40.018 Heuristics and Systems Theory

Project 2 Proposal

CS02

Name	Student ID
Benedict Nai Rong Hui	1008173
Mika Ling	1007901
Muk Chen Eu Zachary	1007848

Introduction

Diabetes and glucose management have always been a significant point of interest in the field of medicine. Type 2 diabetes is especially prominent, given that it accounts for 95% of diabetes patients (World Health Organization, 2024).

Consequently, many experts have derived ways in their fields to improve the methods of diabetes treatment. As diabetes is a multi-faceted condition with different variables involved, biomedical engineers have derived various models to model different aspects of the condition, such as insulin and glucose. Given the prevalence of diabetes and the existence of such models, this project aims to analyse is to define and elaborate on these models to further understand the applications of non-linear control systems in the treatment of diabetes.

Formulation of State-Space Equations

Formulation has been referenced from (B. Candas, 1994). The formulation below follows an intravenous administration of Insulin to a patient:

$$\frac{dG(t)}{dt} = -[k_0 + k(t)]G(t) + RG(t)$$

$$\frac{dk(t)}{dt} = -a_1 * k(t) + a_2 * i(t)$$

$$\frac{di(t)}{dt} = -a_3 * i(t) + a_4 * k(t) + a_6 * i_3(t) + RI(t)$$

$$\frac{di_3(t)}{dt} = -a_6 * i_3(t) + a_5 * i(t)$$

$$where:$$

$$G(t) = plasma\ glucose\ concentration\ (mg/ml)$$

$$k_0 = insulin - independent\ fractional\ removal\ rate\ of\ glucose\ (\mathrm{minute}^{-1})$$

$$k(t) = insulin - dependent\ fractional\ removal\ rate\ of\ glucose\ (minute^{-1})$$

$$i(t) = insulinmass\ in\ the\ central\ compartment\ (\mu U)$$

$$i_3(t) = insulin\ mass\ in\ a\ peripheral\ compartment\ non - active\ in\ glucose\ removal\ (\mu U)$$

$$RG(t) = glucose\ systemic\ appearance\ rate\ (mg/ml * minute)$$

$$RI(t) = insulin\ systemic\ appearance\ rate\ (\mu U/\ minute\ )$$

Here are the associated Discretized Equations:

$$G(t+1) = G(t) + \Delta t * \left(-[k_0 + k(t)] * G(t) + RG(t)\right)$$

$$k(t+1) = k(t) + \Delta t * \left(-a_1 * k(t) + a_2 * i(t)\right)$$

$$i(t+1) = i(t) + \Delta t * \left(-a_3 * i(t) + a_4 * k(t) + a_6 * i_3(t) + RI(t)\right)$$

$$i_3(t+1) = i_3(t) + \Delta t * \left(-a_6 * i_3(t) + a_5 * i(t)\right)$$

These are the equations used to define equilibrium:

$$G^* = G^* - [k_0 + k^*]G^* + (RG)^*$$

$$k^* = k^* - a_1 * k^* + a_2 * i^*$$

$$i^* = i^* - a_3 i^* + a_4 k^* + a_6 i_3^* + (RI)^*$$

$$i_3^* = i_3^* - a_6 i_3^* + a_5 i^*$$

Thus obtaining:

$$k^* = \frac{a_2}{a_1} i^*$$

$$G^* = \frac{1}{k_0 + k^*}(RG)^*$$

$$i_3^* = \frac{a_5}{a_6} i^*$$

Analysis of Equilibrium Points and System Control Plan

$RI(t)$ and $RG(t)$ are treated as external inputs in the analysis of the equilibrium points. Hence, to control the equilibrium values of i , i_3, k and G , RI and RG can be adjusted. The equilibrium points of k and i_3 are dependent on i .

All 4 equations are linearised below. We assume that the initial glucose concentration in the system is ideal. Hence, with reference to the same method used in (B. Candas, 1994) , by using the inverse of $G(t)$, for which $G(t) = G(0)$, the control input is hence, $RG(t)$.

The linearised equations we derived are:

$Glucose\ Equation$:

$$SG(t+1) = (1 - k_0 - k^*)SG(t) - G^* * Sk(t) + SRG(t)$$

$k - equation$:

$$Sk(t+1) = (1 - a_1) * Sk(t) + a_2 Si(t)$$

$i - equation$:

$$Si(t+1) = (1 - a_3) * Si(t) + a_4 * Sk(t) + a_6 * Si_3(t) + SRI(t)$$

$i_3 - equation$:

$$Si_3(t+1) = (1 - a_6) * Si_3(t) + a_5 Si(t)$$

Thus obtaining:

$$x(t) = \begin{bmatrix} SG(t) \\ Sk(t) \\ Si(t) \\ Si_3(t) \end{bmatrix}, \qquad u(t) = \begin{bmatrix} SRG(t) \\ SRI(t) \end{bmatrix}$$

$$x(t+1) = Ax(t) + Bu(t)$$

$$A = \begin{bmatrix} 1 - k_0 - k^* & -G^* & 0 & 0 \\ 0 & 1 - a_1 & a_2 & 0 \\ 0 & a_4 & 1 - a_3 & a_6 \\ 0 & 0 & a_5 & 1 - a_6 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

References

B. Candas, J. R. (1994). An Adaptive Plasma Glucose Controller. *IEEE Transactions on Biomedical Engineering*, 116-124.

World Health Organization. (12 November, 2024). *Diabetes*. Retrieved from World Health Organization: https://www.who.int/news-room/fact-sheets/detail/diabetes