Converting between dates in the Hebrew and Roman Calendars

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Summary. The Hebrew calendar, used for Jewish holidays, does not correspond to the same days of the Roman calendar (e.g., December 25) each year. We explain a new mental algorithm to convert between the two. It involves learning the general quantities required to convert a particular year, deriving those quantities from the corresponding Roman date of Rosh Hashanah (the Jewish New Year's Day) for that year, and finally determining the Roman date of Rosh Hashanah in any year.

John H. Conway was a Professor Emeritus of Mathematics at Princeton University. This article is dedicated to his memory.

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Lesson one: how HE, SHE, and IT are used to convert dates between the calendars

In 2016, the first day of Chanukah coincided with Christmas day. You can confirm this using a formula we describe below. This formula not only works for the major holidays—it helps you translate any date in the Hebrew calendar to its equivalent Roman (namely Julian or Gregorian) date, and vice versa.[1] This task might sound daunting because of the variability of each calendar from year to year: sometimes there are 13 months in the Hebrew year and sometimes there is a 29th of February in the Roman one. Further, the Hebrew and Roman months do not have the same number of days: the Hebrew months have 29 or 30, while the Roman ones have 28, 29, 30, or 31. However, this formulaic method will allow you to perform these conversions either in your head or just with just a pen in hand.

Be careful: since a Hebrew day begins at sundown, the corresponding Roman date is not unique. The Roman date often given for Jewish holidays refers to the evening on which that holiday starts, which only contains about six hours of the holiday. We prefer to use the following Roman day, which contains most of the Hebrew day.

The first step in converting between a Hebrew and a Roman date is calculating the height of a given day, which can be determined from either the Hebrew or the Roman calendar. In the Roman calendar, the height is simply the sum of the number of the

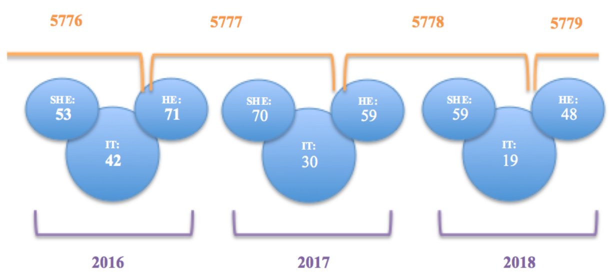
[1] The "Hebrew" and "Roman" nomenclature refers to the cultural significance of these calendar structures without specific holidays. Adding those holidays (e.g., Rosh Hashanah or Easter, respectively) would render the "Hebrew" calendar the "Jewish" calendar and the "Roman" calendar the "Christian" one.

day in the month and the number of the month in the year, except that January and February are counted as the 13th and 14th months of the previous year. For instance, the height of March 18, 2019 would be $3 + 18 = 21$. The year does not matter for calculating the height of a date in the Roman calendar. In the Hebrew calendar, the height is the number of the day in the month plus the appropriate one of three numbers – called HE, SHE, and IT – depending on the Hebrew month in question. HE, SHE, and IT vary by year.

Note that each Roman year, Y, intersects two Hebrew years, Y + 3760 and Y + 3761. For instance, the Roman year 2017 intersects the Hebrew years 5777 and 5778. These intersections, for the years 2016, 2017, and 2018, are displayed in the Mickey Mouse diagrams below.[2] Two adjacent Mickey Mice contain the numbers HE, SHE, and IT (in that order) for a Hebrew year, while a single Mickey Mouse contains the numbers SHE, IT, and HE (in that order) for a Roman year. As you can see from the Mickey Mouse diagrams, IT is always lower than HE and SHE: $12 \leq \text{IT} \leq 44$, while $41 \leq \text{HE} \leq 73$, and $41 \leq \text{SHE} \leq 73$ (for the years 1582, when our current Gregorian Calendar was adopted, to 2200).

Figure 1: HE, SHE, and IT for 2016-2018 (Mickey Mouse)

1



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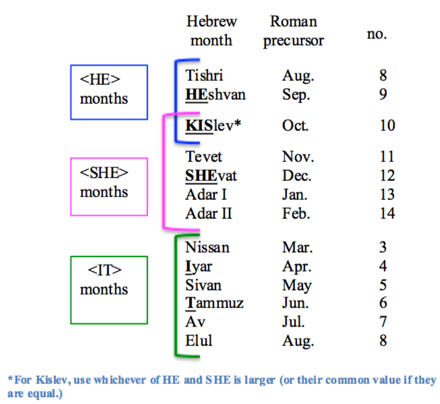
The dates that correspond to each other in the Hebrew and Roman calendars are found from the principle: when referred to partner months, the heights of corresponding dates are equal. Partner months refer to a Hebrew month and its Roman precursor. The following table pairs the Hebrew month with the appropriate one of HE, SHE, or IT, and also indicates its Roman precursor.

Figure 2: Partner Months and Heights

2



	Hebrew month	Roman precursor	no.
<HE> months	Tishri	Aug.	8
	HEshvan	Sep.	9
	KISlev*	Oct.	10
<SHE> months	Tevet	Nov.	11
	SHEvat	Dec.	12
	Adar I	Jan.	13
	Adar II	Feb.	14
<IT> months	Nissan	Mar.	3
	Iyar	Apr.	4
	Sivan	May	5
	Tammuz	Jun.	6
	Av	Jul.	7
	Elul	Aug.	8

*For Kislev, use whichever of HE and SHE is larger (or their common value if they are equal.)

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[2]The name Mickey Mouse comes from the resemblance of this diagram to the head of a particular character in popular culture.

Using the Mickey Mouse diagrams and chart above, we can determine that the height of the 7th of Iyar, 5779 would be 7 + IT, because Iyar is an IT month. For the year 2019, IT = 39, so the height would be 7 + 39 = 46. To convert this Hebrew date to its corresponding Roman date, we set 46 equal to the height of the Roman date. Iyar's precursor month is April, with value 4, so we begin by subtracting 4 from 46. This gives us April 42rd, which does not exist, so we must squash April 42rd to the following month, resulting in May 12rd, 2019.

As seen in the example above, our calculations will often lead to a putative Roman date whose day number exceeds the length, L, of the month. In this case, squash it by subtracting L and putting it into the next month. Alternatively, there are occasions when HE, SHE, or IT is too big to be subtracted. In this case, we stretch the Roman date into the previous month, as in the next example below.

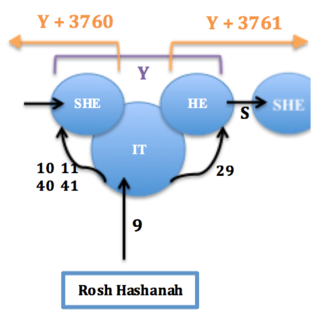
What Hebrew date is April 7th, 2019? April 7th has height 4 + 7 = 11. April is the Roman precursor of Iyar, an IT month, and IT = 39 in 2019, too big to be subtracted from 11. So, we stretch April 7th to March 38th, whose height is 3 + 38 = 41, from which we can subtract 39 (March is the precursor of Nissan, which is still an IT month). 41 – 39 = 2, so April 7th, 2019 = 2nd of Nissan, 5779. Easy, wasn't it?

Lesson two: how to find HE, SHE, and IT from the date of Rosh Hashanah

Calculating HE, SHE, and IT for a given year is simple when we know the date of Rosh Hashanah for that year. The following figure illustrates the procedure.

Figure 3: How to Calculate HE, SHE, and IT

3



3.*pdf*

If Rosh Hashanah falls on the nth day of September, then IT = n + 9 (that is to say, IT is the height of Rosh Hashanah, regarded as a September date). For example, in 2019, the Roman day that begins with Rosh Hashanah is the 30th of September, so IT = 30 + 9 = 39.

To move from IT to HE, we ELevate IT by 29 (which we might call EL, since it is the length of ELul), and to get to SHE, EXtend IT by adding EX, the EXtension Code. EX is one of the numbers 10, 11, 40, or 41, according to the lengths of the Hebrew and Roman years, as follows:

Figure 4: EXtension Codes

4

EX	Hebrew year	Roman year
10	long	ordinary
11	long	leap
40	"4shortened"	ordinary
41	"4shortened"	leap

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To explain the tens digits 1 and 4, regard the 1ong Hebrew year of 13 months as the normal case, from which the 12 month Hebrew year has been foreshortened. For the Roman years, a unit digit of 1 denotes a "1eap" year, while 0 denotes an "0rdinary" year.

The number of days in a Hebrew year is within one of 354 or 384. We call it 354 + S or 384 + S. The year is called abundant, deficient, or regular, depending on whether S is +1, -1, or 0.[3]

Figures 3 and 4 do rather more than enable one to proceed from Rosh Hashanah to HE, SHE, and IT – they also show how these constants change with time. Within a Roman year, we move from SHE to IT by subtracting EX (=10, 11, 40, 41), and then from IT to HE by adding 29. Similarly, next year's Rosh Hashanah yields next year's SHE, from which we can find the S, by which next year's SHE exceeds this year's HE. (Lesson Three, below, completes this series by teaching how to find Rosh Hashanah in any given Roman year.)

With more practice, you will be amazed to find that this method works flawlessly to convert between dates from the very different Roman and Hebrew calendars. With further application, one can even figure out what days of the week these dates fall on, using a formula called The Doomsday Rule (see below). By itself, the conversion formula we present is useful for translating birthdays, holidays, bar mitzvahs, and wedding dates into the other calendar… but you might want to double-check with the Internet that your calculations were correct before printing the invitations.

Lesson three: calculating in your head the date of Rosh Hashanah

The Hebrew calendar is a hybrid of three astronomical phenomena: the time between sunsets (the day), the time between new moons (the lunar month), and the time between autumnal equinoxes (the solar year). This calendar stands in contrast to the Roman calendar (i.e., the Julian and Gregorian calendars, used by Christianity), which has the same day and year but has months with no direct astronomical correspondence, and to the Islamic calendar, which has the same day and month, but has years with no direct astronomical correspondence.

The challenge of the Hebrew calendar is that neither a lunar month (29.53 days) nor a solar year (365.24 days) is a whole number of days. Furthermore, a year of 12 lunar months, at 354.37 days, is shorter than a solar year, whereas a year of 13 lunar months (383.90 days) is longer. The Rabbis of the Middle Ages addressed this dilemma by creating a 19-year cycle containing 12 regular 12-month years and seven 13-month leap years, noticing that 235 lunar months (12 x 12 + 7 x 13) equals 6939.69 days, very close to the 6939.60 days of 19 solar years (which we refer to as the Golden Cycle).

[3] S therefore also corresponds to the number of days in Heshvan and Kislev: If both have 29, then S = - 1, if Heshvan has 29 and Kislev has 30 then S=0, and if both have 30 days, S = +1.

As most modern Jews have noticed, the dates of the Roman calendar that correspond to Jewish holidays vary from year to year. Unfortunately, calculating the Roman calendar date for a specific Jewish holiday in any particular year involves accounting for all of these fractional days and leaps, and is computationally beyond the mental capacity of most individuals. The innovation described here is an algorithm for calculating the Roman Day of Rosh Hashanah and is simple enough that it can be done in one's head.

First, one needs to figure out the year's place in the 19-year Golden Cycle and whether it is a leap year or a regular year, by calculating the following quantities. (Note: J mod K means remainder when J is divided by K. For example, 2 mod 3 = 2; 4 mod 3 = 1; 40 mod 20 = 0.)

$$Y = \text{the Roman Year (e.g. 2019)}$$

$$G \text{ (the Golden number)} = (Y \bmod 19) + 1$$

This is where we are in the 19 year cycle of leap years. It increases by 1 each year until it resets.

Then calculate F:

$$F \text{ (the Finder)} = (12G) \bmod 19$$

This tells us whether the previous year, the next year, or neither is a leap year. If F is small (from 0 to 6), then the year beginning with that Rosh Hashanah will be a leap year. If F is large (from 12 to 18), then the year ending with that Rosh Hashanah was a leap year and the following year will be a regular, short year. If F is in the middle (from 7 to 11) then both the year ending with that Rosh Hashanah and the following year are regular, short years.

The formula for the day of Rosh Hashanah in September is:

$$\text{Rosh Hashanah} = \text{Sep } A+B\text{: } c - d - E\text{, where:}$$

$$A = 1.5F$$

$$B = 6 + (Y \bmod 4)/4 = 6 + (y \bmod 4)/4$$

$$c = F + 1$$

$$d = (2y-1)/35$$

$$E=(F+1)/760 \text{ (can be ignored for 1762-2168)}$$

$$y = Y - 1900$$

A is the Acrobatic term as it jumps around from 0 to 27. It represents approximately how far Rosh Hashanah is from the earliest possible date it can fall on.

B, the Bissextile term, serves two functions. First, it sets the approximate earliest that Rosh Hashanah can be (September 6). Second, it adjusts for Roman leap years by adding a quarter day for each Roman regular year, and subtracting three quarters of a day for a Roman non leap year. B can use either Y or y, as 1900 is evenly divisible by 4—and so doesn't affect the remainder.

The colon preceding c - d indicates that these terms are to be divided by 18. This adjusts the day by adding or subtracting a particular number of long hours of which there are 18 in total in the day. he part that includes F adjusts for the fact that the 1.5 in A is only an approximation. The 1/18 piece adjusts for the fact that the “6” in B is also only an approximation. Finally, E performs an additional small adjustment. All of these deal with the fractions of days in the month and year lengths, described above.

Here is a full example, for 2019 (ignoring E):

$$G = 2019 \bmod 19 + 1 = 119 \bmod 19 + 1 = 5 + 1 = 6$$

$$F = (12G) \bmod 19 = (72) \bmod 19 = 15$$

$$A = 1.5 * 15 = 22.5$$

$$B = 6 + (119 \bmod 4)/4 = 6.75$$

$$c = F + 1 = 16$$

$$d = (2y\text{-}1)/35 = 237/35 = 6.75 \text{ (approximately)}$$

$$(c\text{-}d)/18 = 9.25/18 = 0.5 \text{ (approximately)}$$

$$\text{Rosh Hashanah} = \text{Sep } 22.5 + 6.75 + 0.5 = \text{Sep } 29.5 \rightarrow \text{Sep } 30$$

The following table demonstrates the computation of the Molad, the putative date of Rosh Hashanah, unless this date is a Sunday, Wednesday, or Friday, in which case Rosh Hashanah must be postponed. This table can be called “A Metonic Cycle.” This is named after the Caldean astronomer, Meton, who discovered (around 432 BCE) the fact that 19 solar years very nearly equals 235 lunations. A lunation is the mean difference between the dates of two adjacent new moons, which is around 29.530587 days at the moment, but decreasing by 1 or 2 units in the last decimal place per century. The years in a cycle are parameterized by their Golden numbers. The remaining columns give the various numbers (and consequent weekdays), defined in the previous section.

The rules that that Rabbis determined for Rosh Hashanah included four postponements relative to the exact time of the new moon that would otherwise set the date of the holiday. The first postponement does not apply to our system as we start each day at 12 midnight.[4]

The second postponement is such that Rosh Hashanah cannot fall out on Wednesday, Friday, or Sunday. The Wednesday and Friday postponements are in place so that Yom Kippur does not fall out adjacent to the Sabbath (i.e., on Friday or Sunday). The Sunday postponement is in place so that the minor holiday of Hoshana Rabba, which takes place 20 days after Rosh Hashanah, does not fall out on Saturday which would make it impossible to observe its particular rituals. This is the case in 2019; September 29 is a Sunday, and so Rosh Hashanah is Monday, September 30.

To implement this postponement, one must know the day of the week that the calculated Rosh Hashanah date corresponds to. For this we can use the Doomsday Method that I (John Conway) previously invented. The Doomsday for a given year is defined as the common value of the weekday for the dates:

(early months)	Jan. 30/31, Feb 28/29 = Mar 0
(later even months)	4/4, 6/6, 8/8, 10/10, 12/12
(later odd months)	5/9, 9/5, 7/11, 11/7.

[4] In a system where the day starts at 6 PM of the previous day, when the new moon is after 18 hours (i.e., 12 noon), then Rosh Hashanah goes forward a day.

Since this article was written in part during mathematician Richard Guy's 100th birth-year, we quote his mnemonic verse:

"the last of Feb or of Jan will do (except that in leap years it's Jan 32)
then for even months take the month's own day,
and for odd months, add four or take it away."

To which he added a footnote, "according to length – or simply remember – you only subtract for September or November." However, Americans may find it easier to think of having a 9 to 5 job in a 7/11 store.

There is a second verse that tells us how to find all the doomsdays in any given century, from the doomsday of the starting year of that century.

"Now to work out your doomsday the orthodox way,
three things you should add to the century day:
dozens, remainder, and fours in the latter
(if you alter by sevens, of course it won't matter)."

And there is a third verse to remind you of the centuries:

"In Julian times, lack-a-day, lack-a-day,
zero was Sunday, centuries fell back a day.
But Gregorian four-hundredths are always Tues.
And now centuries extra take us back twos"

In fifty years hence, how will one know that September 29, 2019 was a Sunday?

The doomsday for 2000 (the century doomsday) is a Tuesday, since 2000 is a multiple of 400. Sixteen years contains one dozen, with remainder four, and so just one four in the remainder advances the calendar by 1 + 4 + 1 = 6 days. Six days + Tuesday = Monday. We know that 10/10 is a Monday, as is 10/3, which is one week earlier, and therefore, 10/2 must be one day before that: Sunday.

The third and fourth postponements are arithmetical, and arrange that the Hebrew year length is one of: 353, 354, 355, 383, 384, 385 days. Without them, there would occasionally be 356- and 382-day years. The third postponement copes with the 356-day possibility, by postponing its start. If Rosh Hashanah would fall on a Tuesday and the fraction of days is greater than 0.633, and $F > 6$ (i.e., the starting year is not a leap year), then Rosh Hashanah is postponed by two days to Thursday (since Wednesday is forbidden), reducing the number of days from 356 to 354.

The fourth deals with the 382-day case, by postponing the start of the next year. If Rosh Hashanah would fall on a Monday and the fraction of days is greater than 0.898, and $F > 12$ (i.e., the ending year was a leap year), then Rosh Hashanah is postponed to Tuesday, increasing the length of the year from 382 to 383 days.

Some intuition can be gained by thinking about how F and A change. In a leap year, when F starts out smaller, F increases by 12. In a regular year, when F starts out large, it decreases by 7. This will either indicate a leap year (if F dropped below 7) or another regular year (which will put F below 7).

A, therefore, increases by 18 in a leap year (corresponding to the excess of 383.90 days of a leap year compared to 365.24 days of a solar year) and decreases by 10.5 in a regular year (corresponding to a shortage of 354.37 days in a regular year compared to 365.24 days of a solar year).

Finally, this algorithm is accurate for the years 1900-2099. As the Gregorian calendar skips leap years for years divisible by 100 but not 400 (e.g., 1700, 1800, 1900, 2100, 2200, 2300), B must be adjusted for those centuries. Specifically:

1500-1699: B = 3 + (Y mod 4)/4

1700-1799: B = 4 + (Y mod 4)/4

1800-1899: B = 5 + (Y mod 4)/4

1900-2099: B = 6 + (Y mod 4)/4

2100-2199: B = 7 + (Y mod 4)/4

2200-2299: B = 8 + (Y mod 4)/4

2300-2499: B = 9 + (Y mod 4)/4

This adjustment also accounts for the fact that the Hebrew calendar is slightly longer than the Gregorian calendar (6939.69 days vs. 6939.60 days) and so it is shifting forward ever so slightly.

We can now take apart the algorithm to show that each of the terms accounts for parts of four phenomenon: 1) the gain or loss from a leap or non-leap year (the presence of F in A, C, and E), 2) the drift of the Hebrew calendar vs. the Gregorian calendar (the constant term in B and the "y" term in D), 3) the Gregorian leap and non-leap years (the Y term in B), and 4) the leftover constant difference between the calendars, stemming from the timing of the first Rosh Hashanah, (the 1's in C, D, and E).

The benefit of this algorithm is that instead of calculating the four separate phenomena described in the previous paragraph, one can calculate the quantities A through E (or realistically A through D) and use this algorithm to accurately determine the date of Rosh Hashanah for nearly any Gregorian year.

