Landmarked-based Mapping of Cell Towers near Worcester

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Abstract

The purpose of this study was to compare cell phone coverage of Sprint, Verizon, and AT&T using topological landmarked based mapping. Given the geographic coordinates of both the cell towers and towns around Worcester, MA, we used a method of Ghrist et. al. to create topological structures called simplicial complexes. We used MATLAB in order to visualize the complexes for the three companies, and Javaplex in order to analyze. It was apparent that these three companies had similar topological structure in their complexes, reflecting the areas of high population density in Massachusetts. More specifically, AT&T had superior coverage in Eastern Massachusetts, while Verizon and Sprint had a more balanced coverage across the region.

Introduction

Given the wide-scale dependence on cell phones in the United States, the unreliability of cell phone reception can not only be a small inconvenience for customers, but a serious and dangerous defect of cellular technology. Companies advertise their coverage maps constantly in order to demonstrate that they provide the best and most reliable service. The goal of our project was to create visually useful and accurate approximations to cell phone coverage for Verizon, AT&T and Sprint from Springfield to Boston. We used topological landmark-based mapping in order to visualize simplicial complexes that represent coverage maps for the three companies. Finally, we hoped to analyze the persistent homology of these complexes so we could compare coverages and determine which provider provides superior coverage locally.

This so-called landmark-based mapping was developed by Robert Ghrist et al. in their paper *Topological Landmark-Based Navigation and Mapping.* We hoped to use a variation of their process with our data set in order to approximate the coverages. In their paper, Ghrist et al. used a set of strategically placed observation and landmark points to create approximations to any given domain. Using line-of-sight visibility, they recorded which observers can see which landmarks in order to approximate the underlying space. We will replicate his process, with a few modifications, to approximate coverage maps using *cell phone towers as landmark points* and *towns as observation points*.

Using a function called grabit in MATLAB we were able define our domain and assign each landmark and observation point a set of coordinates in the two-dimensional euclidean plane. From there we used radii of sizes 15, 25, and 35 miles to see which towns were within range of which towers. Once we had this information we could create dual simplicial complexes which approximate the coverage of the region for each individual company.

The results showed a variety of relevant information. First and foremost, due to our restrictions on distance our dual complexes were not necessarily equivalent. Equivalence between the landmark and observer complex was one of the main findings of Ghrist's paper, but due to our modifications to his process (distance restrictions in sight), equivalence did not hold true. It is important to note that while we changed Ghrist's process significantly in this way, it would not have made sense for us to use line-of-sight visibility in the context of cell phone reception. After all, cell phone towers have a defined range (usually about 30 miles), so it clearly does not make sense for a cell tower in Boston to be able to see a cell phone 85 miles away in Springfield. Nevertheless - disregarding equivalence - we were able to draw some important conclusions about each company's coverage.

Background

When making a call on a cell phone, the signal is directed from your phone to the nearest cell tower before it can reach the phone at the other end. Cell towers are everywhere, but unfortunately, a given phone cannot connect to each one. Since different companies have their own towers, the tower that your phone connects with depends on which company's service you pay for. Subsequently, your coverage is a direct result of the of how your company's towers are distributed. Cell towers usually have a range of roughly 20-35 miles. Cell reception is obviously not simply a matter of range. cell phone coverage is a difficult problem because there are a wide variety of variables that affect a given phone's reception. Among others, these factors include cell phone quality, cell tower quality, obstructions (trees, buildings, etc.), cellular traffic in the area and even weather.

For the purposes of our study, we have simplified the problem in a few ways. First, we will only consider three independent variables; company, range, and a single obstructing mountain range. For the variable *company* we will be comparing the coverage of Verizon, AT&T and Sprint. The ranges we will consider are 15 miles, 25 miles and 35 miles around each tower. To complicate the problem a bit we will implement an obstructing mountain ridge that goes through the northern part of our data, voiding some of the connections between tower and phone. Moreover, we will eliminate *signal strength* by regarding reception as binary; either you have service or you don't.

<u>Topology Background</u>

Using landmark-based navigation, as described by Robert Ghrist et al. in his paper *Topological Landmark-Based Navigation and Mapping,* we will create simplicial complexes that will give approximations to the coverage of each company in central/eastern Massachusetts. In his paper, Ghrist introduces landmark based navigation as a tool that can be used to build up an internal map without any reference to coordinates or odometry. In essence, through line-of-sight visibility we can create approximations of the domain.

Given a domain, we introduce landmark points and observation points. Observation points are used to approximate where the landmarks are located. Each observation point records which landmarks it can see. In Grist's paper, he uses infinite line-of-sight visibility meaning distance is not a restricting factor of what an observer can see. The only reason an observer cannot see a landmark is if there is an obstruction between them. Based on this information, he creates a list of ordered pairs (i , j) which represents who-sees-what; if observer i can see landmark j then the ordered pair (i , j) is recorded.

From this data, we can create two simplicial complexes which are homotopically equivalent approximations to the data. The landmark complex uses landmark points as its vertices and edges are added between vertices when two landmark points can see the same observation point. Faces are added when three landmark points can see the same observation point and tetrahedra are added when four landmark points can all see the same observation point. The landmark complex is equivalent to the observation complex which uses the same principles except the observation points are used as the vertices. Ghrist goes on to prove the equivalence of the two complexes. In summary, the dual approximations preserve the key features of the original domain of interest.

We will implement Ghrist's process, with a few modifications, in an effort to approximate cell phone coverage in central and eastern Massachusetts - from Boston to Springfield. In our study cell phone towers constitute landmark points and the relevant townships make up observation points. The simplicial complexes we create will be approximations of the cell coverage each company provides.

Algorithms and Computations

As mentioned previously, the grabit function was used to collect data of the location of cell towers and towns in the usual two-dimensional Euclidean space. We created three functions, complexEdges, complexFaces, and complexTetra that were used to construct an appropriate complex given a certain radius and two sets of data containing the coordinates of landmarks and observation points. These functions were eventually consolidated into one function: celltower_complex.

The functions that created the simplices take the inputs of landmarks and observation points with the selected radius, and organize the data in the appropriate sensing data format ($\mathfrak{R} \subset (\mathscr{O}\times\mathfrak{L})$: sensing data is given as an unordered sequence of pairs of observation-landmark points encoding who-sees-what). For creating edges, the function determines from the sensing data what pairs of landmark points share an observer (or vice-versa, depending on the input parameters). For faces and tetrahedra, the same process was used, except for threesomes and foursomes of landmark points sharing an observer or observation points sharing a landmark.

Another function, mountain, was created that determines if a mountain range is obstructing a connection. This function loads the coordinates of the mountain range and checks if any edges in the complex intersects with the convex hull of the mountain range. Then it removes any tetrahedra, faces, and edges containing the intersecting simplices.

A script, landmark_navigation, utilized the four functions to create a filtration for both the landmark and observation complexes based on the distance between the cell towers and towns. This script loaded javaplex, relevant data, and ran the functions above for our three chosen radii (15, 25, and 35 miles) and created a filtered simplicial complex, graphed the complex and subcomplexes, computed the homologies, and generated persistent barcode diagrams.

Results and Analysis

The way we framed our results was in the context of four independent variables: the company, radius of the towers, observer or landmark complex, and the presence of mountains. Regarding the presence of mountains, we can look at appendix figure 1 and appendix figure 2 in order to demonstrate the effect of the mountain.m function. We ran many simulations with and without the mountain.m function, and the function returned exactly what we expected it to. Specifically, it created a sort of artificial wall between cell towers to the east and west of the mountain ridge in central Massachusetts. This allows us to only consider the complexes that have used mountain.m in order to create a more realistic simulation.

Rather than attempting to preserve a quasi-equivalence of the observation and landmark complex, we accepted the fact that the distance restriction to landmark-based mapping fully deteriorates any equivalence. Consequently, we will simply choose which complex - observer or landmark - to present in order to demonstrate our results most clearly. While considering that one of our main goals of this project was to visualize our data, we chose to present observation complexes. More data points in the observation complexes (the towns) led to a better "map" of the coverage we were trying to approximate.

Now that we have eliminated two of the four presented independent variables, we can focus on the two most important questions: which company are we considering, and what is the radius of the towers? Appendix figures 3 through 5 show the resulting complexes from towers with a 25 mile radius for Verizon, AT&T, and Sprint, respectively. It is clear that at 25 miles, AT&T has the superior coverage in eastern Massachusetts, while Verizon and Sprint have a more spread out coverage. While still having strong coverage around Boston and Worcester, Verizon and Sprint do not match the quality of service of AT&T. Instead, we can see by their complexes that they have a strong connection between eastern and central Massachusetts. One shared aspect of the 25-mile coverage across all three companies was the coverage in 3 parts of Massachusetts - Boston, Worcester, and Springfield.

By limiting the radii of the towers to 25 miles we were able to clearly see this superior coverage by AT&T in eastern Massachusetts. Although there are a lot of edges to consider, the density around Boston and Worcester is noticeably higher in the AT&T complex compared to the Sprint and Verizon complexes in the same area. By restricting the radii even more - to 15 miles - this relationship becomes even more clear. Appendix figures 6 through 8 show the companies' complexes at a 15 mile radius. Although the number of edges in the AT&T complex decreases slightly, the edges in the Sprint and Verizon complexes become much more scarce, illustrating this stronger coverage by AT&T in the east.

The relationship between coverages in eastern Massachusetts versus western Massachusetts also changes when the radii is restricted to 15 miles. As before at 25 miles, we can see Sprint has coverage that is spread throughout Massachusetts. However, while Verizon covered much of Massachusetts at 25 miles, this coverage shrinks considerably at 15 miles. This difference between Verizon and Sprint's ability to hold their coverage across the radii decrease serves as an indication that Sprint has a more complete coverage across Massachusetts.

However, these relationships between coverages gets lost when we allow the radii of the towers to expand to 40 miles - which are shown by appendix figures 9 through 11. Although the AT&T complex still does not connect eastern and western Massachusetts, almost all other relationships are masked by the sheer amount of simplices.

Our barcodes that we hoped to make of the different complexes by using filtration and persistent homology did not turn out the way we had hoped. There were a few factors that came into play here, including dimensionality, computing power, and the nature of our dataset. Naturally, the towns in Massachusetts are grouped relatively close to one another, which causes cellphone companies to strategically place their towers in position to cover these clusters. The problem with analyzing persistent homology of our simplexes was that our method of construction would automatically add a high dimensional simplex. So when considering betti numbers in order to analyze holes or voids in the data, we think that the most interesting outcomes would be at a high dimension. To make matters worse, we simply did not have enough computing power in order to compute these high dimensional simplices.

As shown by appendix figures 12 through 14, the size of the simplices allowed us only to go up to dimension 1 with Verizon and AT&T, and dimension 2 with Sprint. Even at dimension 1, we would have expected to see some holes being born and dying at different stages of the bar code. However, with the filtration values we used - 15, 25, and 35 miles - we found that these tunnels were being born and dying at the same filtration value. In other words, the filtration values we used were too coarse to show tunnels' birth and death times. When we tried to solve this problem by refining the filtration, we had the same problem as before with lack of computing power. The resulting barcode looks like a solid block, not giving us much information about the persistent homology.

Conclusion

Even without the ability to look at higher dimensional persistent homology, we were able to accomplish one of our main goals: visualize the data with simplicial complexes and compare the coverage between the three companies. Comparing the complexes we created with the coverage maps that the companies advertise with, it was a great moment when we realized that they were very similar. Almost all of the comparisons that we discussed above were also present when comparing the "real" coverage maps of Sprint, AT&T, and Verizon.

When considering our project as a whole, we believe there were a number of improvements that we could have made by improving our computing power and by improving our data set. By making our data set larger, we could see the voids on a macro scale that would open up by objects such as large lakes and mountains. By making our data set smarter, we could improve upon the amount of computing power needed to calculate complexes that contain higher dimensions. However, the improvement that would make the most difference would be to simply have access to a bigger computer. With more computing power we would be able to look at a bigger data set and calculate barcodes in higher dimensions. In the context of the problem, observers in higher dimensional simplices could be interpreted as having a better signal, access to 4G rather than 3G, or ability to handle large amount of cellular traffic. With these improvements, there is a possibility that the higher dimensional complex would provide information that even the coverage maps do not.

References

R. Ghrist, D. Lipsky, J. Derenick, and A. Speranzon, \Topological landmark-based navigation and mapping," 2012. [Online]. Available: https://www.math.upenn.edu/~ghrist/preprints/landmarkvisibility.pdf

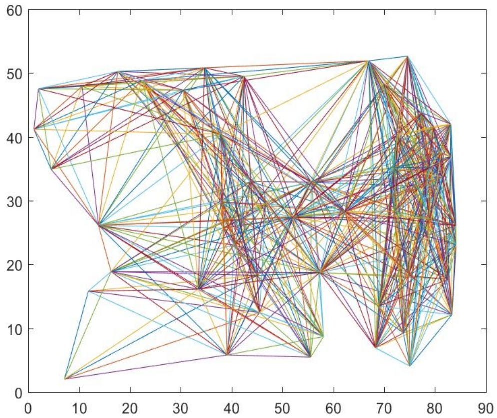
H. Edelsbrunner, J. Harer, *Computational Topology: An Introduction* (American Mathematical Society, Providence, 2010)

http://www.cellreception.com/towers/towers.php?city=Worcester&state_abr=ma

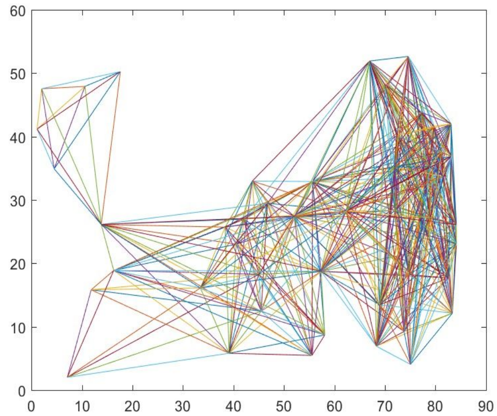
http://opensignal.com/coverage-maps/US/

Professor David Damiano

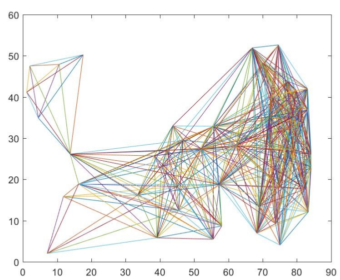
Appendix



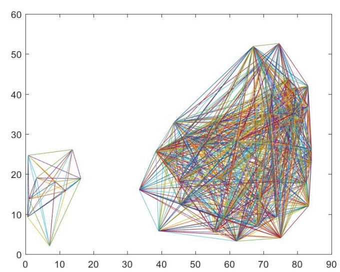
AT&T - Figure 1



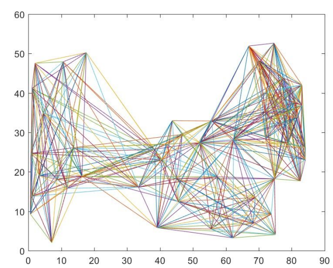
AT&T - Figure 2



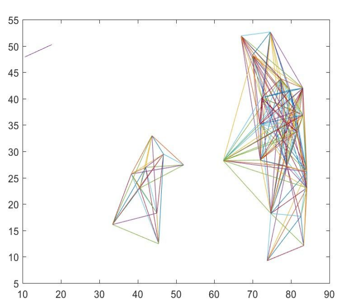
Verizon - Figure 3



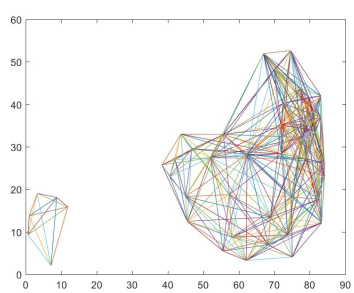
AT&T - Figure 4



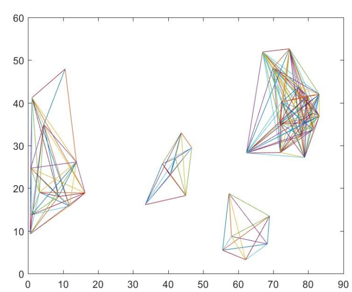
Sprint - Figure 5



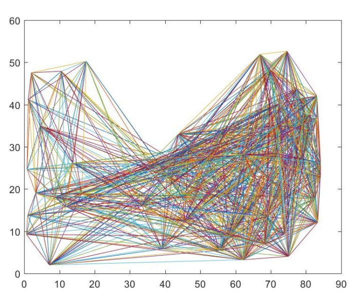
Verizon - Figure 6



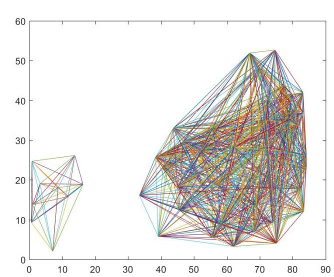
AT&T - Figure 7



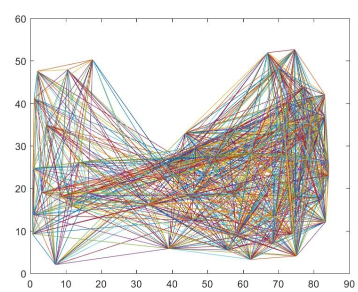
Sprint - Figure 8



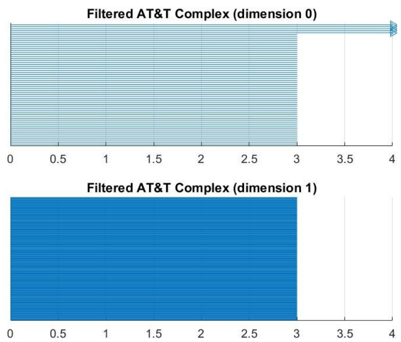
Verizon - Figure 9



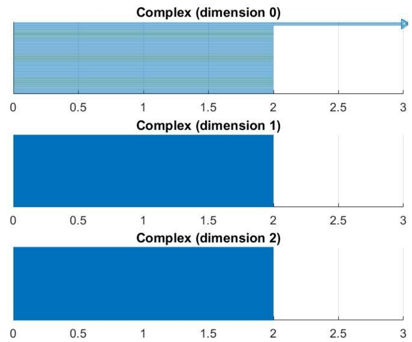
AT&T - Figure 10



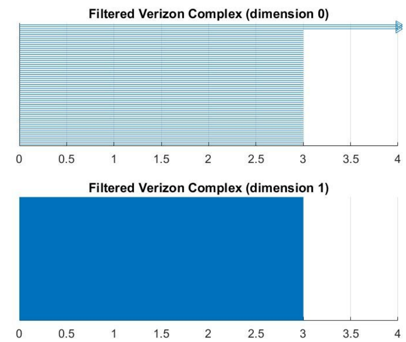
Sprint - Figure 11



AT&T Barcodes - Figure 12



Sprint Barcodes - Figure 13



Verizon Barcodes - Figure 14