Ben Drucker
Douglas Kessel
Ethan Kochav

11

Homework #4
Intermediate Microeconomics
W3211.001

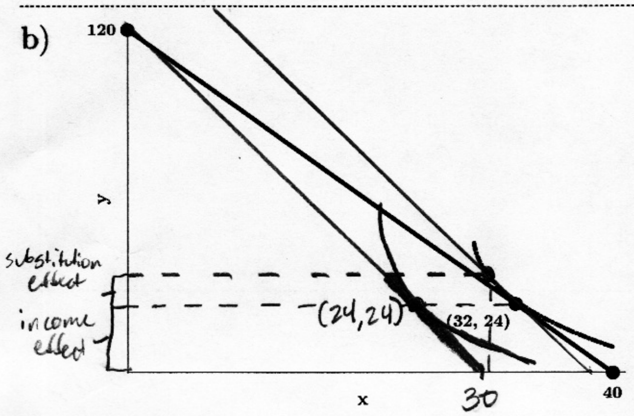
Question 1

a) $$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{4y}{x} = \frac{p_x}{p_y}$$

$$p_x x + p_y y = I$$

$$x = \frac{4I}{5p_x}, y = \frac{I}{5p_y}$$

b)



c) $(x^*, y^*) = (24, 24)$

d) No. Substitution effects equal income effects. Demand for y is unchanged.

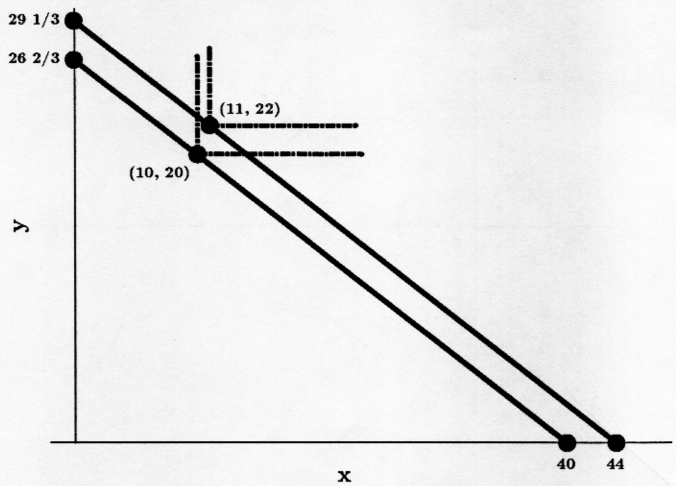
Question 2

a) $y = 2x, xp_x + yp_y = I$

$$x = \frac{I}{p_x + 2p_y}, y = \frac{2I}{p_x + 2p_y}$$

b)

c)



d)

$$\eta_x = \frac{\partial x}{\partial I}\frac{I}{x} = \frac{1}{p_x + 2p_y}\frac{I}{x} = 1$$

$$\epsilon_x = \frac{\partial x}{\partial p_x}\frac{p_x}{x} = \frac{I}{(p_x + 2p_y)^2}\frac{p_x}{\frac{I}{p_x+2p_y}} = \frac{p_x}{p_x + 2p_y} = \frac{1}{4}$$

e) $\eta_y = \eta_x = 1$

$$\frac{11 * 2}{88}(1) + \frac{22 * 3}{88}(1) = 1$$

f)

$$\frac{\partial\left(\frac{p_x x(I)}{I}\right)}{\partial I} = \frac{p_x x'}{I} - \frac{p_x x}{I^2} = \frac{p_x x}{I^2}\left(\frac{\partial x}{\partial I}\frac{I}{x} - 1\right) = \frac{p_x x}{I^2}(\eta_x - 1)$$

g) $\eta_x < 1 \rightarrow (-)$

$\eta_x = 1 \rightarrow (0)$

Falls

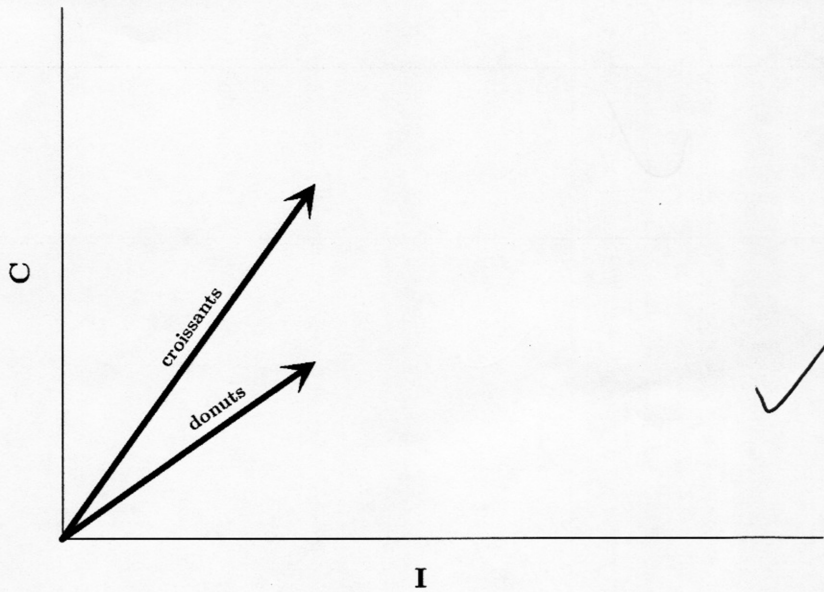
h) 0

i)

$$\theta_x = \frac{p_x}{I}\frac{I}{p_x + 2p_y} = \frac{p_x}{p_x + 2p_y}$$

$$\frac{\partial\left(\frac{p_x}{p_x+2p_y}\right)}{\partial I} = 0$$

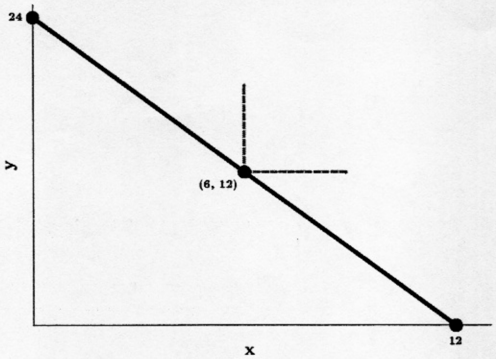
j)



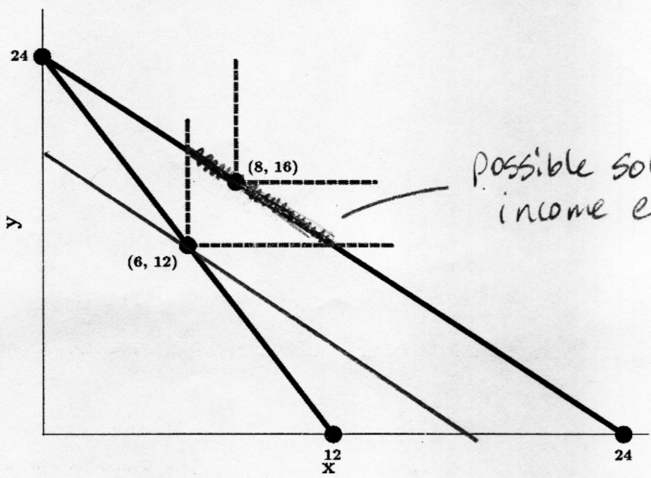
Question 3

a) $$x = \frac{I}{p_x + 2p_y}, y = \frac{2I}{p_x + 2p_y}$$

b)



c)



possible solutions, income effect is greater

d) No. Because the goods are perfect complements, there are no substitution effects. Income effects result in increases in consumption of both goods.

e) $$\frac{\partial x}{\partial p_y}\frac{p_y}{x} = \frac{-2I}{(p_x + 2p_y)^2} < 0$$

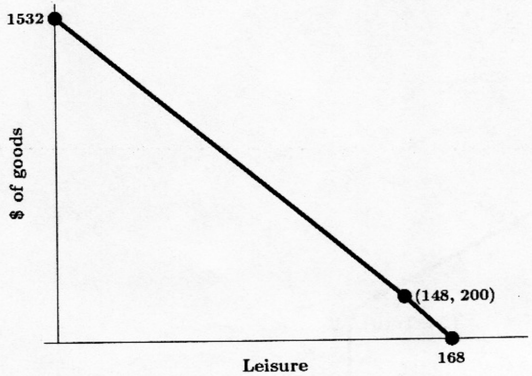
f) No. Income effects could also be responsible for the negative elasticity.

g) $$x^h = \frac{\overline{U}}{2}, y^h = \overline{U}$$

$$\frac{dx^h}{dp_x} = 0$$

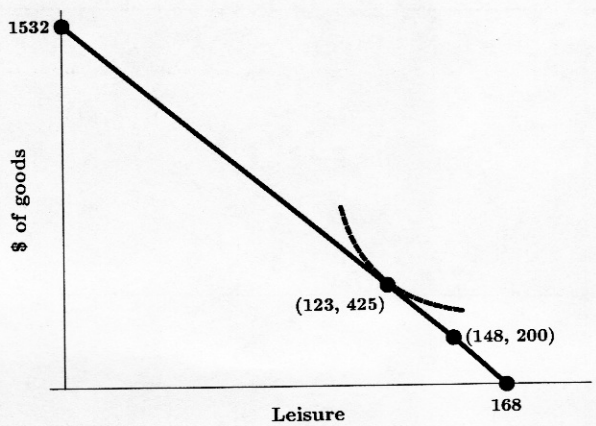
Question 4

a)



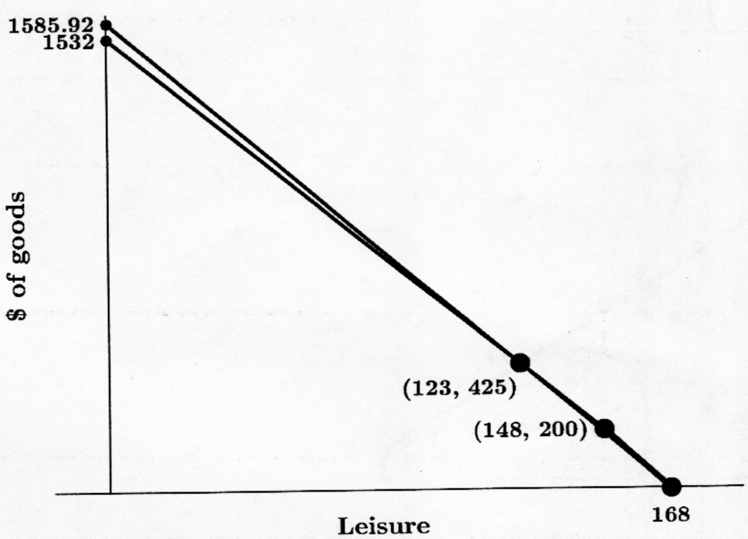
b) \$10 of goods per hour, \$9 of goods per hour

c)



d) $\dfrac{20 * 10 + 25 * 9}{45} = \$9.44, \dfrac{20 * 0 + 25 * .1}{45} = 5.56\%$

e)



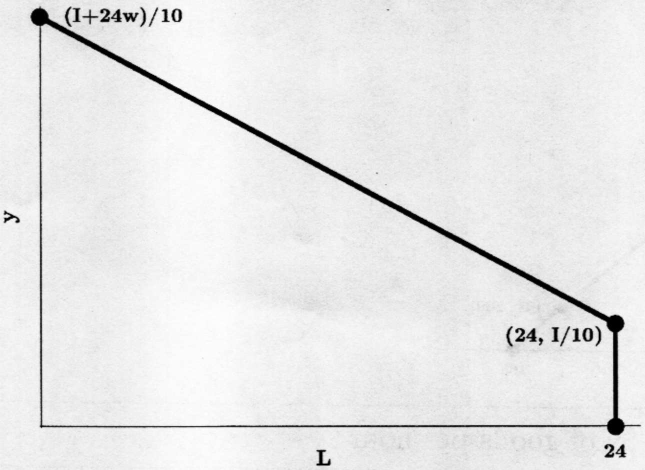
f) No. There will will be strictly better bundles on the new budget line above the original indifference curve. These bundles will be up and to the left, with less than 123 hours of leisure (i.e. more than 45 hours worked) and more dollars of goods.

Question 5

a) $\frac{I}{10}, \frac{I+24w}{10}$

$L \in [0,24) : y = \frac{I+24w}{10} - \frac{\frac{I+24w}{10} - \frac{I}{10}}{24}$

$L = 24 : y \in \left[0, \frac{I}{10}\right]$

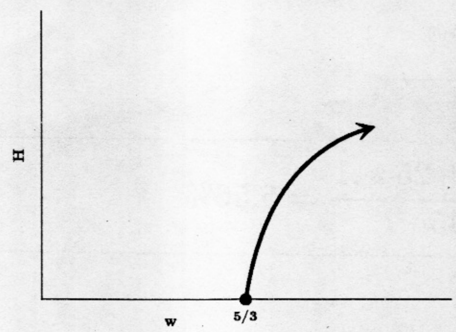


b) $MRS = \frac{w}{p} = \frac{y}{L}, (24-L)w + I = py$

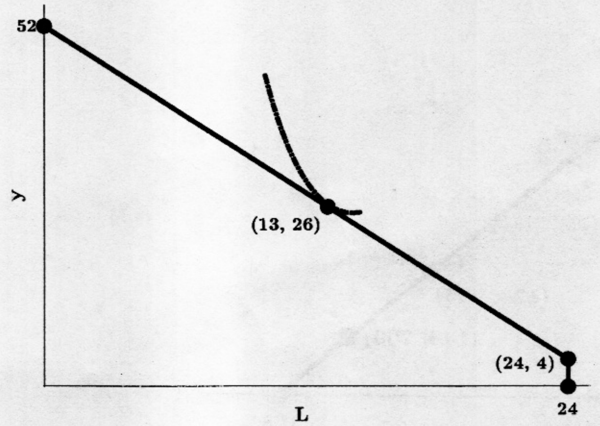
$L = \frac{I}{2w} + 12$

$H = 24 - L = 12 - \frac{I}{2w}$

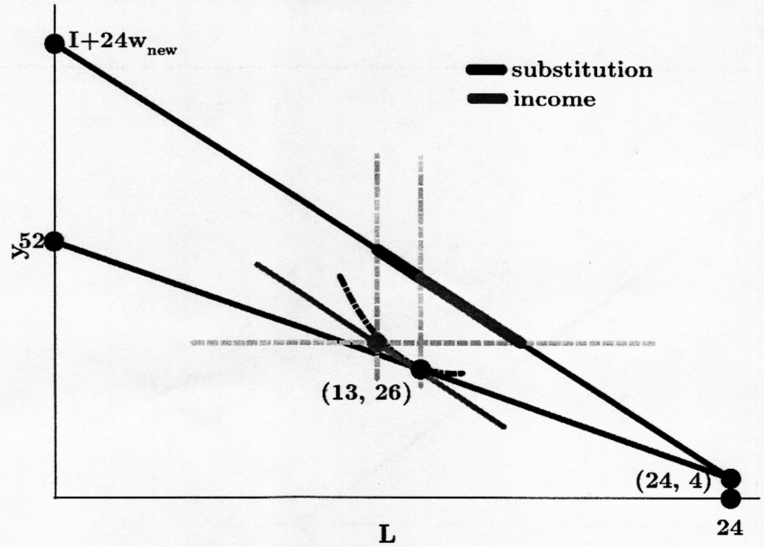
c) $H = 12 - \frac{20}{w}$



d) $(L, y, H) = (13, 26, 11)$



e) As wages increase, the supply of labor increases. Thus, the income effect would be larger.

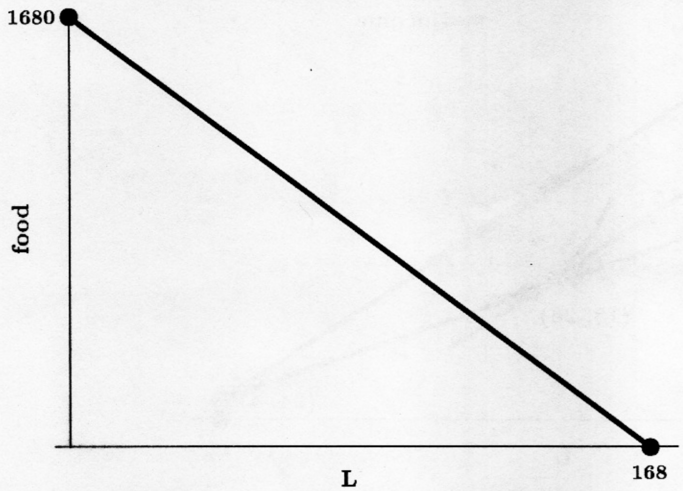


f) $$H = 12 - \frac{I}{40}$$

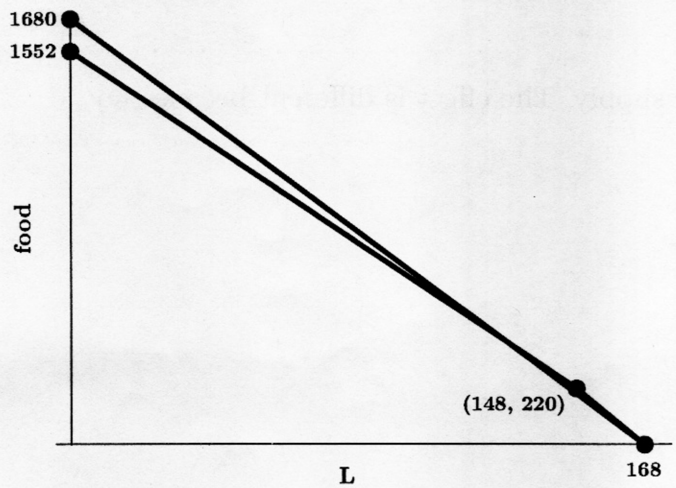
An increase in non-wage income (I) will reduce labor supply. The effect is different because no substitution effect exists, only an income effect.

Question 6

a)

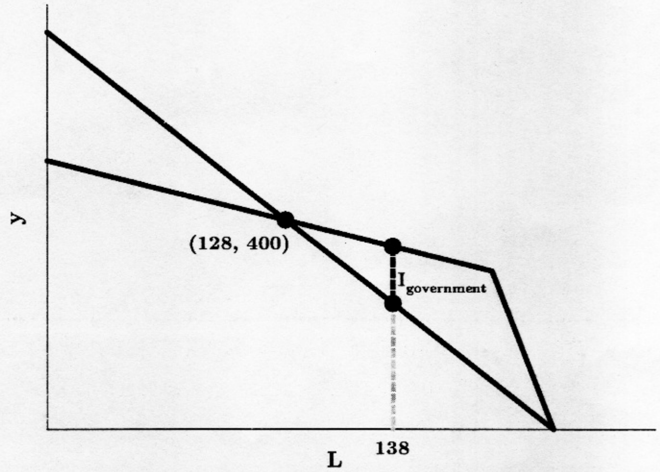


b)

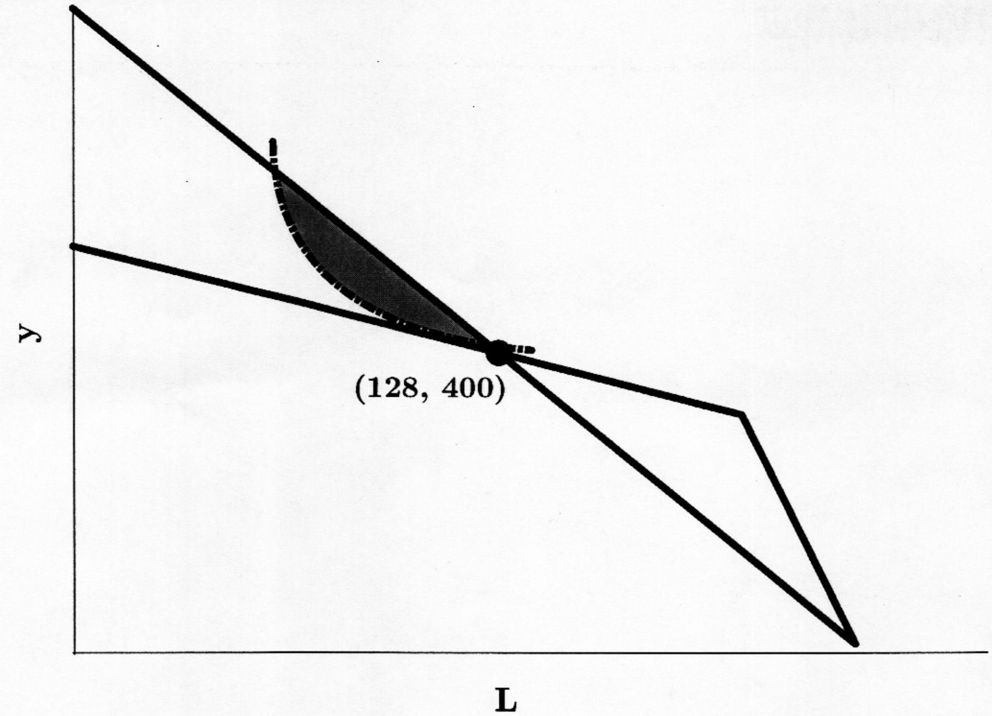


c) $y = 1552 - 9L$
$y = 1680 - 10L$
$(L, y) = (128, 400)$

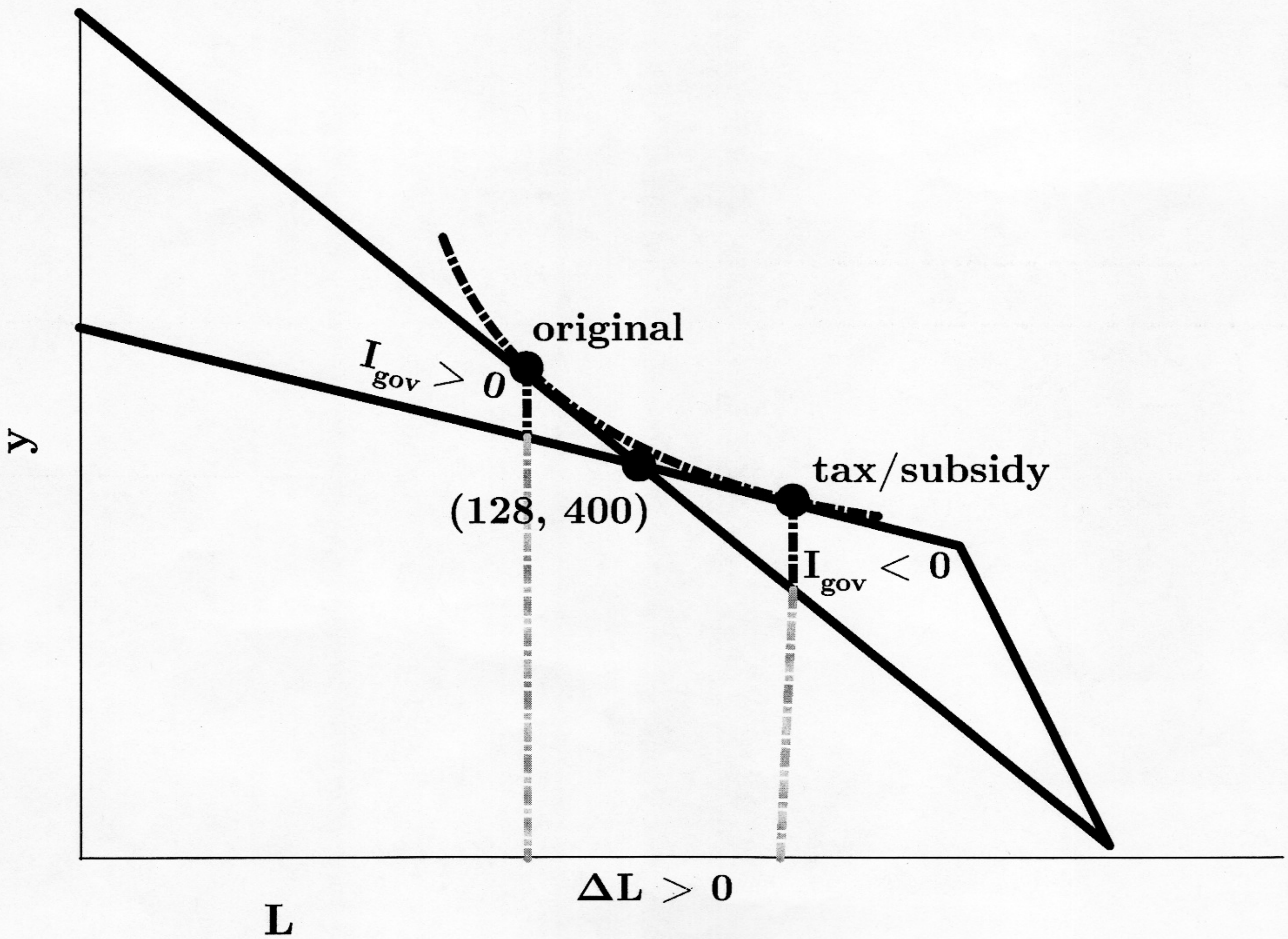
d) Proportions altered for clarity



e) There are strictly better bundles in the dark gray shaded region.



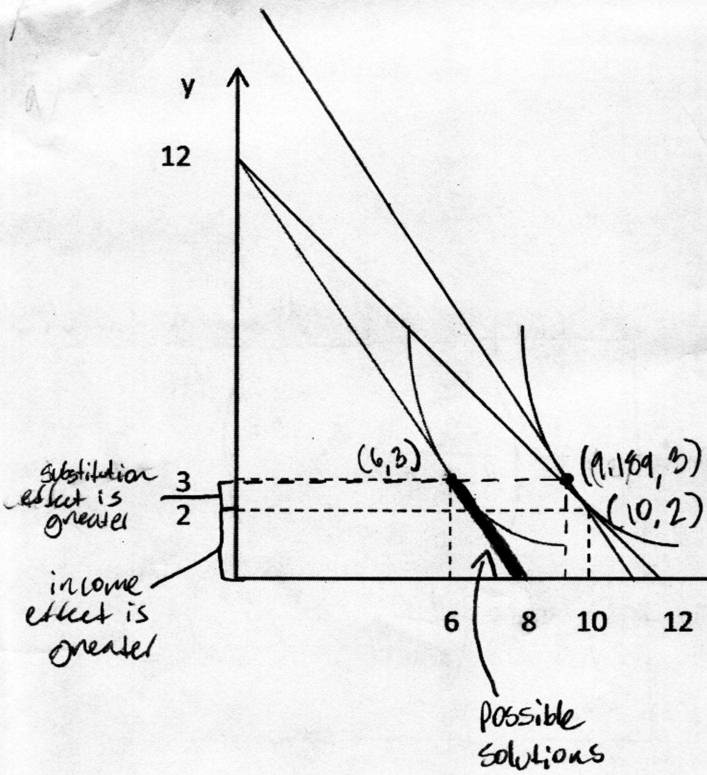
f)



~~Question 7~~

7. a)

Income effect is greater.



(9.189, 3) Hicksian demand at original bundle, new prices

(10, 2) Hicksian demand at original bundle, Original Prices

b)

$$x + 2\ln y = x^* + 2\ln y^*$$

$$\frac{P_x}{P_y} = \frac{y}{2}$$

$$\boxed{y = \frac{2P_x}{P_y}, \quad x = x^* + 2\ln y^* - 2\ln\left(\frac{2P_x}{P_y}\right)}$$

c) Original bundle, original prices

$x^* = 10$ $\quad P_x = 1$

$y^* = 2$ $\quad P_y = 1$

$y = \frac{2P_x}{P_y} = 2$

$x = 10 + 2\ln y^* - 2\ln \frac{2P_x}{P_y}$

$= 10 + 2\ln 2 - 2\ln 2$

$= 10$

Hicksian demand = (10, 2)

Original bundle, new prices

$x^* = 10$ $\quad P_x = 1.5$

$y^* = 2$ $\quad P_y = 1$

$y = \frac{2P_x}{P_y} = 3$

$x = 10 + 2\ln(2) - 2\ln(3)$

$x = 9.189$

Bonus

a) $\ln x + 3\ln y = \ln x^* + 3\ln y^*$

$$\frac{P_x}{P_y} = \frac{y}{3x}, \quad y = \frac{3xP_x}{P_y}$$

$\downarrow$

$$\ln x + 3\ln\left(\frac{3xP_x}{P_y}\right) = \ln x^* + 3\ln y^*$$

$$\ln\left(27x^4\frac{P_x^3}{P_y^3}\right) = \ln xy^3$$

$$27x^4\frac{P_x^3}{P_y^3} = x^*y^{*3} \longrightarrow$$

compensated demand curves

$$\boxed{x = \left(\frac{P_y^3}{27P_x^3}x^*y^{*3}\right)^{1/4} \qquad y = \frac{3P_x}{P_y}\left(\frac{P_y^3}{27P_x^3}x^*y^{*3}\right)^{1/4}}$$

b) uncompensated demand functions

$$\boxed{x^* = \frac{I}{4P_x}, \quad y^* = \frac{3I}{4P_y}}$$

$$\boxed{\frac{\partial x}{\partial P_x} = -\frac{3}{4}P_x^{-7/4}\left(\frac{P_y^3}{27}x^*y^{*3}\right) \text{ [compensated]} \qquad \frac{\partial x}{\partial P_x} = \frac{-I}{4P_x^2} \text{ [uncompensated]}}$$

$$\boxed{\frac{\partial x}{\partial I} = \frac{1}{4P_x}}$$

c) $$\frac{-3}{4}P_x^{-\frac{7}{4}}\left(\frac{P_y^3}{27}x^*y^{*3}\right) - \frac{1}{4P_x}\left(\frac{I}{4P_x}\right) = \frac{-I}{4P_x^2}$$

QED