Landmarked-Based Mapping of Cell Towers near Worcester, MA

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Images from google images

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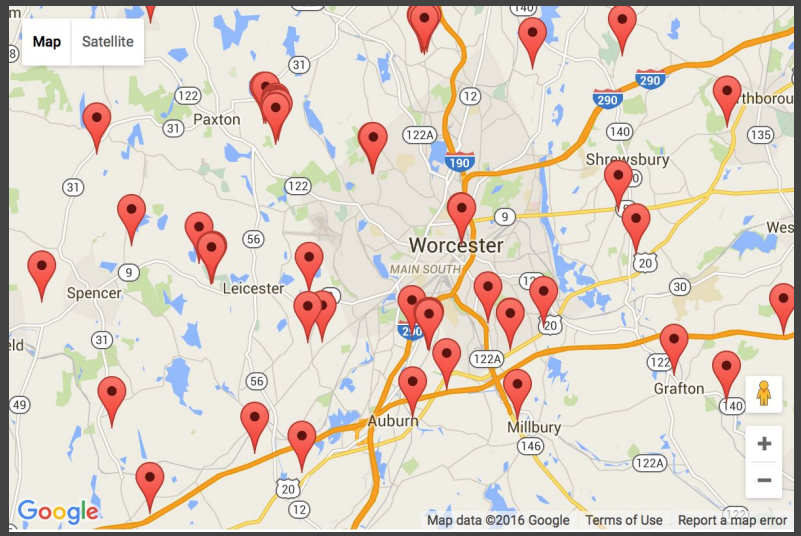
The Problem

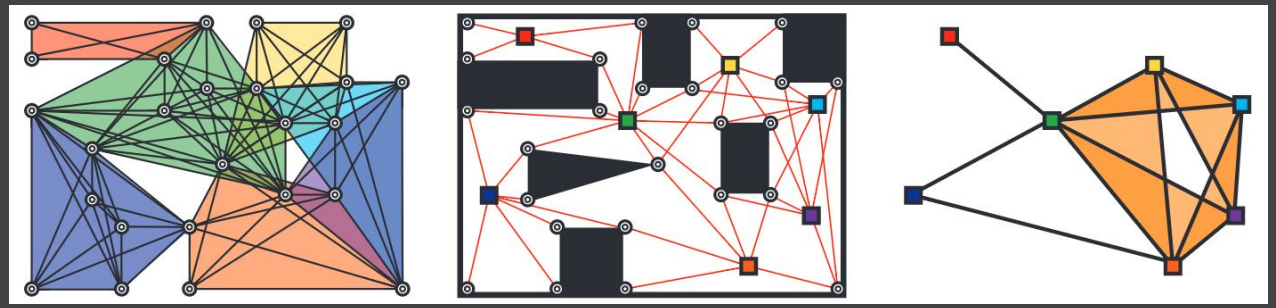


- We want a way to visualize the distribution of cell towers by company.
- Which company provides better cell coverage?

Our Data

- The data we are using is the geographical coordinates of cell towers and towns near Worcester
- Selected towns (60) are our observation points and the cell towers are our landmark points.
- Instead of using line-of-sight visibility to pair observation points and landmarks, we will pair points based on the range of cell towers (15, 25, 35 mi.)

What is Landmarked-based Navigation?

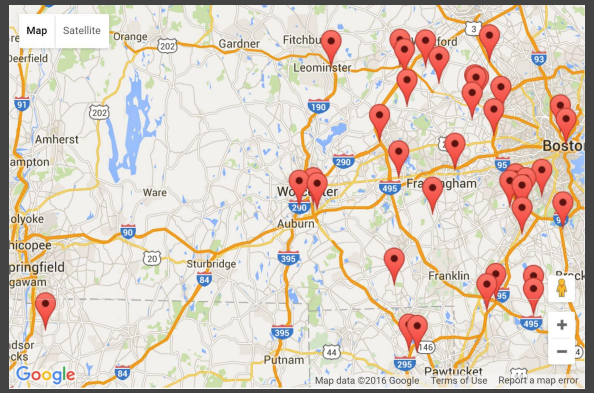


R. Ghrist, D. Lipsky, J. Derenick, and A. Speranzon, \Topological landmark-based navigation and mapping," 2012. [Online]. Available: http://www.math.upenn.edu/ghrist/preprints/landmarkvisibility.pdf

- Landmark complex (left) - vertices are landmarks
- Observation complex (right) - vertices are observation points
- The pairs of nerves are used to construct two *equivalent* simplicial approximations to the domain
- $\mathcal{R} \subset (\mathcal{O} \times \mathcal{L})$: Sensing data is given as an unordered sequence of pairs of observation-landmark points encoding who-sees-what

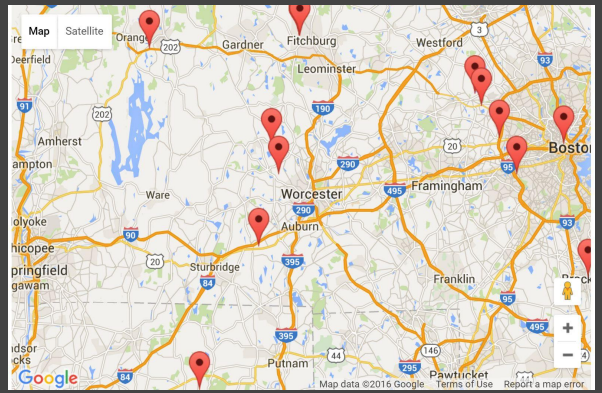
Towers by Company

- The complexes will be simplicial approximations of the cell coverage.
- Based on the maps alone we expect the complexes to vary by company.



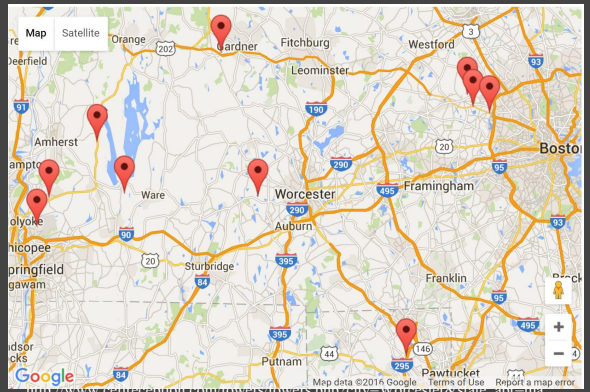
http://www.cellreception.com/towers/towers.php?city=Worcester&state_abr=ma

AT&T



http://www.cellreception.com/towers/towers.php?city=Worcester&state_abr=ma

Verizon



http://www.cellreception.com/towers/towers.php?city=Worcester&state_abr=ma

Sprint

More Expectations

- The landmark complex and the observation complex should give us simplicial approximations to actual cell coverage in the area.
- We can make inferences about which company has superior coverage around Worcester based on their homology.
- Unlike line-of-sight pairing, we found that our observation complexes don’t necessarily have equivalent homology.

Simplifications

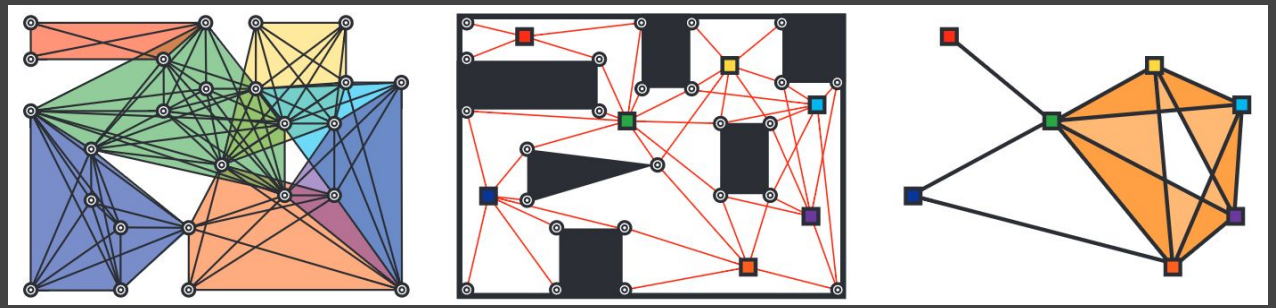
- Cell phone reception depends on multiple factors including
  - Cell phone and cell tower quality
  - Tower Traffic
  - 3G, 4G, LTE, etc.
  - Obstructions (trees, large buildings, hills, mountains etc.)
  - Weather

- For our project we will only be considering proximity to the tower and partial obstruction from high elevation regions
- Cell service is binary

Properties of Landmark-Based Mapping

- Equivalence of Simplicies*
  - Betti numbers

- Only landmarks that are observed are included in the landmark complex (observers must see at least one landmark to be included in the observation complex)



Software and Process

- Using MATLab and javaPlex to create the following types of scripts:
  - The grabit function was used to obtain coordinates of the landmarks and observation points in the 2-D Euclidean plane
  - Three functions were created to determine which landmarks shared observation points (or which observation points shared landmarks) and constructed edges, faces, and tetrahedra: complexEdges, complexFaces and complexTetrahedra to create both landmark and observation complexes
  - A function, mountain.m was created to determine where a mountain range obstructed cell towers and towns, and removed those corresponding simplices
  - A script, landmark_navigation, that utilized the above four functions to create a filtration for both the landmark and observation complexes based on the distance between the cell towers and towns
  -

complexEdges.m, complexFaces.m, complexTetra.m

These three functions take the following input parameters:

- R = Radius of distance
- X/Y = Landmarks/Observations (depending on which complex is constructed)

First, the pdist2 function is used to calculate the distances between landmark points and observation points, then uses the find function to pick pairs of points with distances < R to create a list of (X,Y) pairs. Nested for loops are used to check if points in X have a common point in Y, and adds the relevant simplices.

mountain.m

This functionn takes the input parameters:

- X = Landmarks (or Observations, dependent on which complex is used)
- list1 = edge list
- list2 = face list
- list3 = tetrahedra list
- M = convex hull points of mountain ranges

This determines if an edge intersects the mountain range and creates a list of which rows in the list of edges intersect with the mountain range. Then the function removes any simplices containing the intersecting edges.

landmark_navigation.m

This script creates a filtered simplicial complex. The resulting simplicial is either a landmark or observation complex, depending on which set of points is set as X and Y. (i.e. if X is landmarks, it creates a landmark complex). It utilizes complexEdges.m, complexFaces.m, complexTetra.m, and mountain.m

Then it computes the barcodes and betti numbers.

Computational Issues and Complications

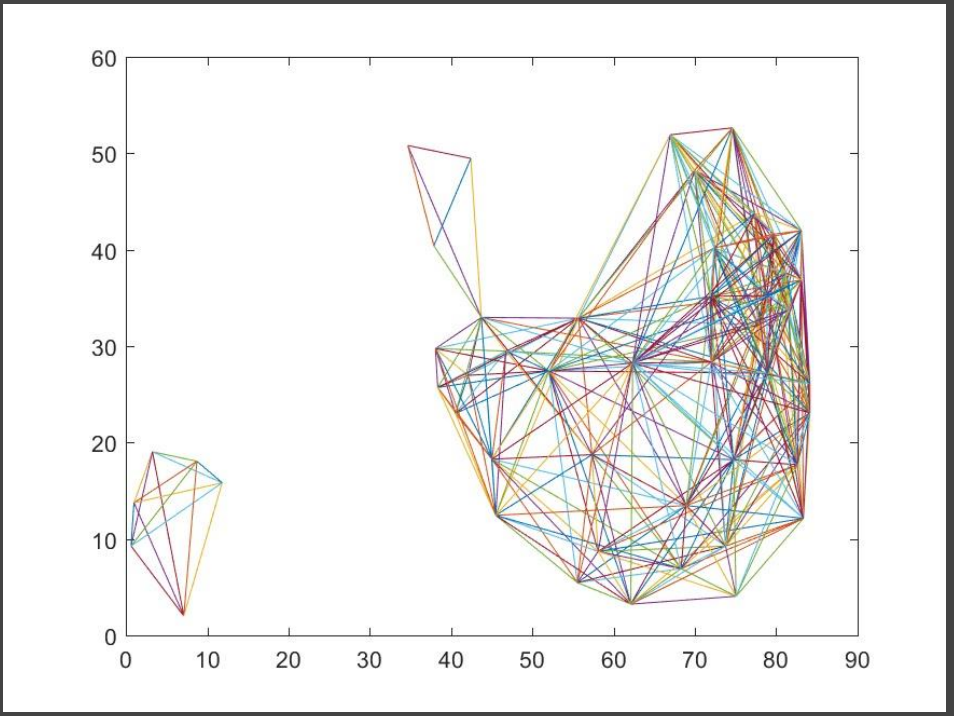
- We had very large simplicial complexes, which created a few issues
  - We could not reasonably compute an area larger than Middle-eastern Massachusetts
  - Our smallest complex was Sprint with over 118,000 simplices
    - We chose to ignore tetrahedra and only computed homology for dimensions 2 and lower for AT&T and Verizon
- More geographical features and a larger map would create a more lively and interesting topological map, but we lacked enough computational power to generate such a complex
  - We hypothesized that there would be more connected components or tunnels in a larger map
  - Analyzing higher dimensional simplices was unreasonable

Results - Complexes and Persistence

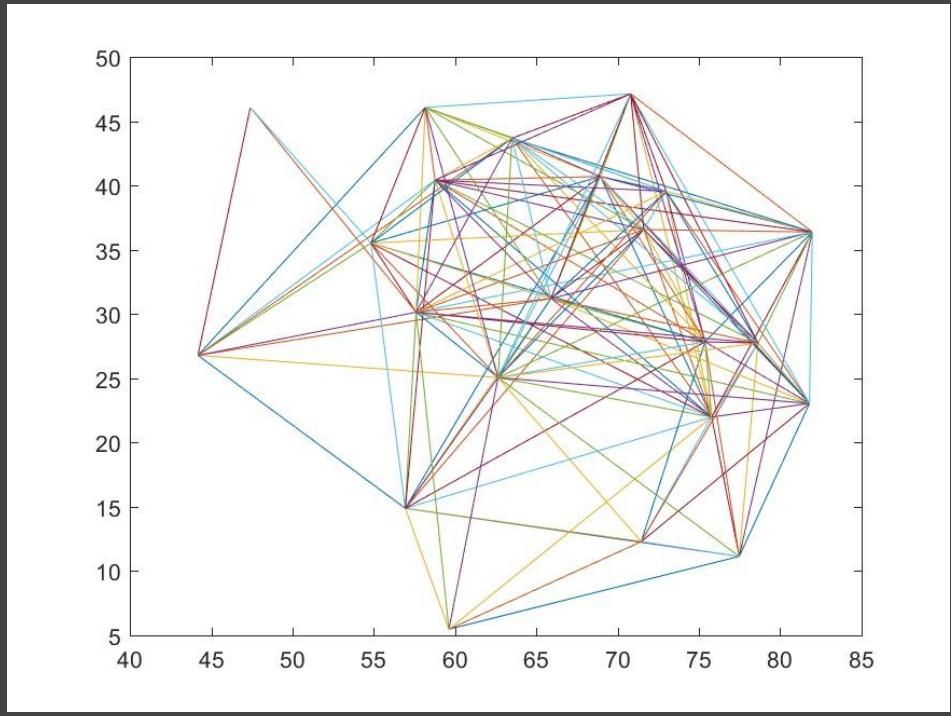
Independent Variables:

1) Company
2) Range of Tower
3) Observer or Landmark
4) Mountain Range Present

Equivalence of Complexes



AT&T
Observer - No Mountains - 15 Miles

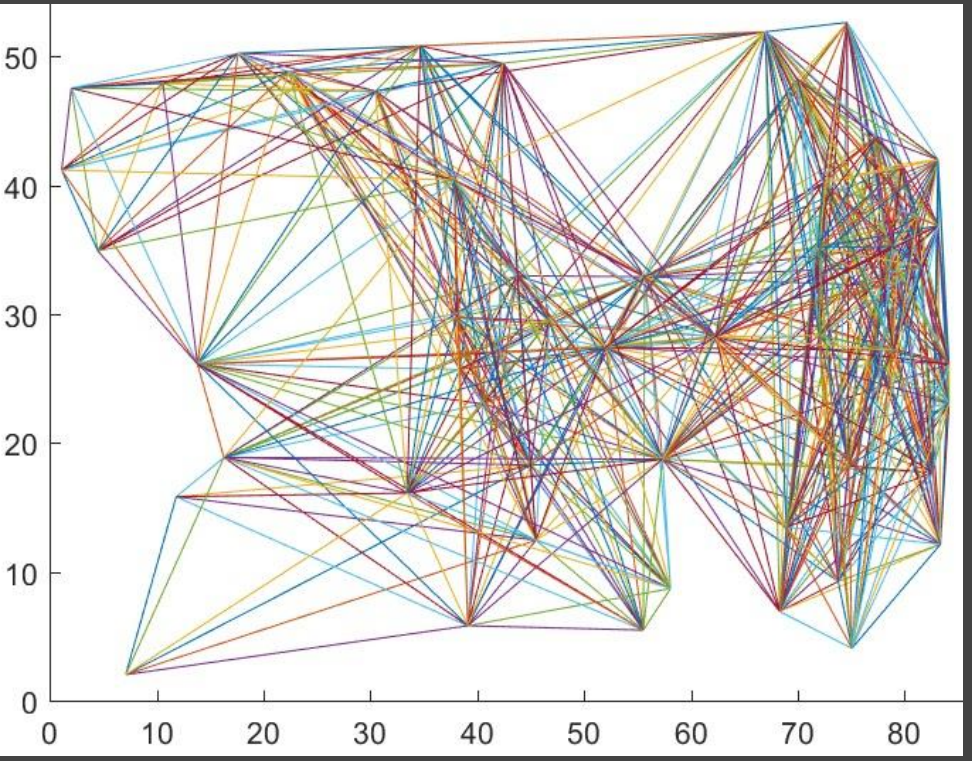


AT&T
Landmark - No Mountains - 15 Miles

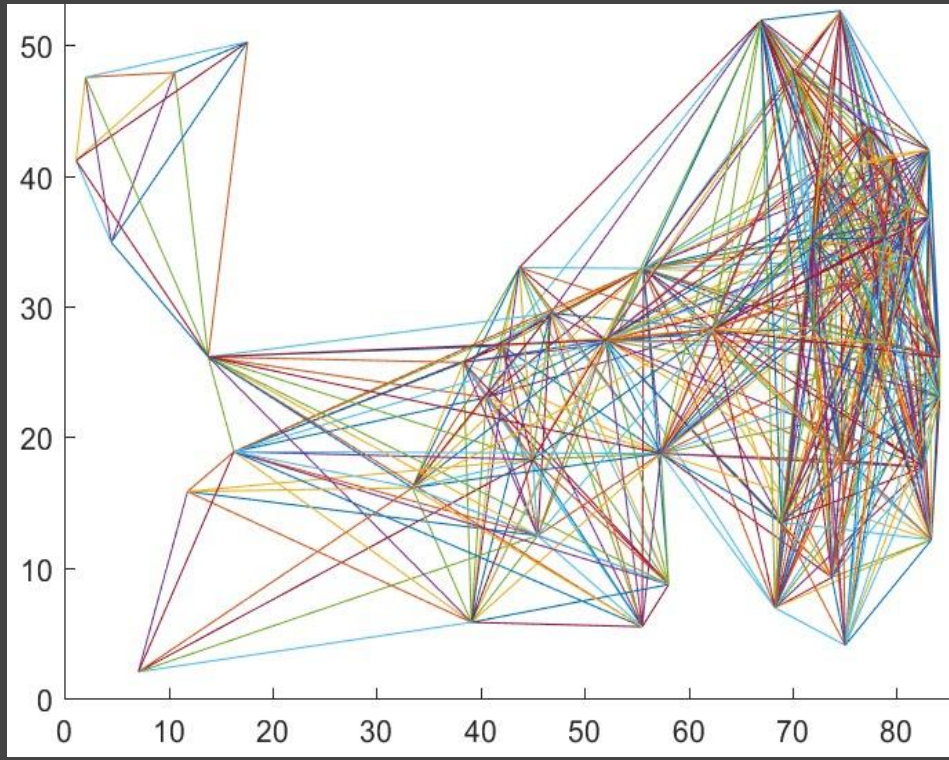
Adding Mountain Ranges

http://www.mappery.com/map-of/Massachusetts-Rivers-Lakes-Mountains-and-Elevations-Map

Adding Mountain Ranges

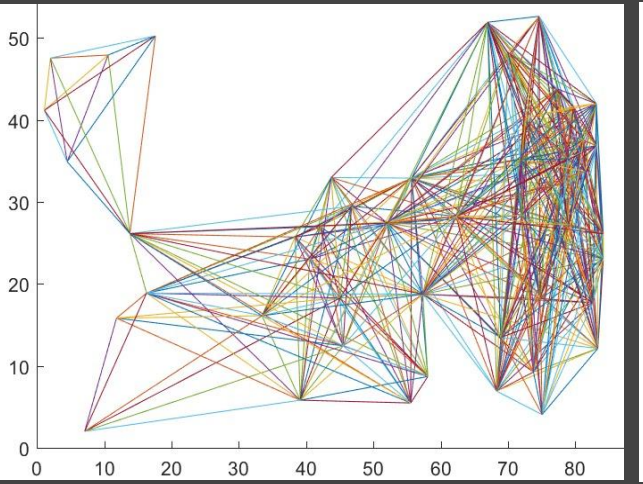


NO MOUNTAINS
Verizon - Observation - 25 miles

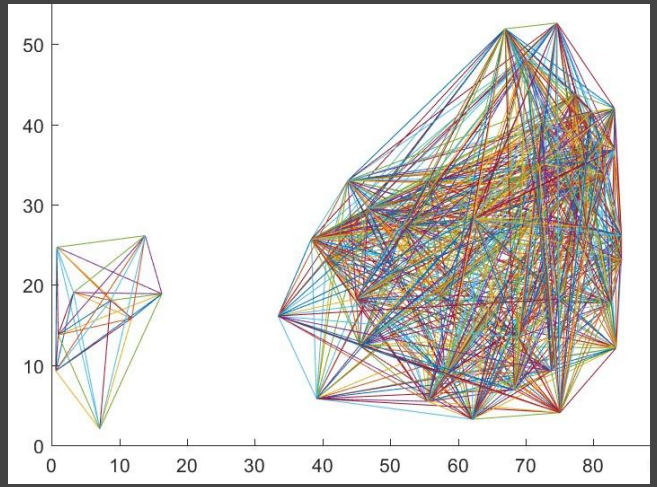


WITH MOUNTAINS
Verizon - Observation - 25 miles

Comparing the Companies at 25 Miles



Verizon



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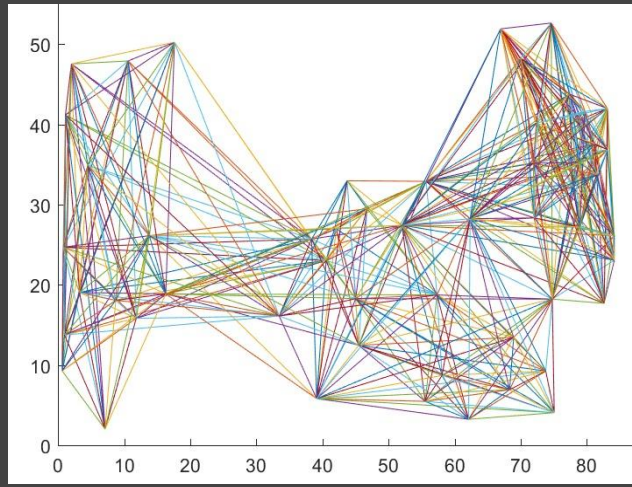
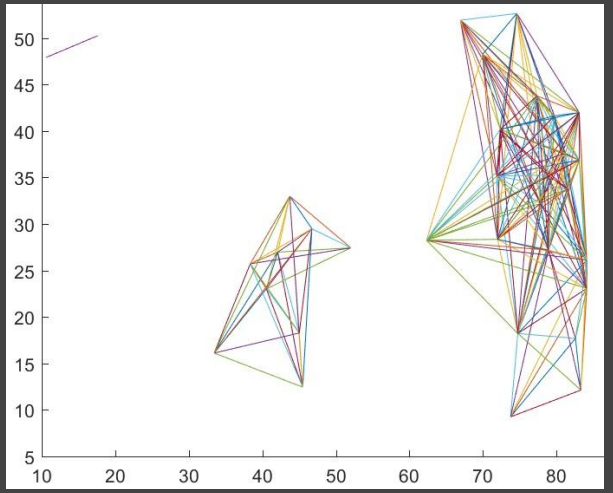
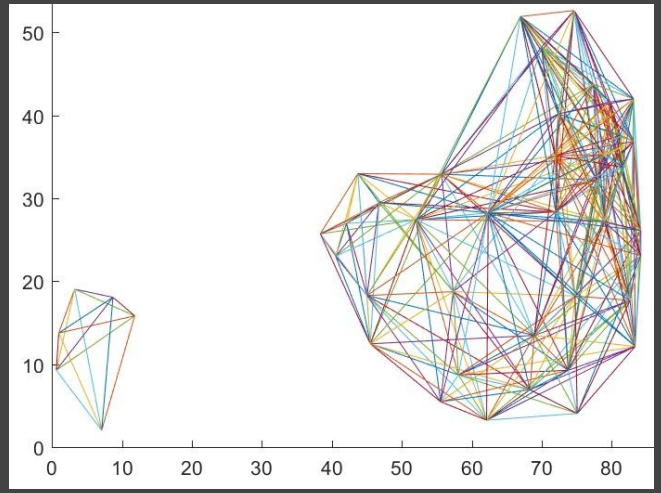


Sprint

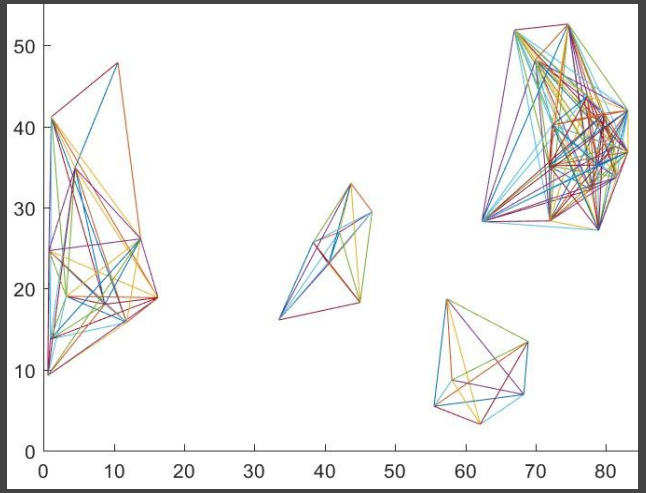
Comparing the Companies at 15 Miles



Verizon



AT&T



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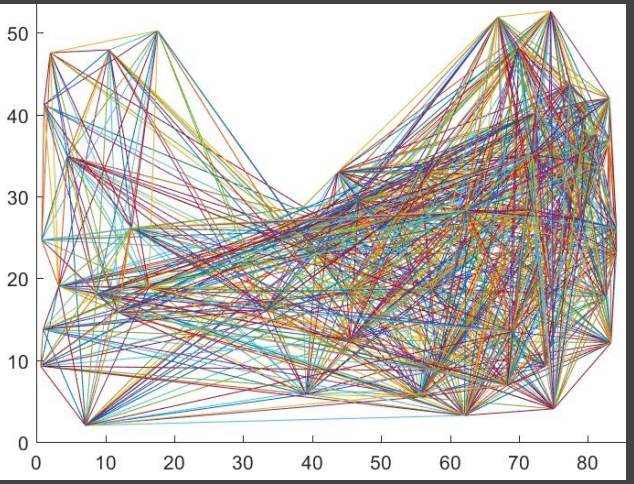
Comparing the Companies at 35 Miles



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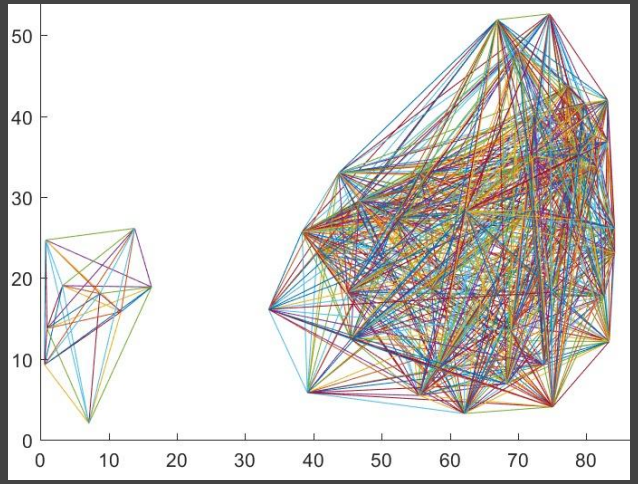
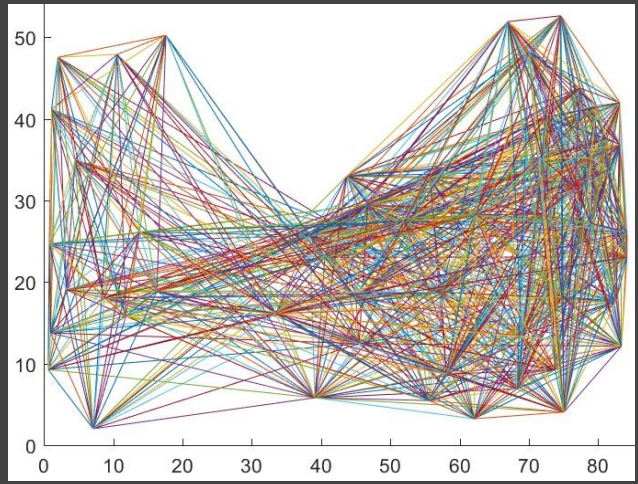


AT&T



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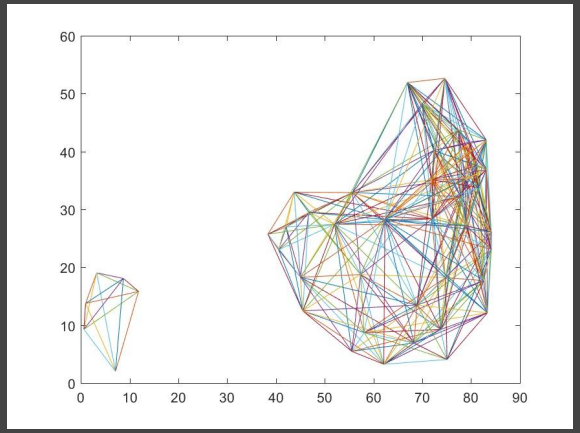
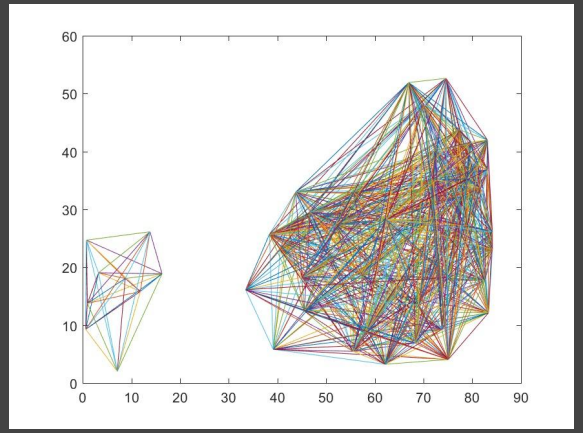
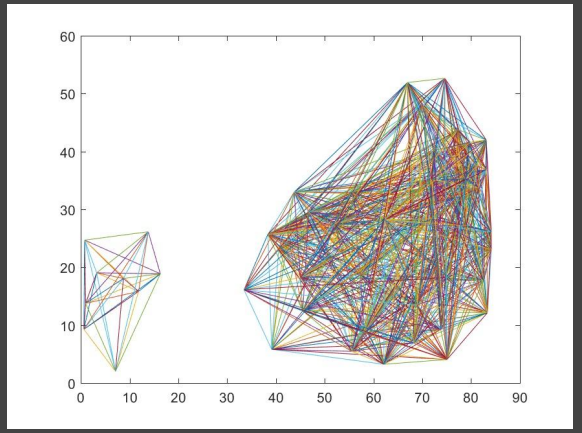
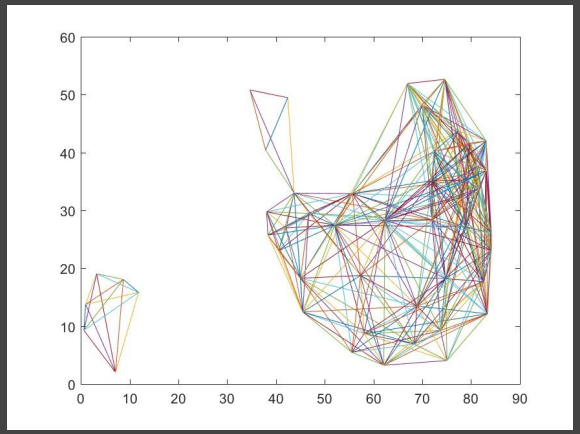
AT&T - Observation Complexes

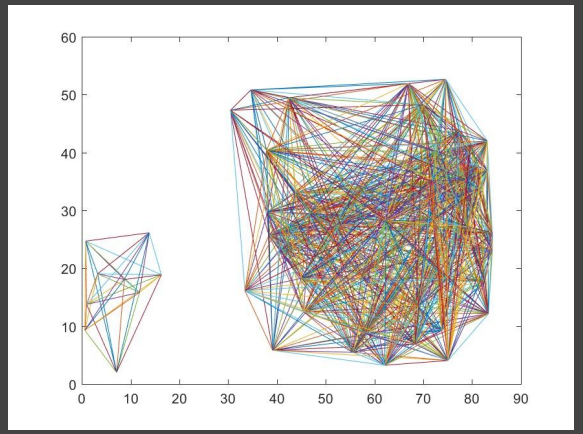


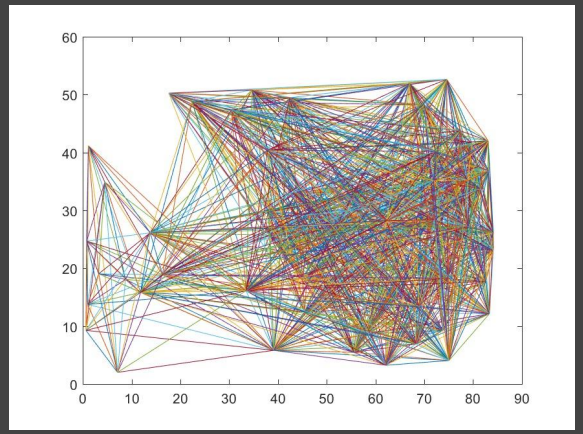




Mountains

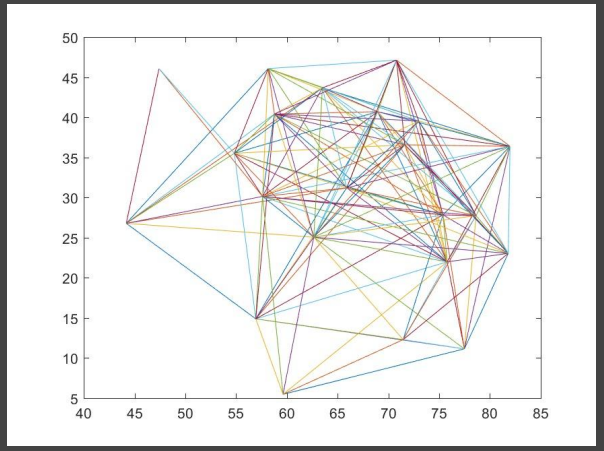


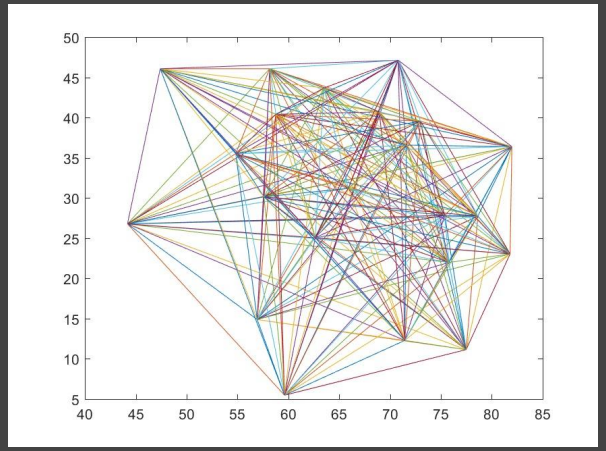


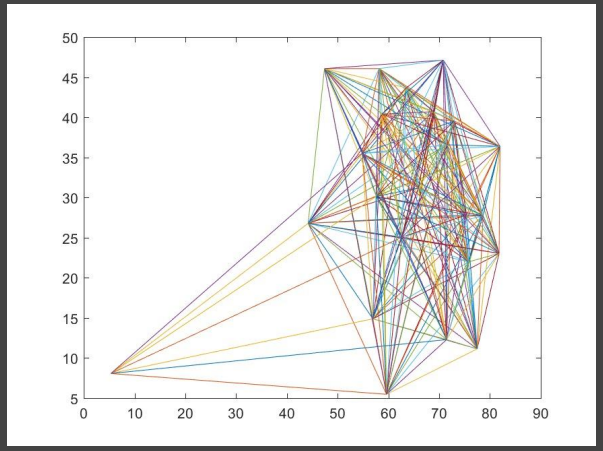


No Mountains

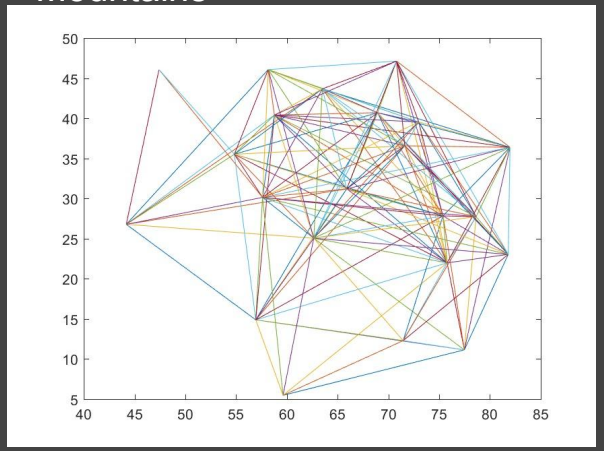
AT&T - Landmark Complexes

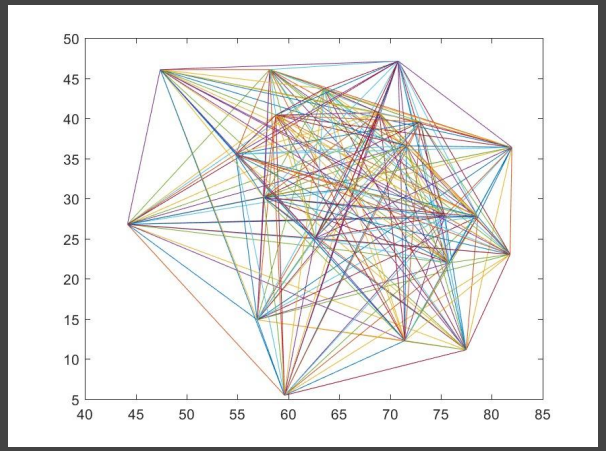


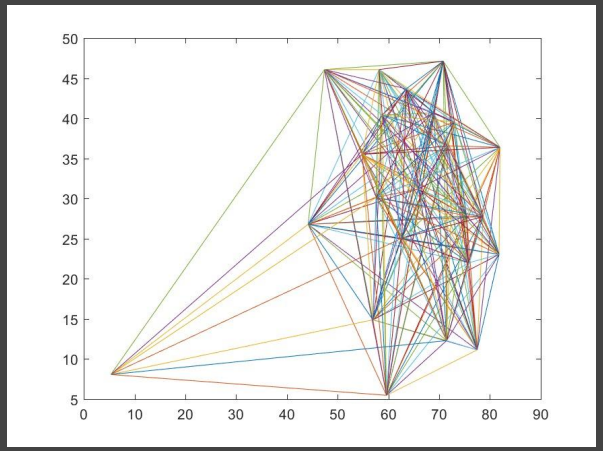




Mountains

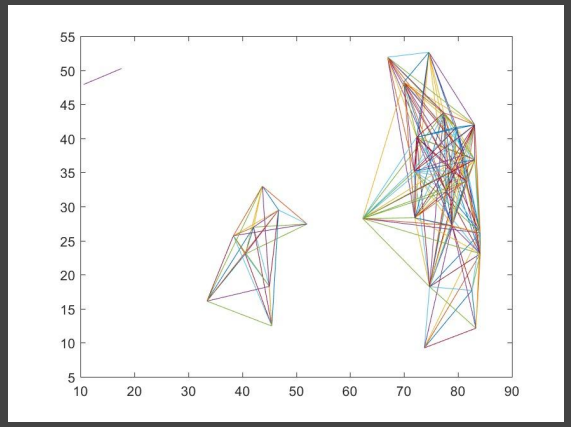


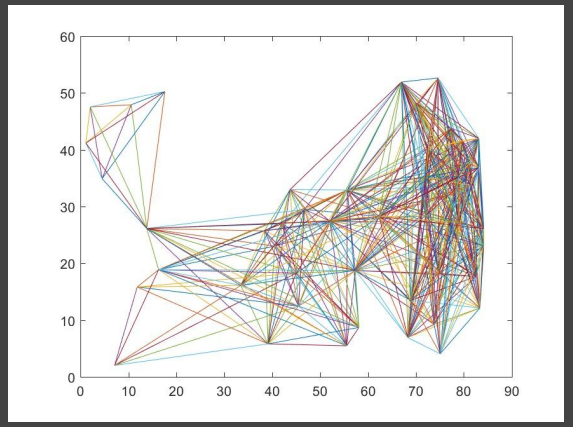


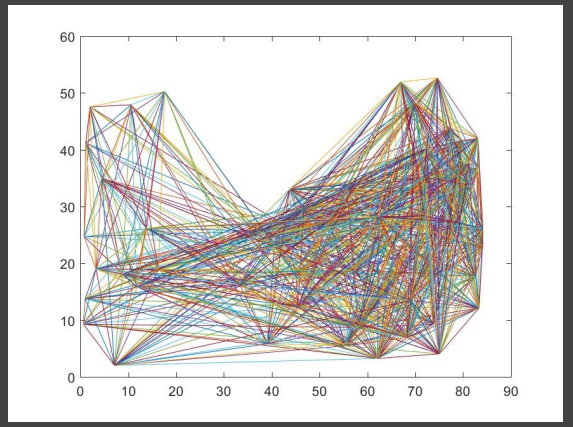


No Mountains

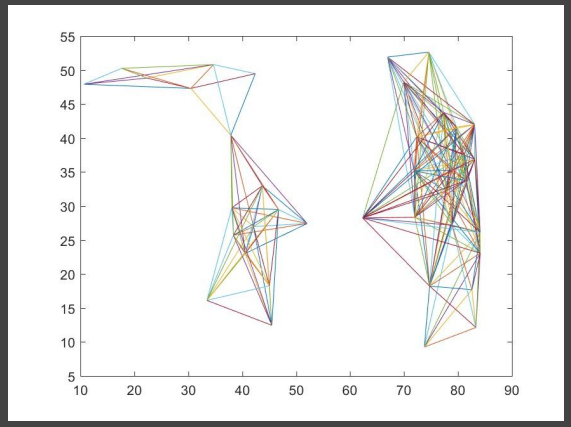
Verizon - Observation Complexes

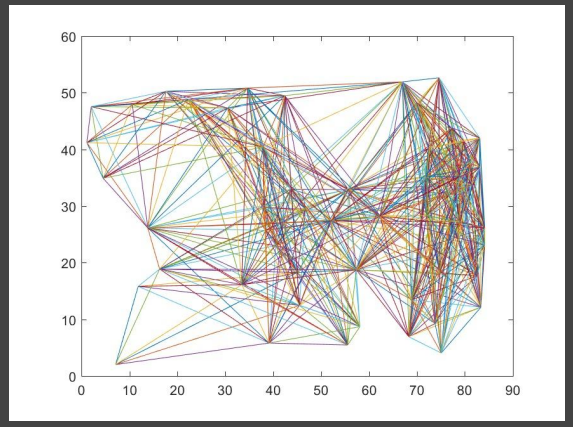


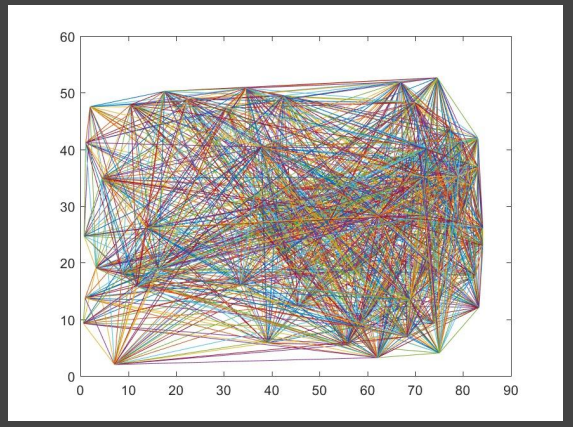




Mountains

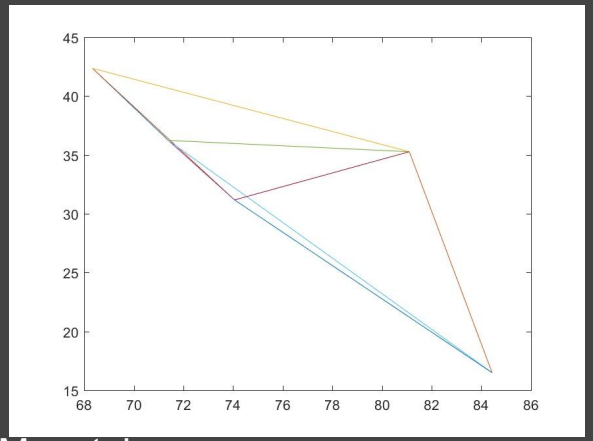


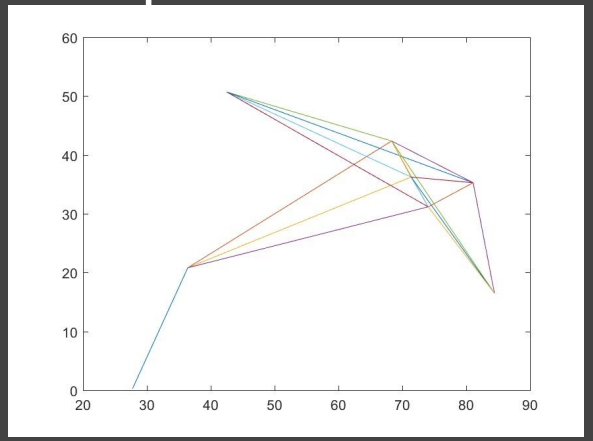


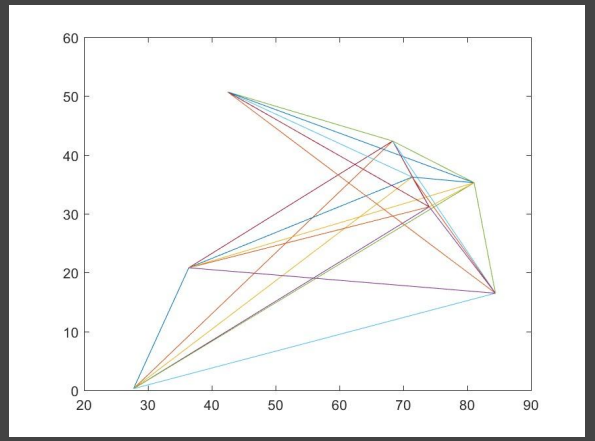


No Mountains

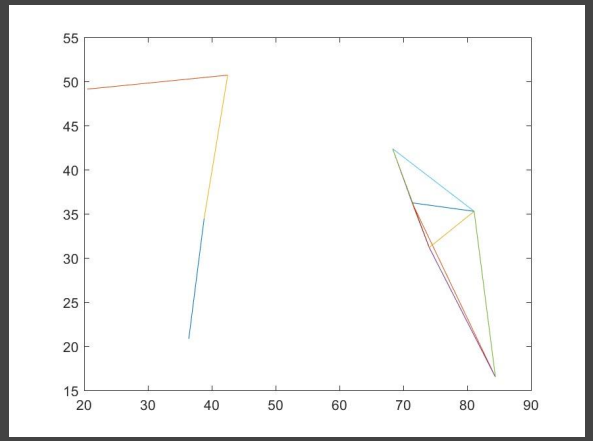
Verizon - Landmark Complexes

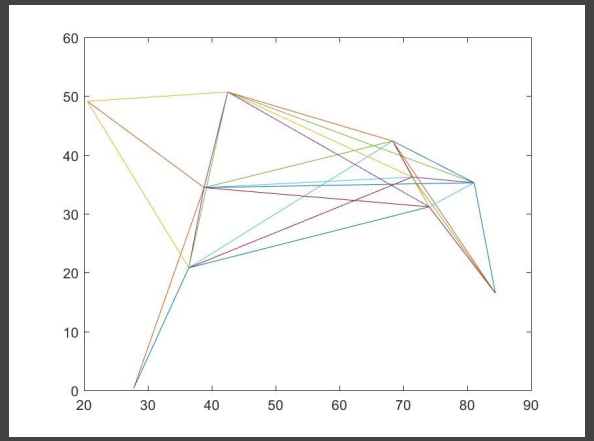


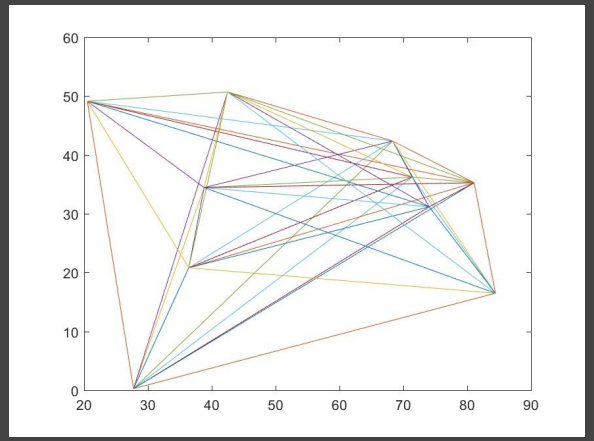




Mountains

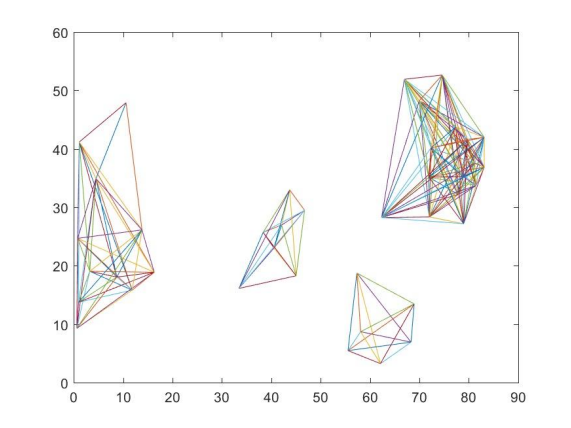


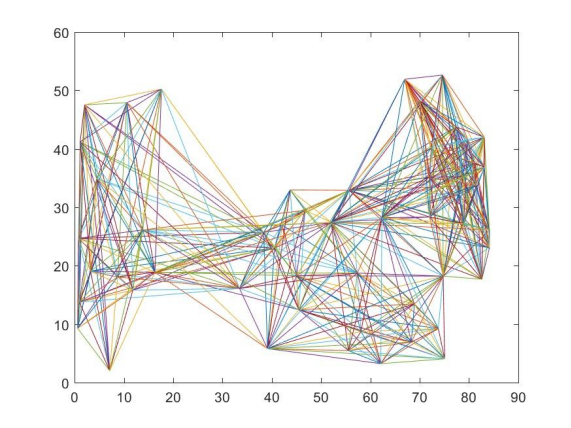


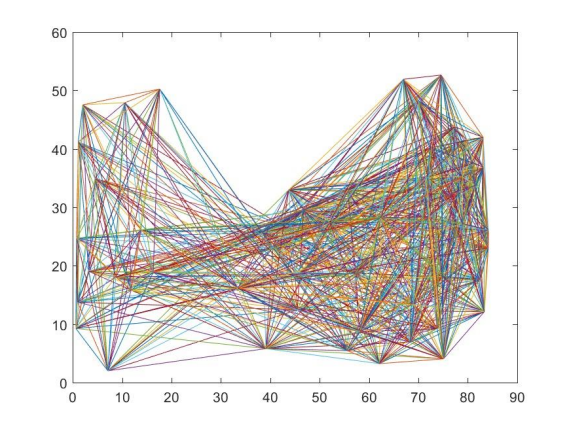


No Mountains

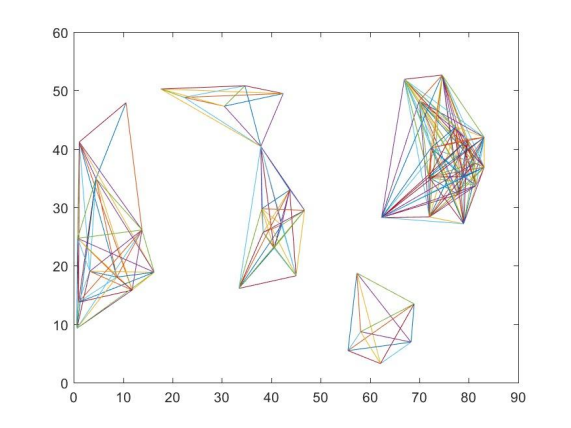
Sprint - Observation Complexes

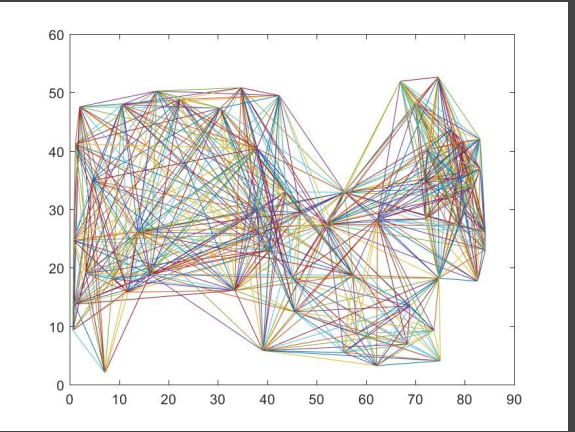


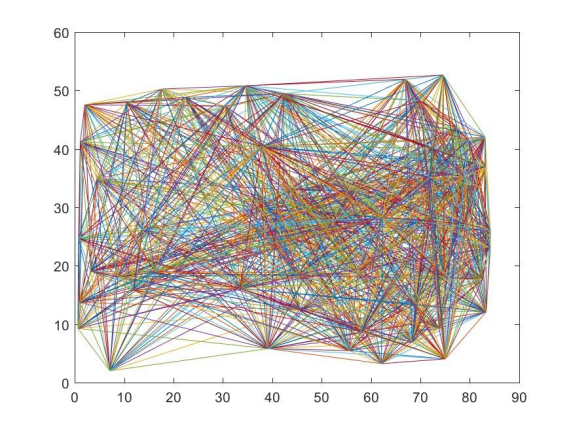




Mountains

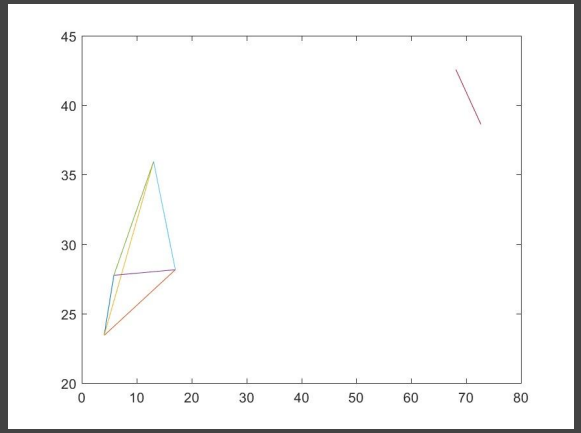


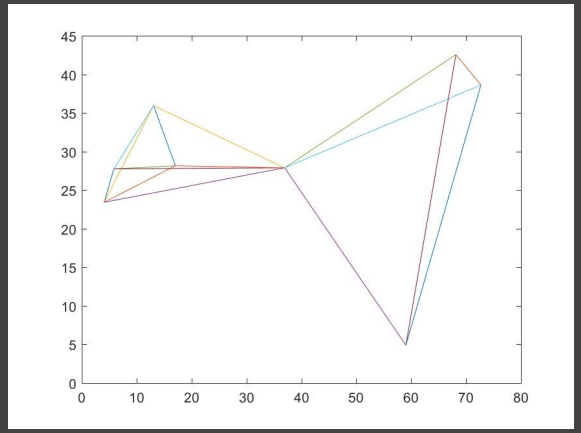


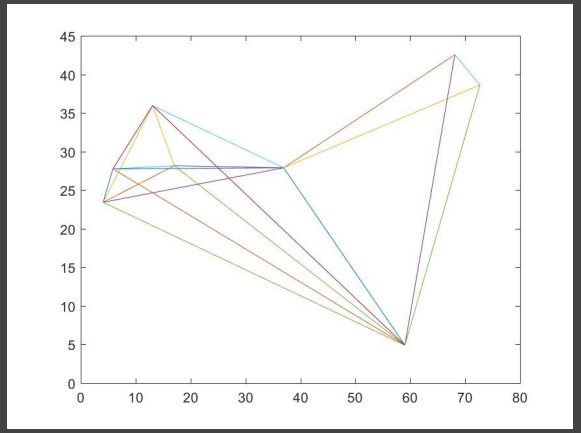


No Mountains

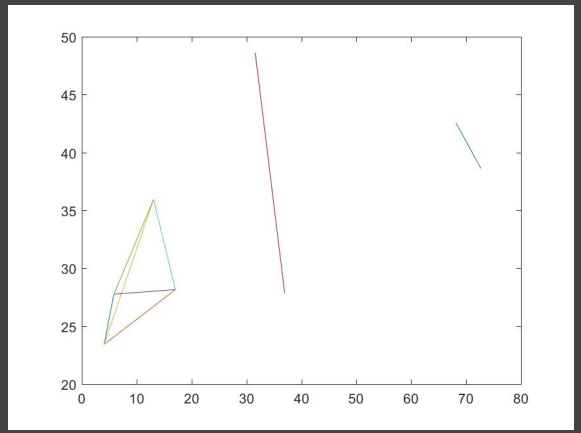
Sprint - Landmark Complexes

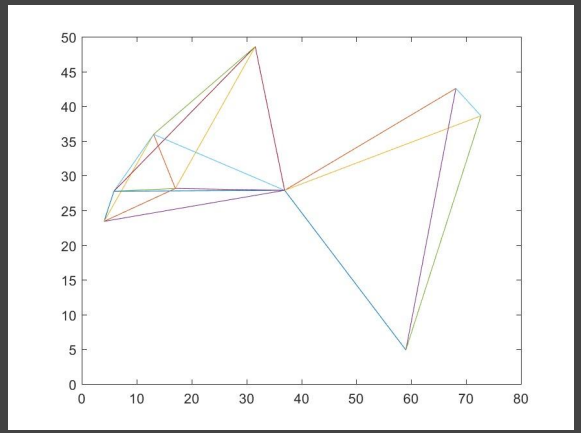


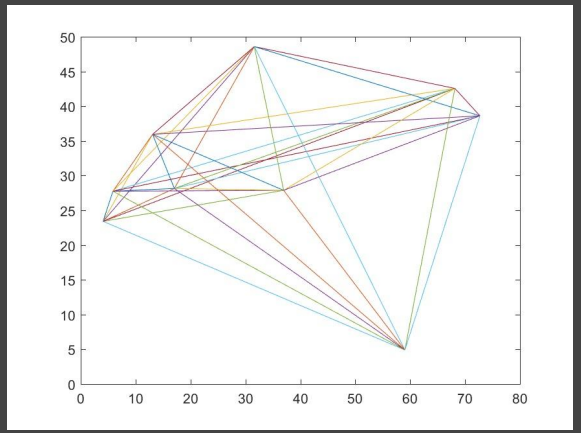




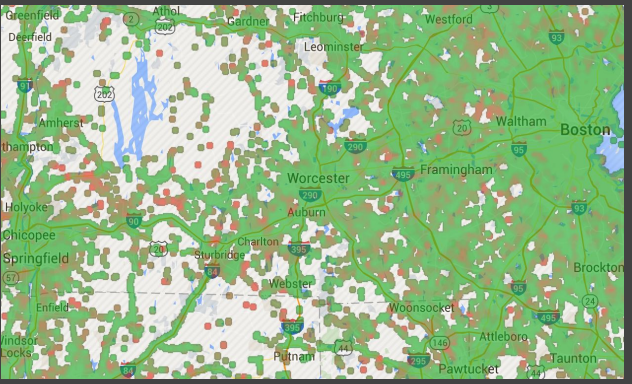
Mountains



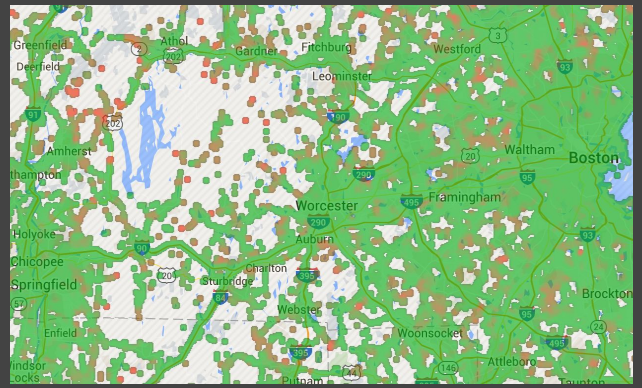




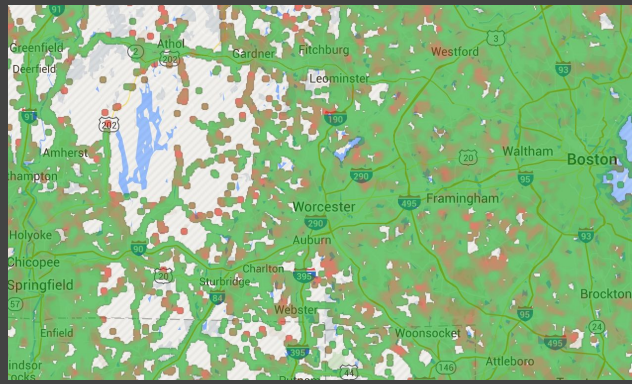
No Mountains



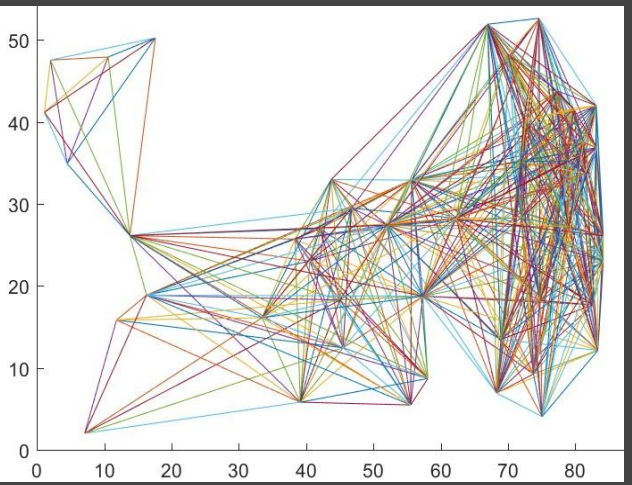
Sprint

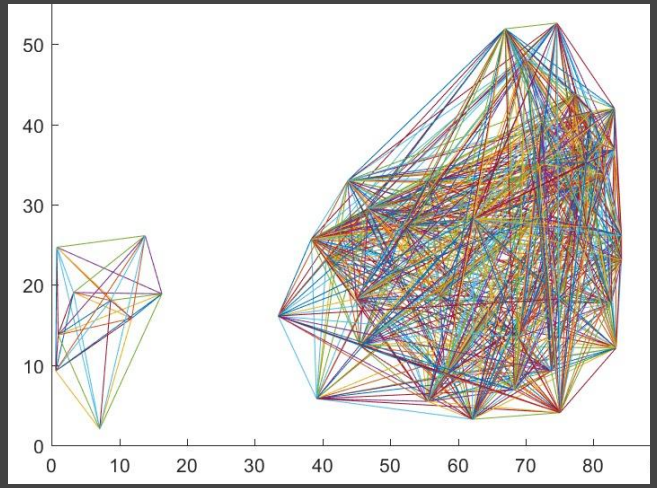


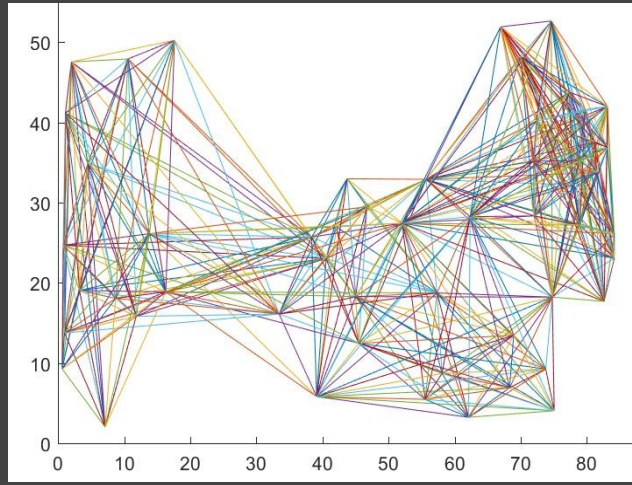
AT&T



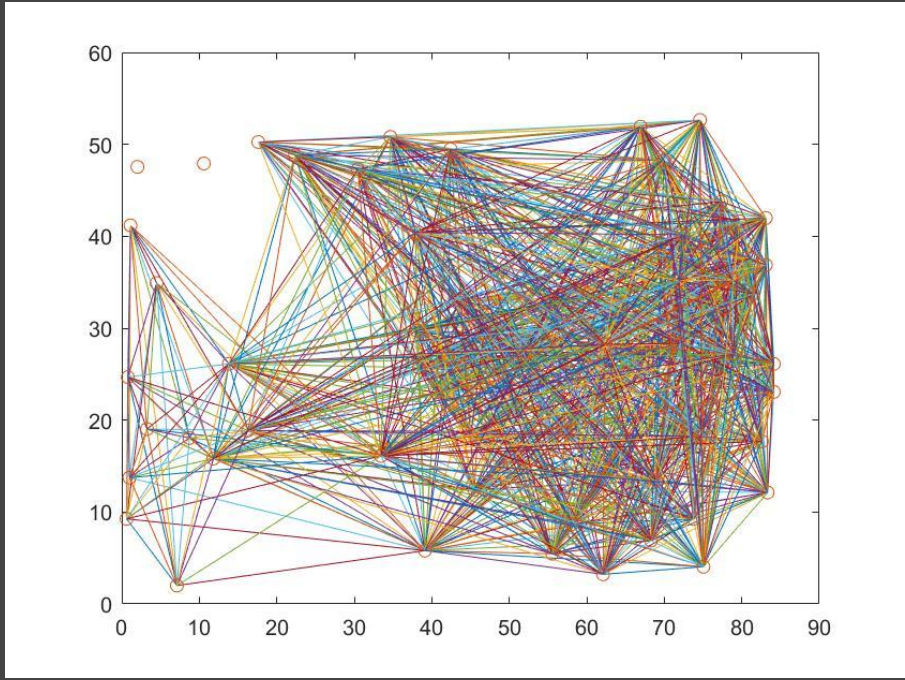
Verizon

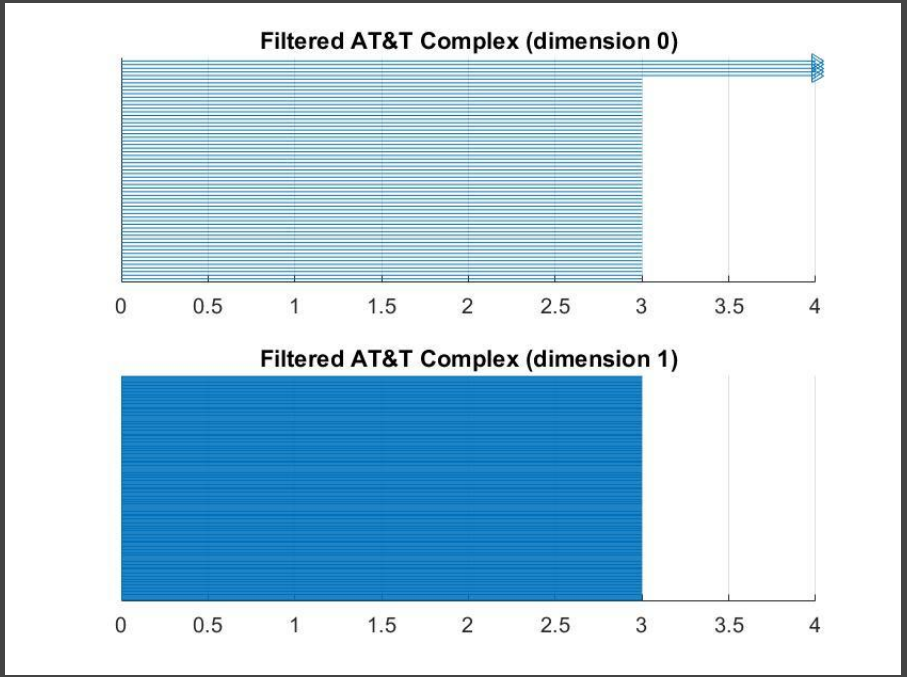






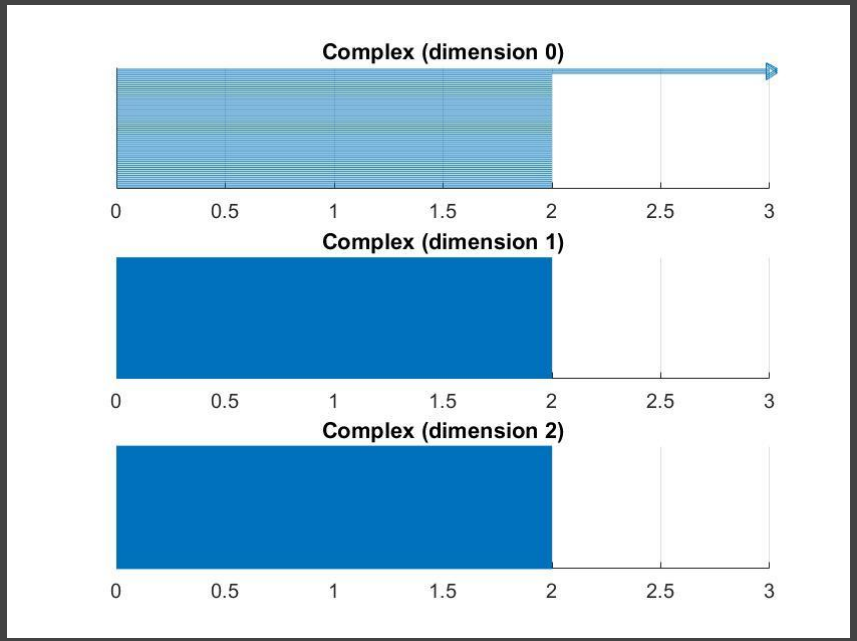
AT&T Barcodes





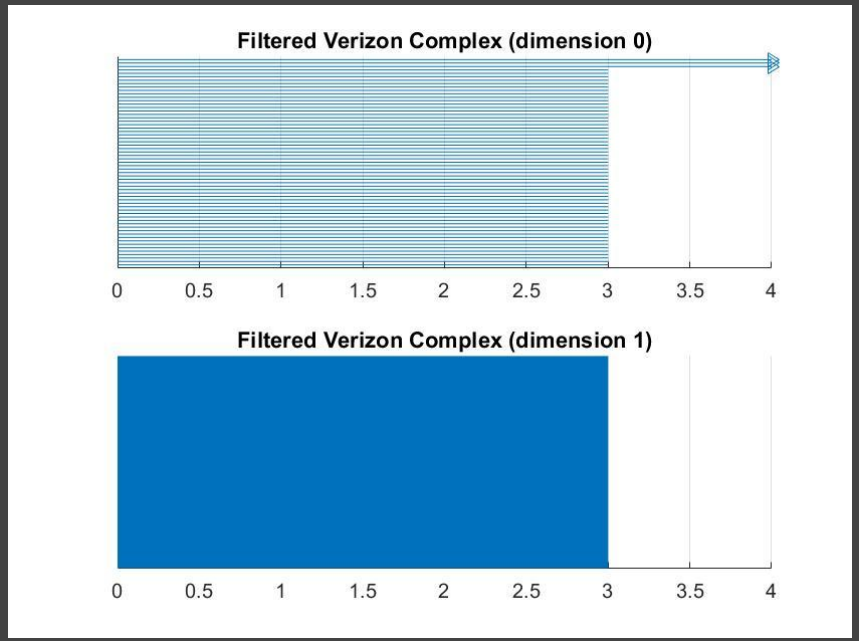
$B_0 = 5$
$B_1 = 0$

Sprint Barcodes



$B_0 = 3$
$B_1 = 0$

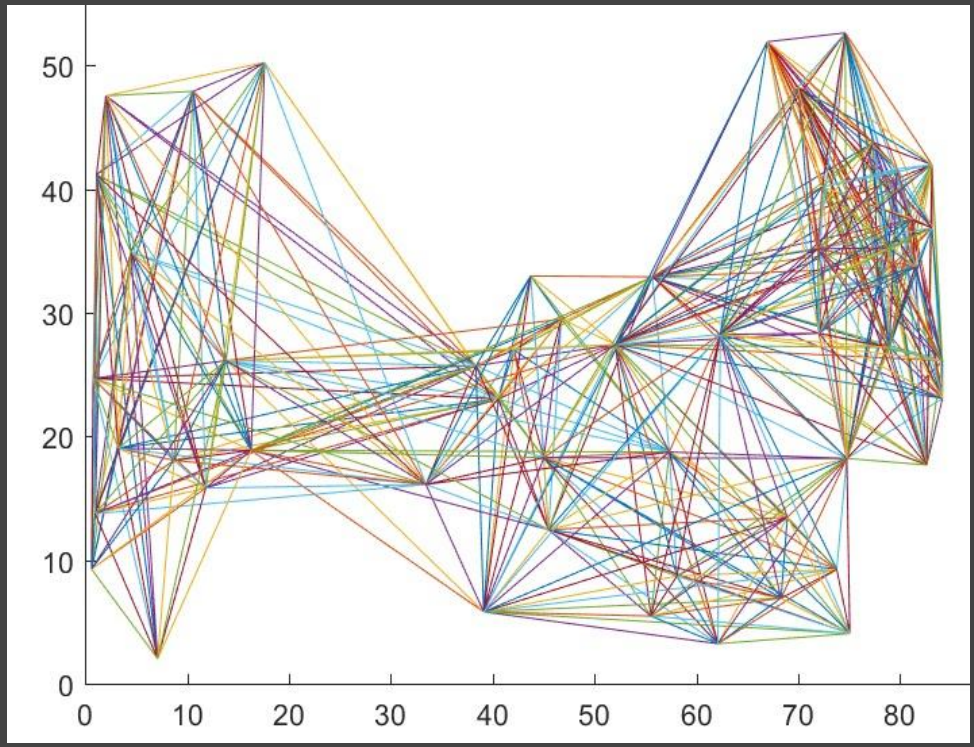
Verizon Barcodes



$B_0 = 3$
$B_1 = 0$

Further Considerations

1. Bigger Data Set
2. Smarter Data Set
3. Higher Dimensions for:
    a. Strength of signal
    b. Tower Traffic



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