TAMÁS BENEDEK

1.1. $\frac{x^{n+2}}{x^{n-2}} = \frac{x^n \cdot x^2 \cdot x^2}{x^n} = \underline{\underline{x^4}}$

1.2. $x^{-1} \cdot 8 = 2$

$\frac{1}{x} = \frac{2}{8} = \frac{1}{4}$

$x = \underline{\underline{4}}$

1.3. $a = 5$

$b = 10$

$(a^5)^0 = (5^{10})^0 = \underline{\underline{1}}$

1.4. $\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2 \cdot \sqrt{x}}{\sqrt{x}} = \underline{\underline{2}}$

1.5. $x^2 + (x+1)^2 = (x+2)^2$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\underline{\underline{x_1 = 3}}$

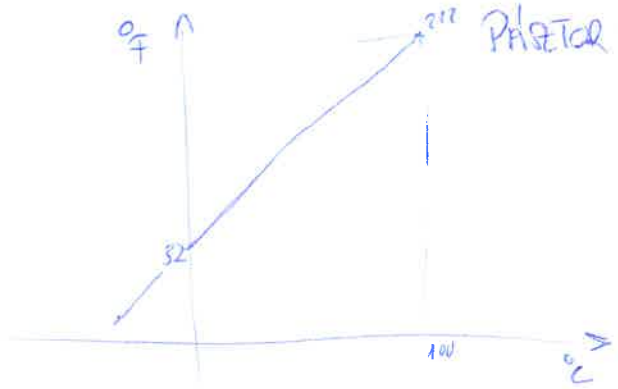
$\underline{\underline{x_2 = -1}}$

1.6. $2^x > 1024 \qquad 2^x > 2^{10}$

$\underline{\underline{x > 10}}$

2.1. $0°C = 32°F$

$100°C = 212°F$



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(1) $32 + x \cdot \frac{212-32}{100} = y$

(2) $x = y$

$32 + x\,\frac{180}{100} = x$

$32 = -0.8x$

$x = -40 \longrightarrow \underline{-40°C = -40°F}$

2.2. $f(x) = 5x + 4$

$f(3) = y$

$y = 5 \cdot 3 + 4 = \underline{19}$

2.3. $x^2 - 4x + 3 = 0$

$(x-2)^2 - 1 = 0$

$x^2 - 4x + 4$

$(x - 2 + 1)(x - 2 - 1) = 0$

$(x-1)(x-3) = 0$

$x_1 = 1$

$\underline{x_2 = 3}$

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2.4. 10 HUF, 90 yrs, 2% | $i = 0.02$ $n = 90$

$$10 \cdot (0.02)^{90} = \underline{59.43}$$

2.5. $e^{\ln 5} = \underline{5}$

3.1. $a = 12$ $b = \frac{1}{6}$

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{a\,b}{1-b} = \frac{12 \cdot \frac{1}{6}}{1-\frac{1}{6}} = \underline{\frac{12}{5}}$$

3.2. $$\lim_{x \to 1} \frac{6^{1-x}}{x} = \lim_{x \to 1} \frac{6}{6^x \cdot x} = \underline{1}$$

3.3. $f(x) = x^5 - 8$ $x = -3$

$$f'(x) = 5x^4 \Big|_{x=-3} = 5(-3)^4 = \underline{405}$$

3.4. $$\frac{d}{dx} \frac{x^3+2x-1}{x-2} = \underline{\frac{(3x^2+2)(x-2) - (x^3+2x-1)}{(x-2)^2}} = \underline{\frac{2x^3-6x^2-3}{x^2-4x+4}}$$

3.5. $$\frac{d^2}{dx^2} 4x^4 + 4x^2 = \frac{d}{dx} 16x^3 + 8x = \underline{48x^2 + 8}$$

3.6. $$\frac{d}{dx} \frac{\ln x}{e^x} = \frac{\frac{1}{x} \cdot e^x - \ln x \cdot e^x}{(e^x)^2} = \underline{\frac{\frac{1}{x} - \ln x}{e^x}}$$

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3.7. $f(x) = 3x^2 - 5x + 2$

$f'(x) = 6x - 5 \longrightarrow 6x - 5 = 0 \longrightarrow x = \frac{5}{6}$

$f''(x) = 6$

x	$x < \frac{5}{6}$	$x = 5/6$	$x > \frac{5}{6}$
$f(x)$	$\lim_{x\to-\infty}(3x^2-5x+2) = \infty$	$-\frac{1}{12}$	$\lim_{x\to\infty}(3x^2-5x+2) = \infty$
$f'(x)$	$-$	0 LOCAL MIN.	$+$
$f''(x)$	$+$ ∪ convex	$+$ ∪ convex	$+$ ∪ convex

3.8. $f(x,y) = x^2 + y^3$

$f(2,3) = 4 + 27 = \underline{\underline{31}}$

3.9. $f(x,y) = \ln(x-y)$

$x - y > 0$

$\underline{\underline{x > y}}$

3.10. $\frac{\partial}{\partial x} x^5 + xy^3 = \underline{\underline{5x^4 + y^3}}$

3.11. $f(x,y) = x^2y^2 + 10$

$f_x(x,y) = 2y^2x \qquad f_y' = 2x^2y \qquad x = 0 \quad y = 0$

$f_x'' = 2y^2 \qquad f_y'' = 2x^2$

Local minima in case of $x = 0$; $y = 0$.

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3.12. $\max(x^2y^2)$ $x+y=10$

$f'_x = 2xy^2 \rightarrow 2xy^2 - \lambda = 0$

$f'_y = 2yx^2 \rightarrow 2yx^2 - \lambda = 0$

$2xy^2 = 2yx^2$

$x = y$

$2x = 10$

$x = 5 = y$

4.1. $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

4.2. $A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$B \cdot A = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$

$\begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$

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4.3. $\begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix}^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$

4.4. $\det(A) = ?$ $A = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$

$\det(A) = 8 - 18 = \underline{-10}$

5.1.

d_1 / d_2	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

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5.2.

Drug user	Test +	Test −
+ (1 %)	99%	1%
− (99%)	0.5%	99.5%

$$P = 0.01 \cdot 0.99 + 0.99 \cdot 0.005 = 0.01485 = 1.485\%$$

5.3.

$$P = \frac{0.01 \cdot 0.99}{0.01 \cdot 0.99 + 0.99 \cdot 0.005} = 0.667 = 66.7\%$$