Ben Durham
Multivariable Calculus

Homework 5/1/20

p. 719 #44, 49, 53

Problem #44

Using the formula for a plane given a normal vector and a point, substitute in the given values.

$$P_0 = (1, 0, 2)$$
$$\vec{n} = \langle -1, 2, 1 \rangle$$
$$-(x-1) + 2(y-0) + (z-2) = 0$$
$$-x + 2y + z - 1 = 0$$

Problem #49

a. Since the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ lie on the plane, taking their cross product will yield a vector normal to the plane.

$$\overrightarrow{PQ} = \langle 2, 3, 4 \rangle$$
$$\overrightarrow{PR} = \langle 1, 3, 0 \rangle$$
$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$$
$$\vec{n} = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 3 & 0 \end{vmatrix}$$
$$\vec{n} = \langle -12, 4, 3 \rangle$$
$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|} = \frac{1}{13}\langle -12, 4, 3 \rangle$$

b. One can find the angle between the vectors using the equation of the dot product

$$\|\overrightarrow{PQ}\|\|\overrightarrow{PR}\| \cos\theta = PQ_x PR_x + PQ_y PR_y + PQ_z PR_z$$
$$\sqrt{(29)(10)} \cos\theta = 11$$
$$\theta = \arccos\left(\frac{11}{\sqrt{290}}\right)$$
$$\theta = 49.7636°$$

c. The cross product yields the area of the parallelogram outlined a set of vectors, so to find the area of the triangle outlined by a set of vectors, the aforementioned value can just be halved.

$$A_{\triangle PQR} = \frac{1}{2}\|\overrightarrow{PQ} \times \overrightarrow{PR}\|$$

$$= \frac{1}{2}\|\langle 2,3,4\rangle \times \langle 1,3,0\rangle\|$$

$$= \frac{1}{2}\|\langle -12,4,3\rangle\|$$

$$= \frac{13}{2}$$

d. To find the distance between R and the line joining P and Q, one can take the projection of the vector $\overrightarrow{PR}$ on the unit vector $\widehat{PQ}$

$$\widehat{PQ} \cdot \overrightarrow{PR} = \frac{1}{\sqrt{29}}\langle 2,3,4\rangle \cdot \langle 1,3,0\rangle$$

$$= \frac{11}{\sqrt{29}}$$

Problem #53

a. For a point to be on the x axis, the y and z value must both be zero. Therefore, setting these two values to zero, one can solve for x.

$$5x = 21$$

$$x = \frac{21}{5}$$

Therefore, the plane intersects the x axis at ($\frac{21}{5}$, 0, 0)

b. $(0,0,3)$; $(0,-21,0)$

c. $\langle 5,-1,7\rangle$

d. $\langle 0,-21,-3\rangle$