ANALISI BIOMECCANICA DEL MOVIMENTO "STAND TO SIT"

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Analisi biomeccanica del movimento “stand to sit”

Il movimento "**stand to sit** " (da in piedi a seduto) è un'azione complessa che coinvolge diversi gruppi muscolari e richiede una coordinazione precisa.

I principali muscoli coinvolti sono:

Muscoli del Core:

- Retto dell'addome
- Obliqui (interni ed esterni)
- Trasverso dell'addome
- Erettori spinali (muscoli della schiena)

Muscoli delle Gambe:

- Quadricipiti
- Glutei
- Bicipiti femorali

Processing del video

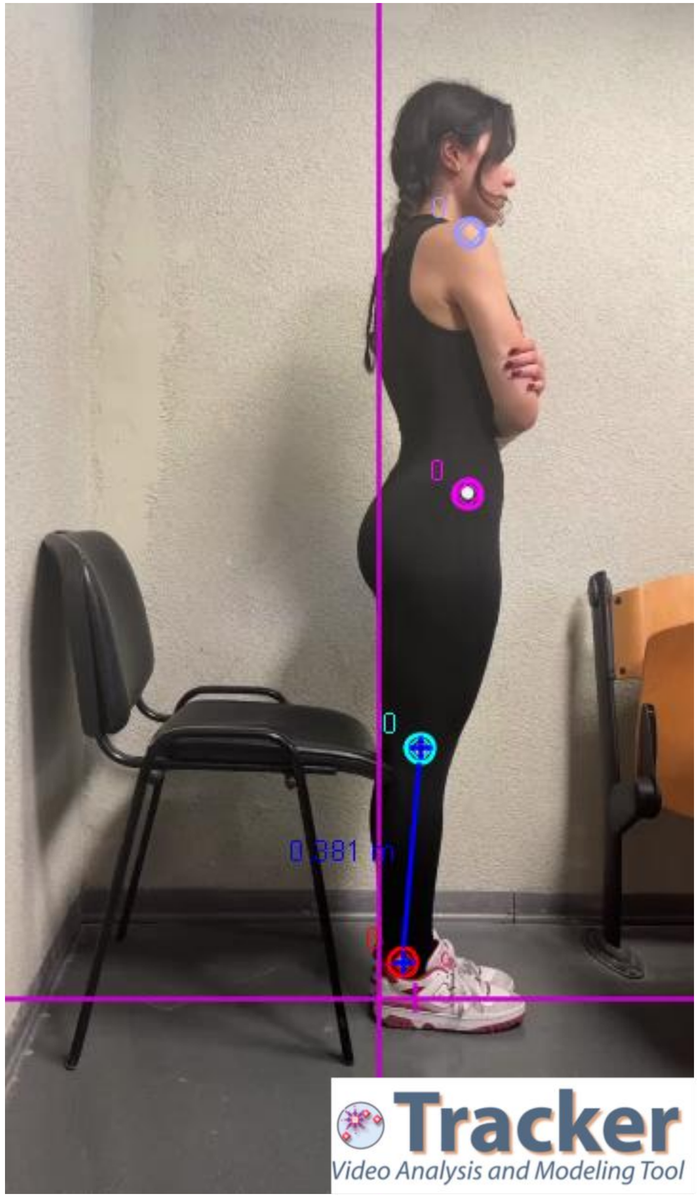
Il video originale durava 31 secondi con una frequenza di 30 FPS.

Il video finale, contenente solo le tre ripetizioni valide, dura poco più di 4 secondi, mantenendo la stessa frequenza di 30 FPS.

Per il tracciamento, abbiamo applicato 4 markers adesivi bianchi su spalla, anca, ginocchio e caviglia del soggetto.

Il sistema di riferimento fisso è stato imposto sul tallone.

Si è definita l'asta di misura tra i markers di caviglia e ginocchio con la rispettiva misura di riferimento:0.381 m ((0.285 – 0.039)×H).



Tracker

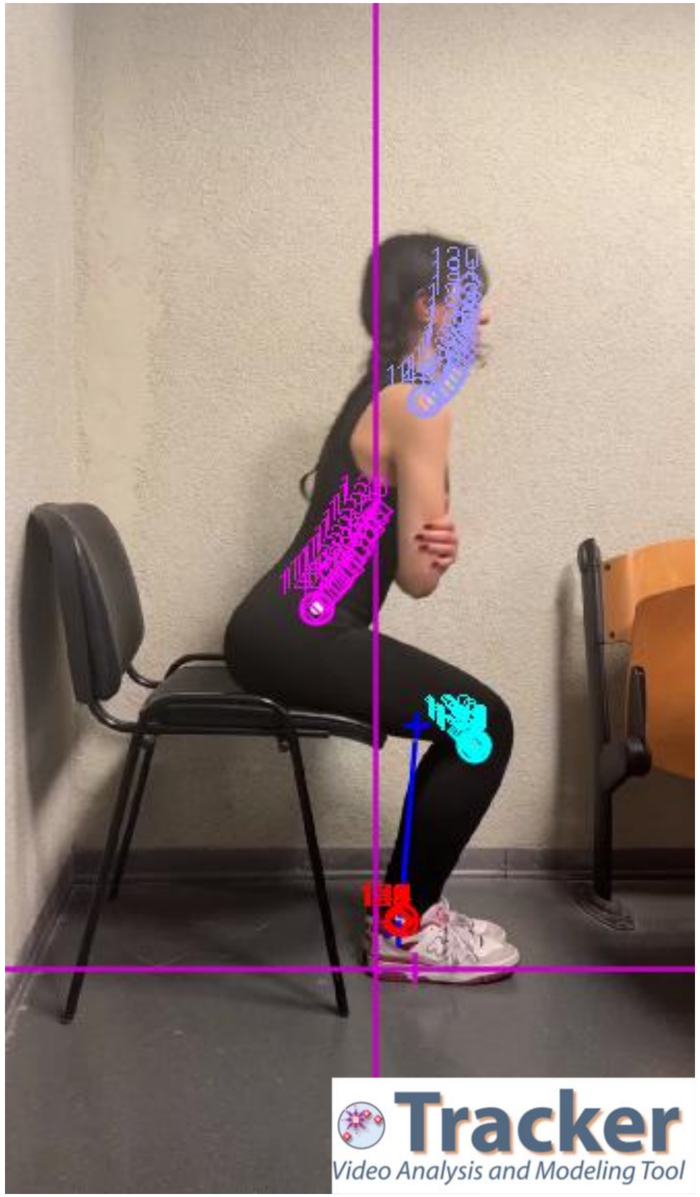
Dopo aver individuato e selezionato i marker su cui lavorare si procede con la traccia con i seguenti parametri settati:

- Frequenza di evoluzione 20%
- Tether 5%
- Automarcamento 4

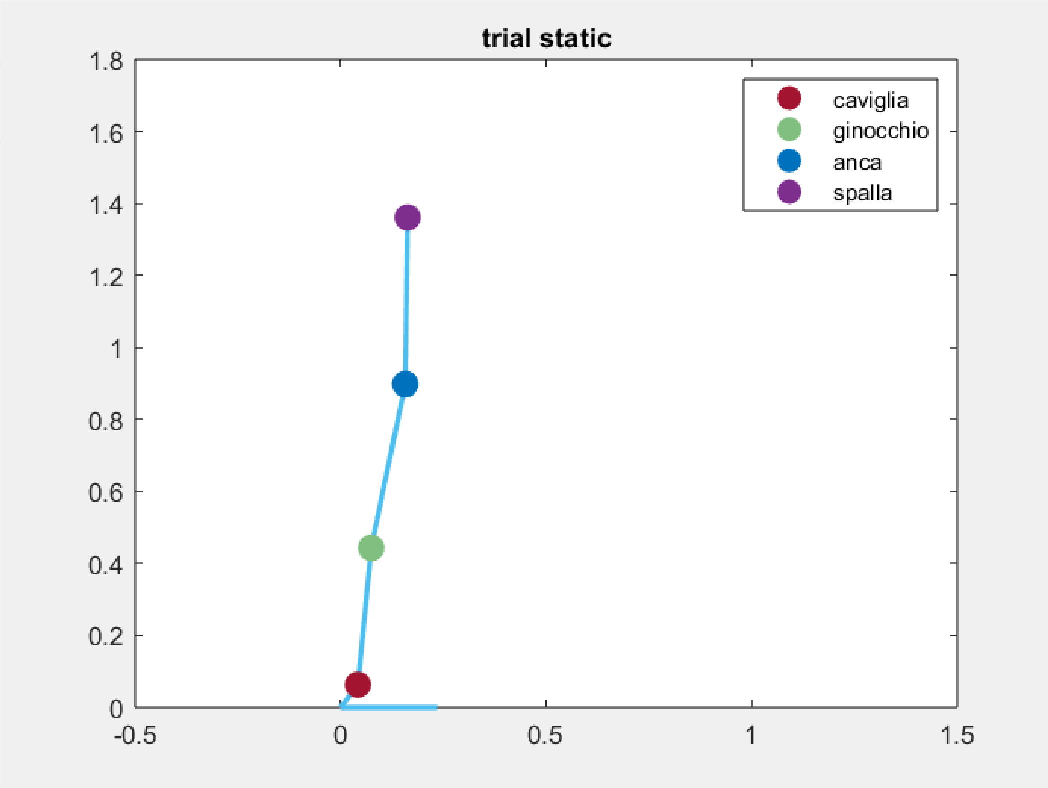
Abbiamo esportato file .txt contenenti le posizioni dei tracker nel tempo per tutti i 144 frames con le seguenti variabili valorizzate come segue:

- Formato Numero: "Precisione Piena"
- Delimitatore: "Tabulatore"

```
%% VERIFYING FRAMES
for i = 1:size(ginocchio,1)
    if caviglia(i,1) ~= ginocchio(i,1) || caviglia(i,1) ~= anca(i,1) ||...
        caviglia(i,1) ~= spalla(i,1) || ginocchio(i,1) ~= anca(i,1) ||...
        ginocchio(i,1) ~= spalla(i,1) || anca(i,1) ~= spalla(i,1)
        error(['I frames acquisiti non sono coerenti in posizione ',num2str(i)]);
    end
end
disp('I frames acquisiti sono coerenti')
```



Modello biomeccanico del movimento "stand to sit": trial static



$H = 1{,}55\ m$

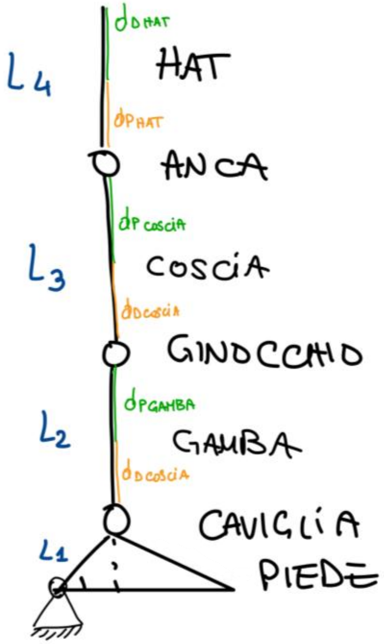
$M = 47\ kg$

$m_{HAT-TOT} = m_{HAT} = M \cdot 0{,}678$

$m_{COSCIA} = M \cdot 0{,}1$

$m_{GAMBA} = M \cdot 0{,}0465$

$m_{PIEDE} = M \cdot 0{,}0145$



DISTANZE PROSSIMALI

$d_{P\,HAT} = 0{,}610\ L_4$

$d_{P\,coscia} = 0{,}433 \cdot L_3$

$d_{P\,GAMBA} = 0{,}433 \cdot L_2$

$d_{P\,PIEDE} = 0{,}5 \cdot L_1$

DISTANZE DISTALI

$d_{D\,HAT} = 0{,}390 \cdot L_4$

$d_{D\,coscia} = 0{,}567 \cdot L_3$

$d_{D\,GAMBA} = 0{,}567 \cdot L_2$

$d_{D\,PIEDE} = 0{,}5\ L_1$

LUNGHEZZE TEORICHE

$L_2 = (0{,}285 - 0{,}039) \cdot H$

$L_3 = (0{,}530 - 0{,}285) \cdot H$

$L_4 = (0{,}818 - 0{,}530) \cdot H$

$L_1 = 0{,}039 \cdot H / mm\alpha$

LUNGHEZZE SPERIMENTALI

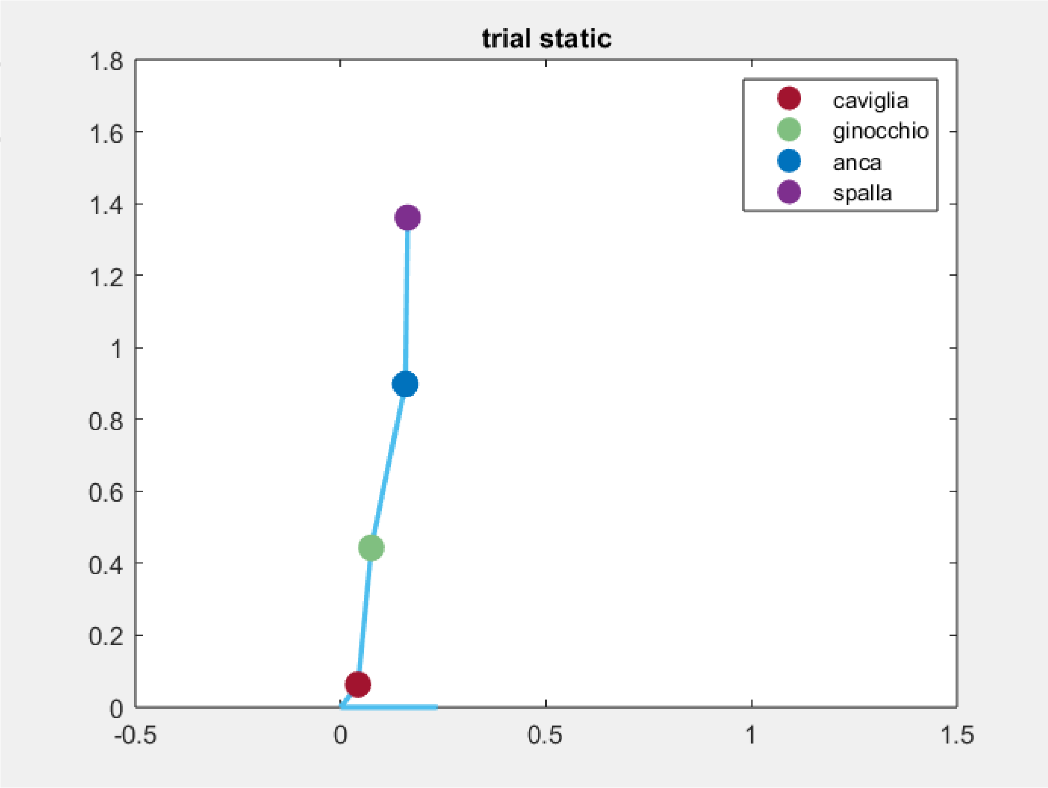
$L_1 = \sqrt{P_{x\,CAVIGLIA}^2 + P_{y\,CAVIGLIA}^2}$

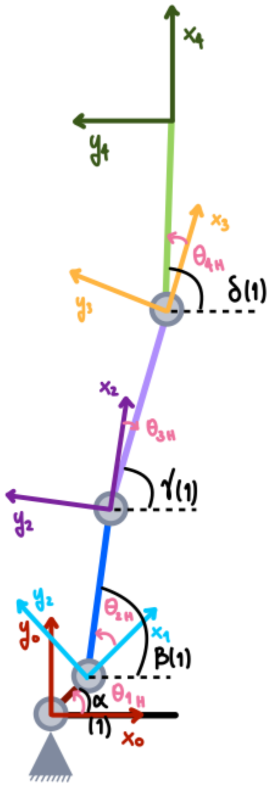
$L_2 = \sqrt{(P_{x\,GINOCCHIO} - P_{x\,CAVIGLIA})^2 + (P_{y\,GINOCCHIO} - P_{y\,CAVIGLIA})^2}$

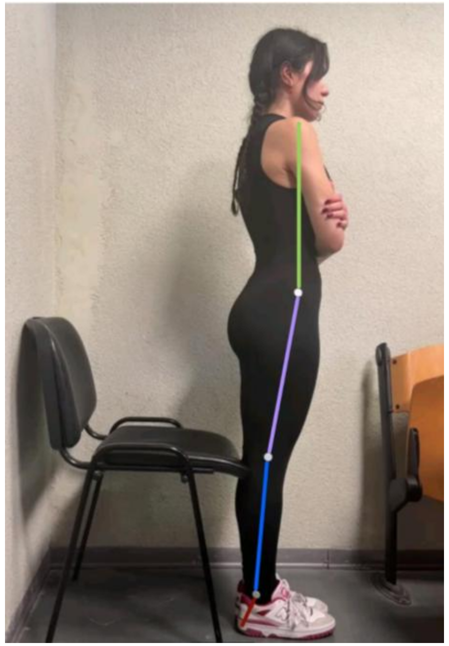
$L_3 = \sqrt{(P_{x\,ANCA} - P_{x\,GINOCCHIO})^2 + (P_{y\,ANCA} - P_{y\,GINOCCHIO})^2}$

$L_4 = \sqrt{(P_{x\,SPALLA} - P_{x\,ANCA})^2 + (P_{y\,SPALLA} - P_{y\,ANCA})^2}$

Modello biomeccanico del movimento “stand to sit”: trial static







	α	a	d	θ_H
0-1	0	l_1	0	$\theta_{1H}=\alpha(1)$
1-2	0	l_2	0	$\theta_{2H}=\beta(1)-\alpha(1)$
2-3	0	l_3	0	$\theta_{3H}=-\beta(1)+\gamma(1)$
3-4	0	l_4	0	$\theta_{4H}=\gamma(1)+\delta(1)$

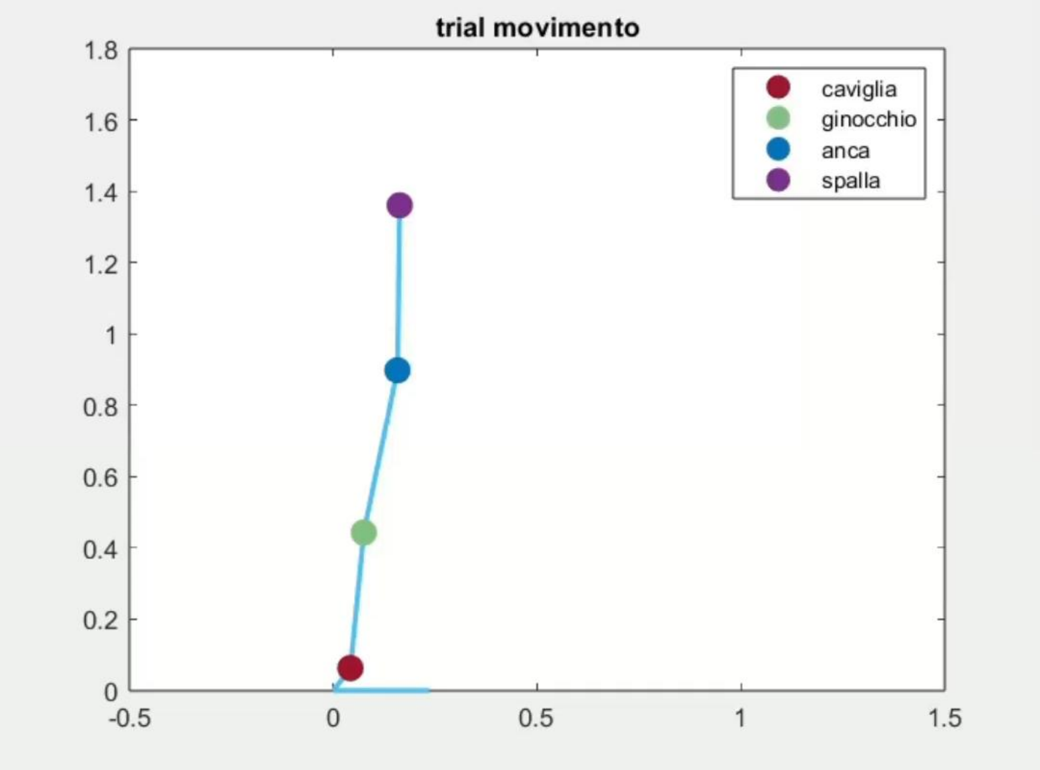
$$q_1=q_2=q_3=q_4=0$$

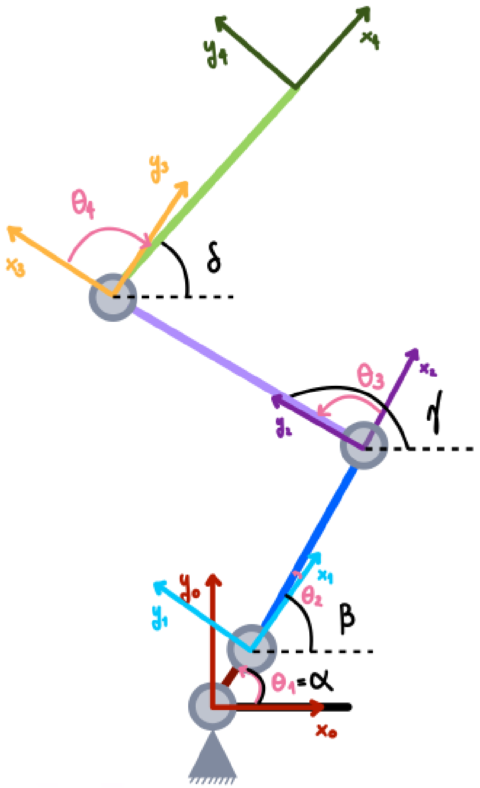
```
% Applicazione della matrice di trasformazione al sdr 1
O1_T_s= T01_s*O1;
X1_T_s= T01_s*X1;
Y1_T_s= T01_s*Y1;

% Applicazione della matrice di trasformazione al sdr 2
O2_T_s= T01_s*T12_s*O2;
X2_T_s= T01_s*T12_s*X2;
Y2_T_s= T01_s*T12_s*Y2;
```

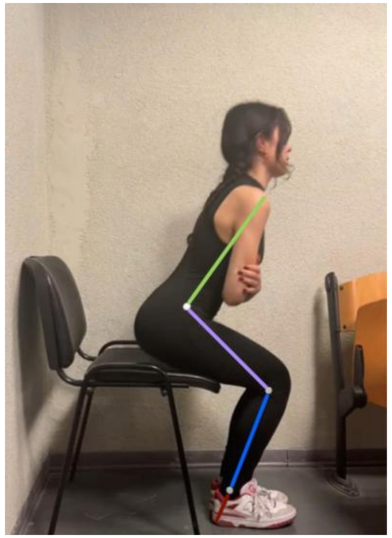
```
% Applicazione della matrice di trasformazione al sdr 3
O3_T_s= T01_s*T12_s*T23_s*O3;
X3_T_s= T01_s*T12_s*T23_s*X3;
Y3_T_s= T01_s*T12_s*T23_s*Y3;

% Applicazione della matrice di trasformazione al sdr 4
O4_T_s= T01_s*T12_s*T23_s*T34_s*O3;
X4_T_s= T01_s*T12_s*T23_s*T34_s*X3;
Y4_T_s= T01_s*T12_s*T23_s*T34_s*Y3;
```

$$^{i-1}T_i=\left[\begin{array}{ccc|c}\cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i\\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i\\ 0 & \sin\alpha_i & \cos\alpha_i & d_i\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

Modello biomeccanico del movimento "stand to sit": trial movimento







	α	a	d	θ	
0-1	0	l_1	0	$\theta_1 = \theta_{1H} + q_1$	$\theta_1 = \alpha$
1-2	0	l_2	0	$\theta_2 = \theta_{2H} + q_2$	$\theta_2 = \beta - \alpha$
2-3	0	l_3	0	$\theta_3 = \theta_{3H} + q_3$	$\theta_3 = \gamma - \beta$
3-4	0	l_4	0	$\theta_4 = \theta_{4H} + q_4$	$\theta_4 = -\gamma + \delta$

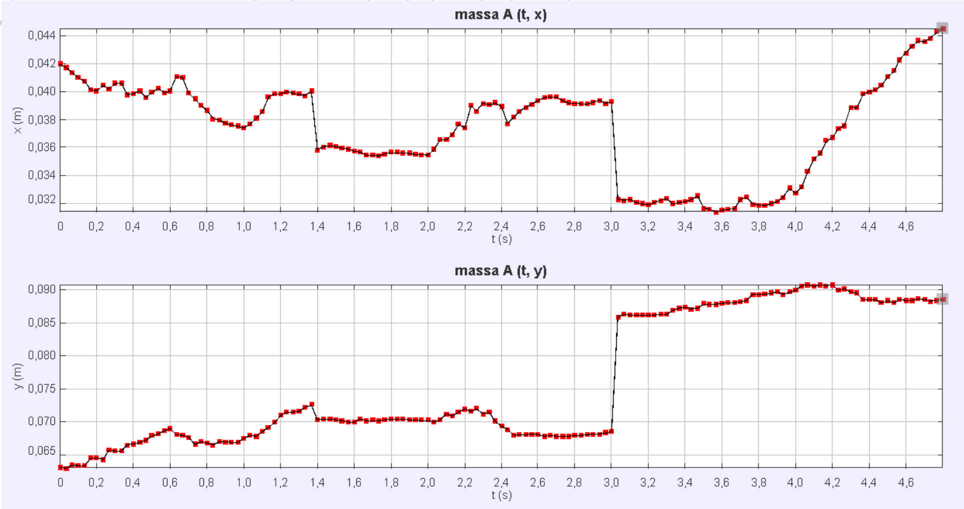
```
%Applicazione della matrice di trasformazione al sdr 1
O1_T= T01*O1;
X1_T= T01*X1;
Y1_T= T01*Y1;

%Applicazione della matrice di trasformazione al sdr 2
O2_T= T01*T12*O2;
X2_T= T01*T12*X2;
Y2_T= T01*T12*Y2;

%Applicazione della matrice di trasformazione al sdr 3
O3_T= T01*T12*T23*O3;
X3_T= T01*T12*T23*X3;
Y3_T= T01*T12*T23*Y3;

%Applicazione della matrice di trasformazione al sdr 4
O4_T= T01*T12*T23*T34*O3;
X4_T= T01*T12*T23*T34*X3;
Y4_T= T01*T12*T23*T34*Y3;
```

$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

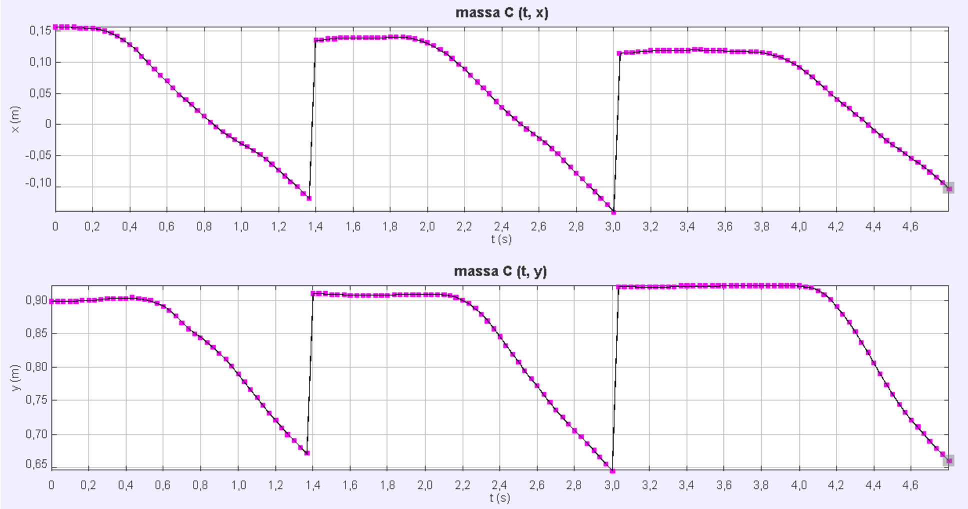
- $\alpha = \alpha(1) + q_1 \rightarrow q_1 = \alpha - \alpha(1)$
- $\beta - \alpha = \beta(1) - \alpha(1) + q_2$
  $\downarrow$
  $q_2 = \beta - \alpha - \beta(1) + \alpha(1)$
- $\gamma - \beta = -\beta(1) + \gamma(1) + q_3$
  $\downarrow$
  $q_3 = \gamma - \beta + \beta(1) - \gamma(1)$
- $-\gamma + \delta = -\gamma(1) + \delta(1) + q_4$
  $\downarrow$
  $q_4 = -\gamma + \delta + \gamma(1) - \delta(1)$

Cinematica

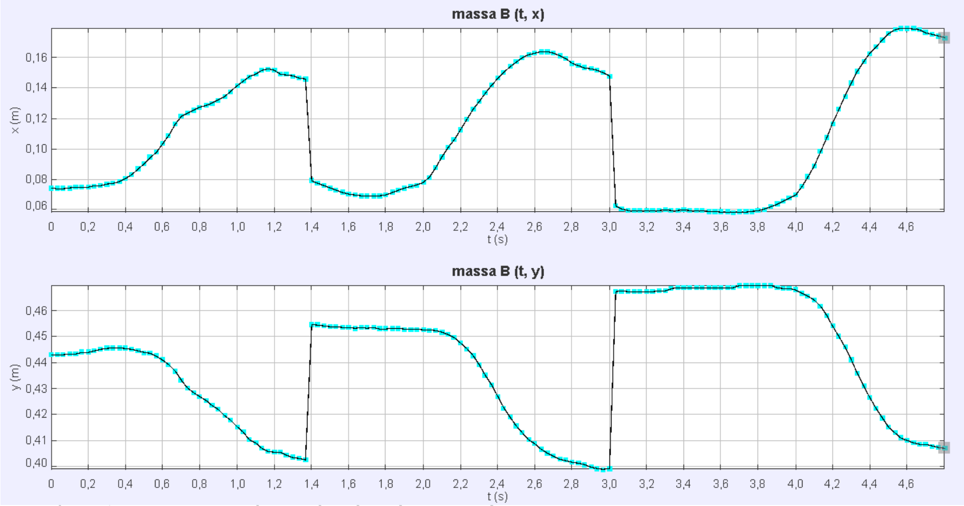
Determinazione delle **posizioni**, velocità lineari e delle accelerazioni lineari



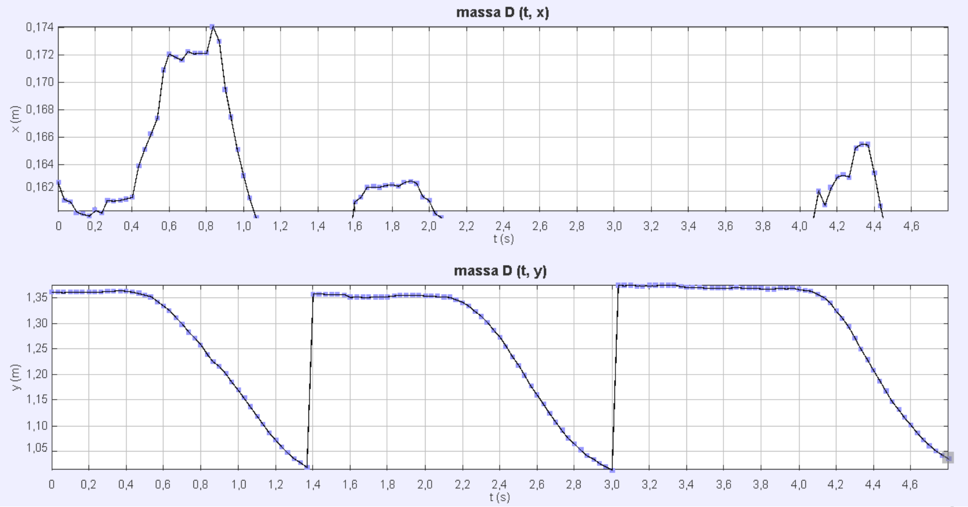
caviglia



anca



ginocchio

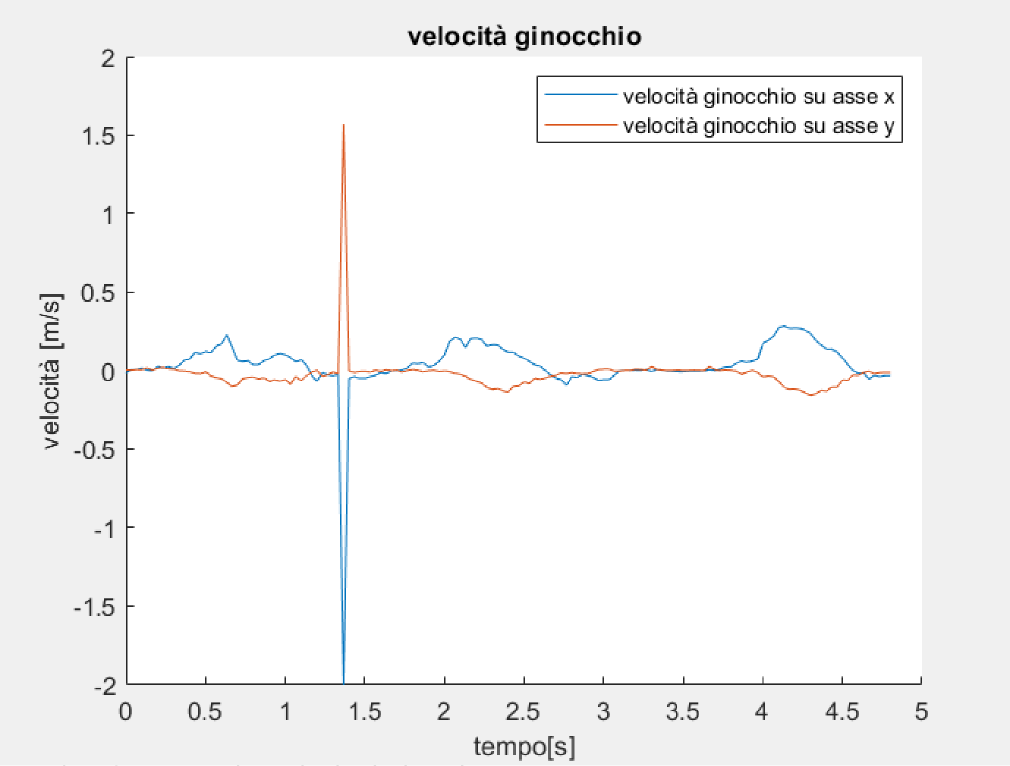


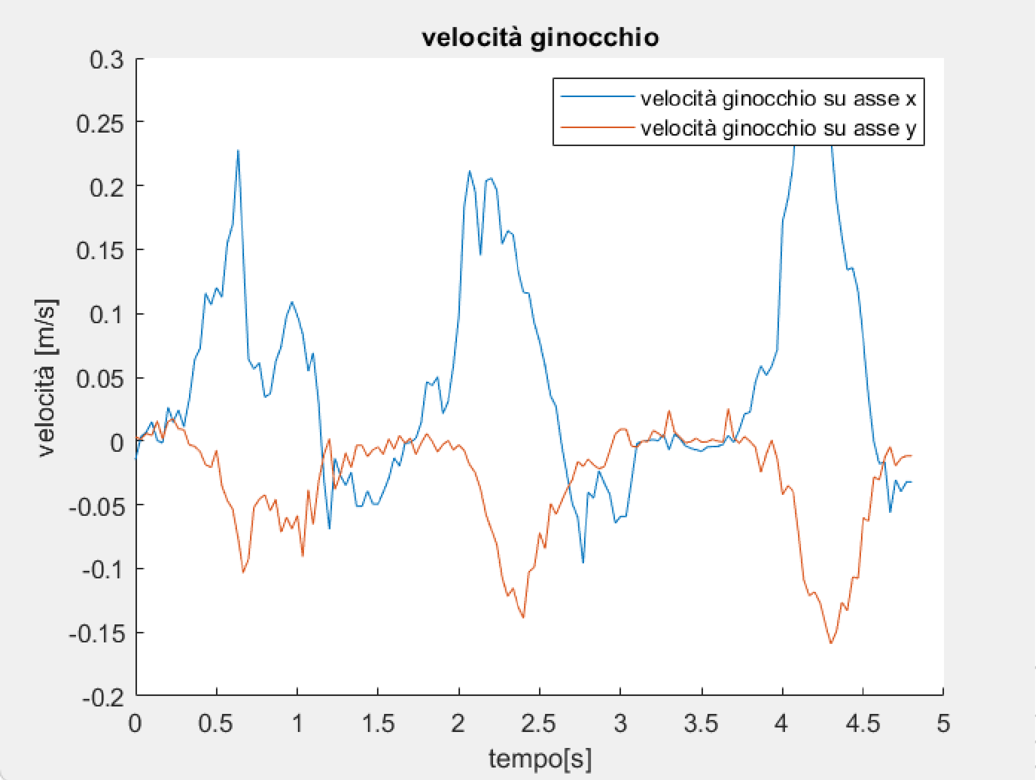
spalla

Cinematica

Determinazione delle posizioni, **velocità lineari** e delle accelerazioni lineari

Per ovviare al problema di visualizzazione dei dati causato dal cambio repentino di posa dovuto alla non continuità del movimento, sono stati elisi quell'istanti di tempo in cui erano presenti i picchi. Qui di seguito, un esempio con la velocità del ginocchio:



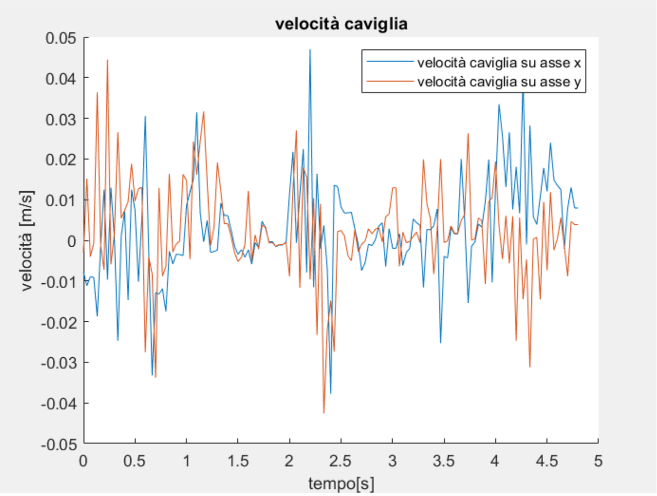


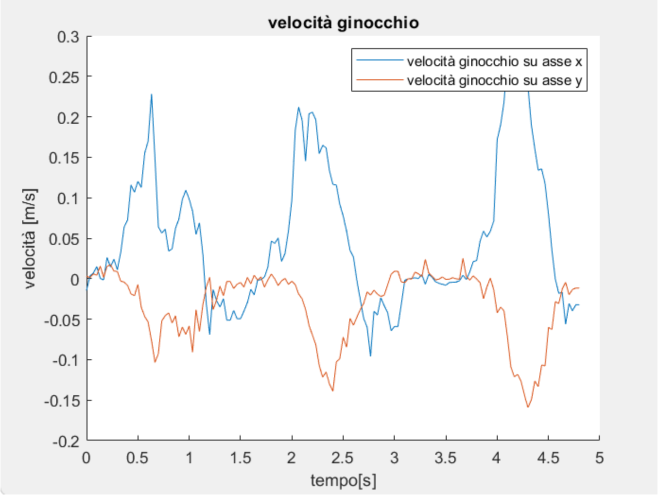
Cinematica

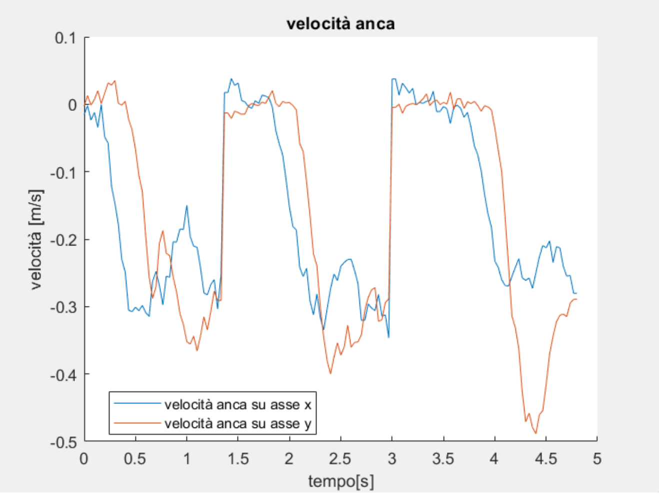
Determinazione delle posizioni, **velocità lineari** e delle accelerazioni lineari

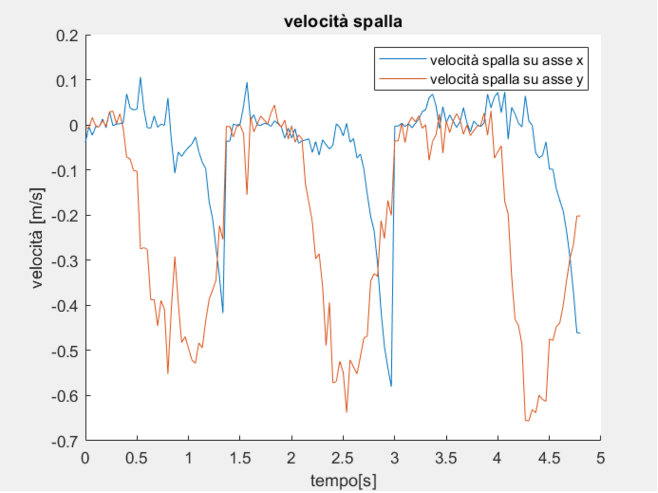
$$V_X = \frac{\Delta S_x}{\Delta T} = \frac{P_{x_{i+1}} - P_{x_i}}{T_{x_{i+1}} - T_{x_i}}$$

$$V_Y = \frac{\Delta S_Y}{\Delta T} = \frac{P_{Y_{i+1}} - P_{Y_i}}{T_{y_{i+1}} - T_{y_i}}$$







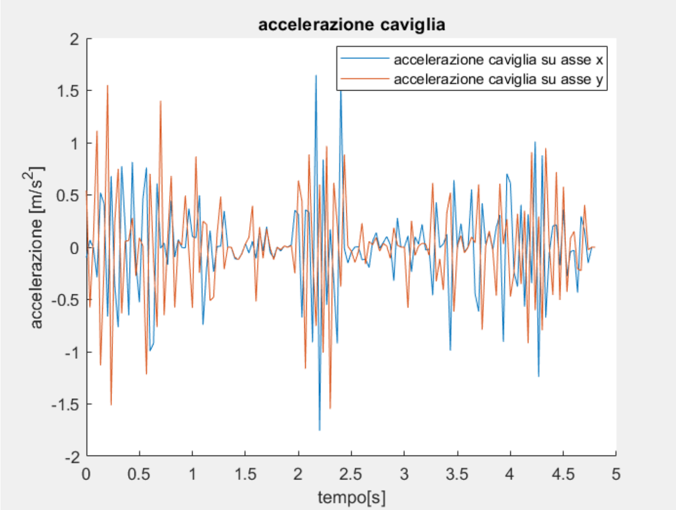


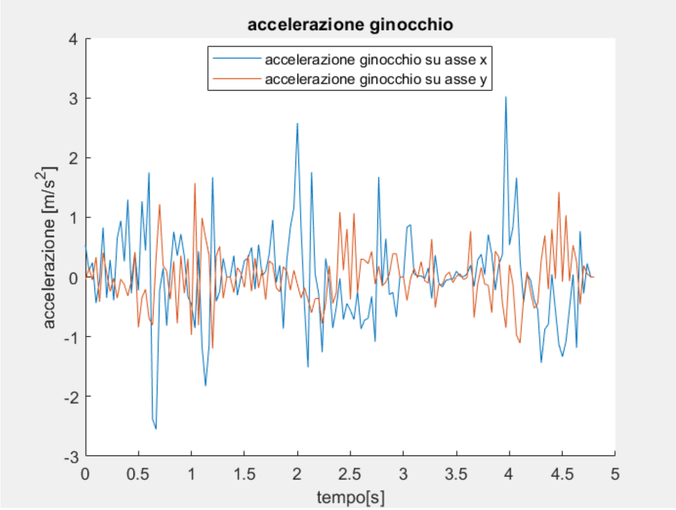
Cinematica

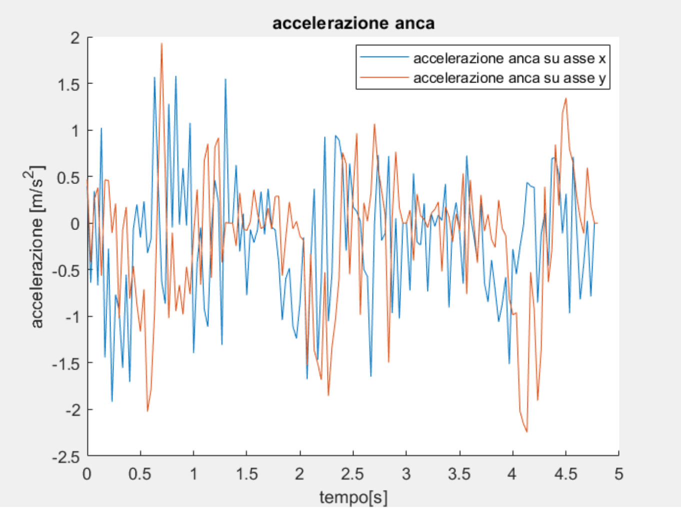
Determinazione delle posizioni, velocità lineari e delle **accelerazioni lineari**

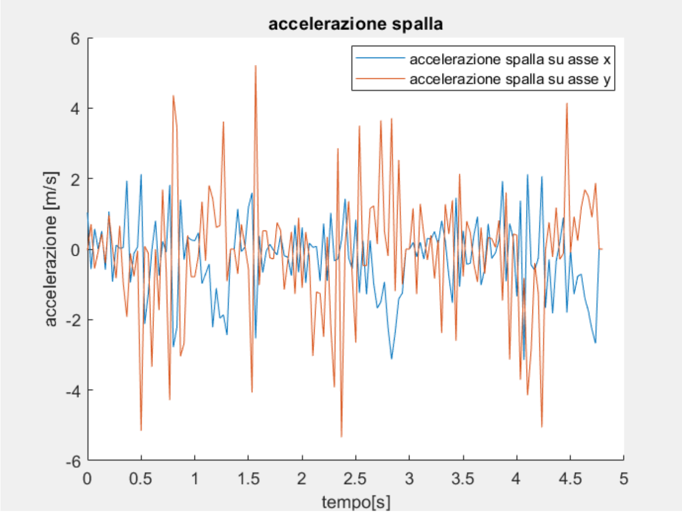
$$a_X = \frac{\Delta V_x}{\Delta T} = \frac{V_{x_{i+1}} - V_{x_i}}{T_{x_{i+1}} - T_{x_i}}$$

$$a_Y = \frac{\Delta V_Y}{\Delta T} = \frac{V_{Y_{i+1}} - V_{Y_i}}{T_{y_{i+1}} - T_{y_i}}$$









Cinematica

Spazio cartesiano → **CINEMATICA INVERSA:** → *Spazio dei giunti*

- Equazione di Galileo:

$$\overline{v_p} = \overline{v_o} + \overline{\omega} \wedge (P - O)$$

- Equazione di Rivals:

$$\overline{a_p} = \overline{a_o} + \dot{\overline{\omega}} \wedge (P - O) + \overline{\omega} \wedge (\overline{\omega} \wedge (P - O))$$

$$V_p = \underbrace{\cancel{V_o}}_{=0} + \omega\,(P-O) \rightarrow \omega = \frac{V_p}{(P-O)}$$

$$\omega_{piede} = \frac{V_{caviglia}}{L_1}$$

$$\omega_{gamba} = \frac{V_{ginocchio}}{L_2}$$

$$\omega_{coscia} = \frac{V_{anca}}{L_3}$$

$$\omega_{HAT} = \frac{V_{spalla}}{L_4}$$

Lavoriamo con il modulo in quanto $(P-O)$, $\overline{V_P}$ e $\overline{a_P}$ si trovano sul piano xy e sappiamo che $\overline{\omega}$ e $\dot{\overline{\omega}}$ sono diretti lungo z.

$$a_p = \underbrace{\cancel{a_o}}_{=0} + \dot{\omega}\,(P-O) - \omega^2 (P-O)$$

$$\dot{\omega} = \frac{a_p + \omega^2 (P-O)}{(P-O)}$$

$$\dot{\omega}_{piede} = \frac{a_{caviglia} + \omega^2_{piede} L_1}{L_1}$$

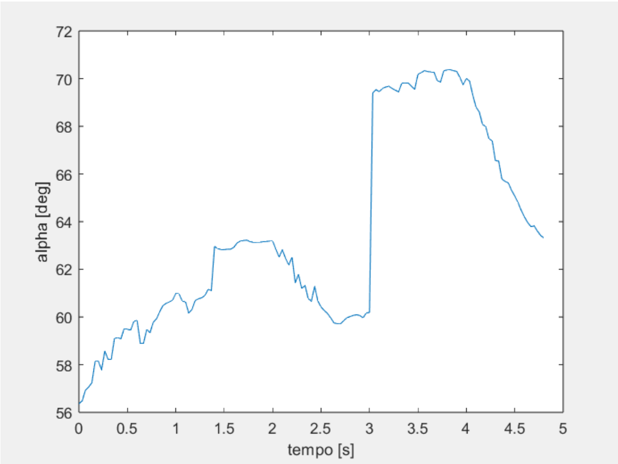
$$\dot{\omega}_{gamba} = \frac{a_{ginocchio} + \omega^2_{gamba} L_2}{L_2}$$

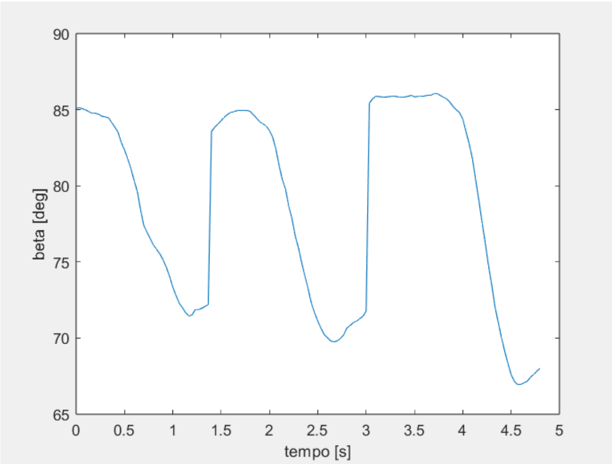
$$\dot{\omega}_{coscia} = \frac{a_{anca} + \omega^2_{coscia} L_3}{L_3}$$

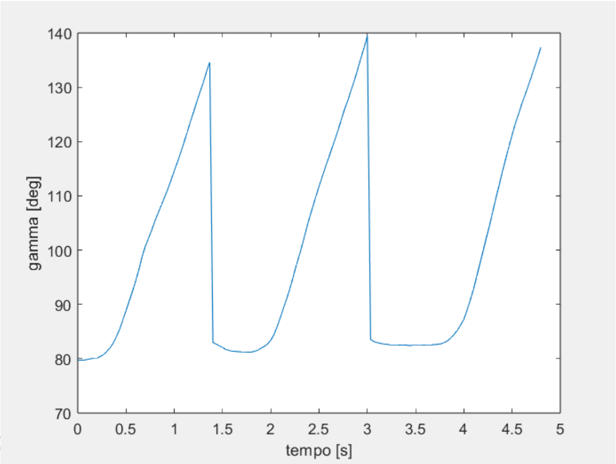
$$\dot{\omega}_{HAT} = \frac{a_{spalla} + \omega^2_{HAT} L_4}{L_4}$$

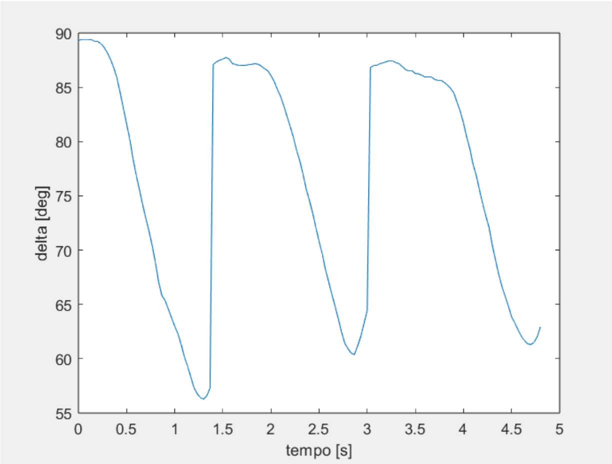
Cinematica

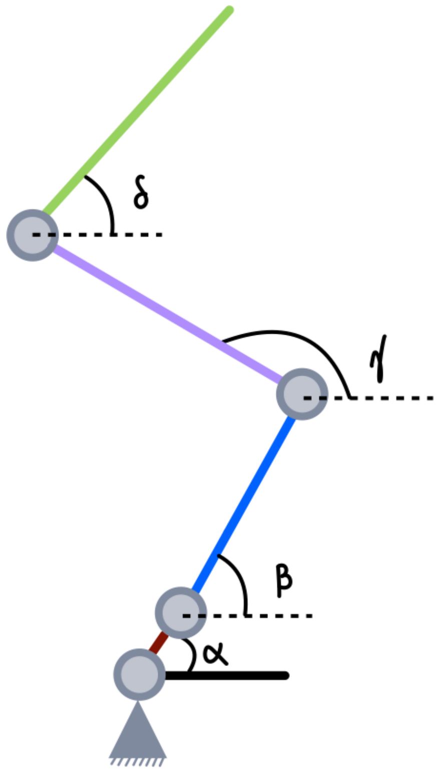
Misurazione degli **angoli articolari** (riferimento ISB)





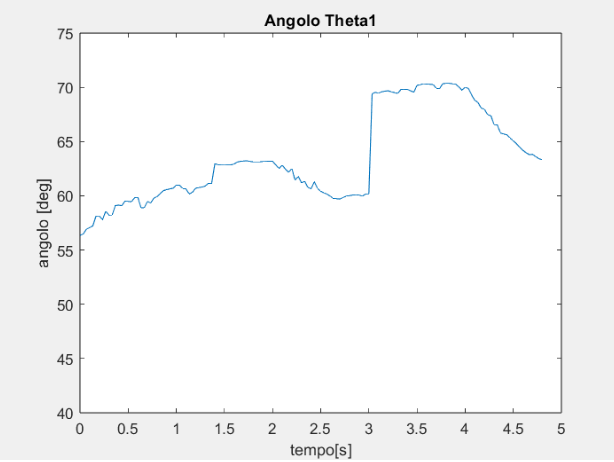


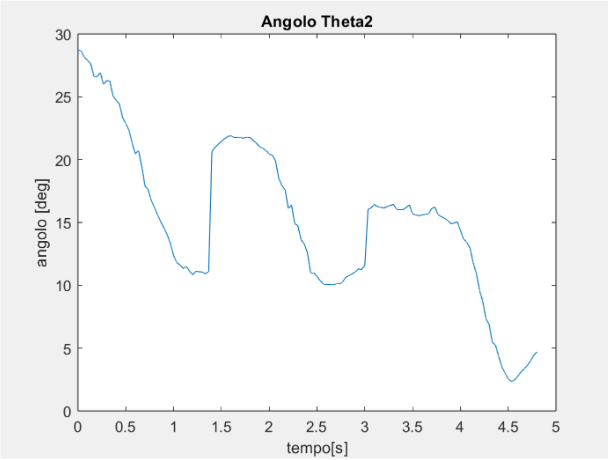


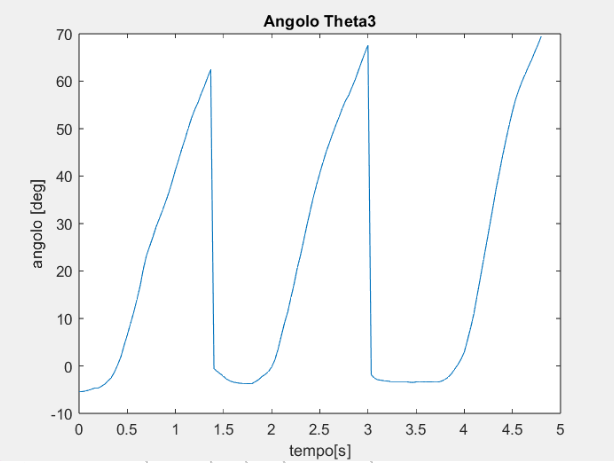


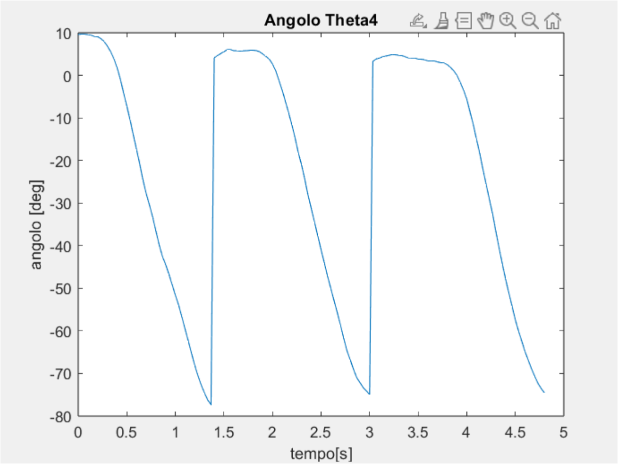
Cinematica

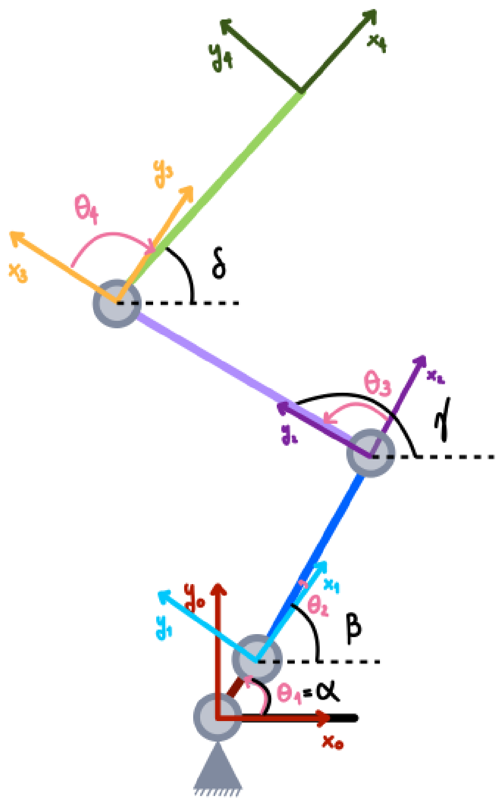
Misurazione degli **angoli articolari** (riferimento DH)









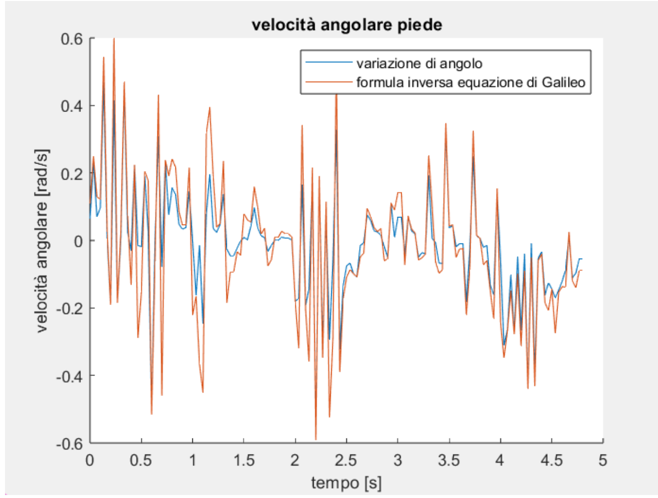


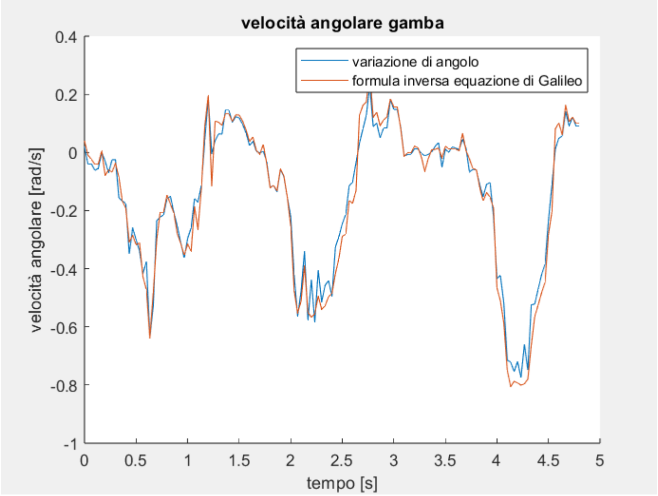
Cinematica

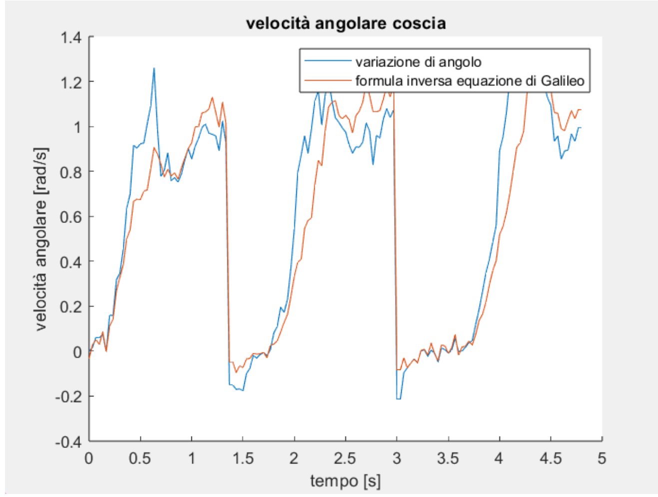
Determinazione delle **velocità angolari** e delle accelerazioni angolari

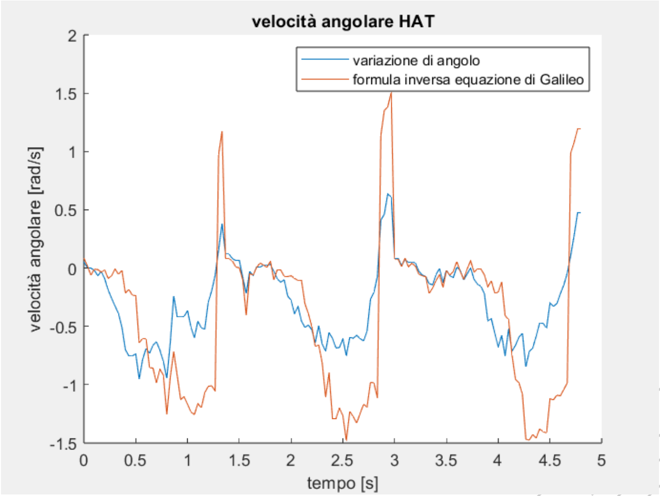
$$w_{\alpha_i} = \frac{\Delta\alpha}{\Delta T} = \frac{\alpha_{i+1} - \alpha_i}{T_{x_{i+1}} - T_{x_i}}$$

$$\overline{V_p} = \overline{V_o} + \overline{w}\ x\ (P - O)$$







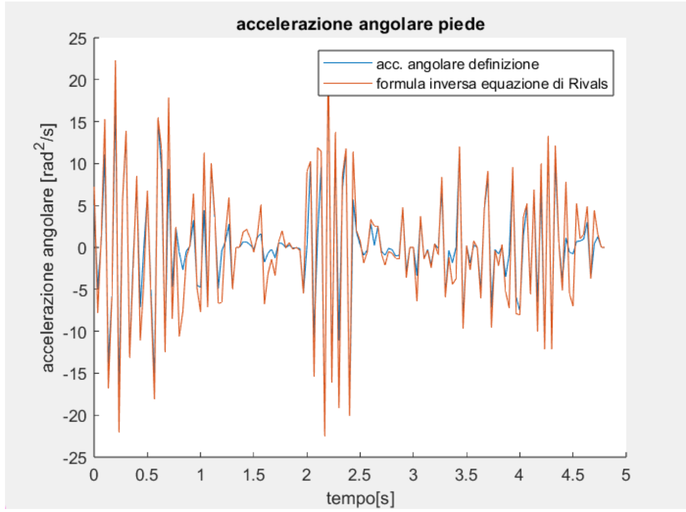


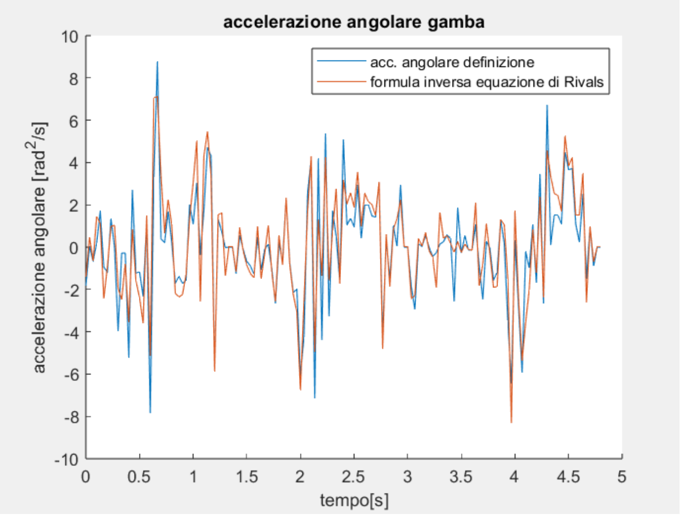
Cinematica

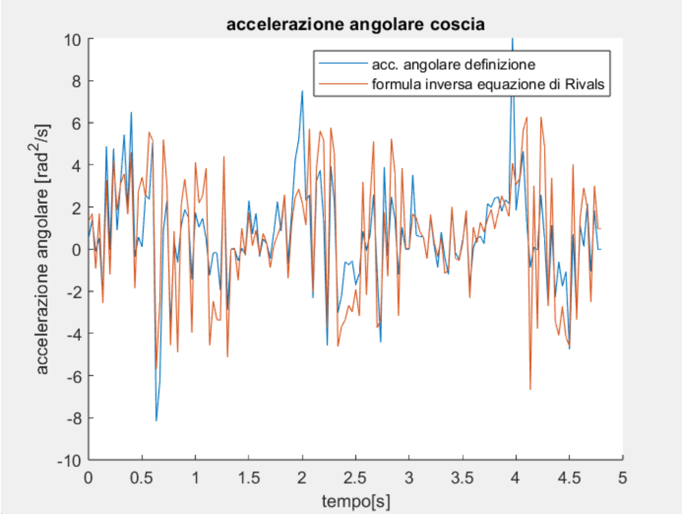
Determinazione delle posizioni, velocità angolari e delle **accelerazioni angolari**

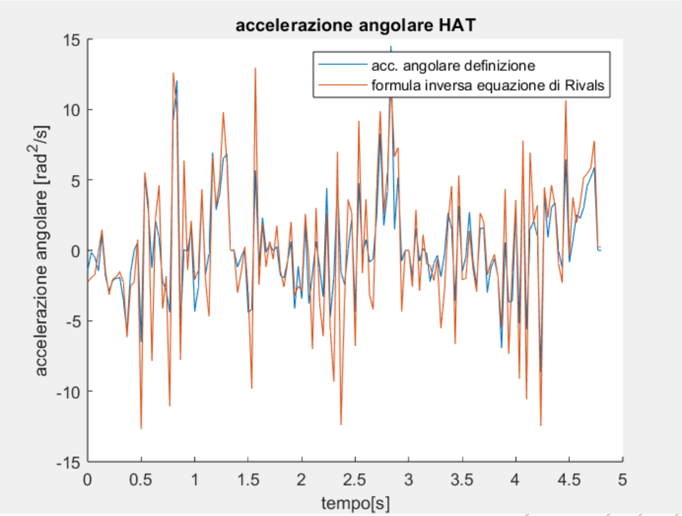
$$\dot{w}_\alpha = \frac{\Delta w_\alpha}{\Delta T} = \frac{w_{\alpha_{i+1}} - w_{\alpha_i}}{T_{x_{i+1}} - T_{x_i}}$$

$$\overline{a_p} = \overline{a_o} + \dot{\overline{w}}\ x\ (P - O) + \overline{w}\ x\ (\overline{w}\ x\ (P - O))$$









Cinematica

Studio del **centro di massa**

massa dell'i-esimo link

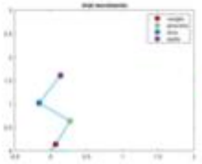
$$cm\_totale = \sum_{1}^{i} \frac{m_i \, cm_i}{M}$$

CoM dell i-esimo link

massa totale

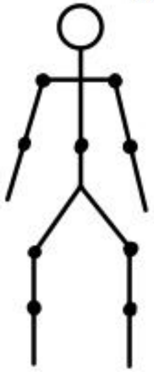
```
cm_piede(:,i) = T01*[-L1(i)*0.5;0; 0; 1];
cm_gamba(:,i)=T01*T12*[-L2(i)*0.433;0; 0; 1];
cm_coscia(:,i)=T01*T12*T23*[-L3(i)*0.433;0; 0; 1];
cm_HAT(:,i)=T01*T12*T23*T34*[-L4(i)*0.390;0; 0; 1];
```

```
vcm_totale = [];
vcm_totale_mod = [];
for i=1:(size(cm_totale,2)-1)
    vcm_totale(:,i) = (cm_totale(:,i+1)-cm_totale(:,i))/(tempo(i+1)-tempo(i));
end
vcm_totale(:,end+1) = vcm_totale(:,end);

 acm_totale = [];
 acm_totale_mod = [];
 for i=1:(size(cm_totale,2)-1)
     acm_totale(:,i) = (vcm_totale(:,i+1)-vcm_totale(:,i))/(tempo(i+1)-tempo(i));
 end
 acm_totale(:,end+1) = acm_totale(:,end);
```



COM

MODELLO SCHELETRICO RIDOTTO



MOCAP

posizioni markers

SOFTWARE

Tracker
Video Analysis and Modeling Tool

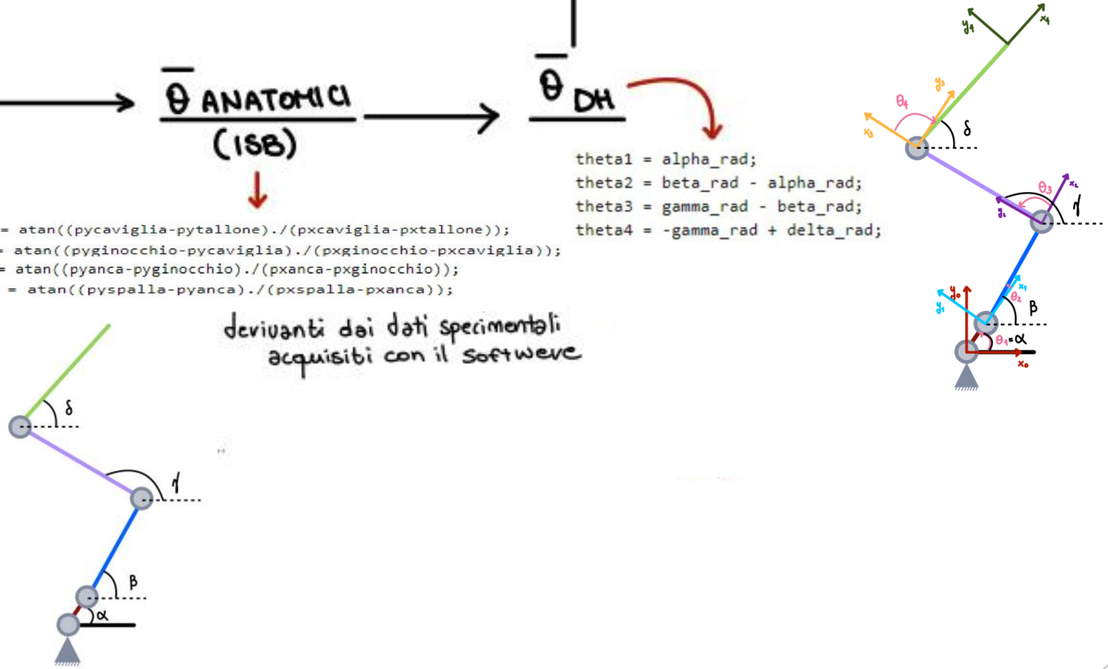
$\bar{\theta}$ ANATOMICI (ISB)

```
alpha_rad = atan((pycaviglia-pytallone)./(pxcaviglia-pxtallone));
beta_rad = atan((pyginocchio-pycaviglia)./(pxginocchio-pxcaviglia));
gamma_rad = atan((pyanca-pyginocchio)./(pxanca-pxginocchio));
delta_rad = atan((pyspalla-pyanca)./(pxspalla-pxanca));
```

derivanti dai dati sperimentali acquisiti con il software

$\bar{\theta}$ DH

```
theta1 = alpha_rad;
theta2 = beta_rad - alpha_rad;
theta3 = gamma_rad - beta_rad;
theta4 = -gamma_rad + delta_rad;
```



Cinematica

Studio del **centro di massa**

posizione

$$
{}^{i-1}T_i = \left[\begin{array}{ccc|c} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

```
cm_piede(:,i) = T01*[-L1(i)*0.5;0; 0; 1];
cm_gamba(:,i)=T01*T12*[-L2(i)*0.433;0; 0; 1];
cm_coscia(:,i)=T01*T12*T23*[-L3(i)*0.433;0; 0; 1];
cm_HAT(:,i)=T01*T12*T23*T34*[-L4(i)*0.390;0; 0; 1];
```

massa dell'i-esimo link

CoM dell i-esimo link

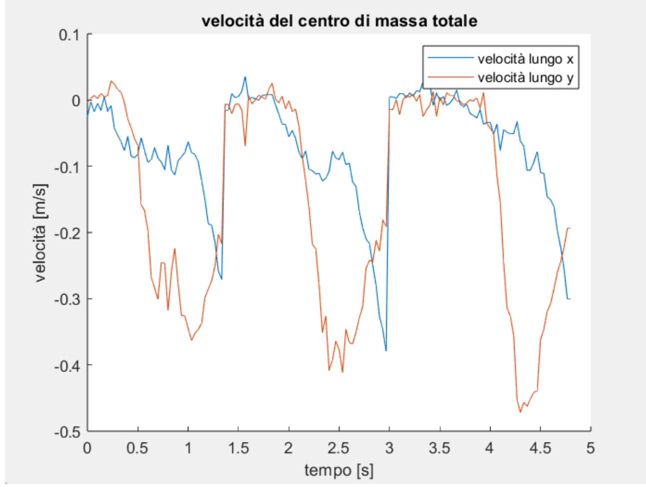
$$
cm\_totale = \sum_{1}^{i} \frac{m_i \, cm_i}{M}
$$

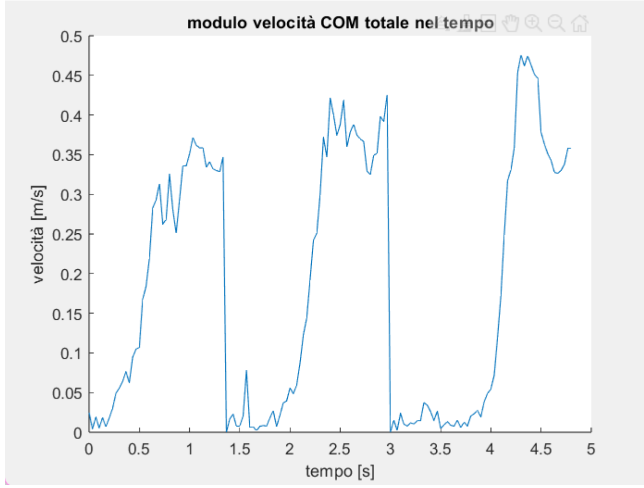
massa totale

velocità lineare

$$v_{cm_{x_i}} = \frac{x_{cm_{i+1}} - x_{cm_i}}{t_{i+1} - t_i}$$

$$v_{cm_{y_i}} = \frac{y_{cm_{i+1}} - y_{cm_i}}{t_{i+1} - t_i}$$

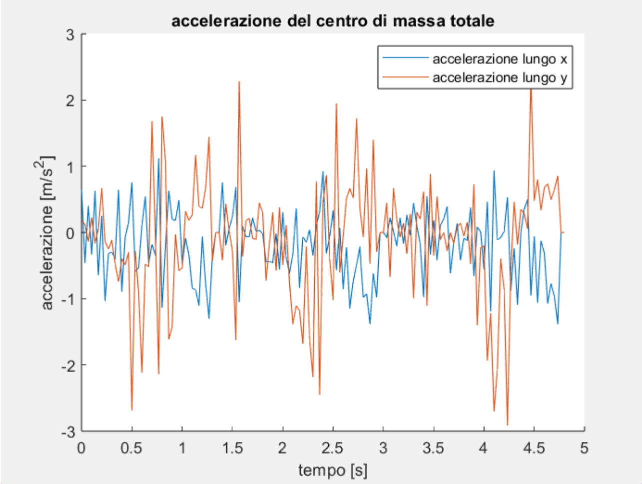


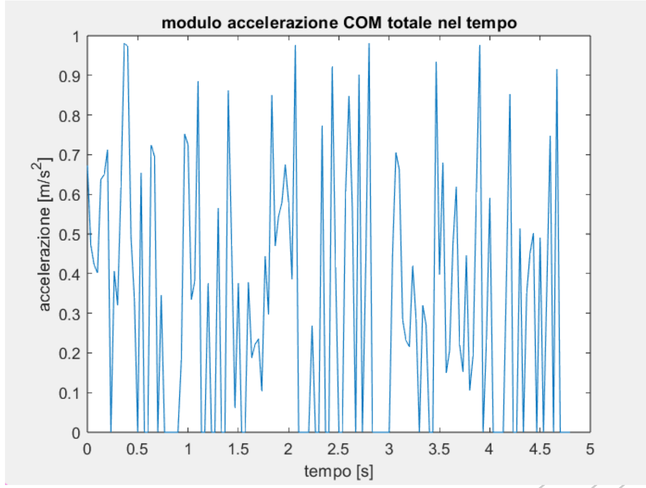


accelerazione lineare

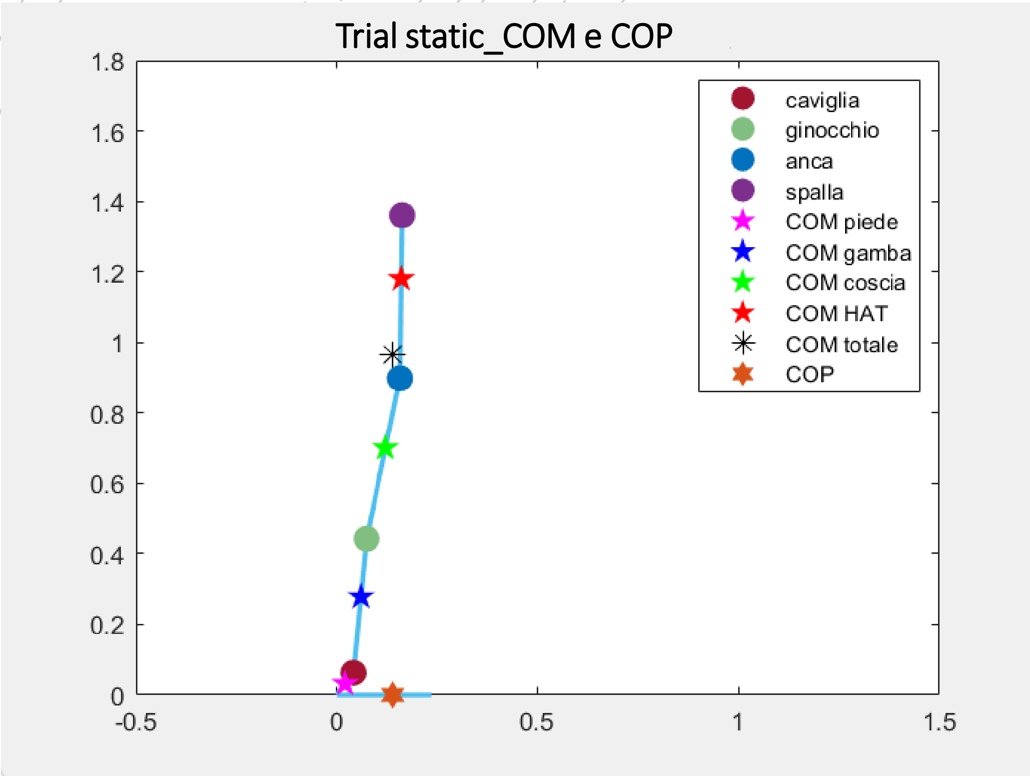
$$a_{cm_{x_i}} = \frac{v_{cm_{i+1}} - v_{cm_i}}{t_{i+1} - t_i}$$

$$a_{cm_{y_i}} = \frac{v_{cm_{i+1}} - v_{cm_i}}{t_{i+1} - t_i}$$





Statica



Date le Equazioni cardinali della statica (ECS)

$$\begin{cases} \text{I} & \sum \overline{F} = 0 \\ \text{II} & \overline{M}_O^{TOT} = 0 \end{cases}$$

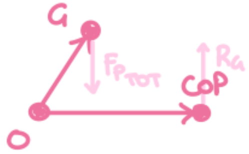
Per la I ECS $\rightarrow$ $-\overline{Fp}_{TOT} + \overline{R}_G = 0$

forza peso totale applicata al COM_TOT

reazione ground

Per la II ECS

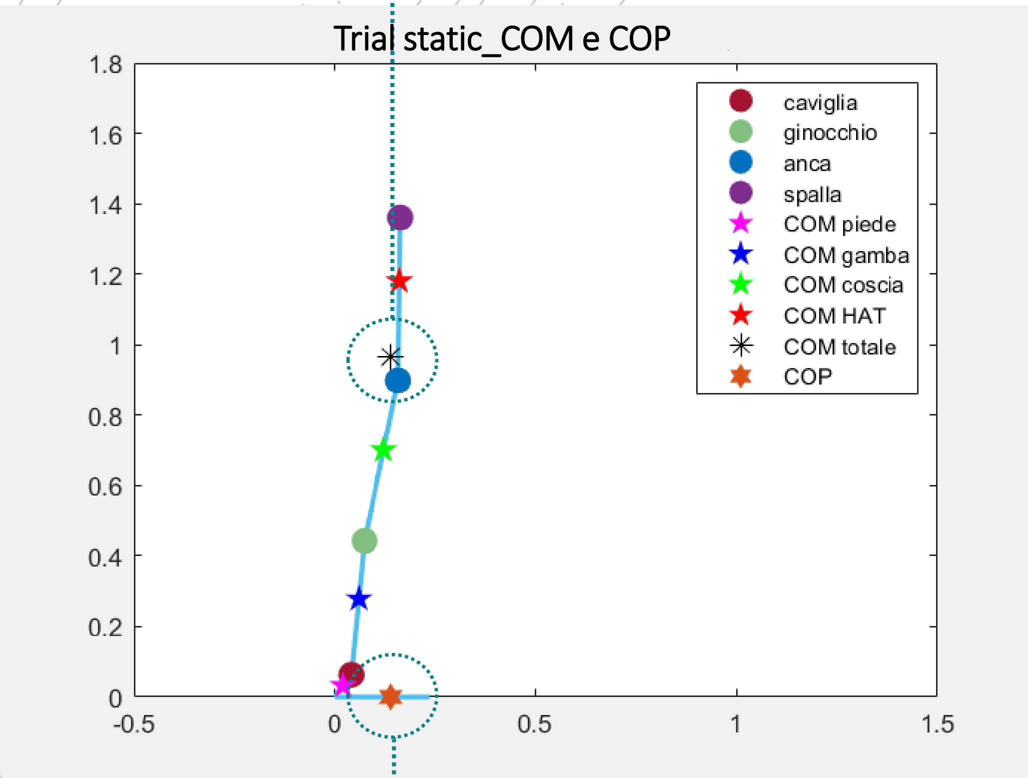
$(G-O) \wedge \overline{Fp}_{TOT} = -Fp_{TOT} \, X_{COM\_TOT}$ $\quad$ $(COP-O) \wedge R_G = +R_G \, X_{COP}$

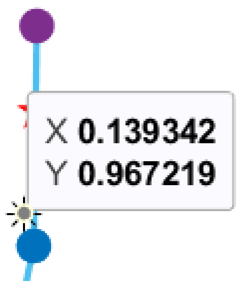


$$-Fp_{TOT} \, X_{COM\_TOT} + R_G \, X_{COP} = 0$$

$$R_G \, X_{COP} = Fp_{TOT} \, X_{COM\_TOT} \rightarrow \boxed{X_{COP} = \frac{Fp_{TOT} \, X_{COM\_TOT}}{R_G}}$$

Statica





Date le Equazioni cardinali della statica (ECS)

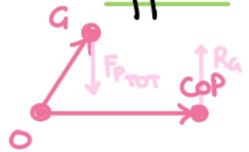
$$\begin{cases} \sum \overline{F} = 0 & \text{(I)} \\ \overline{M}_O^{TOT} = 0 & \text{(II)} \end{cases}$$

forza peso totale applicata al COM_TOT

reazione ground

Per la (I) ECS → $-\overline{F}_{p_{TOT}} + \overline{R}_G = 0$ → Per rispettare (I) ECS $\overline{F}_{p_{TOT}}$ ed $\overline{R}_G$ sono vettori di ugual modulo e verso opposto *

Per la (II) ECS



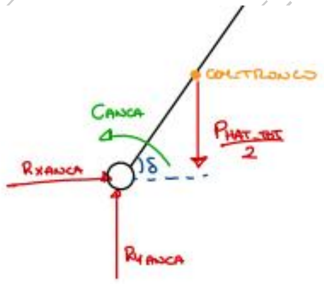
$(G-O) \wedge \overline{F}_{p_{TOT}} = -F_{p_{TOT}} X_{COM\_TOT}$

$(COP-O) \wedge R_G = +R_G X_{COP}$

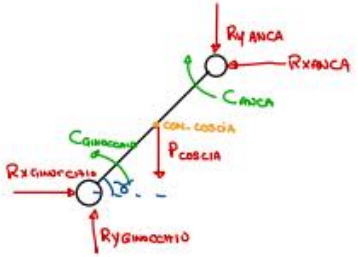
$$-F_{p_{TOT}} X_{COM\_TOT} + R_G X_{COP} = 0$$

$$* \quad \cancel{R_G} X_{COP} = \cancel{F_{p_{TOT}}} X_{COM\_TOT} \rightarrow \boxed{X_{COP} = X_{COM\_TOT}}$$

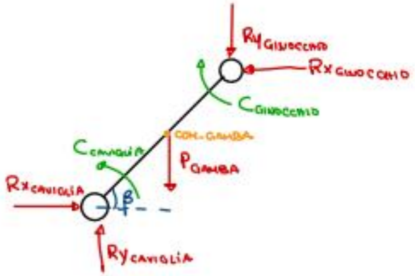
Statica



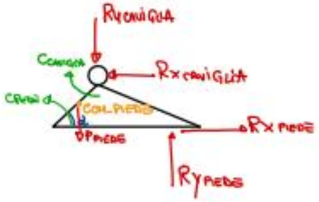
$$
\begin{cases}
R_{x\,ANCA} = 0 \\
R_{y\,ANCA} = \dfrac{P_{HAT\_TOT}}{2} = \dfrac{m_{HAT\_TOT} \cdot g}{2} \\
\Sigma M(com\_tronco) = C_{ANCA} - R_{y\,ANCA} \cdot d_{HAT} \cdot \cos\delta + R_{x\,ANCA} \cdot d_{HAT} \cdot \sin\delta = 0
\end{cases}
$$



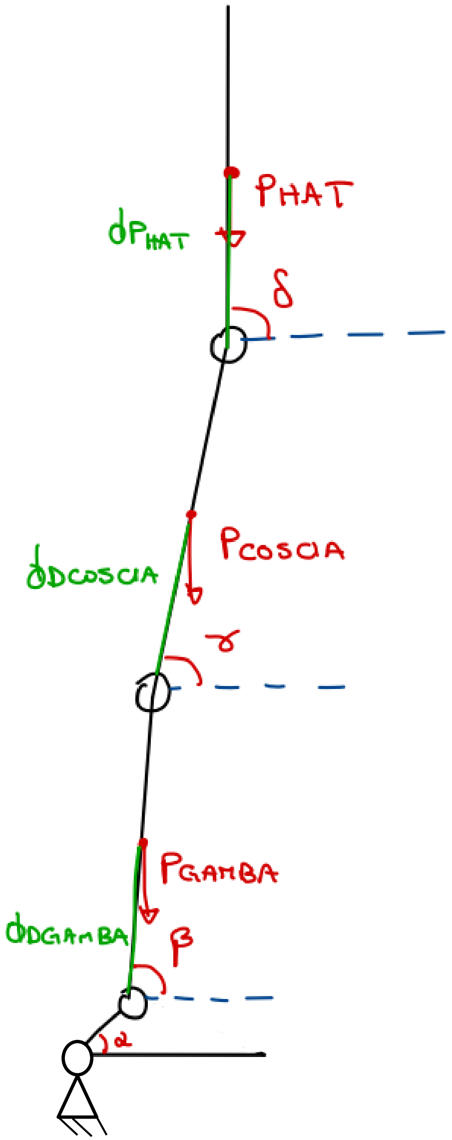
$$
\begin{cases}
R_{x\,GINOCCHIO} = 0 \\
R_{y\,GINOCCHIO} = R_{y\,ANCA} + P_{COSCIA} = R_{y\,ANCA} + m_{COSCIA} \cdot g \\
\Sigma M(com\_coscia) = C_{GINOCCHIO} - C_{ANCA} + R_{x\,GINOCCHIO}\, d_{COSCIA} \sin\gamma - R_{y\,GINOCCHIO}\, d_{COSCIA} \cos\gamma + \\
\quad + R_{x\,ANCA}\, d_{P\,COSCIA} \sin\gamma - R_{y\,ANCA}\, d_{P\,COSCIA} \cdot \cos\gamma = 0
\end{cases}
$$



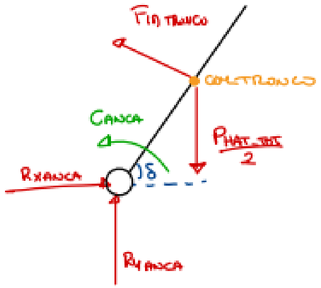
$$
\begin{cases}
R_{x\,CAVIGLIA} = 0 \\
R_{y\,CAVIGLIA} = R_{y\,GINOCCHIO} + P_{GAMBA} = R_{y\,GINOCCHIO} + m_{GAMBA} \cdot g \\
\Sigma M(com\_gamba) = C_{CAVIGLIA} - C_{GINOCCHIO} + R_{x\,CAVIGLIA} \cdot d_{GAMBA} \sin\beta - R_{y\,CAVIGLIA}\, d_{GAMBA} \cos\beta + \\
\quad + R_{x\,GINOCCHIO}\, d_{P\,GAMBA} \sin\beta - R_{y\,GINOCCHIO}\, d_{P\,GAMBA} \cdot \cos\beta = 0
\end{cases}
$$



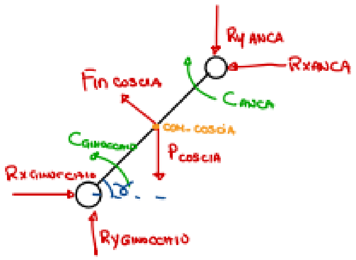
$$
\begin{cases}
R_{x\,PIEDE} = 0 \\
R_{y\,PIEDE} = R_{y\,CAVIGLIA} + P_{PIEDE} = R_{y\,CAVIGLIA} + m_{PIEDE} \cdot g \\
\Sigma M(com\_piede) = C_{PIEDE} - C_{CAVIGLIA} + R_{x\,PIEDE} \cdot y_{COM} + R_{y\,PIEDE}(x_{COP} - x_{COM}) + \\
\quad + R_{x\,CAVIGLIA}\, d_{PIEDE} \sin\alpha - R_{y\,CAVIGLIA}\, d_{PIEDE} \cdot \cos\alpha = 0
\end{cases}
$$



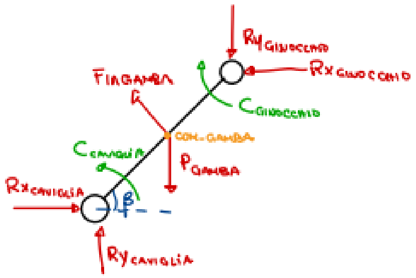
Dinamica



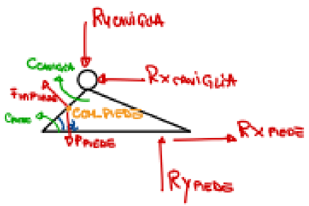
$$\begin{cases} R_{x\,ANCA} = F_{im\,x\,TRONCO} = \dfrac{m_{HAT-TOT}}{2} \cdot a_{COM\,TRONCO\,x} \\ R_{y\,ANCA} = \dfrac{P_{HAT-TOT}}{2} + F_{im\,y\,TRONCO} = \dfrac{m_{HAT-TOT}}{2} \cdot g + \dfrac{m_{HAT-TOT}}{2} \cdot a_{COM\,TRONCO\,y} \\ \Sigma M(COM\text{-}TRONCO) = C_{ANCA} - R_{y\,ANCA} \cdot d_{HAT} \cdot \cos\delta + R_{x\,ANCA} \cdot d_{HAT} \cdot \sin\delta = I_G \cdot \dot{\omega} = 0 \ (\text{POICHÉ IL MODELLO È SEMPLIFICATO}) \end{cases}$$



$$\begin{cases} R_{x\,GINOCCHIO} = R_{x\,ANCA} + F_{im\,x\,COSCIA} = R_{x\,ANCA} + m_{COSCIA} \cdot a_{COM\,COSCIA\,x} \\ R_{y\,GINOCCHIO} = R_{y\,ANCA} + P_{COSCIA} + F_{im\,y\,COSCIA} = R_{y\,ANCA} + m_{COSCIA} \cdot g + m_{COSCIA} \cdot a_{COM\,COSCIA\,y} \\ \Sigma M(COM\text{-}COSCIA) = C_{GINOCCHIO} - C_{ANCA} + R_{x\,GINOCCHIO}\, d_{D\,COSCIA} \sin\gamma - R_{y\,GINOCCHIO}\, d_{D\,COSCIA} \cos\gamma + \\ \quad + R_{x\,ANCA}\, d_{P\,COSCIA} \sin\gamma - R_{y\,ANCA}\, d_{P\,COSCIA} \cdot \cos\gamma = I_G \cdot \dot{\omega} = 0 \ (\text{POICHÉ IL MODELLO È SEMPLIFICATO}) \end{cases}$$



$$\begin{cases} R_{x\,CAVIGLIA} = R_{x\,GINOCCHIO} + F_{im\,x\,GAMBA} = R_{x\,GINOCCHIO} + m_{GAMBA} \cdot a_{COM\,GAMBA\,x} \\ R_{y\,CAVIGLIA} = R_{y\,GINOCCHIO} + P_{GAMBA} + F_{im\,y\,GAMBA} = R_{y\,GINOCCHIO} + m_{GAMBA} \cdot g + m_{GAMBA} \cdot a_{COM\,GAMBA\,y} \\ \Sigma M(COM\text{-}GAMBA) = C_{CAVIGLIA} - C_{GINOCCHIO} + R_{x\,CAVIGLIA} \cdot d_{D\,GAMBA} \sin\beta - R_{y\,CAVIGLIA}\, d_{D\,GAMBA} \cos\beta + \\ \quad + R_{x\,GINOCCHIO}\, d_{P\,GAMBA} \sin\beta - R_{y\,GINOCCHIO}\, d_{P\,GAMBA} \cdot \cos\beta = I_G \cdot \dot{\omega} = 0 \ (\text{POICHÉ IL MODELLO È SEMPLIFICATO}) \end{cases}$$

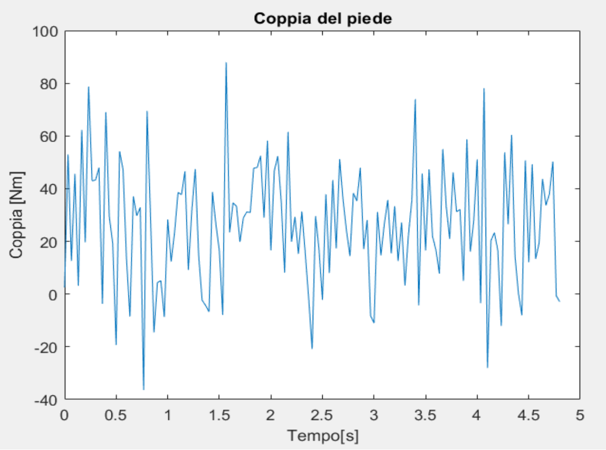


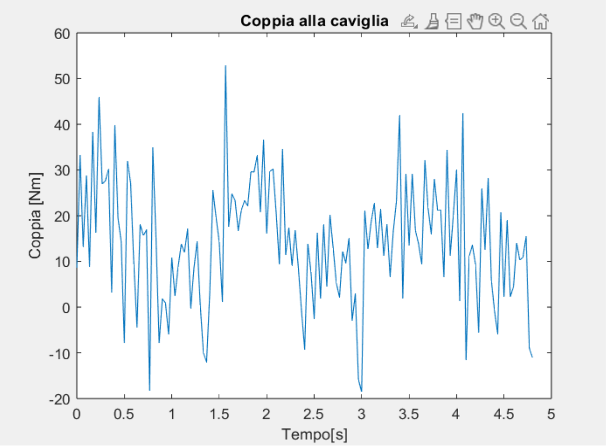
$$\begin{cases} R_{x\,PIEDE} = R_{x\,CAVIGLIA} + F_{im\,x\,PIEDE} = R_{x\,CAVIGLIA} + m_{PIEDE} \cdot a_{COM\,PIEDE\,x} \\ R_{y\,PIEDE} = R_{y\,CAVIGLIA} + P_{PIEDE} + F_{im\,y\,PIEDE} = R_{y\,CAVIGLIA} + m_{PIEDE} \cdot g + m_{PIEDE} \cdot a_{COM\,PIEDE\,y} \\ \Sigma M(COM\text{-}PIEDE) = C_{PIEDE} - C_{CAVIGLIA} + R_{x\,PIEDE} \cdot y_{COM} + R_{y\,PIEDE}(x_{COP} - x_{COM}) + \\ \quad + R_{x\,CAVIGLIA}\, d_{P\,PIEDE} \sin\alpha - R_{y\,CAVIGLIA}\, d_{P\,PIEDE} \cdot \cos\alpha = I_G \cdot \dot{\omega} = 0 \ (\text{POICHÉ IL MODELLO È SEMPLIFICATO}) \end{cases}$$

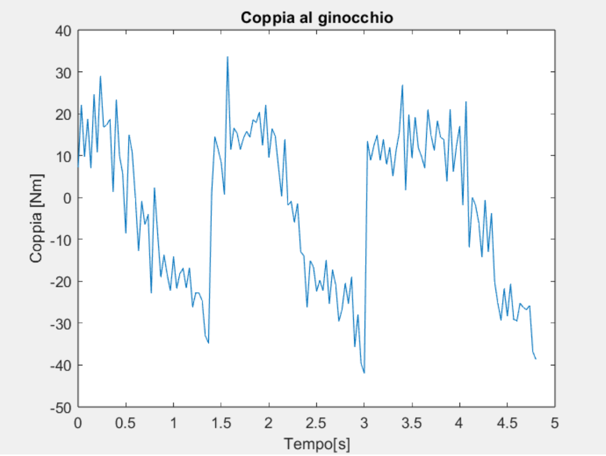
Dinamica

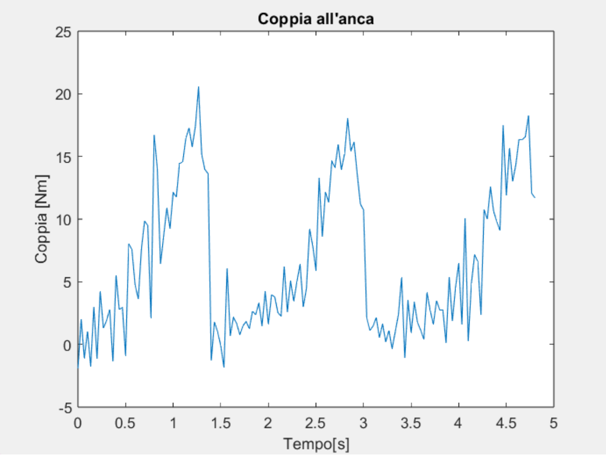
Determinazione delle **coppie articolari**

	C_anca	C_ginocchio	C_caviglia	C_piede
x	0	0	0	0
y	0	0	0	0
z	[-1.91 20.52]	[-41.90 33.67]	[-18.55 52.84]	[-36.40 87.95]





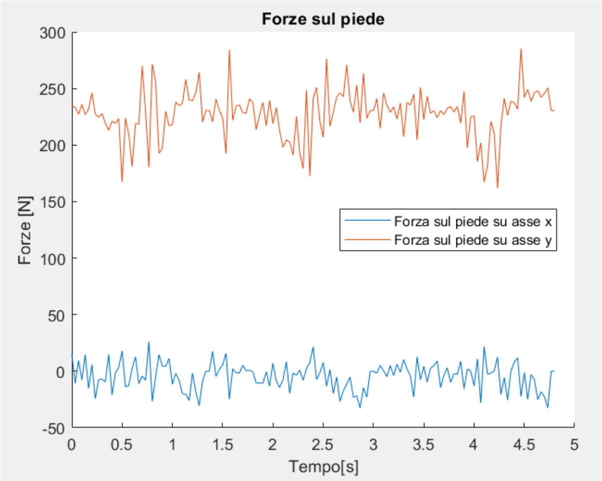


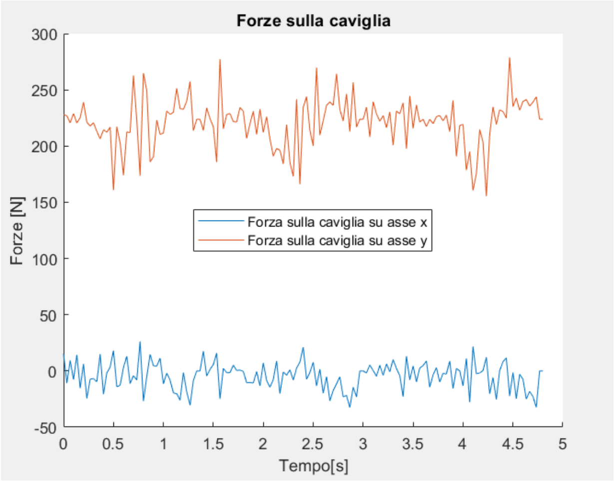


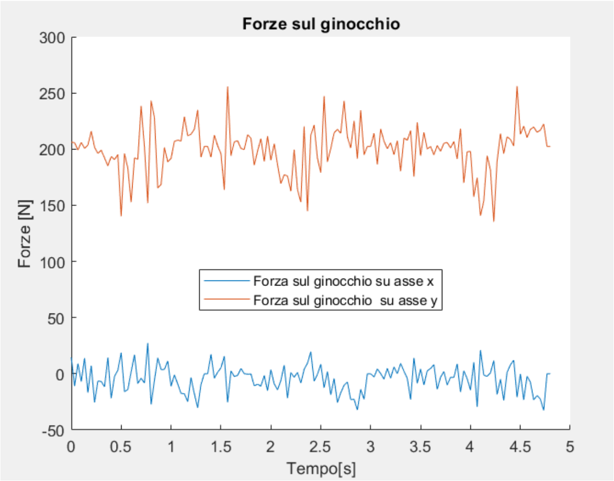
Dinamica

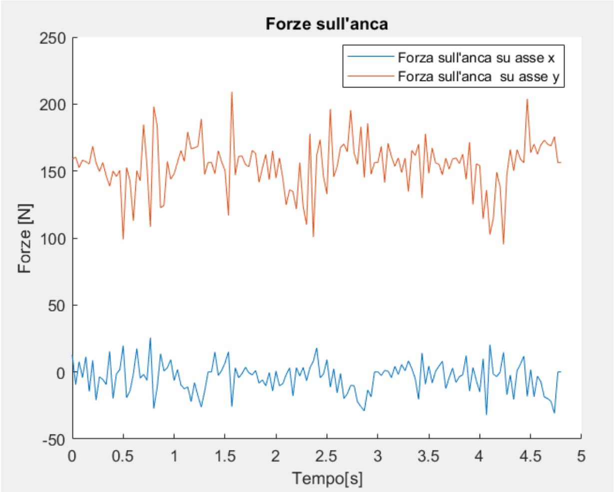
Determinazione delle **forze articolari**

	R_anca	R_ginocchio	R_caviglia	R_piede
x	[-32.15 25.62]	[-32.52 27.37]	[-32.52 26.21]	[-32.52 26.15]
y	[95.32 209.20]	[135.28 256.06]	[155.50 278.79]	[161.98 285.30]
z	0	0	0	0

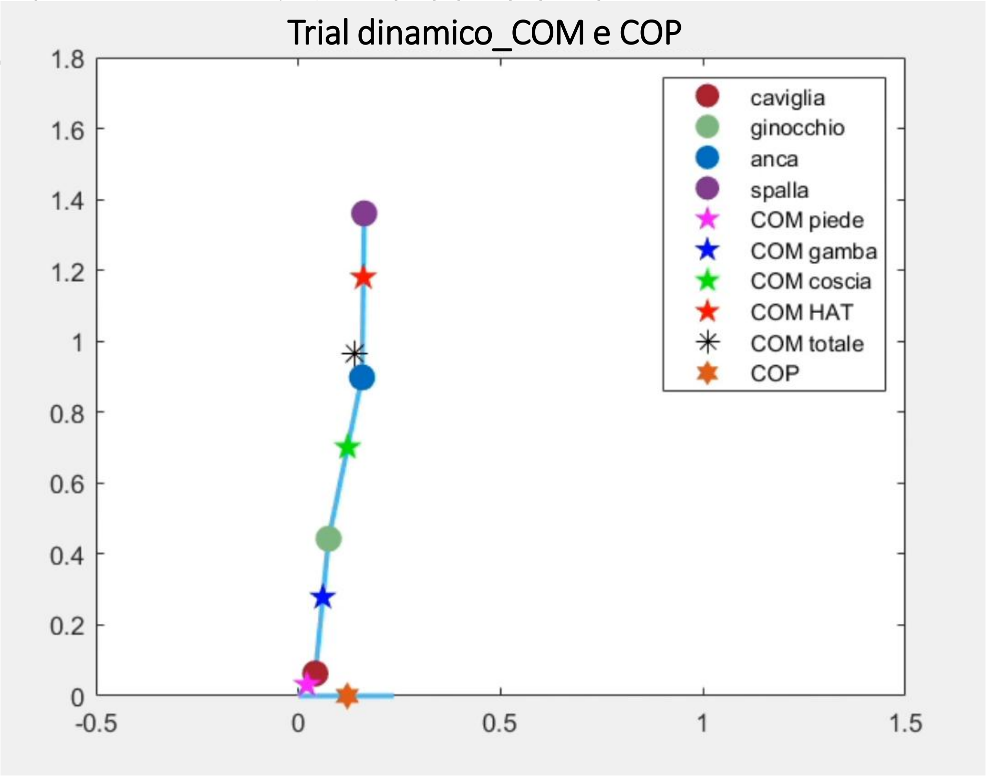








Dinamica



Data la II Equazione cardinale della dinamica (ECD) per un sistema di particelle

$$\underline{\overline{M}(a) = \dot{\overline{H}}_a}$$

Particolarizzando $\dot{\overline{H}}_a$ per un sistema di corpi rigidi la formula é applicabile al nostro modello multicorpo

Adottando un modello semplificato i termini in $\dot{\overline{H}}_a$ contenenti I_a si annullano

$$\underline{\overline{R}_{a_x}\, y_{COM} + \overline{R}_{a_y}(x_{COP} - x_{COM}) = \dot{\overline{H}}_a}$$

Dal momento che vale $\overline{R}_a = -\overline{F}_{gi}$

$$\overline{R}_{a_x} = -\overline{F}_{x\,gi} \qquad \overline{R}_{a_y} = -\overline{F}_{y\,gi}$$

$$-\overline{F}^{gi}_x\, y_{COM} - \overline{F}^{gi}_y (x_{COP} - x_{COM}) = \dot{\overline{H}}_a \rightarrow$$

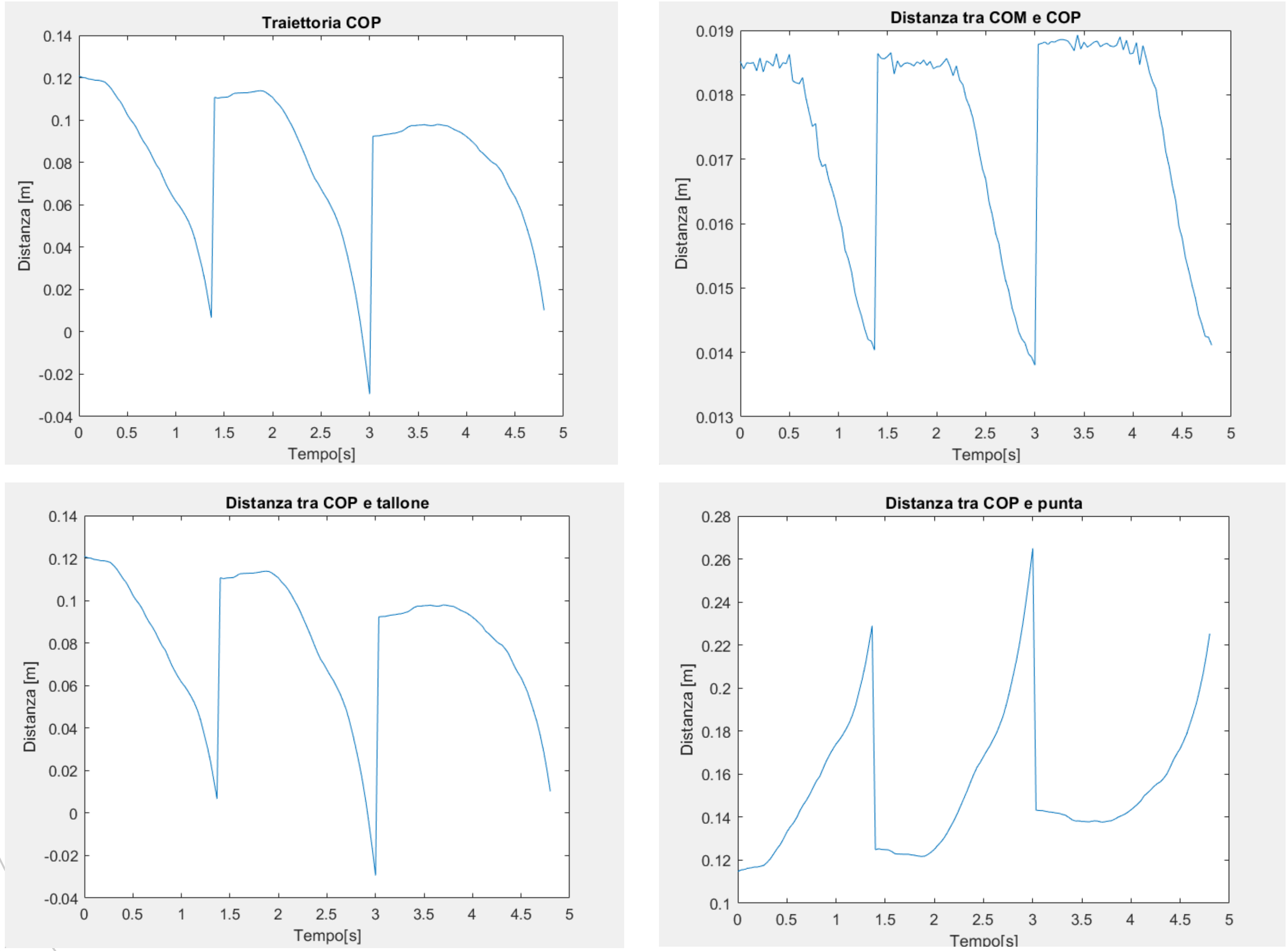
$$\rightarrow -\overline{F}^{gi}_x\, y_{COM} - \overline{F}^{gi}_y\, x_{COP} + \overline{F}^{gi}_y\, x_{COM} = \dot{\overline{H}}_a$$

$$x_{COP} = \frac{-\dot{\overline{H}}_a + \overline{F}^{gi}_y x_{COM} - F^{gi}_x y_{COM}}{\overline{F}^{gi}_y} \rightarrow$$

$$\rightarrow \boxed{x_{COP} = x_{COM} - \frac{\dot{\overline{H}}_a + F^{gi}_x y_{COM}}{\overline{F}^{gi}_y}}$$

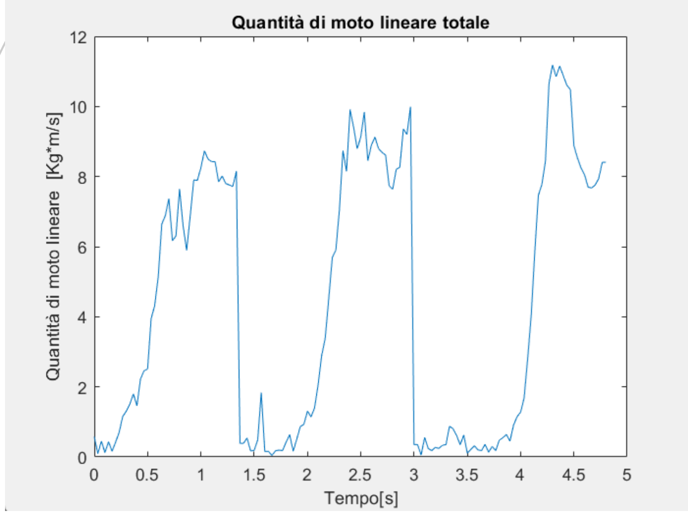
Dinamica

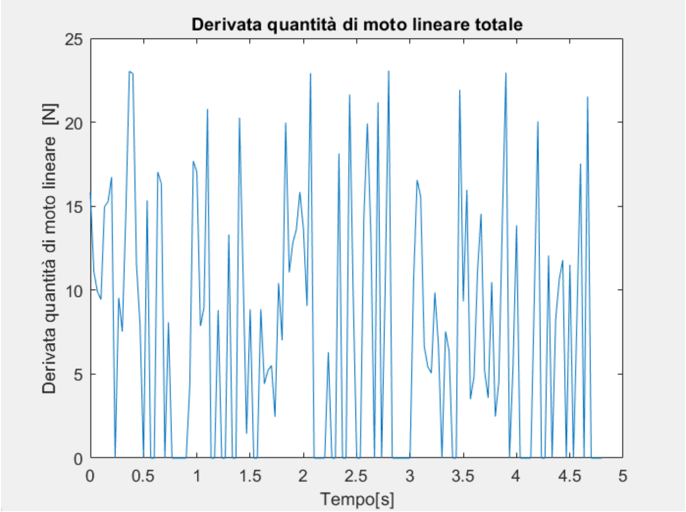
Determinazione della **traiettoria del COP**, della **distanza tra COM e COP** (lungo x), della **distanza tra COP e tallone** e della **distanza tra COP e punta del piede**

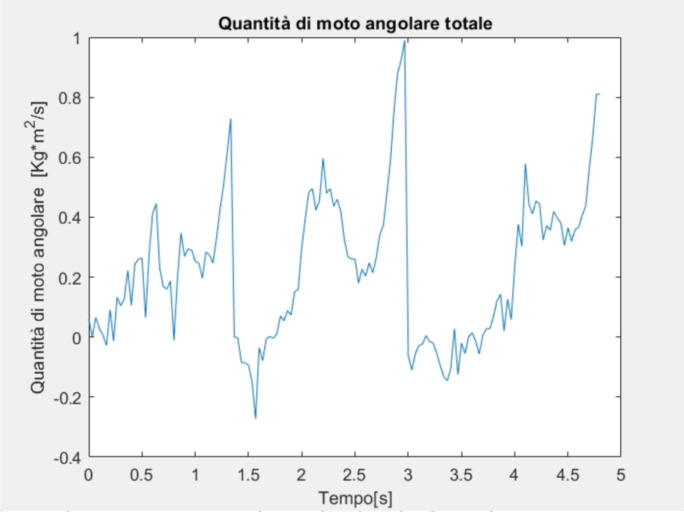


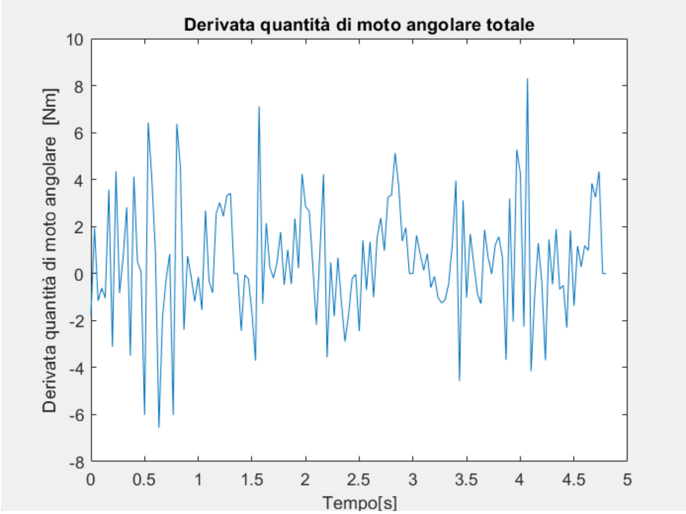
Dinamica

Quantità di moto lineare, quantità di moto angolare e loro **derivate**









$$\bar{Q} = M\bar{V}_G$$

$$\dot{\bar{Q}} = M\bar{a}_G$$

$$\bar{H}_G = \sum_i (\bar{P}_i - G) \wedge m_i (\bar{V}_i - \bar{V}_G)$$

$$\dot{\bar{H}}_G = \sum_i (\bar{P}_i - G) \wedge m_i (\bar{a}_i - \bar{a}_G)$$

GRAZIE PER L'ATTENZIONE