In [2]:
```
import pandas as pd
import numpy as np

from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import train_test_split, GridSearchCV
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import TimeSeriesSplit
from sklearn.model_selection import cross_val_score

import matplotlib.pyplot as plt
from matplotlib.lines import Line2D
```

In [4]:
```
# Load the dataset with Date parsed as datetime
df = pd.read_csv('data\\daily_expenses.csv', parse_dates=['Date'])
```

In [5]:
```
# Exclude 'Income' to focus on daily expenses
expense_df = df[df['Category'] != 'Income']

# Group by date to get total daily expense
daily_expense = expense_df.groupby('Date')['Amount_NTD'].sum().reset_index()
daily_expense.rename(columns={'Amount_NTD': 'DailyExpense'}, inplace=True)

# Sort by date (important if you want rolling features)
daily_expense.sort_values('Date', inplace=True)

# Drop daily expenses larger than 1000 NTD
daily_expense = daily_expense[daily_expense['DailyExpense'] <= 1000]
```

Feature Engineering

In [6]:
```
# (a) Day of week (Monday=0, Sunday=6)
daily_expense['DayOfWeek'] = daily_expense['Date'].dt.dayofweek

# (b) Weekend indicator
daily_expense['IsWeekend'] = daily_expense['DayOfWeek'].isin([5, 6]).astype(int)

# (c) Month and Day of month
daily_expense['Month'] = daily_expense['Date'].dt.month
daily_expense['Day'] = daily_expense['Date'].dt.day

# (d) Lag features (Will be used later)
# Create a lag feature (previous day's expense)
daily_expense['Lag1'] = daily_expense['DailyExpense'].shift(1)
# Fill missing values (e.g., first day) with the mean or zero
daily_expense['Lag1'].fillna(daily_expense['DailyExpense'].mean(), inplace=True)


# (e) Rolling 7-day average of expenses (Will be used later)
# This can help the model learn from recent spending trends
daily_expense['Rolling7'] = (
    daily_expense['DailyExpense']
    .rolling(window=7, min_periods=1)
    .mean()
)
```

```
# (f) Log transform the target to reduce the impact of large spikes
daily_expense['LogExpense'] = np.log1p(daily_expense['DailyExpense'])
```

Random Forest

Base Function

In [6]:

```
# Function to train model and evaluate RMSE for a given feature set
def evaluate_feature_set(X, y, tscv=None, use_tscv=True):
    # Split data into training and test sets
    X_train, X_test, y_train, y_test = train_test_split(
        X, y, test_size=0.2, random_state=42
    )

    # Set up a simple parameter grid (or use your existing grid)
    rf = RandomForestRegressor(random_state=42)
    param_grid = {
        'n_estimators': [100, 200, 300, 400, 500],
        'max_depth': [10, 20, 30, 40, 50],
        'min_samples_split': [2, 5, 10, 15, 20],
        'min_samples_leaf': [1, 2, 4, 8, 16],
        'max_features': ['sqrt', 'log2'],
        'bootstrap': [True, False]
    }

    if use_tscv:
        grid_search = GridSearchCV(
            rf, param_grid, cv=tscv, scoring='neg_mean_squared_error', n_jobs=-1
        )
    else:
        grid_search = GridSearchCV(
            rf, param_grid, cv=5, scoring='neg_mean_squared_error', n_jobs=-1
        )

    grid_search.fit(X_train, y_train)
    print("Best Params from Grid Search:", grid_search.best_params_)

    best_rf = grid_search.best_estimator_
    y_pred_log = best_rf.predict(X_test)
    y_pred = np.expm1(y_pred_log)
    y_test_exp = np.expm1(y_test)

    mse = mean_squared_error(y_test_exp, y_pred)
    rmse = np.sqrt(mse)
    return {'rmse': rmse, 'grid_search': grid_search, 'y_pred': y_pred, 'y_test': y_te
```

Evaluate

Without Lag1 and Rolling7

In [7]:

```
tscv = TimeSeriesSplit(n_splits=5)

features_A = ['DayOfWeek', 'IsWeekend', 'Month', 'Day']  # without Lag1 and Rolling7
```

```
X = daily_expense[features_A]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_A, best_params_A = result['rmse'], result['grid_search'].best_params_
best_rf_A = result['grid_search'].best_estimator_
y_pred_A, y_test_A = result['y_pred'], result['y_test']
feature_importance_A = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set A (no Lag1, no Rolling7) RMSE:", rmse_A)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 20, 'max_features': 's
qrt', 'min_samples_leaf': 4, 'min_samples_split': 2, 'n_estimators': 100}
Feature Set A (no Lag1, no Rolling7) RMSE: 38.66275658792042
```

With Lag1

In [8]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_B = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Lag1']  # with Lag1 only

X = daily_expense[features_B]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_B, best_params_B = result['rmse'], result['grid_search'].best_params_
best_rf_B = result['grid_search'].best_estimator_
y_pred_B, y_test_B = result['y_pred'], result['y_test']
feature_importance_B = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set B (with Lag1, no Rolling7) RMSE:", rmse_B)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 's
qrt', 'min_samples_leaf': 8, 'min_samples_split': 2, 'n_estimators': 100}
Feature Set B (with Lag1, no Rolling7) RMSE: 39.3385956464684
```

With Rolling7

In [9]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_C = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7']  # with Rolling7 o

X = daily_expense[features_C]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_C, best_params_C = result['rmse'], result['grid_search'].best_params_
best_rf_C = result['grid_search'].best_estimator_
y_pred_C, y_test_C = result['y_pred'], result['y_test']
feature_importance_C = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set C (no Lag1, with Rolling7) RMSE:", rmse_C)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 's
qrt', 'min_samples_leaf': 4, 'min_samples_split': 2, 'n_estimators': 100}
Feature Set C (no Lag1, with Rolling7) RMSE: 37.275187253048614
```

With Lag1 & Rolling7

In [10]:

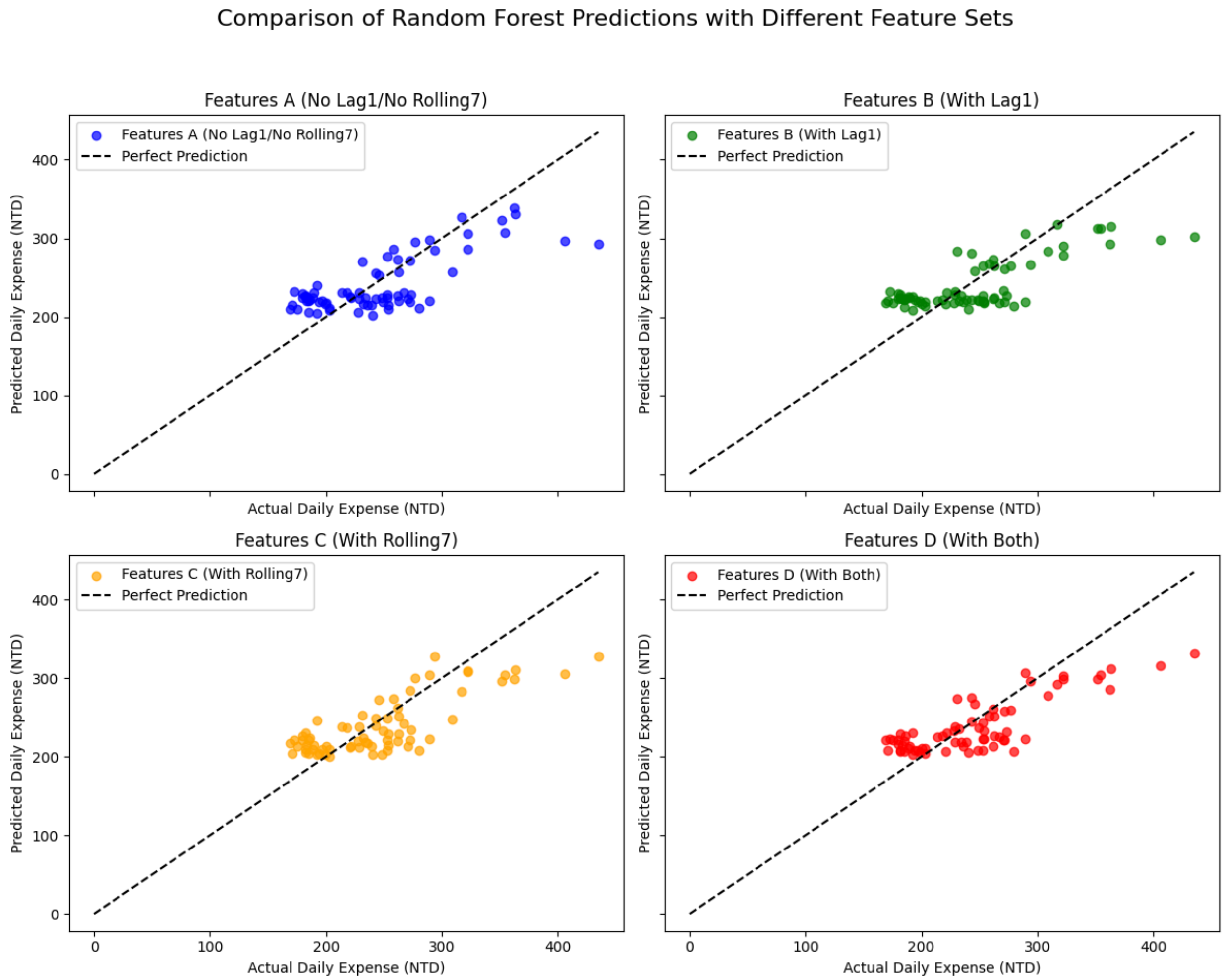
```
tscv = TimeSeriesSplit(n_splits=5)

features_D = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Lag1', 'Rolling7']  # with bo

X = daily_expense[features_D]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_D, best_params_D = result['rmse'], result['grid_search'].best_params_
best_rf_D = result['grid_search'].best_estimator_
y_pred_D, y_test_D = result['y_pred'], result['y_test']
feature_importance_D = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set D (with Lag1, with Rolling7) RMSE:", rmse_D)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 's
qrt', 'min_samples_leaf': 4, 'min_samples_split': 2, 'n_estimators': 200}
Feature Set D (with Lag1, with Rolling7) RMSE: 36.19773875656066
```

Result

In [20]:

```
# Create a 2x2 grid of subplots
fig, axes = plt.subplots(2, 2, figsize=(12, 10), sharex=True, sharey=True)
axes = axes.flatten()

# List of (true values, predictions, label, color) for each feature set
plot_data = [
    (y_test_A, y_pred_A, 'Features A (No Lag1/No Rolling7)', 'blue'),
    (y_test_B, y_pred_B, 'Features B (With Lag1)', 'green'),
    (y_test_C, y_pred_C, 'Features C (With Rolling7)', 'orange'),
    (y_test_D, y_pred_D, 'Features D (With Both)', 'red'),
]

for ax, (y_test, y_pred, title, color) in zip(axes, plot_data):
    ax.scatter(y_test, y_pred, color=color, alpha=0.7, label=title)
    # Determine maximum value for perfect prediction line in each subplot
    max_val = max(y_test.max(), y_pred.max())
    ax.plot([0, max_val], [0, max_val], 'k--', label='Perfect Prediction')
    ax.set_xlabel('Actual Daily Expense (NTD)')
    ax.set_ylabel('Predicted Daily Expense (NTD)')
    ax.set_title(title)
    ax.legend()

fig.suptitle('Comparison of Random Forest Predictions with Different Feature Sets', fo
plt.savefig('data\\experiment_result\\random_forest_comparison.png')
plt.tight_layout(rect=[0, 0.03, 1, 0.95])
plt.show()
```



Feature Importance

In [12]:

```
# Feature Set A: Without Lag1 and Rolling7
feature_importance_df_A = pd.DataFrame({'Feature': features_A, 'Importance': feature_i
print("Feature Importances for Feature Set A (No Lag1/No Rolling7):")
print(feature_importance_df_A)
print("\n" + "="*50 + "\n")

# Feature Set B: With Lag1 only
feature_importance_df_B = pd.DataFrame({'Feature': features_B, 'Importance': feature_i
print("Feature Importances for Feature Set B (With Lag1):")
print(feature_importance_df_B)
print("\n" + "="*50 + "\n")

# Feature Set C: With Rolling7 only
feature_importance_df_C = pd.DataFrame({'Feature': features_C, 'Importance': feature_i
print("Feature Importances for Feature Set C (With Rolling7):")
print(feature_importance_df_C)
print("\n" + "="*50 + "\n")

# Feature Set D: With both Lag1 and Rolling7
feature_importance_df_D = pd.DataFrame({'Feature': features_D, 'Importance': feature_i
print("Feature Importances for Feature Set D (With Both Lag1 & Rolling7):")
print(feature_importance_df_D)
```

```
Feature Importances for Feature Set A (No Lag1/No Rolling7):
     Feature  Importance
0  DayOfWeek    0.406320
1  IsWeekend    0.282965
2      Month    0.144396
3        Day    0.166320

==================================================

Feature Importances for Feature Set B (With Lag1):
     Feature  Importance
0  DayOfWeek    0.406350
1  IsWeekend    0.314623
2      Month    0.064919
3        Day    0.073103
4       Lag1    0.141005

==================================================

Feature Importances for Feature Set C (With Rolling7):
     Feature  Importance
0  DayOfWeek    0.335043
1  IsWeekend    0.239466
2      Month    0.078241
3        Day    0.108969
4   Rolling7    0.238281

==================================================

Feature Importances for Feature Set D (With Both Lag1 & Rolling7):
     Feature  Importance
0  DayOfWeek    0.292754
1  IsWeekend    0.229379
2      Month    0.054489
3        Day    0.079247
4       Lag1    0.130432
5   Rolling7    0.213698
```

Cross Validation

In [13]:
```python
# Function to perform cross-validation for a given feature set and best estimator
def cross_val_feature_set(features, best_rf):
    X_set = daily_expense[features]
    y_set = daily_expense['LogExpense']

    cv_scores = cross_val_score(
        best_rf,
        X_set,
        y_set,
        cv=5,
        scoring='neg_mean_squared_error',
        n_jobs=-1
    )
    mse_scores = -cv_scores
    rmse_scores = np.sqrt(mse_scores)
    return rmse_scores, rmse_scores.mean(), rmse_scores.std()
```

In [15]:
```
# Perform cross-validation on each feature set using the best estimator from grid sear
rmse_scores_A, mean_rmse_A, std_rmse_A = cross_val_feature_set(features_A, best_rf_A)
rmse_scores_B, mean_rmse_B, std_rmse_B = cross_val_feature_set(features_B, best_rf_B)
rmse_scores_C, mean_rmse_C, std_rmse_C = cross_val_feature_set(features_C, best_rf_C)
rmse_scores_D, mean_rmse_D, std_rmse_D = cross_val_feature_set(features_D, best_rf_D)
```

In [16]:
```
print("Cross-Validation RMSE for Feature Set A (No Lag1/No Rolling7):")
print(rmse_scores_A)
print("Mean RMSE:", mean_rmse_A)
print("Standard Deviation of RMSE:", std_rmse_A)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set B (With Lag1):")
print(rmse_scores_B)
print("Mean RMSE:", mean_rmse_B)
print("Standard Deviation of RMSE:", std_rmse_B)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set C (With Rolling7):")
print(rmse_scores_C)
print("Mean RMSE:", mean_rmse_C)
print("Standard Deviation of RMSE:", std_rmse_C)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set D (With Both Lag1 & Rolling7):")
print(rmse_scores_D)
print("Mean RMSE:", mean_rmse_D)
print("Standard Deviation of RMSE:", std_rmse_D)
```

```
Cross-Validation RMSE for Feature Set A (No Lag1/No Rolling7):
[0.14123133 0.18620571 0.15099621 0.19368341 0.16927219]
Mean RMSE: 0.16827776916692772
Standard Deviation of RMSE: 0.019990040177141778

==================================================

Cross-Validation RMSE for Feature Set B (With Lag1):
[0.13739528 0.17689063 0.15231579 0.19879546 0.16225005]
Mean RMSE: 0.16552944218569143
Standard Deviation of RMSE: 0.02103611466426487

==================================================

Cross-Validation RMSE for Feature Set C (With Rolling7):
[0.12797353 0.17395989 0.14638658 0.17412378 0.15168051]
Mean RMSE: 0.15482485750497763
Standard Deviation of RMSE: 0.017553818642204157

==================================================

Cross-Validation RMSE for Feature Set D (With Both Lag1 & Rolling7):
[0.12514251 0.16964064 0.14900217 0.17844627 0.1477058 ]
Mean RMSE: 0.1539874791836075
Standard Deviation of RMSE: 0.018652610616576897
```

In [ ]:
```
result_df = pd.DataFrame({
    'feature_set': ['A', 'B', 'C', 'D'],
    'RMSE': [rmse_A, rmse_B, rmse_C, rmse_D],
    'best_params': [best_params_A, best_params_B, best_params_C, best_params_D],
    'best_rf': [best_rf_A, best_rf_B, best_rf_C, best_rf_D],
    'features': [features_A, features_B, features_C, features_D],
    'feature_importance': [feature_importance_A, feature_importance_B, feature_importa
```

```
    'cross_validation_RMSE': [rmse_scores_A, rmse_scores_B, rmse_scores_C, rmse_scores
    'cross_validation_RMSE_mean': [mean_rmse_A, mean_rmse_B, mean_rmse_C, mean_rmse_D]
    'cross_validation_RMSE_std': [std_rmse_A, std_rmse_B, std_rmse_C, std_rmse_D],
})

result_df.to_csv('data\\experiment_result\\random_forest_result.csv', index=False)
```

K-Means Clustering

In [63]:

```
from sklearn.cluster import KMeans
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import silhouette_score, davies_bouldin_score, calinski_harabasz_
```

In [25]:

```
# Selecting features for clustering.
# We include some of the engineered features that capture temporal and trend informati
cluster_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7', 'Lag1']
```

In [26]:

```
# Create a subset of data for clustering.
X_cluster = daily_expense[cluster_features].copy()

# Standardize the features (recommended for K-Means).
scaler = StandardScaler()
X_cluster_scaled = scaler.fit_transform(X_cluster)
```

In [64]:

```
# Determine the optimal number of clusters using the Elbow Method.
sse = []  # Sum of Squared Errors for each k
sil_scores = []
db_scores = []
ch_scores = []

k_values = range(2, 31)
for k in k_values:
    # Note: In scikit-learn 1.4+ use n_init='auto'
    kmeans = KMeans(n_clusters=k, random_state=42, n_init='auto')
    labels = kmeans.fit_predict(X_cluster_scaled)

    sil = silhouette_score(X_cluster_scaled, labels)
    db = davies_bouldin_score(X_cluster_scaled, labels)
    ch = calinski_harabasz_score(X_cluster_scaled, labels)

    sse.append(kmeans.inertia_)
    sil_scores.append(sil)
    db_scores.append(db)
    ch_scores.append(ch)
```

In [66]:

```
# Get the corresponding SSE value for optimal_k_elbow:
optimal_k_elbow = 6
sse_optimal = sse[optimal_k_elbow - k_values[0]]  # adjust index if k_values does not

# Compute optimal k based on Silhouette Score and Davies-Bouldin Index
optimal_k_sil = k_values[sil_scores.index(max(sil_scores))]
sil_optimal = max(sil_scores)

optimal_k_db = k_values[db_scores.index(min(db_scores))]
db_optimal = min(db_scores)
```

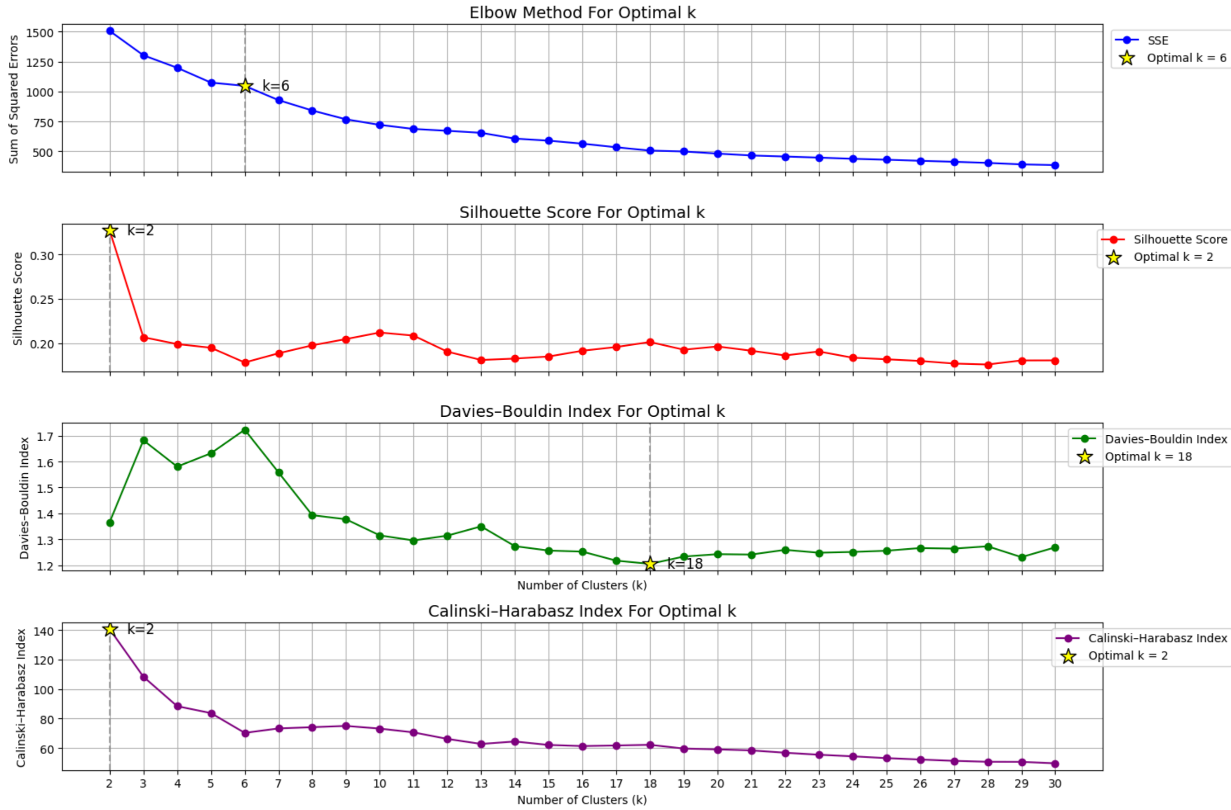
```
optimal_k_ch = k_values[np.argmax(ch_scores)]
ch_optimal_value = max(ch_scores)

print(f"Optimal k based on Elbow Method: {optimal_k_elbow} (SSE={sse_optimal:.2f})")
print(f"Optimal k based on Silhouette Score: {optimal_k_sil} (Score={sil_optimal:.2f})
print(f"Optimal k based on Davies-Bouldin Index: {optimal_k_db} (Score={db_optimal:.2f
print(f"Optimal k based on Calinski-Harabasz Index: {optimal_k_ch} (Score={ch_optimal_
```

```
Optimal k based on Elbow Method: 6 (SSE=1047.89)
Optimal k based on Silhouette Score: 2 (Score=0.33)
Optimal k based on Davies-Bouldin Index: 18 (Score=1.20)
Optimal k based on Calinski-Harabasz Index: 2 (Score=140.54)
```

In [68]:

```
fig, axes = plt.subplots(4, 1, figsize=(15, 10), sharex=True)

# --- 1) Elbow Method (SSE) ---
axes[0].plot(k_values, sse, marker='o', color='b', label="SSE")
axes[0].scatter(
    optimal_k_elbow, sse_optimal,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_elbow}'
)
axes[0].axvline(optimal_k_elbow, color='gray', linestyle='--', alpha=0.7)
axes[0].text(
    optimal_k_elbow + 0.5, sse_optimal,
    f"k={optimal_k_elbow}",
    fontsize=12,
    verticalalignment='center'
)
axes[0].set_ylabel("Sum of Squared Errors")
axes[0].set_title("Elbow Method For Optimal k", fontsize=14)
axes[0].grid(True)
axes[0].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# --- 2) Silhouette Score ---
axes[1].plot(k_values, sil_scores, marker='o', color='r', label="Silhouette Score")
axes[1].scatter(
    optimal_k_sil, sil_optimal,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_sil}'
)
axes[1].axvline(optimal_k_sil, color='gray', linestyle='--', alpha=0.7)
axes[1].text(
    optimal_k_sil + 0.5, sil_optimal,
    f"k={optimal_k_sil}",
    fontsize=12,
    verticalalignment='center'
)
axes[1].set_ylabel("Silhouette Score")
axes[1].set_title("Silhouette Score For Optimal k", fontsize=14)
axes[1].grid(True)
axes[1].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# --- 3) Davies-Bouldin Index ---
axes[2].plot(k_values, db_scores, marker='o', color='g', label="Davies-Bouldin Index")
axes[2].scatter(
    optimal_k_db, db_optimal,
```

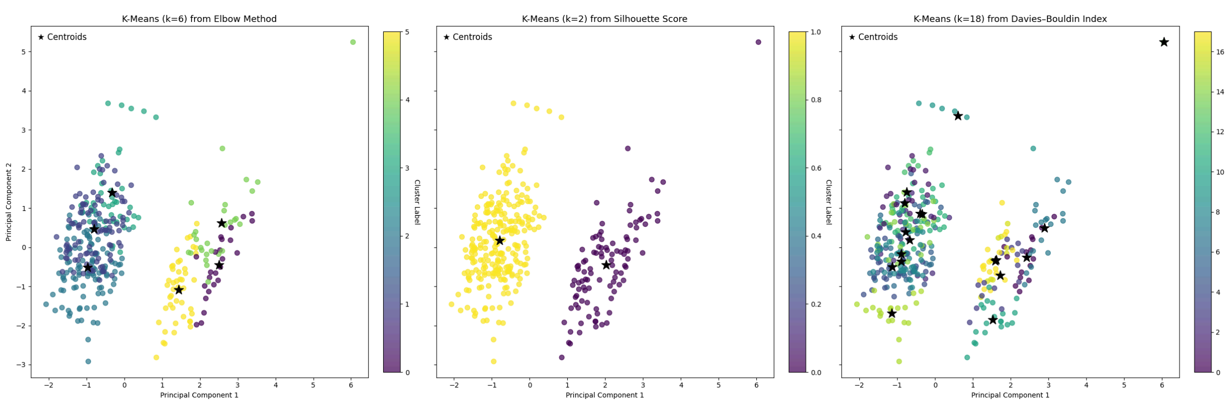
```
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_db}'
)
axes[2].axvline(optimal_k_db, color='gray', linestyle='--', alpha=0.7)
axes[2].text(
    optimal_k_db + 0.5, db_optimal,
    f"k={optimal_k_db}",
    fontsize=12,
    verticalalignment='center'
)
axes[2].set_xlabel("Number of Clusters (k)")
axes[2].set_ylabel("Davies-Bouldin Index")
axes[2].set_title("Davies-Bouldin Index For Optimal k", fontsize=14)
axes[2].grid(True)
axes[2].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

axes[3].plot(k_values, ch_scores, marker='o', color='purple', label="Calinski-Harabasz
axes[3].scatter(
    optimal_k_ch, ch_optimal_value,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_ch}'
)
axes[3].axvline(optimal_k_ch, color='gray', linestyle='--', alpha=0.7)
axes[3].text(
    optimal_k_ch + 0.5, ch_optimal_value,
    f"k={optimal_k_ch}",
    fontsize=12,
    verticalalignment='center'
)

axes[3].set_xlabel("Number of Clusters (k)")
axes[3].set_ylabel("Calinski-Harabasz Index")
axes[3].set_title("Calinski-Harabasz Index For Optimal k", fontsize=14)
axes[3].grid(True)
axes[3].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# Set common x-ticks across subplots
axes[3].set_xticks(k_values)

plt.tight_layout()
plt.savefig('data/experiment_result/kmeans_comparison_chart.png', dpi=150)
plt.show()
```



In [61]:

```
# Based on the elbow plot, choose an optimal number of clusters.
k_methods = [
    (optimal_k_elbow, "Elbow Method"),
    (optimal_k_sil, "Silhouette Score"),
    (optimal_k_db, "Davies-Bouldin Index")
]

# X_cluster_scaled is your scaled data for clustering
# Perform PCA once for consistent axes
pca = PCA(n_components=2, random_state=42)
X_pca_all = pca.fit_transform(X_cluster_scaled)

fig, axes = plt.subplots(1, 3, figsize=(25, 8), sharex=True, sharey=True)

for i, (k, method_name) in enumerate(k_methods):
    ax = axes[i]

    # Fit KMeans for the current k
    kmeans_model = KMeans(n_clusters=k, random_state=42, n_init='auto')
    clusters = kmeans_model.fit_predict(X_cluster_scaled)

    # Scatter plot of PCA-transformed data
    scatter = ax.scatter(
        X_pca_all[:, 0],
        X_pca_all[:, 1],
        c=clusters,
        cmap='viridis',
        alpha=0.7,
        s=50
    )

    # Plot cluster centroids in PCA space
    centers_2d = pca.transform(kmeans_model.cluster_centers_)
    ax.scatter(
```

```
        centers_2d[:, 0],
        centers_2d[:, 1],
        c='black',
        marker='*',
        s=200
    )

    # Title & labels
    ax.set_title(f"K-Means (k={k}) from {method_name}", fontsize=13)
    ax.set_xlabel("Principal Component 1")
    if i == 0:
        ax.set_ylabel("Principal Component 2")

    # Add a colorbar for cluster labels
    cbar = plt.colorbar(scatter, ax=ax, fraction=0.046, pad=0.04)
    cbar.set_label('Cluster Label', rotation=270, labelpad=15)

    # Place a small text label in the top-left corner indicating the black star is for
    # Using Unicode star (U+2605) for a small star symbol
    ax.text(
        0.02, 0.98,  # x,y in Axes fraction
        u"\u2605 Centroids",
        transform=ax.transAxes,
        color='black',
        fontsize=12,
        ha='left',
        va='top',
        bbox=dict(facecolor='white', alpha=0.5, edgecolor='none')  # optional backgrou
    )

plt.tight_layout()
plt.savefig('kmeans_pca_comparison.png', dpi=150, bbox_inches='tight')
plt.show()
```



In [81]:
```
features_to_describe = ['DailyExpense', 'Rolling7', 'Lag1']

for k, model in k_methods:
    kmeans_model = KMeans(n_clusters=k, random_state=42, n_init='auto')
    clusters = kmeans_model.fit_predict(X_cluster_scaled)
    col_name = f'Cluster_{model}_{k}'

    print(f'\n{col_name}')
    # Print counts of samples per cluster
    print(pd.Series(clusters).value_counts())

    # Optionally add the cluster labels to your DataFrame for further analysis
    daily_expense[col_name] = clusters
```

```
    # Use .describe() to calculate count, mean, std, min, 25%, 50%, 75%, and max
    stats = daily_expense.groupby(col_name)[features_to_describe].describe().round(2)
    print("\nDescriptive statistics:")
    print(stats)

    # Alternatively, if you want to compute specific metrics using .agg():
    custom_stats = daily_expense.groupby(col_name)[features_to_describe].agg({
        'DailyExpense': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'mec
        'Rolling7': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'median'
        'Lag1': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'median', la
    })
    # Rename the lambda columns for clarity
    custom_stats.columns = ['_'.join(col).strip() for col in custom_stats.columns.valu
    print("\nCustom statistics:")
    print(custom_stats)
```

```
Cluster_Elbow Method_6
2    115
1     96
5     46
3     42
4     28
0     25
Name: count, dtype: int64

Descriptive statistics:
                       DailyExpense                                         \
                              count    mean     std    min     25%    50%   
Cluster_Elbow Method_6                                                       
0                              25.0  260.92   50.41  195.0  218.00  251.0   
1                              96.0  220.35   34.43  168.0  190.75  221.0   
2                             115.0  220.47   31.00  168.0  193.50  221.0   
3                              42.0  231.38   30.98  170.0  210.00  239.5   
4                              28.0  309.14  103.65  195.0  256.75  296.5   
5                              46.0  320.24   59.16  191.0  283.00  316.0   

                                     Rolling7          ...                  \
                           75%    max    count    mean  ...     75%     max   
Cluster_Elbow Method_6                                  ...                   
0                       298.00  396.0     25.0  245.36  ...  253.29  270.14   
1                       242.25  304.0     96.0  239.99  ...  249.43  266.29   
2                       240.50  303.0    115.0  237.07  ...  245.79  262.57   
3                       249.75  289.0     42.0  270.13  ...  271.07  323.71   
4                       322.75  759.0     28.0  254.42  ...  261.43  320.57   
5                       354.00  451.0     46.0  242.07  ...  252.04  284.14   

                         Lag1                                                \
                        count    mean     std    min     25%    50%     75%   
Cluster_Elbow Method_6                                                        
0                        25.0  334.76   47.95  272.0  300.00  322.0  354.00   
1                        96.0  224.66   37.49  168.0  194.00  219.0  240.50   
2                       115.0  228.25   37.86  168.0  196.50  230.0  250.00   
3                        42.0  266.19   45.20  185.0  242.25  253.5  282.25   
4                        28.0  323.96  104.11  193.0  257.25  299.0  354.50   
5                        46.0  210.54   28.72  169.0  187.25  200.5  239.00   


                          max  
Cluster_Elbow Method_6         
0                       451.0  
1                       369.0  
2                       377.0  
3                       403.0  
4                       759.0  
5                       268.0  

[6 rows x 24 columns]

Custom statistics:
                        DailyExpense_mean  DailyExpense_std  DailyExpense_min  \
Cluster_Elbow Method_6                                                          
0                              260.920000         50.414052               195   
1                              220.354167         34.434065               168   
2                              220.469565         31.002072               168   
3                              231.380952         30.976744               170   
4                              309.142857        103.653166               195   
```

```
5                             320.239130         59.159740              191

                        DailyExpense_max  DailyExpense_<lambda_0>  \
Cluster_Elbow Method_6
0                                    396                   218.00
1                                    304                   190.75
2                                    303                   193.50
3                                    289                   210.00
4                                    759                   256.75
5                                    451                   283.00

                        DailyExpense_median  DailyExpense_<lambda_1>  \
Cluster_Elbow Method_6
0                                     251.0                   298.00
1                                     221.0                   242.25
2                                     221.0                   240.50
3                                     239.5                   249.75
4                                     296.5                   322.75
5                                     316.0                   354.00

                        Rolling7_mean  Rolling7_std  Rolling7_min  ...  \
Cluster_Elbow Method_6                                             ...
0                          245.355714     12.410378    217.857143  ...
1                          239.988095     12.779745    215.000000  ...
2                          237.072588     12.794384    200.500000  ...
3                          270.129252     20.261182    241.714286  ...
4                          254.418367     23.361982    220.714286  ...
5                          242.065217     18.056241    202.428571  ...

                        Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Elbow Method_6
0                                239.428571       247.714286
1                                229.928571       240.142857
2                                228.285714       238.428571
3                                260.035714       265.000000
4                                238.071429       253.500000
5                                231.035714       237.500000

                        Rolling7_<lambda_1>   Lag1_mean    Lag1_std  Lag1_min  \
Cluster_Elbow Method_6
0                                253.285714  334.760000   47.946394     272.0
1                                249.428571  224.656250   37.492688     168.0
2                                245.785714  228.249605   37.860765     168.0
3                                271.071429  266.190476   45.203193     185.0
4                                261.428571  323.964286  104.108733     193.0
5                                252.035714  210.543478   28.721426     169.0

                        Lag1_max  Lag1_<lambda_0>  Lag1_median  \
Cluster_Elbow Method_6
0                          451.0           300.00        322.0
1                          369.0           194.00        219.0
2                          377.0           196.50        230.0
3                          403.0           242.25        253.5
4                          759.0           257.25        299.0
5                          268.0           187.25        200.5

                        Lag1_<lambda_1>
Cluster_Elbow Method_6
0                                354.00
1                                240.50
```

```
2                                250.00
3                                282.25
4                                354.50
5                                239.00

[6 rows x 21 columns]

Cluster_Silhouette Score_2
1    253
0     99
Name: count, dtype: int64

Descriptive statistics:
                           DailyExpense                                    \
                                  count    mean    std    min    25%    50%
Cluster_Silhouette Score_2
0                                  99.0  302.12  76.10  191.0  252.0  297.0
1                                 253.0  222.24  32.47  168.0  192.0  222.0

                                         Rolling7          ...          \
                              75%    max    count    mean  ...     75%
Cluster_Silhouette Score_2                                  ...
0                           326.0  759.0     99.0  246.39  ...  254.79
1                           244.0  304.0    253.0  243.67  ...  253.71

                                     Lag1                                    \
                               max  count    mean    std    min    25%    50%
Cluster_Silhouette Score_2
0                           320.57   99.0  273.99  86.45  169.0  202.0  253.0
1                           323.71  253.0  233.18  41.61  168.0  201.0  231.0


                              75%    max
Cluster_Silhouette Score_2
0                           319.5  759.0
1                           253.0  403.0

[2 rows x 24 columns]

Custom statistics:
                            DailyExpense_mean  DailyExpense_std  \
Cluster_Silhouette Score_2
0                                  302.121212         76.097316
1                                  222.237154         32.474885

                            DailyExpense_min  DailyExpense_max  \
Cluster_Silhouette Score_2
0                                        191               759
1                                        168               304

                            DailyExpense_<lambda_0>  DailyExpense_median  \
Cluster_Silhouette Score_2
0                                             252.0                297.0
1                                             192.0                222.0

                            DailyExpense_<lambda_1>  Rolling7_mean  \
Cluster_Silhouette Score_2
0                                             326.0     246.389971
1                                             244.0     243.666535
```

```
                        Rolling7_std  Rolling7_min  ...  \
Cluster_Silhouette Score_2                          ...
0                          19.111819    202.428571  ...
1                          18.550536    200.500000  ...

                        Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Silhouette Score_2
0                                234.357143       245.571429
1                                231.571429       242.714286

                        Rolling7_<lambda_1>   Lag1_mean   Lag1_std  \
Cluster_Silhouette Score_2
0                                254.785714  273.989899  86.446928
1                                253.714286  233.184603  41.607929

                        Lag1_min  Lag1_max  Lag1_<lambda_0>  Lag1_median  \
Cluster_Silhouette Score_2
0                          169.0     759.0            202.0        253.0
1                          168.0     403.0            201.0        231.0

                        Lag1_<lambda_1>
Cluster_Silhouette Score_2
0                                 319.5
1                                 253.0

[2 rows x 21 columns]

Cluster_Davies-Bouldin Index_18
8     41
6     34
4     32
11    26
14    24
15    20
12    20
10    20
7     19
17    18
1     18
2     18
0     15
3     15
5     15
16    10
9      6
13     1
Name: count, dtype: int64

Descriptive statistics:
                            DailyExpense                                \
                                   count    mean    std    min     25%
Cluster_Davies-Bouldin Index_18
0                                   15.0  282.87  61.97  203.0  233.50
1                                   18.0  234.83  34.50  180.0  214.25
2                                   18.0  234.83  28.21  170.0  231.00
3                                   15.0  294.93  40.14  234.0  263.00
4                                   32.0  222.78  28.97  171.0  202.00
5                                   15.0  226.53  34.19  178.0  201.50
6                                   34.0  222.82  34.13  168.0  193.00
7                                   19.0  276.26  54.27  196.0  233.00
```

```
8                                  41.0  212.34   33.34  168.0  189.00
9                                   6.0  333.50  208.53  243.0  244.50
10                                 20.0  281.75   49.56  195.0  250.50
11                                 26.0  231.31   32.14  169.0  206.50
12                                 20.0  214.50   30.09  179.0  190.00
13                                  1.0  253.00     NaN  253.0  253.00
14                                 24.0  221.29   34.76  173.0  187.25
15                                 20.0  205.30   26.35  175.0  187.25
16                                 10.0  366.20   55.09  290.0  325.25
17                                 18.0  315.83   69.13  191.0  278.00

                                                       Rolling7          ...  \
                                  50%     75%    max    count    mean  ...
Cluster_Davies-Bouldin Index_18                                        ...
0                               280.0  320.50  400.0     15.0  248.71  ...
1                               236.5  253.50  297.0     18.0  249.15  ...
2                               234.5  248.75  278.0     18.0  257.21  ...
3                               297.0  319.50  362.0     15.0  235.21  ...
4                               230.0  241.50  280.0     32.0  236.32  ...
5                               224.0  244.00  289.0     15.0  262.09  ...
6                               226.0  243.25  303.0     34.0  240.13  ...
7                               277.0  318.50  403.0     19.0  254.11  ...
8                               201.0  223.00  304.0     41.0  233.30  ...
9                               248.0  256.75  759.0      6.0  321.07  ...
10                              279.0  312.50  377.0     20.0  231.76  ...
11                              236.0  253.50  276.0     26.0  258.28  ...
12                              211.0  227.75  289.0     20.0  247.88  ...
13                              253.0  253.00  253.0      1.0  311.29  ...
14                              225.0  244.00  288.0     24.0  236.31  ...
15                              199.0  215.50  268.0     20.0  218.00  ...
16                              370.0  405.00  436.0     10.0  267.54  ...
17                              316.5  365.25  451.0     18.0  242.40  ...

                                                   Lag1                       \
                                   75%     max count    mean    std    min
Cluster_Davies-Bouldin Index_18
0                               253.93  269.14  15.0  318.27  41.56  236.0
1                               255.75  261.86  18.0  255.28  31.78  209.0
2                               268.32  278.00  18.0  256.17  31.02  218.0
3                               240.79  261.14  15.0  242.53  47.92  174.0
4                               242.18  250.86  32.0  238.34  27.86  175.0
5                               266.00  272.86  15.0  254.93  39.15  185.0
6                               248.86  262.57  34.0  211.32  27.78  168.0
7                               261.43  283.57  19.0  368.11  45.77  301.0
8                               239.43  253.29  41.0  209.32  27.94  168.0
9                               322.54  323.71   6.0  251.17   5.64  244.0
10                              240.42  249.86  20.0  235.55  39.09  185.0
11                              264.04  267.86  26.0  234.27  27.93  187.0
12                              252.64  271.14  20.0  306.50  46.95  245.0
13                              311.29  311.29   1.0  759.00    NaN  759.0
14                              243.36  253.71  24.0  213.46  30.76  173.0
15                              222.25  233.14  20.0  198.79  22.20  171.0
16                              275.82  284.14  10.0  205.10  26.90  170.0
17                              250.46  274.29  18.0  219.06  37.03  169.0


                                   25%    50%     75%    max
Cluster_Davies-Bouldin Index_18
0                               295.00  318.0  342.00  400.0
1                               235.25  249.0  279.75  322.0
```

```
2                              232.50  247.5  267.00  324.0
3                              196.00  245.0  282.50  317.0
4                              229.75  240.0  253.00  309.0
5                              231.00  248.0  286.00  318.0
6                              190.00  210.0  229.25  271.0
7                              328.00  362.0  400.00  451.0
8                              189.00  203.0  223.00  314.0
9                              247.00  251.5  254.50  259.0
10                             200.00  233.5  256.75  314.0
11                             213.25  238.0  258.50  276.0
12                             279.25  295.0  324.25  403.0
13                             759.00  759.0  759.00  759.0
14                             190.25  211.5  228.25  288.0
15                             181.50  196.0  206.75  248.0
16                             191.00  193.5  225.00  250.0
17                             186.50  219.0  247.25  296.0

[18 rows x 24 columns]

Custom statistics:
                               DailyExpense_mean  DailyExpense_std  \
Cluster_Davies-Bouldin Index_18
0                                     282.866667         61.974496
1                                     234.833333         34.496803
2                                     234.833333         28.213993
3                                     294.933333         40.143433
4                                     222.781250         28.974666
5                                     226.533333         34.188274
6                                     222.823529         34.129407
7                                     276.263158         54.267693
8                                     212.341463         33.336624
9                                     333.500000        208.532731
10                                    281.750000         49.558976
11                                    231.307692         32.138163
12                                    214.500000         30.091964
13                                    253.000000               NaN
14                                    221.291667         34.758145
15                                    205.300000         26.352070
16                                    366.200000         55.087405
17                                    315.833333         69.130099

                               DailyExpense_min  DailyExpense_max  \
Cluster_Davies-Bouldin Index_18
0                                           203               400
1                                           180               297
2                                           170               278
3                                           234               362
4                                           171               280
5                                           178               289
6                                           168               303
7                                           196               403
8                                           168               304
9                                           243               759
10                                          195               377
11                                          169               276
12                                          179               289
13                                          253               253
14                                          173               288
15                                          175               268
16                                          290               436
```

```
17                                              191                451

                                  DailyExpense_<lambda_0>  DailyExpense_median  \
Cluster_Davies-Bouldin Index_18
0                                                  233.50                280.0
1                                                  214.25                236.5
2                                                  231.00                234.5
3                                                  263.00                297.0
4                                                  202.00                230.0
5                                                  201.50                224.0
6                                                  193.00                226.0
7                                                  233.00                277.0
8                                                  189.00                201.0
9                                                  244.50                248.0
10                                                 250.50                279.0
11                                                 206.50                236.0
12                                                 190.00                211.0
13                                                 253.00                253.0
14                                                 187.25                225.0
15                                                 187.25                199.0
16                                                 325.25                370.0
17                                                 278.00                316.5

                                  DailyExpense_<lambda_1>  Rolling7_mean  \
Cluster_Davies-Bouldin Index_18
0                                                  320.50     248.714286
1                                                  253.50     249.150794
2                                                  248.75     257.214286
3                                                  319.50     235.209524
4                                                  241.50     236.316667
5                                                  244.00     262.085714
6                                                  243.25     240.126050
7                                                  318.50     254.112782
8                                                  223.00     233.299652
9                                                  256.75     321.071429
10                                                 312.50     231.758929
11                                                 253.50     258.280220
12                                                 227.75     247.878571
13                                                 253.00     311.285714
14                                                 244.00     236.309524
15                                                 215.50     217.996429
16                                                 405.00     267.542857
17                                                 365.25     242.396825

                                  Rolling7_std  Rolling7_min  ...  \
Cluster_Davies-Bouldin Index_18                               ...
0                                    10.785748    230.714286  ...
1                                     9.507370    224.428571  ...
2                                    11.907532    242.714286  ...
3                                    11.739007    213.428571  ...
4                                     7.830816    223.000000  ...
5                                     6.108542    251.142857  ...
6                                    10.610945    217.857143  ...
7                                    14.499791    222.428571  ...
8                                     9.881636    215.000000  ...
9                                     2.110711    318.285714  ...
10                                   12.252890    202.428571  ...
11                                    6.475383    242.857143  ...
12                                   11.412703    229.571429  ...
13                                         NaN    311.285714  ...
```

```
14                                  9.853647    216.428571  ...
15                                  7.373439    200.500000  ...
16                                 10.467269    252.285714  ...
17                                 12.760943    221.142857  ...

                               Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Davies-Bouldin Index_18
0                                       240.642857       247.857143
1                                       246.107143       248.785714
2                                       247.642857       253.500000
3                                       230.357143       235.571429
4                                       229.250000       237.285714
5                                       258.857143       262.714286
6                                       232.678571       239.857143
7                                       247.071429       254.000000
8                                       224.857143       234.714286
9                                       319.500000       321.285714
10                                      223.464286       231.928571
11                                      254.642857       258.642857
12                                      240.750000       244.714286
13                                      311.285714       311.285714
14                                      228.857143       235.500000
15                                      214.535714       219.000000
16                                      259.964286       267.071429
17                                      234.892857       240.214286

                               Rolling7_<lambda_1>   Lag1_mean   Lag1_std  \
Cluster_Davies-Bouldin Index_18
0                                       253.928571  318.266667  41.561429
1                                       255.750000  255.277778  31.783839
2                                       268.321429  256.166667  31.020392
3                                       240.785714  242.533333  47.917886
4                                       242.178571  238.343750  27.862114
5                                       266.000000  254.933333  39.147097
6                                       248.857143  211.323529  27.781311
7                                       261.428571  368.105263  45.771042
8                                       239.428571  209.317073  27.938718
9                                       322.535714  251.166667   5.636193
10                                      240.419643  235.550000  39.088934
11                                      264.035714  234.269231  27.930711
12                                      252.642857  306.500000  46.948460
13                                      311.285714  759.000000        NaN
14                                      243.357143  213.458333  30.764151
15                                      222.250000  198.785227  22.202398
16                                      275.821429  205.100000  26.904977
17                                      250.464286  219.055556  37.025384

                               Lag1_min  Lag1_max  Lag1_<lambda_0>  \
Cluster_Davies-Bouldin Index_18
0                                 236.0     400.0           295.00
1                                 209.0     322.0           235.25
2                                 218.0     324.0           232.50
3                                 174.0     317.0           196.00
4                                 175.0     309.0           229.75
5                                 185.0     318.0           231.00
6                                 168.0     271.0           190.00
7                                 301.0     451.0           328.00
8                                 168.0     314.0           189.00
9                                 244.0     259.0           247.00
10                                185.0     314.0           200.00
```

```
11                                   187.0     276.0           213.25
12                                   245.0     403.0           279.25
13                                   759.0     759.0           759.00
14                                   173.0     288.0           190.25
15                                   171.0     248.0           181.50
16                                   170.0     250.0           191.00
17                                   169.0     296.0           186.50

                                Lag1_median  Lag1_<lambda_1>
Cluster_Davies-Bouldin Index_18
0                                     318.0           342.00
1                                     249.0           279.75
2                                     247.5           267.00
3                                     245.0           282.50
4                                     240.0           253.00
5                                     248.0           286.00
6                                     210.0           229.25
7                                     362.0           400.00
8                                     203.0           223.00
9                                     251.5           254.50
10                                    233.5           256.75
11                                    238.0           258.50
12                                    295.0           324.25
13                                    759.0           759.00
14                                    211.5           228.25
15                                    196.0           206.75
16                                    193.5           225.00
17                                    219.0           247.25

[18 rows x 21 columns]
```

Comparing How Cluster Assignments Change from k to k+1

In [69]:

```python
assignment_changes = []
previous_labels = None
previous_k = None

for k in k_values:
    kmeans = KMeans(n_clusters=k, random_state=42, n_init='auto')
    labels = kmeans.fit_predict(X_cluster_scaled)

    if previous_labels is not None:
        # Count how many labels differ
        changes = np.sum(labels != previous_labels)
        fraction_changed = changes / len(labels)
        assignment_changes.append((previous_k, k, fraction_changed))

    previous_labels = labels
    previous_k = k

# Convert to a DataFrame for easier reading
changes_df = pd.DataFrame(assignment_changes, columns=["k_old", "k_new", "fraction_cha
print("Fraction of points changing clusters when going from k to k+1:")
print(changes_df)
```

```
Fraction of points changing clusters when going from k to k+1:
    k_old  k_new  fraction_changed
0       2      3          0.352273
1       3      4          0.116477
2       4      5          0.116477
3       5      6          0.221591
4       6      7          0.301136
5       7      8          0.735795
6       8      9          0.187500
7       9     10          0.090909
8      10     11          0.116477
9      11     12          0.076705
10     12     13          0.150568
11     13     14          0.025568
12     14     15          0.142045
13     15     16          0.110795
14     16     17          0.036932
15     17     18          0.073864
16     18     19          0.079545
17     19     20          0.056818
18     20     21          0.792614
19     21     22          0.068182
20     22     23          0.076705
21     23     24          0.085227
22     24     25          0.105114
23     25     26          0.059659
24     26     27          0.062500
25     27     28          0.088068
26     28     29          0.065341
27     29     30          0.076705
```

In [70]:

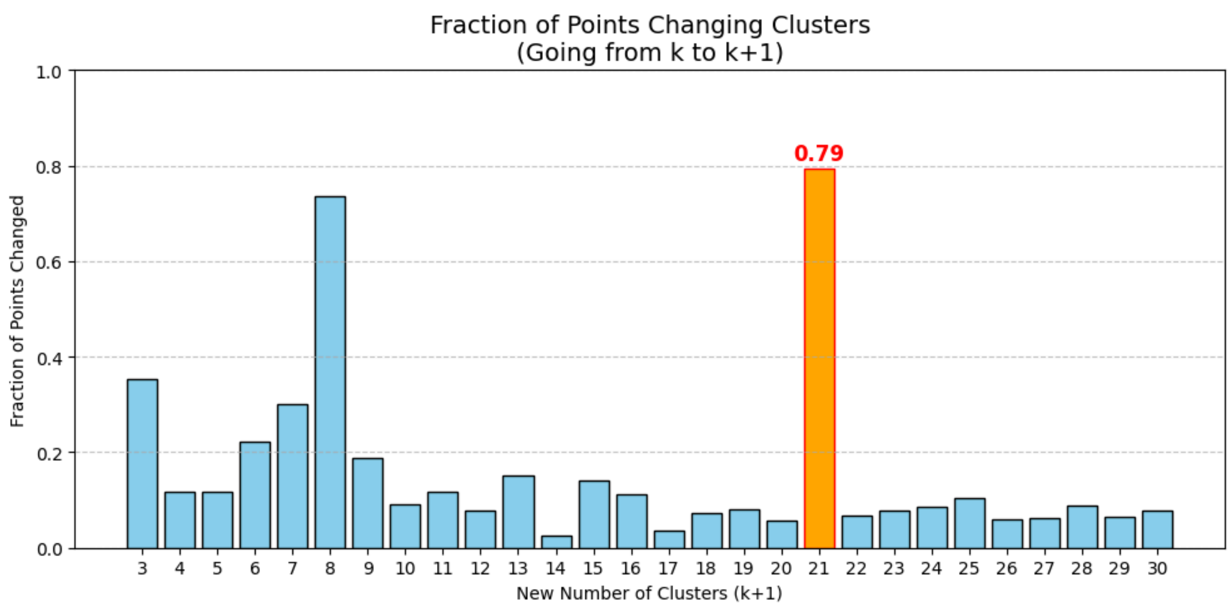
```python
# Identify the row with the maximum fraction_changed
max_idx = changes_df['fraction_changed'].idxmax()
max_k = changes_df.loc[max_idx, 'k_new']
max_val = changes_df.loc[max_idx, 'fraction_changed']
print(f"Maximum change in cluster assignments occurs when going from k={max_k-1} to k=
```

```
Maximum change in cluster assignments occurs when going from k=20 to k=21 (79.26% cha
nge)
```

In [105…

```python
plt.figure(figsize=(10, 5))

# Create a bar chart
bars = plt.bar(
    changes_df['k_new'],
    changes_df['fraction_changed'],
    color='skyblue',
    edgecolor='black'
)

# Highlight the bar with the maximum fraction changed
for bar in bars:
    bar_center = bar.get_x() + bar.get_width()/2
    if np.isclose(bar_center, max_k, atol=0.1):
        bar.set_color('orange')
        bar.set_edgecolor('red')

# Annotate the highest bar
plt.text(
    max_k,
```

```
    max_val + 0.02,  # Slightly above the bar
    f"{max_val:.2f}",
    ha='center',
    color='red',
    fontsize=12,
    fontweight='bold'
)

plt.title("Fraction of Points Changing Clusters\n(Going from k to k+1)", fontsize=14)
plt.xlabel("New Number of Clusters (k+1)")
plt.ylabel("Fraction of Points Changed")
plt.ylim(0, 1)  # Fractions range from 0 to 1
plt.xticks(changes_df['k_new'])
plt.grid(axis='y', linestyle='--', alpha=0.7)
plt.tight_layout()
plt.savefig('data/experiment_result/cluster_assignment_changes.png', dpi=150)
plt.show()
```



Cluster on First X Month, Then See How The Data Fits

In [82]:

```
cutoff_date = pd.to_datetime("2025-06-01")
train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

# Scale them separately or together, depending on your approach
scaler = StandardScaler()
train_features = train_data[cluster_features]
test_features = test_data[cluster_features]

X_train_scaled = scaler.fit_transform(train_features)
X_test_scaled = scaler.transform(test_features)

k = 6  # or your chosen k
kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
kmeans_split.fit(X_train_scaled)

# Assign clusters to training data
train_clusters = kmeans_split.predict(X_train_scaled)
train_data['Cluster'] = train_clusters
```

```
# Assign clusters to test data
test_clusters = kmeans_split.predict(X_test_scaled)
test_data['Cluster'] = test_clusters

print("Training cluster distribution:")
print(train_data['Cluster'].value_counts())

print("\nTest cluster distribution:")
print(test_data['Cluster'].value_counts())
```

```
Training cluster distribution:
Cluster
0    31
5    26
3    26
1    24
4    22
2    16
Name: count, dtype: int64

Test cluster distribution:
Cluster
4    115
3     47
1     23
2     22
Name: count, dtype: int64
```

Check for 2, 4, 6, 8, 10 Months

In [102…

```
offsets = [2, 4, 6, 8, 10]

start_date = daily_expense['Date'].min()
split_results = {}
```

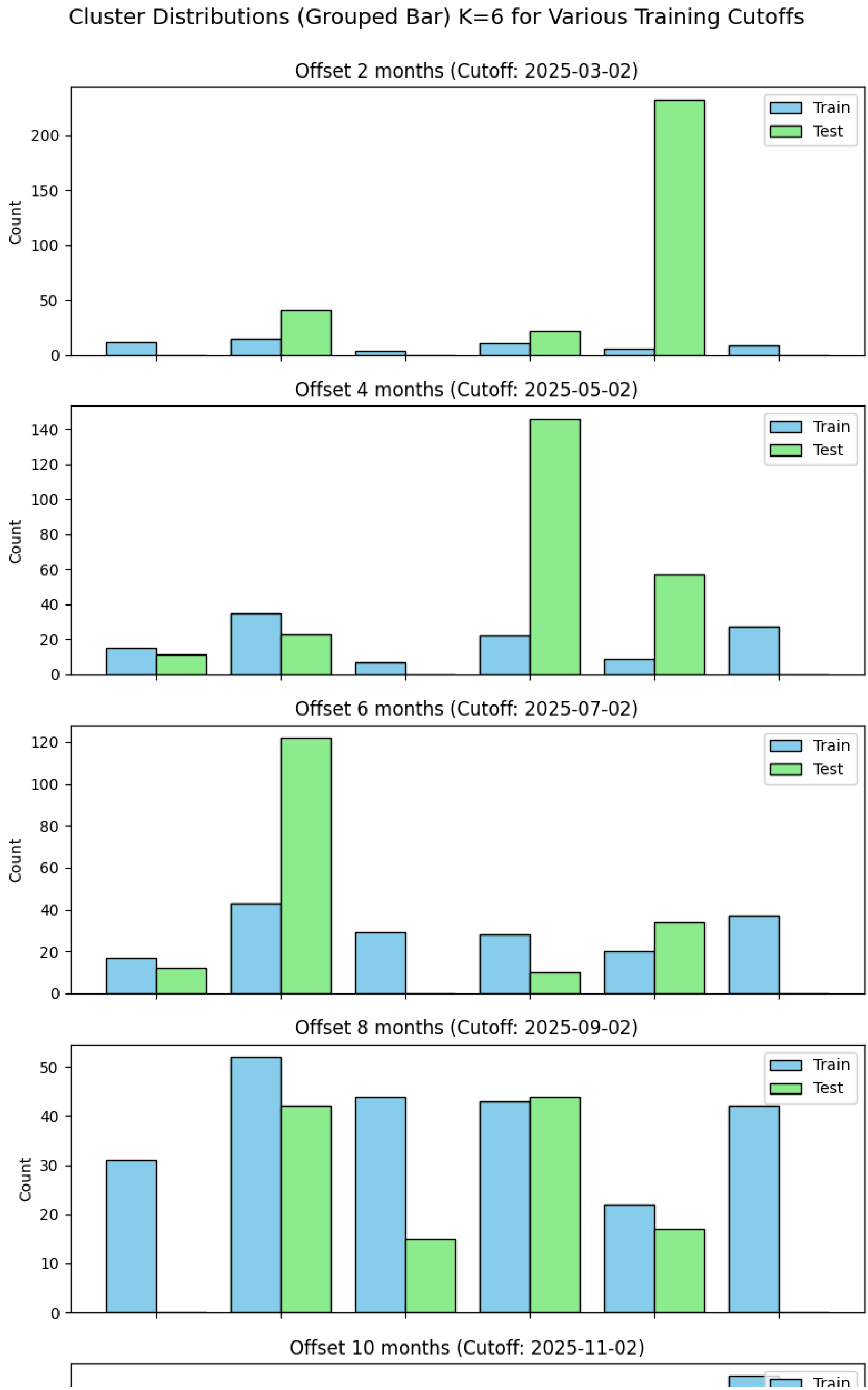
In [103…

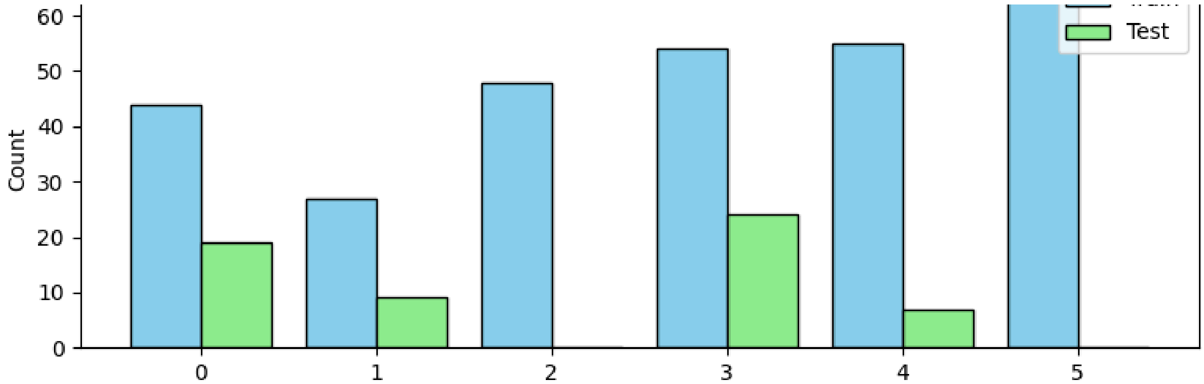
```
# Loop over each offset to compute cluster assignments and count samples per cluster
for offset in offsets:
    cutoff_date = start_date + pd.DateOffset(months=offset)

    # Split the data
    train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
    test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

    # Scale the features (fit scaler on train and transform test)
    scaler = StandardScaler()
    train_features = train_data[cluster_features]
    test_features = test_data[cluster_features]

    X_train_scaled = scaler.fit_transform(train_features)
    X_test_scaled = scaler.transform(test_features)

    # Fit K-Means on training data with chosen k (e.g., 6)
    k = 6
    kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
    kmeans_split.fit(X_train_scaled)

    # Predict clusters for training and test data
    train_clusters = kmeans_split.predict(X_train_scaled)
```

```
    test_clusters = kmeans_split.predict(X_test_scaled)

    train_data['Cluster'] = train_clusters
    test_data['Cluster'] = test_clusters

    # Count samples per cluster
    train_counts = train_data['Cluster'].value_counts().sort_index()
    test_counts = test_data['Cluster'].value_counts().sort_index()

    # Store results for this offset
    split_results[offset] = {
        'cutoff_date': cutoff_date,
        'train_counts': train_counts,
        'test_counts': test_counts
    }
```

In [104…

```
fig, axes = plt.subplots(nrows=len(offsets), ncols=1, figsize=(8, 3 * len(offsets)), s

x_positions = np.arange(k)  # k is the number of clusters, e.g. 6
bar_width = 0.4

for i, offset in enumerate(offsets):
    result = split_results[offset]
    cutoff_date = result['cutoff_date']
    train_counts = result['train_counts']
    test_counts = result['test_counts']

    # Ensure both Series cover all clusters [0..k-1], fill missing with 0
    train_counts = train_counts.reindex(range(k), fill_value=0)
    test_counts = test_counts.reindex(range(k), fill_value=0)

    ax = axes[i] if len(offsets) > 1 else axes  # handle single-subplot case

    # Plot grouped bars
    ax.bar(x_positions - bar_width/2, train_counts.values, width=bar_width,
           color='skyblue', edgecolor='black', label='Train')
    ax.bar(x_positions + bar_width/2, test_counts.values, width=bar_width,
           color='lightgreen', edgecolor='black', label='Test')

    ax.set_xticks(x_positions)
    ax.set_xticklabels([str(c) for c in range(k)])
    ax.set_ylabel("Count")
    ax.set_title(f"Offset {offset} months (Cutoff: {cutoff_date.date()})")
    ax.legend(loc='upper right')

plt.suptitle("Cluster Distributions (Grouped Bar) K=6 for Various Training Cutoffs", f
plt.savefig('data/experiment_result/train_test_cluster_distribution_k_6.png', dpi=150,
plt.tight_layout()
plt.show()
```


Cluster Distributions (Grouped Bar) K=6 for Various Training Cutoffs
Offset 2 months (Cutoff: 2025-03-02)
Count
0
50
100
150
200
Train
Test
Offset 4 months (Cutoff: 2025-05-02)
Count
0
20
40
60
80
100
120
140
Train
Test
Offset 6 months (Cutoff: 2025-07-02)
Count
0
20
40
60
80
100
120
Train
Test
Offset 8 months (Cutoff: 2025-09-02)
Count
0
10
20
30
40
50
Train
Test
Offset 10 months (Cutoff: 2025-11-02)
Train



In [91]:
```
# Distribution of DayOfWeek within each cluster
dayofweek_distribution = (
    daily_expense
    .groupby('Cluster')['DayOfWeek']
    .value_counts(normalize=True)
    .mul(100)
    .rename("percentage")
    .reset_index()
)

print("DayOfWeek distribution by cluster (in %):")
print(dayofweek_distribution)
```

```
---------------------------------------------------------------------------
KeyError                                  Traceback (most recent call last)
Cell In[91], line 4
      1 # Distribution of DayOfWeek within each cluster
      2 dayofweek_distribution = (
      3     daily_expense
----> 4     .groupby('Cluster')['DayOfWeek']
      5     .value_counts(normalize=True)
      6     .mul(100)
      7     .rename("percentage")
      8     .reset_index()
      9 )
     11 print("DayOfWeek distribution by cluster (in %):")
     12 print(dayofweek_distribution)

File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\frame.py:8252, in DataFrame.groupby(self, by, axis, level, as_index, sort, group_keys, observed, dropna)
   8249     raise TypeError("You have to supply one of 'by' and 'level'")
   8250 axis = self._get_axis_number(axis)
-> 8252 return DataFrameGroupBy(
   8253     obj=self,
   8254     keys=by,
   8255     axis=axis,
   8256     level=level,
   8257     as_index=as_index,
   8258     sort=sort,
   8259     group_keys=group_keys,
   8260     observed=observed,
   8261     dropna=dropna,
   8262 )

File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\groupby\groupby.py:931, in GroupBy.__init__(self, obj, keys, axis, level, grouper, exclusions, selection, as_index, sort, group_keys, observed, dropna)
    928 self.dropna = dropna
    930 if grouper is None:
--> 931     grouper, exclusions, obj = get_grouper(
    932         obj,
    933         keys,
    934         axis=axis,
    935         level=level,
    936         sort=sort,
    937         observed=observed,
    938         dropna=self.dropna,
    939     )
    941 self.obj = obj
    942 self.axis = obj._get_axis_number(axis)

File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\groupby\grouper.py:985, in get_grouper(obj, key, axis, level, sort, observed, validate, dropna)
    983         in_axis, level, gpr = False, gpr, None
    984     else:
--> 985         raise KeyError(gpr)
    986 elif isinstance(gpr, Grouper) and gpr.key is not None:
    987     # Add key to exclusions
    988     exclusions.add(gpr.key)
```

```
KeyError: 'Cluster'
```

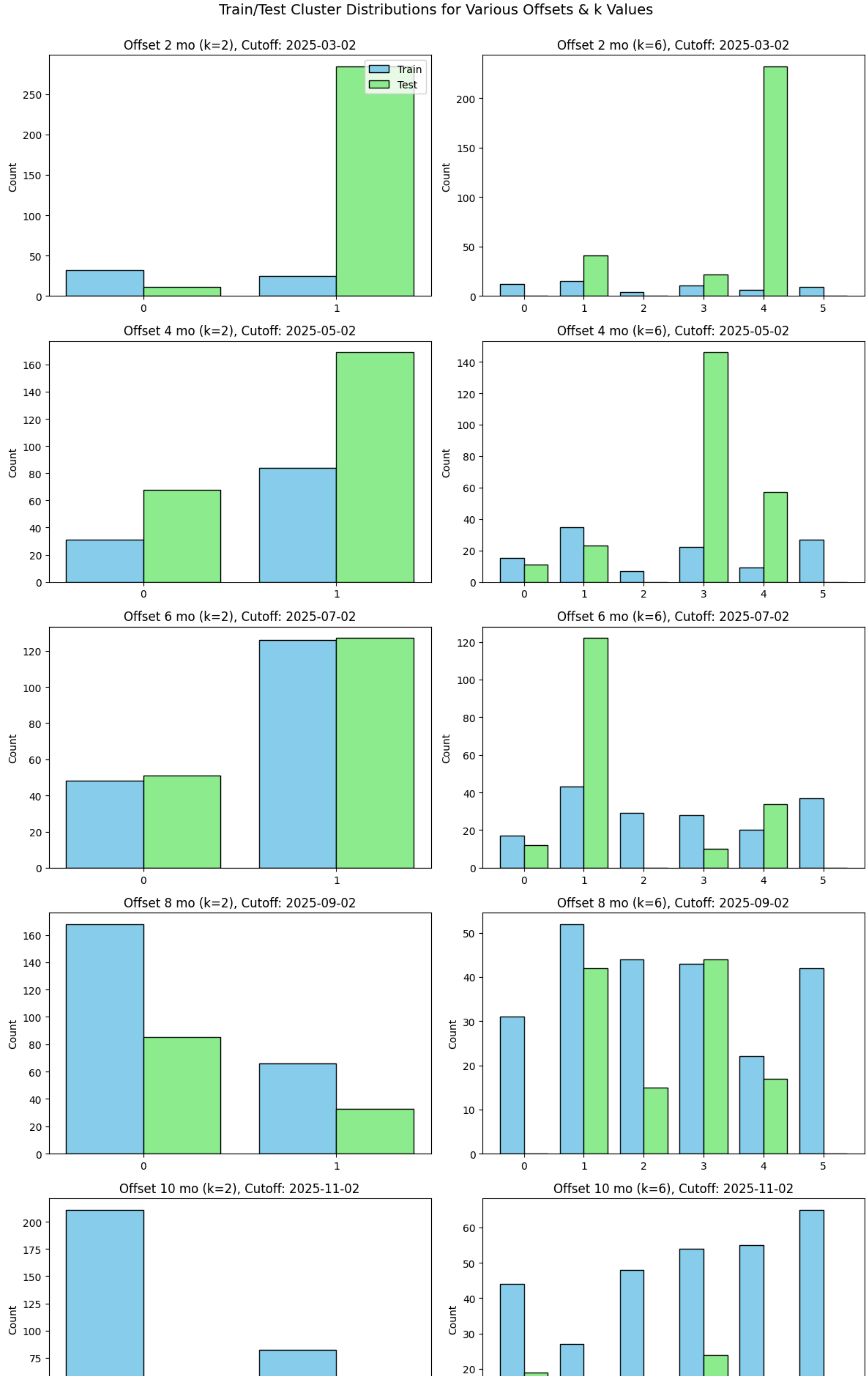
Check for 2, 4, 6, 8, 10 Months Using K=6 and K=2

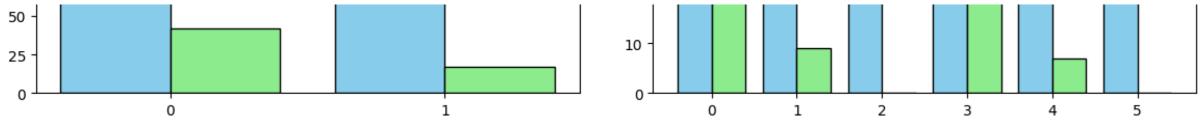
In [96]:
```
offsets = [2, 4, 6, 8, 10]
k_vals = [2, 6]
start_date = daily_expense['Date'].min()

split_results = {offset: {} for offset in offsets}
```

In [97]:
```
for offset in offsets:
    cutoff_date = start_date + pd.DateOffset(months=offset)

    # Split the data
    train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
    test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

    # Scale the features (fit scaler on train, then transform test)
    scaler = StandardScaler()
    train_features = train_data[cluster_features]
    test_features = test_data[cluster_features]

    X_train_scaled = scaler.fit_transform(train_features)
    X_test_scaled = scaler.transform(test_features)

    for k in k_vals:
        # Fit K-Means on training data
        kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
        kmeans_split.fit(X_train_scaled)

        # Predict clusters for training and test data
        train_clusters = kmeans_split.predict(X_train_scaled)
        test_clusters = kmeans_split.predict(X_test_scaled)

        # Count samples per cluster
        train_counts = pd.Series(train_clusters).value_counts().sort_index()
        test_counts = pd.Series(test_clusters).value_counts().sort_index()

        # Store the results
        split_results[offset][k] = {
            'cutoff_date': cutoff_date,
            'train_counts': train_counts,
            'test_counts': test_counts
        }
```

In [100…
```
n_offsets = len(offsets)
n_kvals = len(k_vals)
fig, axes = plt.subplots(
    nrows=n_offsets, ncols=n_kvals,
    figsize=(12, 4 * n_offsets),
    sharex=False, sharey=False
)

bar_width = 0.4

for i, offset in enumerate(offsets):
    for j, k in enumerate(k_vals):
```

```
        ax = axes[i, j] if n_offsets > 1 else axes[j]  # handle single-row case
        results = split_results[offset][k]

        cutoff_date = results['cutoff_date']
        train_counts = results['train_counts']
        test_counts = results['test_counts']

        # Ensure we have a count for each cluster index [0..k-1]
        train_counts = train_counts.reindex(range(k), fill_value=0)
        test_counts = test_counts.reindex(range(k), fill_value=0)

        x_positions = np.arange(k)

        # Grouped bar chart: train vs test
        ax.bar(
            x_positions - bar_width/2,
            train_counts.values,
            width=bar_width,
            color='skyblue',
            edgecolor='black',
            label='Train'
        )
        ax.bar(
            x_positions + bar_width/2,
            test_counts.values,
            width=bar_width,
            color='lightgreen',
            edgecolor='black',
            label='Test'
        )

        ax.set_xticks(x_positions)
        ax.set_xticklabels([str(c) for c in range(k)])
        ax.set_ylabel("Count")
        ax.set_title(f"Offset {offset} mo (k={k}), Cutoff: {cutoff_date.date()}")

        if i == 0 and j == 0:
            ax.legend(loc='upper right')

plt.suptitle("Train/Test Cluster Distributions for Various Offsets & k Values", fontsi
plt.savefig('data/experiment_result/train_test_cluster_distribution_multiple_offsets_a
plt.tight_layout()
plt.show()
```


Train/Test Cluster Distributions for Various Offsets & k Values
Offset 2 mo (k=2), Cutoff: 2025-03-02
Offset 2 mo (k=6), Cutoff: 2025-03-02
Train
Test
Offset 4 mo (k=2), Cutoff: 2025-05-02
Offset 4 mo (k=6), Cutoff: 2025-05-02
Offset 6 mo (k=2), Cutoff: 2025-07-02
Offset 6 mo (k=6), Cutoff: 2025-07-02
Offset 8 mo (k=2), Cutoff: 2025-09-02
Offset 8 mo (k=6), Cutoff: 2025-09-02
Offset 10 mo (k=2), Cutoff: 2025-11-02
Offset 10 mo (k=6), Cutoff: 2025-11-02
Count



In [ ]:
```
# Distribution of DayOfWeek within each cluster
dayofweek_distribution = (
    daily_expense
    .groupby('Cluster')['DayOfWeek']
    .value_counts(normalize=True)
    .mul(100)
    .rename("percentage")
    .reset_index()
)

print("DayOfWeek distribution by cluster (in %):")
print(dayofweek_distribution)
```

```
---------------------------------------------------------------------------
KeyError                                  Traceback (most recent call last)
Cell In[91], line 4
      1 # Distribution of DayOfWeek within each cluster
      2 dayofweek_distribution = (
      3     daily_expense
----> 4     .groupby('Cluster')['DayOfWeek']
      5     .value_counts(normalize=True)
      6     .mul(100)
      7     .rename("percentage")
      8     .reset_index()
      9 )
     11 print("DayOfWeek distribution by cluster (in %):")
     12 print(dayofweek_distribution)

File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\frame.py:8252, in DataFrame.groupby(self, by, axis, level, as_index, sort, group_keys, observed, dropna)
   8249     raise TypeError("You have to supply one of 'by' and 'level'")
   8250 axis = self._get_axis_number(axis)
-> 8252 return DataFrameGroupBy(
   8253     obj=self,
   8254     keys=by,
   8255     axis=axis,
   8256     level=level,
   8257     as_index=as_index,
   8258     sort=sort,
   8259     group_keys=group_keys,
   8260     observed=observed,
   8261     dropna=dropna,
   8262 )
```

```
File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\groupby\groupby.py:931, in GroupBy.__init__(self, obj, keys, axis, level, grouper, exclusions, selection, as_index, sort, group_keys, observed, dropna)

    928 self.dropna = dropna

    930 if grouper is None:

--> 931     grouper, exclusions, obj = get_grouper(

    932         obj,

    933         keys,

    934         axis=axis,

    935         level=level,

    936         sort=sort,

    937         observed=observed,

    938         dropna=self.dropna,

    939     )

    941 self.obj = obj

    942 self.axis = obj._get_axis_number(axis)


File ~\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages\pandas\core\groupby\grouper.py:985, in get_grouper(obj, key, axis, level, sort, observed, validate, dropna)

    983         in_axis, level, gpr = False, gpr, None

    984     else:

--> 985         raise KeyError(gpr)

    986 elif isinstance(gpr, Grouper) and gpr.key is not None:

    987     # Add key to exclusions

    988     exclusions.add(gpr.key)


KeyError: 'Cluster'
```

GMM (Gaussian Mixture Model)

In [16]:
```
from sklearn.mixture import GaussianMixture
from sklearn.preprocessing import StandardScaler
import seaborn as sns
```

In [8]:
```
cluster_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7', 'Lag1']
```

In [9]:
```
scaler = StandardScaler()
X_cluster_scaled = scaler.fit_transform(daily_expense[cluster_features])
```

In [10]:
```
n_components = 6
```

In [11]:
```
gmm = GaussianMixture(n_components=n_components, random_state=42)
gmm.fit(X_cluster_scaled)
```

Out[11]:
```
▾ GaussianMixture
GaussianMixture(n_components=6, random_state=42)
```

In [12]:
```
# Predict cluster labels and get the membership probabilities
gmm_labels = gmm.predict(X_cluster_scaled)
gmm_probabilities = gmm.predict_proba(X_cluster_scaled)

# Add the GMM cluster assignments and probabilities to your DataFrame
daily_expense['GMM_Cluster'] = gmm_labels
# For example, you might store the maximum probability (confidence) for each point
daily_expense['GMM_Confidence'] = gmm_probabilities.max(axis=1)

print("GMM Cluster distribution:")
print(daily_expense['GMM_Cluster'].value_counts())

# Optionally, display the first few rows with cluster labels and confidence
print(daily_expense[['Date', 'DailyExpense', 'GMM_Cluster', 'GMM_Confidence']].head())
```

```
GMM Cluster distribution:
GMM_Cluster
2    132
1     99
0     49
5     47
3     22
4      3
Name: count, dtype: int64
        Date  DailyExpense  GMM_Cluster  GMM_Confidence
1 2025-01-02           206            2        0.999993
2 2025-01-03           195            2        0.999993
3 2025-01-04           295            5        1.000000
4 2025-01-05           263            0        1.000000
5 2025-01-06           230            2        0.999994
```

Descriptive Statistics

In [13]:
```
features_numeric = ['DailyExpense', 'Rolling7', 'Lag1']
print("=== Numeric Descriptive Statistics by Cluster ===")
profile_stats = daily_expense.groupby('GMM_Cluster')[features_numeric].describe().rour
print(profile_stats)
```

```
=== Numeric Descriptive Statistics by Cluster ===
            DailyExpense                                                    \
                   count    mean     std    min     25%    50%     75%    max
GMM_Cluster
0                   49.0  268.43   54.05  191.0  222.00  258.0  309.00  403.0
1                   99.0  220.91   34.25  169.0  191.00  221.0  243.00  304.0
2                  132.0  223.13   31.51  168.0  194.75  227.0  246.25  303.0
3                   22.0  222.86   31.15  168.0  200.50  217.5  245.75  289.0
4                    3.0  442.00  276.22  253.0  283.50  314.0  536.50  759.0
5                   47.0  328.32   52.77  245.0  292.50  321.0  358.00  451.0

            Rolling7          ...                   Lag1                    \
               count    mean  ...     75%     max  count    mean     std
GMM_Cluster                   ...
0               49.0  244.03  ...  254.00  283.57   49.0  322.84   57.82
1               99.0  242.64  ...  254.57  268.57   99.0  220.87   29.11
2              132.0  240.41  ...  249.50  278.00  132.0  230.63   36.74
3               22.0  267.84  ...  272.43  323.71   22.0  303.95   48.94
4                3.0  297.67  ...  315.93  320.57    3.0  419.33  294.21
5               47.0  245.58  ...  254.64  284.14   47.0  213.79   32.34


               min     25%    50%     75%    max
GMM_Cluster
0            185.0  289.00  318.0  354.00  451.0
1            168.0  194.50  220.0  240.00  288.0
2            168.0  201.25  231.5  253.00  324.0
3            244.0  254.50  300.5  321.25  403.0
4            244.0  249.50  255.0  507.00  759.0
5            169.0  191.00  201.0  243.50  288.0

[6 rows x 24 columns]
```

In [14]:

```
# For categorical features, calculate percentages (e.g., DayOfWeek, IsWeekend)
categorical_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day']
print("\n=== Categorical Distribution by Cluster (in %): ===")
cat_profile = daily_expense.groupby('GMM_Cluster')[categorical_features].agg(
    lambda x: x.value_counts(normalize=True).mul(100).round(2).to_dict()
)
print(cat_profile)
```

```
=== Categorical Distribution by Cluster (in %): ===
                                                     DayOfWeek   IsWeekend  \
GMM_Cluster
0                                                  {6: 100.0}  {1: 100.0}
1             {4: 24.24, 3: 23.23, 1: 21.21, 2: 20.2, 0: 11.11}  {0: 100.0}
2               {3: 20.45, 2: 20.45, 4: 19.7, 0: 19.7, 1: 19.7}  {0: 100.0}
3              {0: 59.09, 2: 18.18, 1: 13.64, 3: 4.55, 4: 4.55}  {0: 100.0}
4                                        {6: 66.67, 5: 33.33}  {1: 100.0}
5                                                  {5: 100.0}  {1: 100.0}

                                                         Month  \
GMM_Cluster
0             {3: 10.2, 8: 10.2, 1: 8.16, 2: 8.16, 4: 8.16, ...
1             {11: 19.19, 12: 19.19, 9: 18.18, 8: 17.17, 10:...
2             {1: 16.67, 3: 15.91, 4: 15.91, 2: 15.15, 5: 15...
3             {10: 36.36, 8: 13.64, 9: 13.64, 12: 13.64, 7: ...
4                                        {10: 66.67, 11: 33.33}
5             {5: 10.64, 8: 10.64, 1: 8.51, 4: 8.51, 6: 8.51...

                                                           Day
GMM_Cluster
0             {9: 6.12, 16: 6.12, 23: 6.12, 5: 4.08, 13: 4.0...
1             {4: 5.05, 10: 5.05, 24: 5.05, 25: 5.05, 30: 4....
2             {3: 4.55, 21: 4.55, 14: 4.55, 23: 3.79, 17: 3....
3             {22: 9.09, 2: 9.09, 13: 9.09, 3: 9.09, 16: 9.0...
4                           {11: 33.33, 12: 33.33, 2: 33.33}
5             {8: 6.38, 15: 6.38, 4: 4.26, 12: 4.26, 20: 4.2...
```

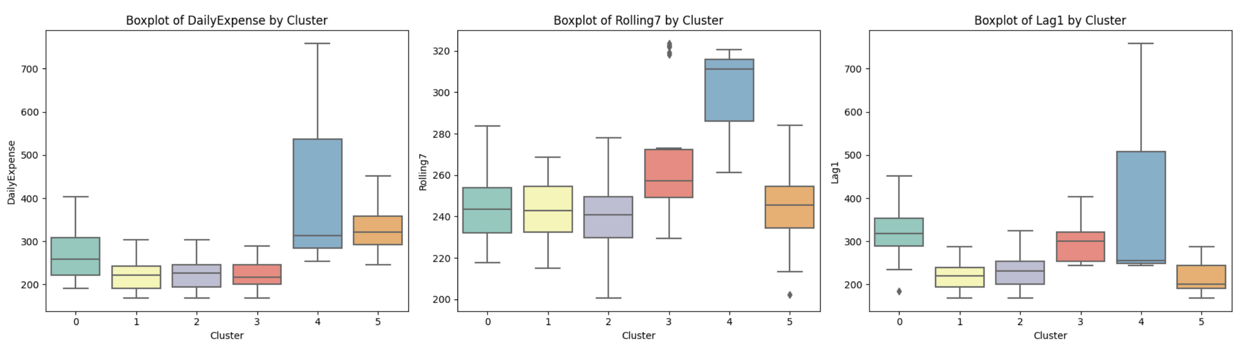
In [15]:

```
# Investigate a small cluster (e.g., Cluster 4)
print("\n=== Detailed Profile for Cluster 4 ===")
cluster_4 = daily_expense[daily_expense['GMM_Cluster'] == 4]
print(cluster_4.describe())
```

```
=== Detailed Profile for Cluster 4 ===
                      Date  DailyExpense  DayOfWeek  IsWeekend      Month  \
count                    3      3.000000   3.000000        3.0   3.000000
mean   2025-10-18 16:00:00    442.000000   5.666667        1.0  10.333333
min    2025-10-11 00:00:00    253.000000   5.000000        1.0  10.000000
25%    2025-10-11 12:00:00    283.500000   5.500000        1.0  10.000000
50%    2025-10-12 00:00:00    314.000000   6.000000        1.0  10.000000
75%    2025-10-22 12:00:00    536.500000   6.000000        1.0  10.500000
max    2025-11-02 00:00:00    759.000000   6.000000        1.0  11.000000
std                    NaN    276.219116   0.577350        0.0   0.577350

             Day        Lag1   Rolling7  LogExpense  GMM_Cluster  \
count   3.000000    3.000000   3.000000    3.000000          3.0
mean    8.333333  419.333333 297.666667    5.974408          4.0
min     2.000000  244.000000 261.142857    5.537334          4.0
25%     6.500000  249.500000 286.214286    5.644953          4.0
50%    11.000000  255.000000 311.285714    5.752573          4.0
75%    11.500000  507.000000 315.928571    6.192946          4.0
max    12.000000  759.000000 320.571429    6.633318          4.0
std     5.507571  294.211375  31.969479    0.580692          0.0

       GMM_Confidence
count    3.000000e+00
mean     9.999999e-01
min      9.999997e-01
25%      9.999998e-01
50%      1.000000e+00
75%      1.000000e+00
max      1.000000e+00
std      1.893129e-07
```

In [17]:

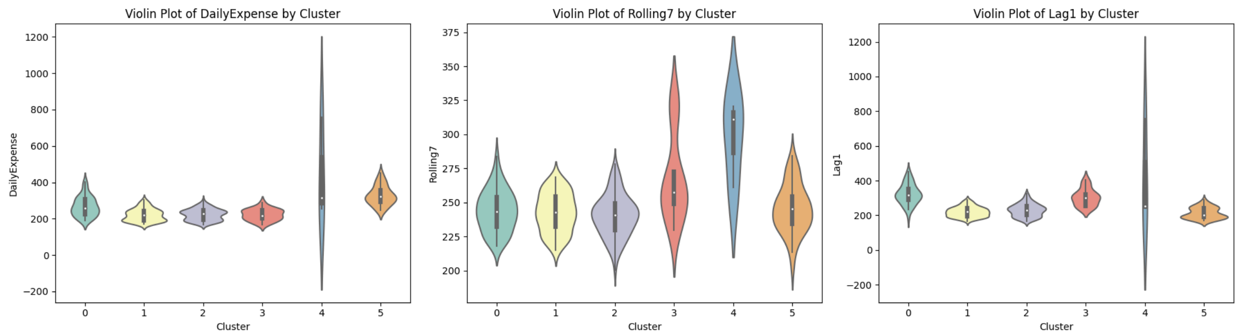
```python
plt.figure(figsize=(18, 5))
for i, feat in enumerate(features_numeric):
    plt.subplot(1, 3, i+1)
    sns.boxplot(x='GMM_Cluster', y=feat, data=daily_expense, palette='Set3')
    plt.title(f"Boxplot of {feat} by Cluster")
    plt.xlabel("Cluster")
    plt.ylabel(feat)
plt.tight_layout()
plt.show()
```



In [18]:

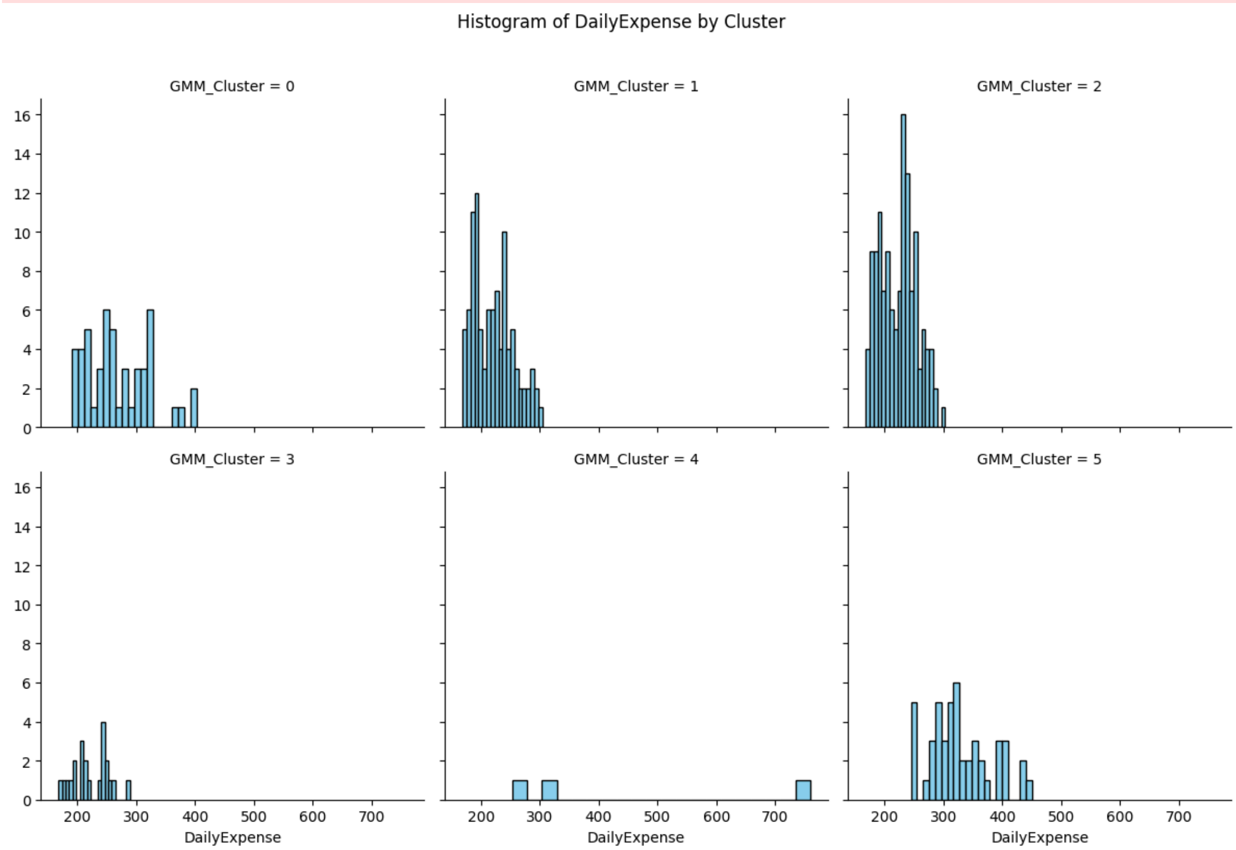
```python
plt.figure(figsize=(18, 5))
for i, feat in enumerate(features_numeric):
    plt.subplot(1, 3, i+1)
    sns.violinplot(x='GMM_Cluster', y=feat, data=daily_expense, palette='Set3')
    plt.title(f"Violin Plot of {feat} by Cluster")
    plt.xlabel("Cluster")
    plt.ylabel(feat)
```

```
plt.tight_layout()
plt.show()
```



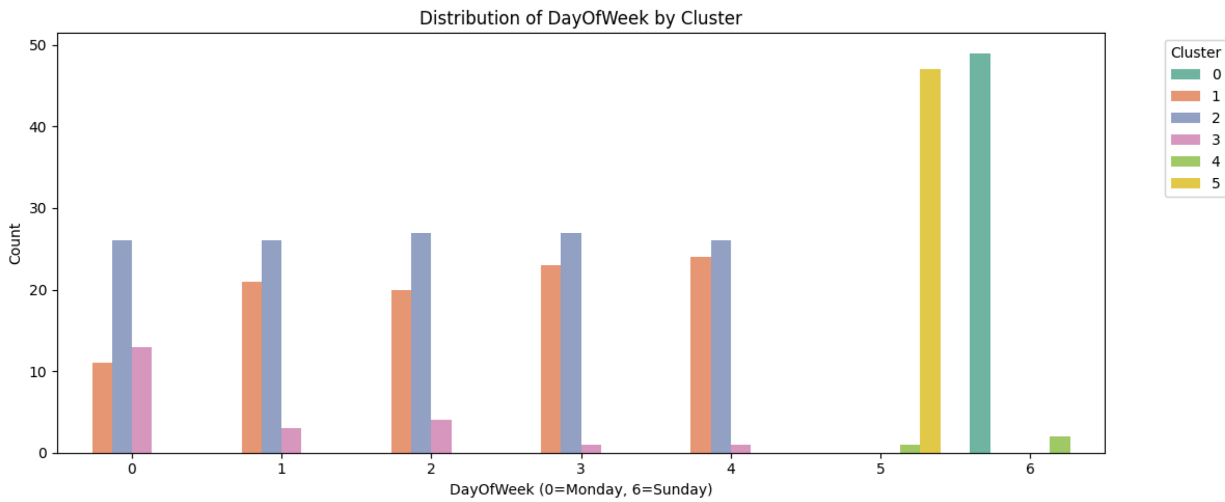
In [19]:
```
g = sns.FacetGrid(daily_expense, col="GMM_Cluster", col_wrap=3, height=4)
g.map(plt.hist, "DailyExpense", bins=20, color='skyblue', edgecolor='black')
g.fig.suptitle("Histogram of DailyExpense by Cluster", y=1.02)
plt.tight_layout()
plt.show()
```

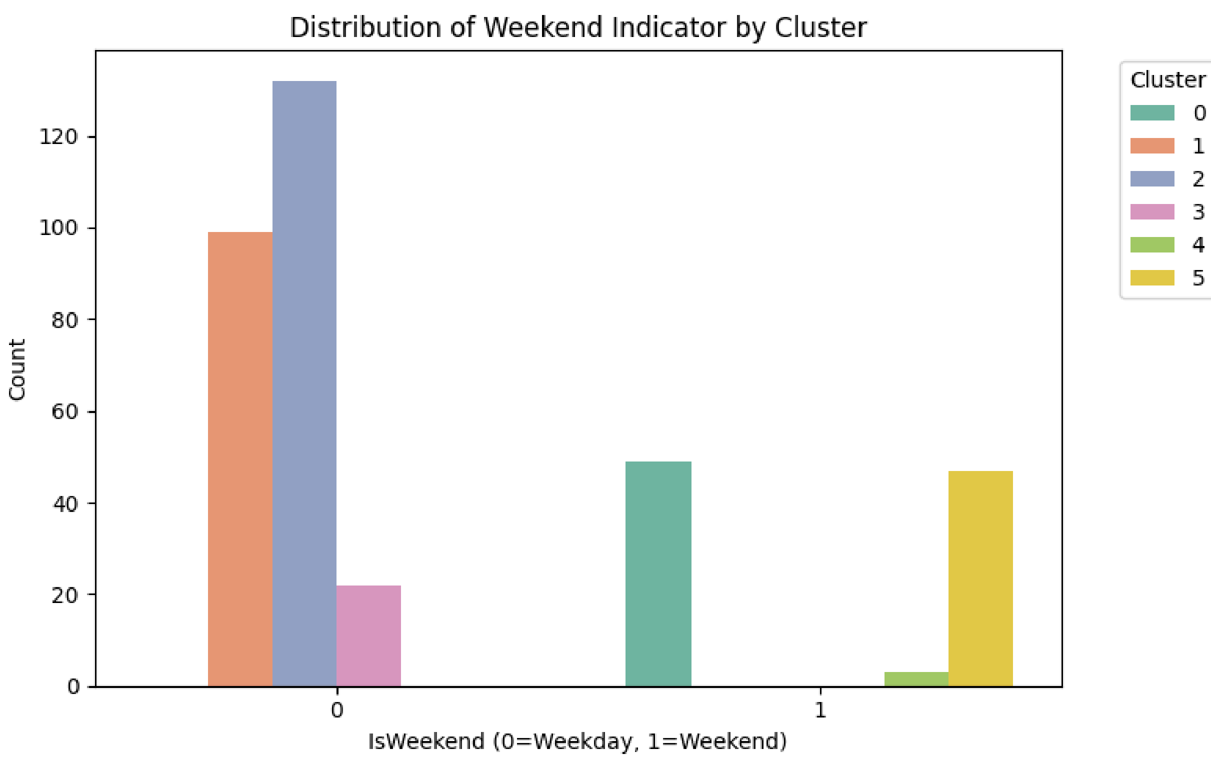
```
C:\Users\Davon\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra
8p0\LocalCache\local-packages\Python311\site-packages\seaborn\axisgrid.py:118: UserWa
rning: The figure layout has changed to tight
  self._figure.tight_layout(*args, **kwargs)
C:\Users\Davon\AppData\Local\Temp\ipykernel_17252\3102834032.py:4: UserWarning: The f
igure layout has changed to tight
  plt.tight_layout()
```



In [20]:
```
plt.figure(figsize=(12, 5))
sns.countplot(x='DayOfWeek', hue='GMM_Cluster', data=daily_expense, palette='Set2')
plt.title("Distribution of DayOfWeek by Cluster")
plt.xlabel("DayOfWeek (0=Monday, 6=Sunday)")
plt.ylabel("Count")
```

```
plt.legend(title="Cluster", bbox_to_anchor=(1.05, 1), loc='upper left')
plt.tight_layout()
plt.show()

plt.figure(figsize=(8, 5))
sns.countplot(x='IsWeekend', hue='GMM_Cluster', data=daily_expense, palette='Set2')
plt.title("Distribution of Weekend Indicator by Cluster")
plt.xlabel("IsWeekend (0=Weekday, 1=Weekend)")
plt.ylabel("Count")
plt.legend(title="Cluster", bbox_to_anchor=(1.05, 1), loc='upper left')
plt.tight_layout()
plt.show()
```





Radar Chart for Cluster Mean Profiles

In [21]:
```
features_radar = ['DailyExpense', 'Rolling7', 'Lag1']
cluster_means = daily_expense.groupby('GMM_Cluster')[features_radar].mean().reset_inde
```

```
# Normalize each feature (min-max normalization) for fair comparison
def normalize(series):
    return (series - series.min()) / (series.max() - series.min())

for feat in features_radar:
    cluster_means[feat] = normalize(cluster_means[feat])

# Prepare the angles for the radar chart
num_vars = len(features_radar)
angles = np.linspace(0, 2 * np.pi, num_vars, endpoint=False).tolist()
angles += angles[:1]

# Plot radar chart
fig, ax = plt.subplots(figsize=(6, 6), subplot_kw=dict(polar=True))
for i, row in cluster_means.iterrows():
    values = row[features_radar].tolist()
    values += values[:1]  # close the circle
    ax.plot(angles, values, label=f"Cluster {int(row['GMM_Cluster'])}")
    ax.fill(angles, values, alpha=0.25)
ax.set_thetagrids(np.degrees(angles[:-1]), features_radar)
ax.set_title("Radar Chart of Cluster Mean Profiles (Normalized)", fontsize=14)
ax.legend(loc='upper right', bbox_to_anchor=(1.3, 1.1))
plt.tight_layout()
plt.show()
```