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Homework #2
Intermediate Microeconomics
W3211.001

100/100

Question 1

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No. An income elasticity greater than 1 implies that the percent change in quantity change in demand will be greater than the percent change in real income. After the subsidy they will purchase relatively more books as a percentage of their total income.

Question 2

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a) $U(x,y) = \ln x + 3\ln y$

$$MRS(x,y) = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y}{3x} = \frac{p_x}{p_y}, p_x x + p_y y = I$$

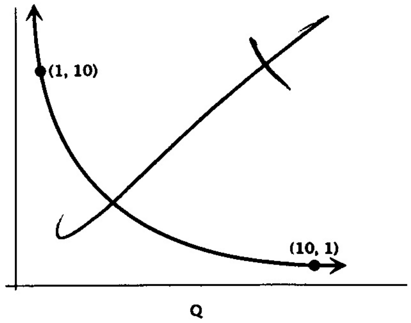
$$y = \frac{3xp_x}{p_y}, p_x x + 3xp_x = 4xp_x = I$$

$$\mathbf{x} = \frac{\mathbf{I}}{\mathbf{4p_x}}, \mathbf{y} = \frac{\mathbf{3I}}{\mathbf{4p_y}}$$

b) $p_y = 1, I = 40$

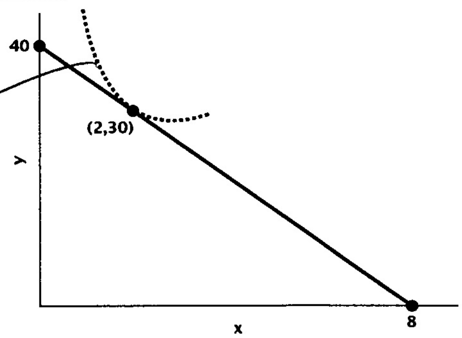
$$\mathbf{x} = \frac{\mathbf{I}}{\mathbf{4p_x}} = \frac{\mathbf{1}}{\mathbf{10p_x}}$$

$\frac{10}{p_x}$



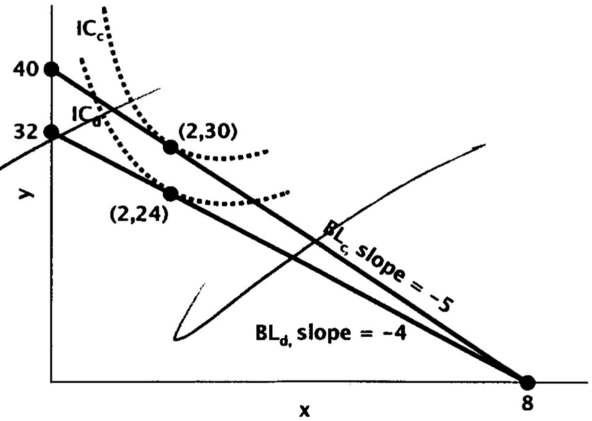
c) $p_x = 5, p_y = 1, I = 40$

$$(\mathbf{x^*}, \mathbf{y^*}) = (\mathbf{2}, \mathbf{30})$$



d) $p_x = 5, p_y = 1.25, I = 40$

$\mathbf{(x^*, y^*) = (2, 24)}$



e) 0. A change in the price of pretzels has no effect on demand for hot dogs. In mathematical terms:

$$E_{x,y} = \frac{\partial x}{\partial p_y}\frac{p_y}{x} = 0\,\frac{p_y}{x} = 0.$$

Question 3

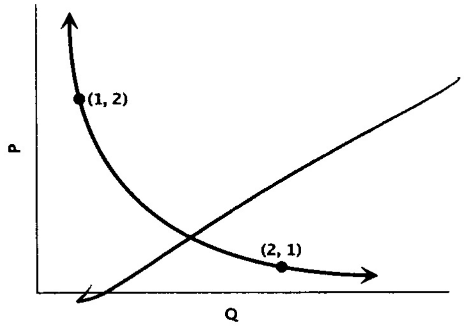
a) $MRS(x,y) = \dfrac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \dfrac{y}{2} = \dfrac{p_x}{p_y}\,, p_x x + p_y y = I$

$y = 2\,\dfrac{p_x}{p_y}\,, 2p_x + xp_x = I = (x+2)p_x \rightarrow x = \dfrac{I}{p_x} - 2$

$\mathbf{y = 2\,\dfrac{p_x}{p_y}\,, x = \dfrac{I}{p_x} - 2}$

b) $p_x = 1, I = 12$

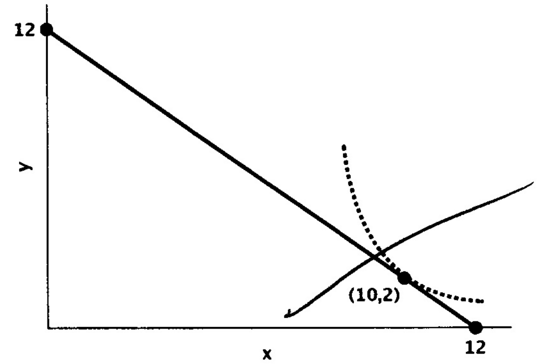
$y = 2\,\dfrac{p_x}{p_y} = \dfrac{2}{p_y}$



c) $p_x = 1, p_y = 1, I = 12$

$x = \dfrac{12}{1} - 2 = 10, y = 2\,\dfrac{1}{1} = 2$

$\mathbf{(x^*, y^*) = (10, 2)}$

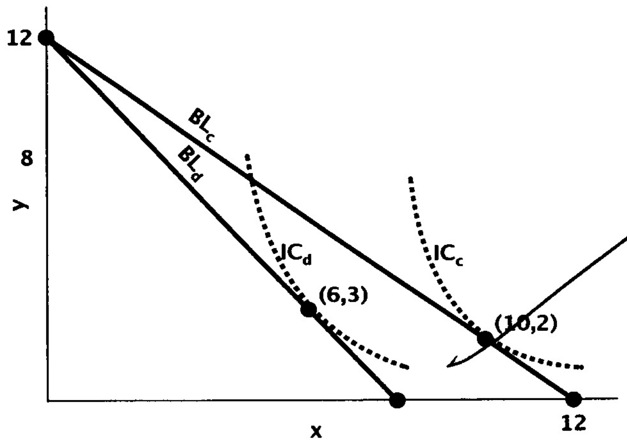


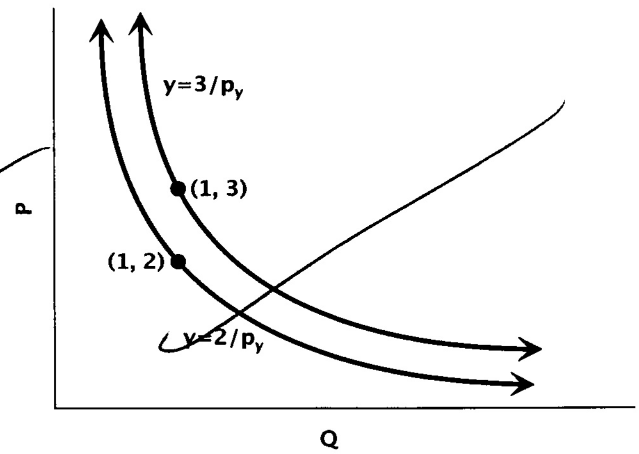
d) $p_x = 1.5, p_y = 1, I = 12$

$\mathbf{y} = \dfrac{\mathbf{3}}{\mathbf{p_y}}, x = \dfrac{12}{p_x} - 2$

$\mathbf{(x^*, y^*) = (6, 3)}$

e)





Question 4

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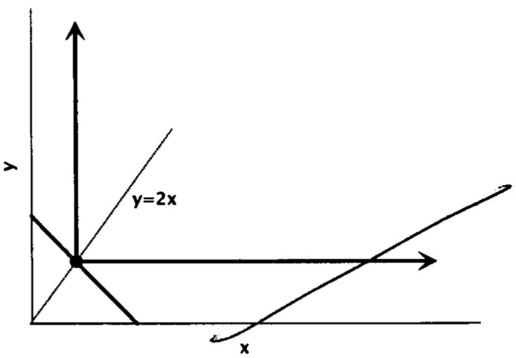
a)

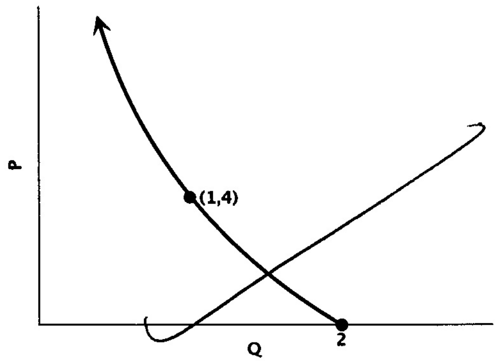
$$y = 2x = \frac{I}{p_y} - \frac{p_x}{p_y}$$

$$x = \frac{I}{p_x + 2p_y}, y = \frac{2I}{p_x + 2p_y}$$



b) $p_x = 8, I = 8$

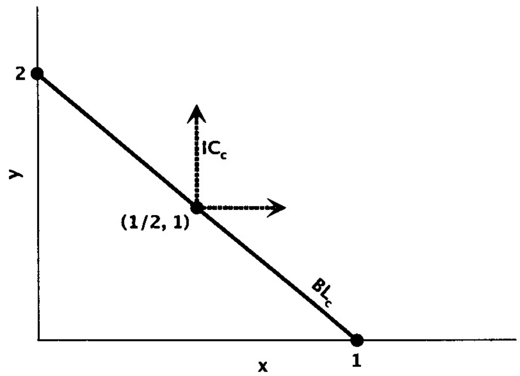
$$y = \frac{16}{8 + 2p_y}$$



c) $p_x = 8, p_y = 4, I = 8$

$$y = 2x = \frac{16}{8+8} = 1$$

$$(\mathbf{x^*}, \mathbf{y^*}) = \left(\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1}\right)$$

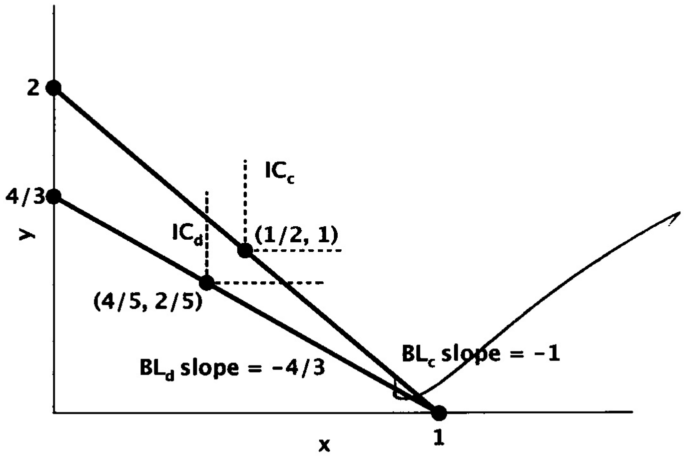


d)

$$y = 2x = \frac{(2)(8)}{8 + (2)(6)} = \frac{4}{5}$$

$$(\mathbf{x^*}, \mathbf{y^*}) = \left(\frac{\mathbf{2}}{\mathbf{5}}, \frac{\mathbf{4}}{\mathbf{5}}\right)$$

e)



f) $$E_{x,y} = \frac{dx}{dp_y}\frac{p_y}{x} = -\frac{4}{(4+p_y)^2}\frac{p_y}{x}$$

$$\mathbf{E_{x,y} = -.6}$$

g) $$E_y = \frac{dy}{dp_y}\frac{p_y}{y} = -.6$$

Question 5

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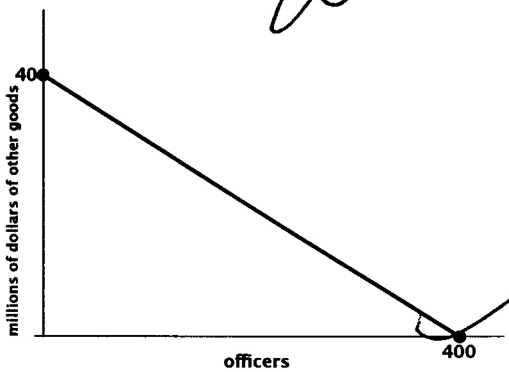
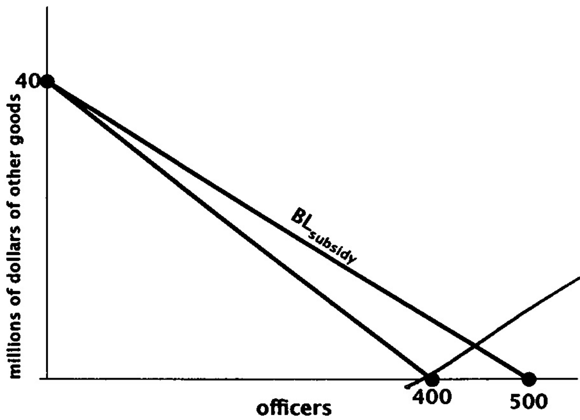
a)



Opportunity cost = \$100k of other goods

b)

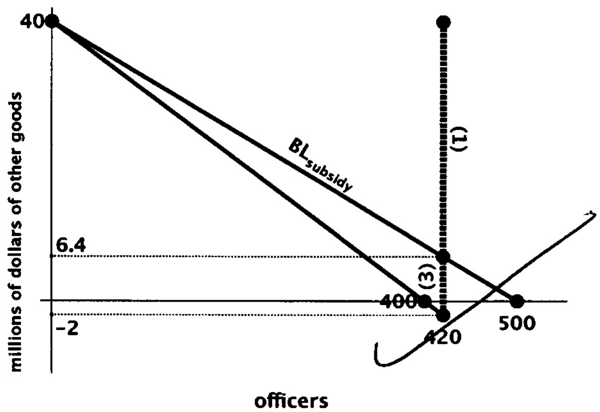


c)

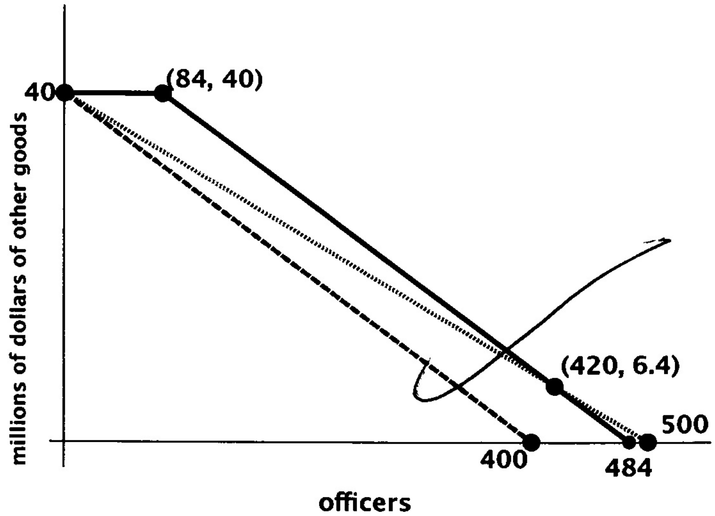
1) $40,000,000 - (420)(80,000) = \$6.4$ million

2) $MRS = \dfrac{p_x}{p_y} = \$80,000$

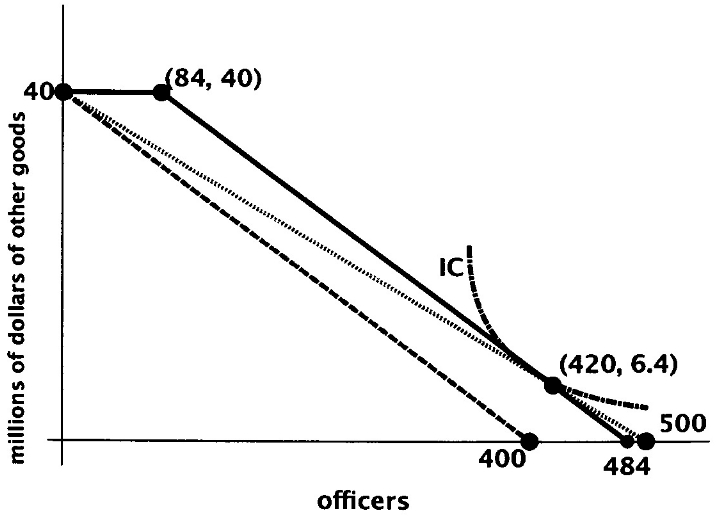
3) $TC = (20,000)(420) = \$8.4$ million



d) The opportunity cost for the first 84 officers is zero. After that, the opportunity cost of an additional officer is \$100k of other expenditures. The dashed line and the solid line after the point (84, 40) are parallel.



e) If the indifference curve is tangent to the original budget line at (420, 6.4), then it is not tangent to the new budget line, which has a different slope (p_x/p_y). Thus, this bundle is not the optimal solution for the lump sum, which means the local government will choose that bundle, which means they will hire a different amount of officers. They will hire fewer officers because more officers would be more inexpensive if they had the subsidy.



f) Because the bundle of (420, 6.4) is the optimal solution for the subsidy proposal and not for the lump sum proposal, this means that there is a better bundle (optimal solution) for the local government if they receive a lump sum. Therefore, they would prefer the lump sum as they can get a better bundle than if they received a subsidy.

Which one will have more police officer?

Bonus

The only way such solutions *x>0, y=0* or *x=0, y>0* can exist is if the utility function resolves to linear indifference curves. In this case, the points of maximum utility occur at the *x* or *y* intercepts respectively, as any point of intersection between the indifference curve and the budget line otherwise would occur with the utility function crossing the budget line, which inherently means that the solution is not optimal at the point of intersection.

Thus, the first order equations that characterize the solution in the case *x>0, y=0* would be:

$$x = \frac{I}{p_x}, y = 0 \text{ where } \frac{U_x}{U_y} > \frac{p_x}{p_y}$$

And the first order equations that characterize the solution in the case *x=0, y>0* would be:

$$x = 0, y = \frac{I}{p_y} \text{ where } \frac{U_x}{U_y} > \frac{p_x}{p_y}$$

For the case *U(x,y)=2x+3y*: $U(x,y) = x + 3y$.

$$\frac{U_x}{U_y} = \frac{2}{3}$$

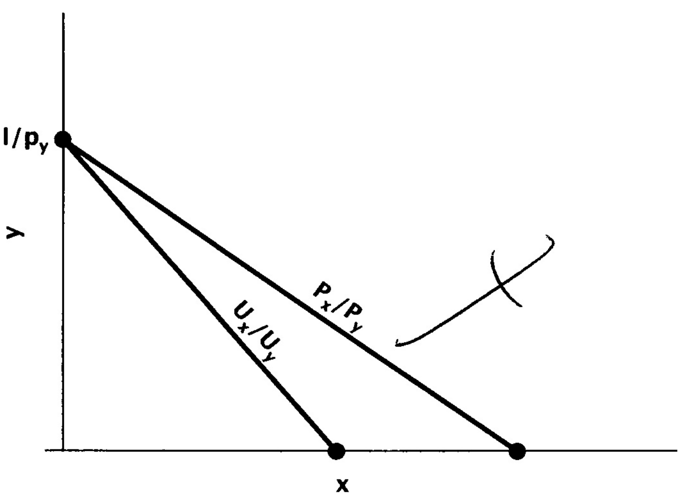
$$x = \frac{I}{p_x} \text{ if } \frac{p_x}{p_y} > \frac{2}{3}$$

$$y = \frac{I}{p_y} \text{ if } \frac{p_x}{p_y} < \frac{2}{3}$$

y and *x* can be any point on the budget line if:

$$\frac{p_x}{p_y} = \frac{2}{3}$$

x=0, y>0



x>0, y=0