some synthetic data with known parameters and see if you can retrieve them using this method.
- Try without the ODE term – perhaps it is better without?
- Compare with other standard methods (see e.g  
<https://docs.scipy.org/doc/scipy/reference/tutorial/optimize.html>)
- Try adding some noise
- Look at parameter identifiability
- Experiment with settings for the neural network (Johannes)
- <https://kent-and.github.io/OnWriting.pdf>

**simula**