AI Capstone Homework 1 – Spring 2025
周恭麟 **110550201**
[Github Link]

Personal Finance Daily Expenses Tracking Prediction Using Machine Learning

I. Introduction

This project investigates the use of machine learning for predicting daily expenses. Although no singular hypothesis was defined at the outset, the objective is to evaluate how accurately a model can forecast daily expenditures using a custom‑generated dataset. The experiments focus on exploring the predictive power of engineered temporal features—such as day of the week, weekend indicators, lag features, and rolling averages—and assessing the impact of these features on model performance. In addition, unsupervised clustering methods are applied to reveal underlying patterns in spending behavior.

II. Dataset Documentation

A. Dataset Generation

The dataset was generated using a custom Python script (see *generate_data.py* in the appendix) that simulates a full year (365 days) of financial transactions for a student. The simulation captures both income and various expense categories, such as:

- **Income:** Includes scholarship payments, parental support, part-time job income (e.g., from Burger King and office assistant work).
- **Expenses:** Includes fixed monthly costs (e.g., transport, entertainment subscriptions, groceries) and daily variable expenses (meals, coffee, dinner, and laundry), as well as occasional special events (e.g., weekend dining out) and gym-related spending.

III. Data Composition and Characteristics

- **Time Frame:** January 1, 2025, to December 31, 2025.
- **Features:**

Date	The transaction date.
Category	Labels such as: Income, Transport, Entertainment, Groceries, Meal, Coffee, Food & Drink, Gym, and Laundry.
Amount_NTD	Transaction amounts in New Taiwan Dollars.
Description, Payment_Method, Time	Additional metadata.

- **Generation Process:**
A biased random function was used to simulate realistic values (e.g., amounts ending in 0 or 5 are favored). The script includes fixed monthly incomes and

recurring expenses, while daily expenditures are generated with variability based on conditions (e.g., gym days, weekends).

- **Dataset Access:**
  The dataset is stored in a CSV file (daily_expenses.csv) and is available on my GitHub repository.

IV. Methodology

A. Supervised Learning

To predict daily expenses, we employed a Random Forest regression model using the open-source scikit-learn library (scikit-learn.org). Our supervised learning pipeline was implemented in Python and leveraged other widely adopted libraries such as Pandas and NumPy for data manipulation, and Matplotlib and Seaborn for visualization (pandas.pydata.org, numpy.org, matplotlib.org, seaborn.pydata.org).

- **Feature Engineering:**
  Several features were derived from the raw date:
  1. **DayOfWeek:** Integer representing the day of the week (Monday=0, …, Sunday=6).
  2. **IsWeekend:** Binary indicator (1 if Saturday or Sunday, else 0).
  3. **Month and Day:** Extracted from the date.
  4. **Lag1:** The expense value from the previous day, capturing short-term dependencies.
  5. **Rolling7:** A 7-day moving average of expenses to capture trends.
  6. **LogExpense:** The logarithm of daily expenses (after adding 1) to reduce the impact of outliers.
- **Model Training and Evaluation:**
  The Random Forest model was trained using different feature sets:
  1. **Feature Set A:** Without Lag1 and Rolling7.
  2. **Feature Set B:** With Lag1 only.
  3. **Feature Set C:** With Rolling7 only.
  4. **Feature Set D:** With both Lag1 and Rolling7.
- A grid search with cross-validation (using TimeSeriesSplit) was performed for hyperparameter tuning. Evaluation metrics included RMSE, with additional analysis of feature importance.

Additionally, our pipeline incorporated standard data resampling techniques to ensure a balanced representation of the temporal data. Dimensionality reduction, specifically Principal Component Analysis (PCA), was applied to visualize high-dimensional features in two dimensions. This aided in both exploratory data analysis and in validating the clustering results.

B. Unsupervised Learning

Clustering techniques were employed to explore spending behavior patterns:

- **K-Means Clustering:**
  Multiple metrics (Elbow method, Silhouette score, Davies–Bouldin Index, Calinski–Harabasz Index) were used to evaluate the optimal number of clusters. Experiments revealed variability in the optimal k, with the Elbow method suggesting k = 6.
- **Gaussian Mixture Model (GMM):**
  GMM was applied to obtain probabilistic cluster assignments and confidence scores, providing another perspective on the grouping of daily expenses.

The use of PCA for dimensionality reduction was critical in visualizing both the K-Means and GMM clustering results, as it enabled us to project the multi-dimensional feature space onto two principal components.

V. Experiments Results & Discussion

A. Supervised Learning Experiments

Features	**RMSE**	**Cross-Validation Mean RMSE**
Feature Set A (No Lag1/No Rolling7)	38.67	0.1683
Feature Set B (With Lag1 Only)	39.34	0.1655
Feature Set C (With Rolling7 Only)	37.28	0.1548
Feature Set D (With Both Lag1 & Rolling7)	36.20	0.1540

Table 1. Random Forest Regression Performance by Different Feature Set

In this experiments, I evaluated a Random Forest regression model using four distinct feature sets to predict daily expenses. Feature Set A, which excluded both lagged and rolling features, produced an RMSE of approximately 38.66, while incorporating a lag feature in Feature Set B resulted in a slight improvement with an RMSE near 39.34. Notably, Feature Set C—using only a 7-day rolling average—yielded a lower RMSE of about 37.28, and the combination of both lag (Lag1) and rolling average (Rolling7) in Feature **Set D** achieved the best performance with an **RMSE** of roughly **36.20**. Cross-validation confirmed the model's stability, and feature importance analysis (Table 2) consistently highlighted **DayOfWeek** and **IsWeekend** as the most influential predictors, with the added features enhancing the prediction by capturing temporal trends.

Feature	Set A	Set B	Set C	Set D
DayOfWeek	0.406320	0.406350	0.335043	0.292754
IsWeekend	0.282965	0.314623	0.239466	0.229379
Month	0.144396	0.064919	0.078241	0.054489
Day	0.166320	0.073103	0.108969	0.079247
Lag1	-	0.141005	-	0.130432
Rolling7	-	-	0.238281	0.213698

Table 2. Feature Importances for Random Forest with Different Feature Sets

In addition, unsupervised learning experiments using K-Means clustering and Gaussian Mixture Models revealed that the dataset naturally segmented into distinct groups; while the Elbow method suggested an optimal k of 6, alternative metrics provided varied insights, and GMM clustering further confirmed the presence of six distinct clusters with high confidence, each exhibiting unique spending profiles. These findings underscore that careful feature engineering—particularly the integration of temporal dynamics—significantly improves predictive accuracy and offers valuable insights into underlying spending behaviors.

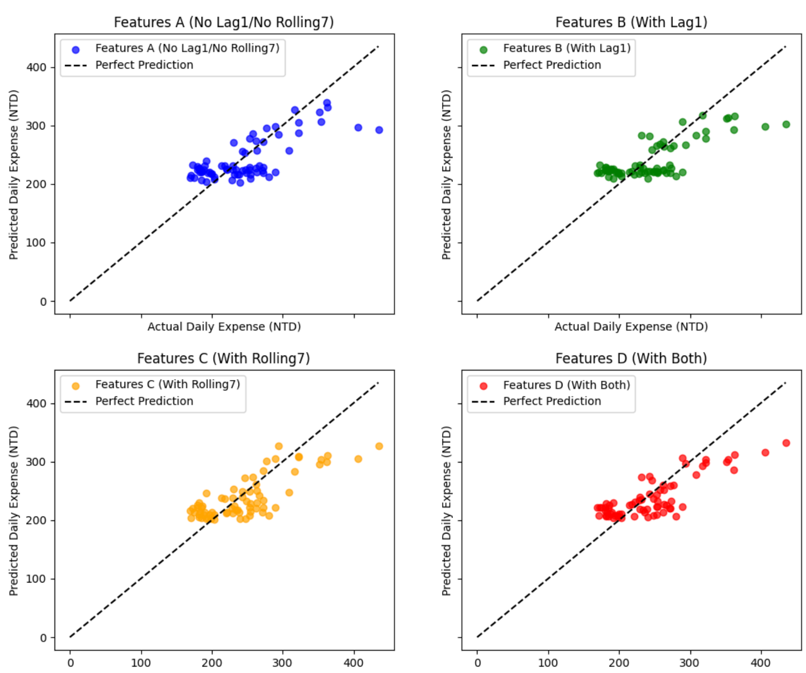


Figure 1. Comparison of Random Forest Predictions with Different Feature Sets

As illustrated in Figure 1, each subplot compares the model's predicted daily expenses (y-axis) to the actual values (x-axis), with the diagonal dashed line representing perfect predictions. Feature Set A exhibits a relatively wide scatter around this line, reflecting higher prediction errors. Feature Set B shows a slightly narrower spread, while Feature Set C appears even more tightly clustered around the diagonal, indicating improved accuracy. Notably, Feature Set D demonstrates the closest alignment of points with the perfect prediction line, visually confirming that the addition of both the lagged feature and rolling average yields the best predictive performance among the four configurations.

B. Unsupervised Learning Experiments

Unsupervised learning experiments using K-Means clustering and Gaussian Mixture Models revealed that the dataset naturally segmented into distinct groups; while the Elbow method suggested an optimal k of 6, alternative metrics provided varied insights, and GMM clustering further confirmed the presence of six distinct clusters with high confidence, each exhibiting unique spending profiles. These findings underscore that careful feature engineering—particularly the integration of temporal dynamics—significantly improves predictive accuracy and offers valuable insights into underlying spending behaviors.

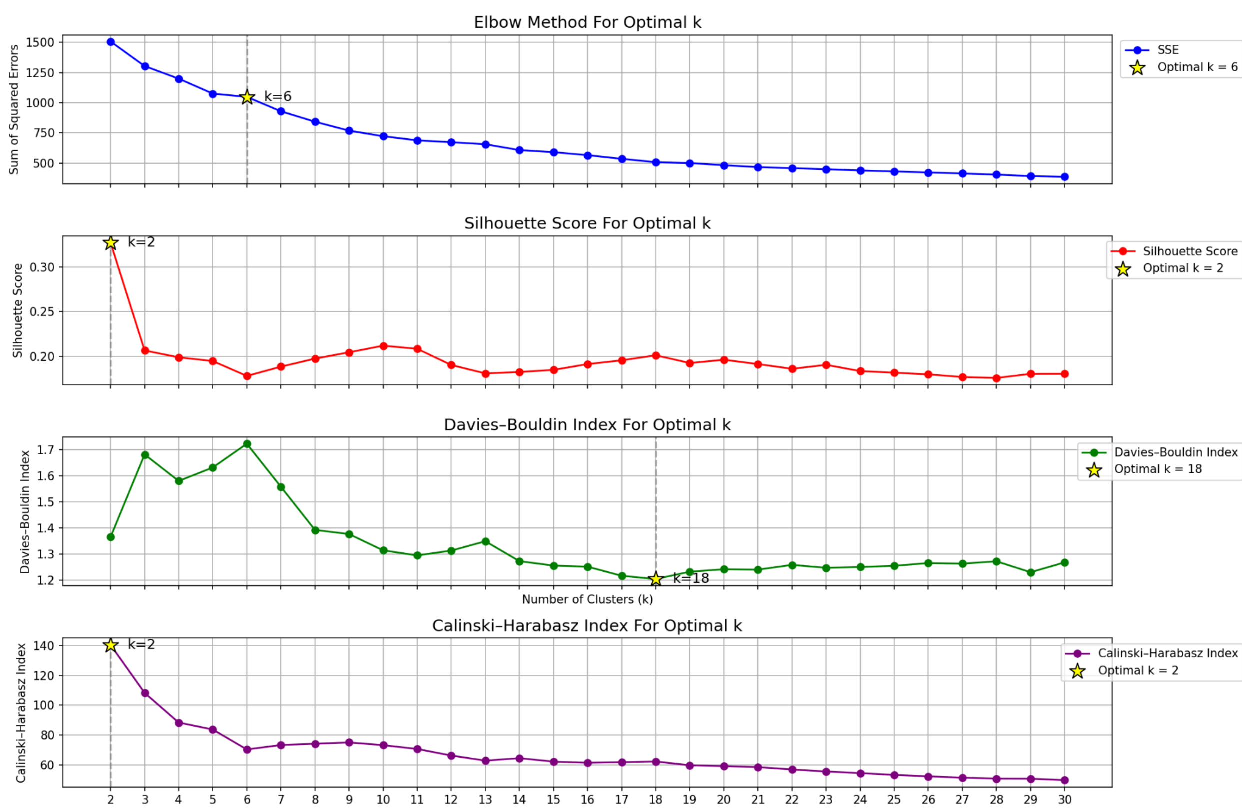


Figure 2. Optimal k Value Using Four Different Algorithms

To determine the optimal value of k, I use four algorithms—Elbow Method, Silhouette Score, Davies-Bouldin Index, and Calinski-Harabasz—and get three unique values of k=2, 6, and 18. As depicted in Figure 3, the data has been projected onto two principal components for visualization, with each color indicating a cluster and black stars marking the cluster centroids. The left subplot illustrates the K-Means solution for k=6 as suggested by the Elbow Method, yielding moderately sized, distinct clusters. In contrast, the middle subplot uses k=2 from the Silhouette Score, resulting in just two large clusters that capture broad groupings of points. Meanwhile, the right subplot with k=18 from the Davies–Bouldin Index partitions the data into numerous smaller clusters, reflecting a more granular segmentation. These differing cluster configurations underscore the inherent variability in selecting an optimal k, further emphasizing the importance of examining multiple clustering metrics to gain a comprehensive understanding of the dataset's structure.

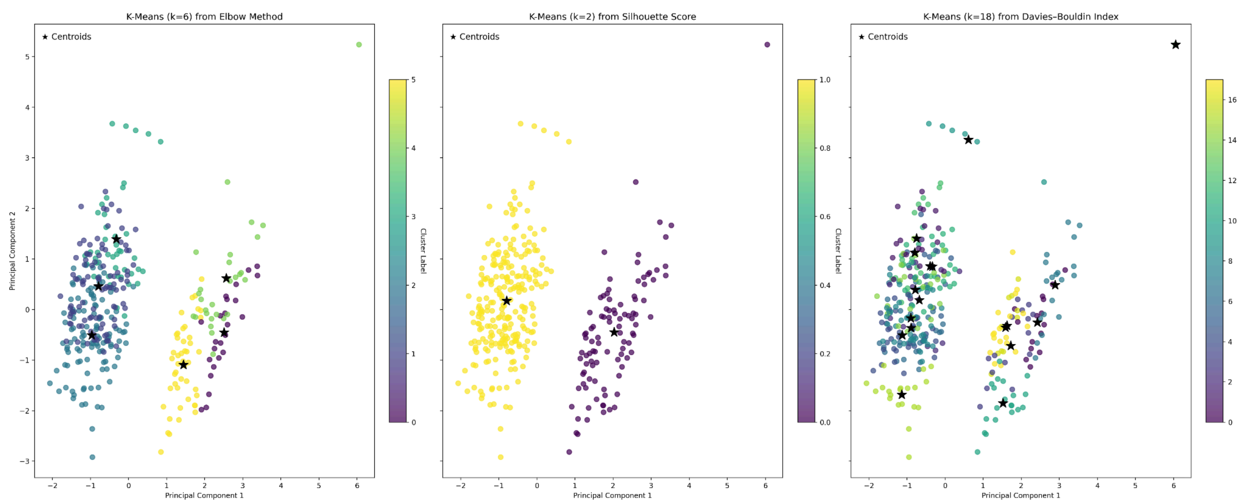


Figure 3. K-Means Clustering Solutions for k=2, k=6, and k=18

Cluster	Count	Mean Daily Expense (NTD)	Std. Dev (NTD)
0	25	260.92	50.41
1	96	220.35	34.43
2	115	220.47	31.00
3	42	231.38	30.98
4	28	309.14	103.65
5	46	320.24	59.16

Table 3. Key Statistics for Clusters (Elbow Method, k=6)

Building on these clustering solutions and metrics, we now turn our attention to a more detailed analysis of the clusters—particularly those derived from the Elbow method with k=6. This in-depth examination allows us to better understand the underlying spending behaviors captured by each cluster. The clustering analysis reveals that the dataset naturally segments into distinct groups with unique spending profiles. For example, Cluster 2 is the largest with 115 samples and shows a consistent spending pattern, with a mean daily expense of approximately 220.47 NTD and a low standard deviation of 31.00 NTD. In contrast, Cluster 5, although smaller with 46 samples, has a significantly higher mean daily expense of around 320.24 NTD and a greater variability (std ≈ 59.16 NTD), indicating more heterogeneous spending behavior. Cluster 4 also stands out, with a high mean expense of 309.14 NTD, but its small size (28 samples) and very high standard deviation (103.65 NTD) suggest the presence of extreme spending values. These differences illustrate that spending behaviors differ markedly across segments, supporting the value of tailored financial strategies for each group. Table 3 highlights the distinct financial behaviors across clusters and reinforces the conclusion that incorporating unsupervised learning provides valuable insights into spending patterns.

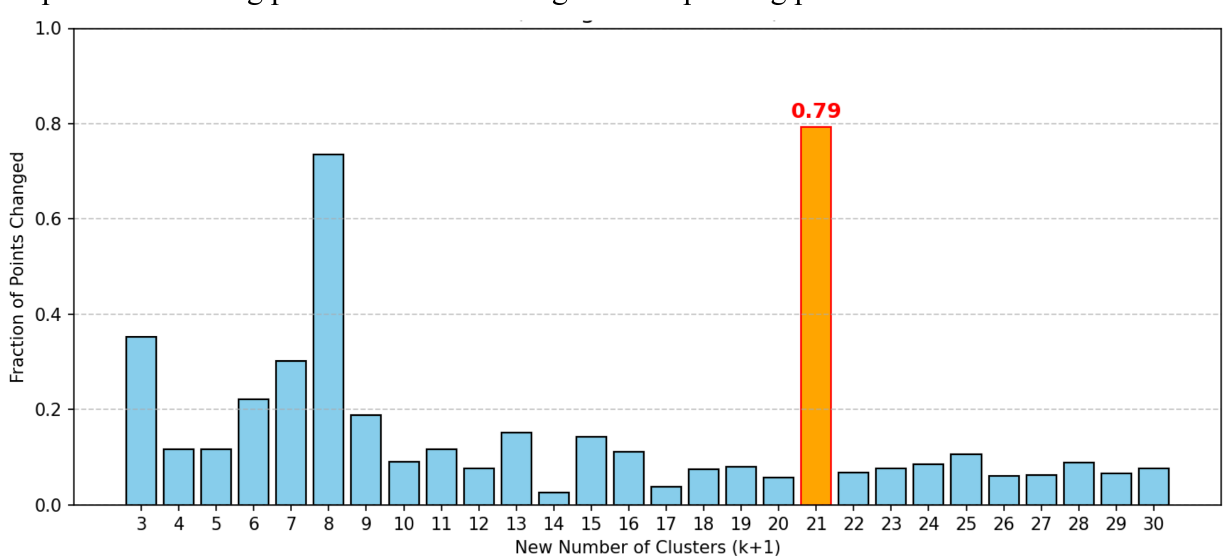


Figure 4. Fraction of Points Changing Clusters (From k to k+1)

Beyond examining fixed clustering solutions, I also investigated how cluster assignments change as we increment the number of clusters from k to k+1. By computing the fraction of points that switch cluster membership between successive k values, we can gauge the stability of the clustering structure. In most cases, the fraction of points that change clusters is relatively low, indicating that the underlying groupings are robust. However, my analysis revealed that there are specific transitions—most notably from k=7 to k=8 and k=20 to k=21—where nearly 80% of the points are reassigned. This abrupt change suggests that the data structure becomes highly unstable when attempting to over-segment the dataset, reinforcing the idea that the optimal number of clusters should be chosen carefully to capture the inherent patterns without fragmenting them excessively.

Besides that, I also experimented by clustering the data using only the initial X months as the training set and then evaluated how well the learned cluster structure fit the remaining data. By varying the cutoff—using offsets of 2, 4, 6, 8, and 10 months—I examined whether the clusters derived from a smaller historical window remain representative when applied to future data. If the spending patterns are consistent over time, I expect the cluster distributions in both the training and subsequent test sets to be similar. However, noticeable differences in these distributions could indicate evolving consumer behavior or structural shifts in the data. This analysis not only serves to validate the robustness of the clustering approach but also provides insight into the temporal dynamics of daily expenses.

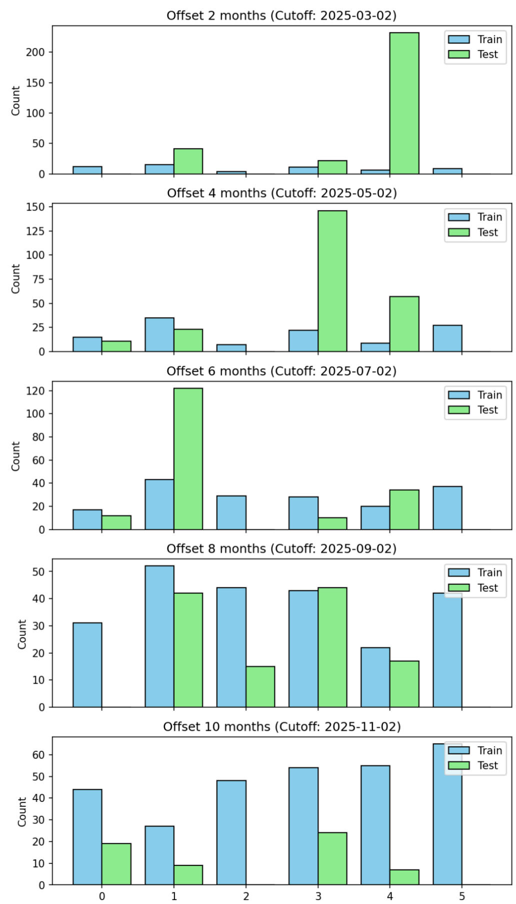


Figure 5. Cluster Distributions for k=6 for Various Training Cutoffs

When I plotted the cluster assignments for various offsets (2, 4, 6, 8, and 10 months) and different k values, several interesting patterns emerged. For smaller offsets, the training data is relatively limited and may not capture the full diversity of spending behaviors. As a result, the distribution of clusters in the test set can differ substantially from that in the training set, indicating that additional months of data are necessary to learn stable, representative clusters. Conversely, for larger offsets, the training set grows and the resulting cluster distributions between training and test sets often become more aligned, suggesting that the learned clusters generalize better over time. Notably, when k=6, certain clusters remain fairly consistent across train and test splits, whereas others display significant shifts—particularly in smaller training windows—implying that new spending patterns or evolving behaviors appear in the latter months. These observations reinforce the importance of choosing an appropriate historical window when applying clustering models to time-dependent data, as an insufficient training period may overlook critical variations in consumer spending and lead to clusters that do not accurately represent future patterns.

Cluster	Count	Mean Daily Expense	Std. Dev (NTD)	Mean Confidence
0	49	268.43	54.05	0.99
1	99	220.91	34.25	0.98
2	132	223.13	31.51	0.97
3	22	222.86	31.15	0.98
4	3	442.00	276.22	0.95
5	47	328.32	52.77	0.99

Table 4. GMM Cluster's Size and Key Statistics

We also applied a Gaussian Mixture Model (GMM) to gain a probabilistic perspective on the data's cluster structure. Using six components (in line with the Elbow method's suggestion for K-Means), GMM produced clusters of varying sizes, with Cluster 2 being the largest (132 data points) and Cluster 4 containing only three data points. The presence of such a small cluster suggests that a handful of transactions were extreme outliers in terms of spending patterns. One advantage of GMM over K-Means is its soft assignment of points to clusters, reflected in each record's GMM_Confidence value; most points displayed high confidence scores, indicating that they fit well within their assigned cluster's distribution. However, for points on the boundaries or in rare, high-expense categories (e.g., Cluster 4), the confidence may be lower or heavily skewed, highlighting the model's ability to capture nuanced variations in the data. Overall, the GMM approach confirms the existence of multiple spending profiles in the dataset, aligning with the K-Means results but offering additional insight into how firmly each transaction belongs to its respective cluster.

VI. Conclusion

This experiments largely produced results that aligned with my expectations, while also uncovering some unexpected nuances. For instance, the improvement in predictive accuracy when incorporating both lag and rolling features confirmed that recent spending behavior plays a critical role in forecasting daily expenses. The clustering analyses further revealed distinct spending patterns—consistent with the heterogeneous nature of our simulated financial data—yet also highlighted sensitivity to outliers and shifts in cluster assignments when over-segmenting the dataset. Factors such as the variability inherent in daily expenditures, the presence of extreme values, and temporal influences (e.g., weekend effects, gym days) were found to significantly impact model performance. If more time were available, additional experiments could include testing advanced time-series models (like

LSTM networks) and exploring alternative clustering techniques such as hierarchical clustering to better capture non-linear and temporal dependencies. I also plan to use my real life spending notes if I have more time instead of generating the dataset. These experiments have underscored the importance of robust feature engineering and careful model selection, while raising further questions about the optimal balance between training window length and clustering stability, particularly in the face of rare, high-spending events.

Appendix

generate_data.py

```
import pandas as pd
import random
from datetime import datetime, timedelta, date

# Set random seed for reproducibility
random.seed(42)

# Helper function to generate a whole number with bias:
def biased_amount(min_val, max_val):
    """
    Generate a whole number between min_val and max_val (inclusive)
    with bias: numbers ending in 0 or 5 have higher probability.
    """
    values = list(range(min_val, max_val + 1))
    weights = [2 if (val % 10 == 0 or val % 10 == 5) else 1 for val in values]
    return random.choices(values, weights=weights, k=1)[0]

# Simulation period: one year (365 days)
start_date = date(2025, 1, 1)
num_days = 365
end_date = start_date + timedelta(days=num_days - 1)

records = []

# Create a list of month start dates for monthly records
months = pd.date_range(start=start_date, end=end_date, freq='MS')

# --- Revised Monthly Income ---

for m in months:
    m_date = m.date()

    # Scholarship: fixed 6000 NTD paid on the 20th of the same month.
    try:
        scholarship_date = date(m_date.year, m_date.month, 20)
```

```
    except ValueError:
        # In case the month doesn't have 20 days (shouldn't occur)
        scholarship_date = date(m_date.year, m_date.month, 20)
    records.append({
        'Date': scholarship_date,
        'Category': 'Income',
        'Amount_NTD': 6000,
        'Description': f"Scholarship income for {m_date.strftime('%B %Y')}",
        'Payment_Method': None,
        'Time': None
    })

    # Parents: fixed 9000 NTD paid on a random day between 1st and 3rd of the
same month.
    parent_pay_day = random.randint(1, 3)
    parents_date = date(m_date.year, m_date.month, parent_pay_day)
    records.append({
        'Date': parents_date,
        'Category': 'Income',
        'Amount_NTD': 9000,
        'Description': f"Parents' income for {m_date.strftime('%B %Y')}",
        'Payment_Method': None,
        'Time': None
    })

    # Burger King income: paid on the 5th of the same month.
    bk_amount = biased_amount(1500, 4000)
    bk_pay_date = date(m_date.year, m_date.month, 5)
    records.append({
        'Date': bk_pay_date,
        'Category': 'Income',
        'Amount_NTD': bk_amount,
        'Description': f"Burger King income for {m_date.strftime('%B %Y')}",
        'Payment_Method': None,
        'Time': None
    })

    # Office Assistant income: paid on a random day between 25 and 30 of the next
month.
```

```
    next_month_year = m_date.year
    next_month = m_date.month + 1
    if next_month > 12:
        next_month = 1
        next_month_year += 1
    oa_pay_day = random.randint(25, 30)
    try:
        oa_pay_date = date(next_month_year, next_month, oa_pay_day)
    except ValueError:
        # Adjust if the day is invalid (e.g., February 30)
        oa_pay_date = date(next_month_year, next_month, 28)
    # Only add office assistant income if it falls within the simulation period.
    if start_date <= oa_pay_date <= end_date:
        oa_amount = biased_amount(4500, 5500)
        records.append({
            'Date': oa_pay_date,
            'Category': 'Income',
            'Amount_NTD': oa_amount,
            'Description': f"Office assistant income for {m_date.strftime('%B
%Y')}",
            'Payment_Method': None,
            'Time': None
        })

    # Fixed monthly expenses (Transport, Entertainment, Groceries)
    # Transport expense: between 200 and 500 NTD
    records.append({
        'Date': m_date,
        'Category': 'Transport',
        'Amount_NTD': biased_amount(200, 500),
        'Description': 'Monthly transport expense',
        'Payment_Method': None,
        'Time': None
    })

    # Entertainment subscriptions: fixed monthly amounts
    records.append({
        'Date': m_date,
        'Category': 'Entertainment',
```

```
        'Amount_NTD': 99,
        'Description': 'Spotify subscription (student)',
        'Payment_Method': None,
        'Time': None
    })
    records.append({
        'Date': m_date,
        'Category': 'Entertainment',
        'Amount_NTD': 600,
        'Description': 'ChatGPT Plus subscription',
        'Payment_Method': None,
        'Time': None
    })
    records.append({
        'Date': m_date,
        'Category': 'Entertainment',
        'Amount_NTD': 200,
        'Description': 'WuxiaWorld subscription',
        'Payment_Method': None,
        'Time': None
    })

    # Groceries (household items): between 500 and 1000 NTD
    records.append({
        'Date': m_date,
        'Category': 'Groceries',
        'Amount_NTD': biased_amount(500, 1000),
        'Description': 'Monthly groceries/household items',
        'Payment_Method': None,
        'Time': None
    })

# Occasional special weekend events: one per semester (2 per year)
def get_random_weekend(start, end):
    """Return a random weekend date (Saturday or Sunday) between start and
end."""
    delta = (end - start).days
    while True:
        rand_day = start + timedelta(days=random.randint(0, delta))
```

```python
        if rand_day.weekday() >= 5:
            return rand_day

first_half = (start_date, date(start_date.year, 6, 30))
second_half = (date(start_date.year, 7, 1), end_date)
special_events = [
    {'Description': 'Korean BBQ with friends', 'Amount_NTD': 700},
    {'Description': 'Haidilao with friends', 'Amount_NTD': 500}
]
special_date1 = get_random_weekend(first_half[0], first_half[1])
special_date2 = get_random_weekend(second_half[0], second_half[1])
special_dates = [special_date1, special_date2]

for i, event in enumerate(special_events):
    records.append({
        'Date': special_dates[i],
        'Category': 'Food & Drink',
        'Amount_NTD': event['Amount_NTD'],
        'Description': event['Description'],
        'Payment_Method': None,
        'Time': 'Evening'
    })

# Gym supplements
protein_months = random.sample(list(months), 2)
for m in protein_months:
    records.append({
        'Date': m.date(),
        'Category': 'Gym',
        'Amount_NTD': 1500,
        'Description': 'Protein powder purchase (2.5kg)',
        'Payment_Method': None,
        'Time': None
    })
creatine_months = random.sample(list(months), 4)
for m in creatine_months:
    records.append({
        'Date': m.date(),
        'Category': 'Gym',
```

```python
        'Amount_NTD': 600,
        'Description': 'Creatine purchase',
        'Payment_Method': None,
        'Time': None
    })

# Define gym days: roughly 3 days per week
gym_days = set()
for day in range(num_days):
    current_date = start_date + timedelta(days=day)
    if random.random() < 3/7:
        gym_days.add(current_date)

# Simulate daily expenses for meals, coffee, and laundry.
for day in range(num_days):
    current_date = start_date + timedelta(days=day)
    weekday = current_date.weekday()  # Monday=0, Sunday=6

    # Lunch at around 12:00 PM (Meal expense: 80-105 NTD)
    lunch_time = "12:00"
    lunch_amount = biased_amount(80, 105)
    lunch_payment = 'Cash' if random.random() < 0.7 else 'Card'
    records.append({
        'Date': current_date,
        'Category': 'Meal',
        'Amount_NTD': lunch_amount,
        'Description': 'Lunch meal',
        'Payment_Method': lunch_payment,
        'Time': lunch_time
    })

    # Coffee: approximately 3 times per week around 12:20-13:00 PM (40-60 NTD)
    if random.random() < (3/7):
        coffee_time = f"12:{random.randint(20, 59):02d}"
        coffee_amount = biased_amount(40, 60)
        coffee_payment = 'Cash' if random.random() < 0.7 else 'Card'
        records.append({
            'Date': current_date,
            'Category': 'Coffee',
```

```
            'Amount_NTD': coffee_amount,
            'Description': 'Coffee purchase',
            'Payment_Method': coffee_payment,
            'Time': coffee_time
        })

    # Dinner: between 5 PM and 8 PM.
    # On gym days, dinner is more expensive (100-150 NTD);
    # on weekends, moderately higher (100-250 NTD);
    # otherwise, normal dinner: 80-110 NTD.
    dinner_hour = random.randint(17, 20)
    dinner_minute = random.randint(0, 59)
    dinner_time = f"{dinner_hour}:{dinner_minute:02d}"
    if current_date in gym_days:
        dinner_amount = biased_amount(100, 150)
    elif weekday >= 5:  # weekend
        dinner_amount = biased_amount(100, 250)
    else:
        dinner_amount = biased_amount(80, 110)
    dinner_payment = 'Cash' if random.random() < 0.7 else 'Card'
    records.append({
        'Date': current_date,
        'Category': 'Meal',
        'Amount_NTD': dinner_amount,
        'Description': 'Dinner meal',
        'Payment_Method': dinner_payment,
        'Time': dinner_time
    })

    # Laundry: 60 NTD per week on Saturday
    if current_date.weekday() == 5:
        records.append({
            'Date': current_date,
            'Category': 'Laundry',
            'Amount_NTD': 60,
            'Description': 'Weekly laundry expense',
            'Payment_Method': None,
            'Time': None
        })
```

```python
# Create DataFrame and sort by Date and Time
df = pd.DataFrame(records)
df['Date'] = pd.to_datetime(df['Date'])
df.sort_values(by=['Date', 'Time'], inplace=True)

# Save the dataset to a CSV file and preview the first 20 records.
df.to_csv('daily_expenses.csv', index=False)
print("Data generation complete. Preview of the first 20 records:")
print(df.head(20))
```

```python
In [23]: import pandas as pd
import numpy as np

from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import train_test_split, GridSearchCV
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import TimeSeriesSplit
from sklearn.model_selection import cross_val_score

import matplotlib.pyplot as plt
from matplotlib.lines import Line2D
```

```python
In [24]: # Load the dataset with Date parsed as datetime
df = pd.read_csv('data\\daily_expenses.csv', parse_dates=['Date'])
```

```python
In [25]: # Exclude 'Income' to focus on daily expenses
expense_df = df[df['Category'] != 'Income']

# Group by date to get total daily expense
daily_expense = expense_df.groupby('Date')['Amount_NTD'].sum().reset_index()
daily_expense.rename(columns={'Amount_NTD': 'DailyExpense'}, inplace=True)

# Sort by date (important if you want rolling features)
daily_expense.sort_values('Date', inplace=True)

# Drop daily expenses larger than 1000 NTD
daily_expense = daily_expense[daily_expense['DailyExpense'] <= 1000]
```

Feature Engineering

```python
In [26]: # (a) Day of week (Monday=0, Sunday=6)
daily_expense['DayOfWeek'] = daily_expense['Date'].dt.dayofweek

# (b) Weekend indicator
daily_expense['IsWeekend'] = daily_expense['DayOfWeek'].isin([5, 6]).astype(int)

# (c) Month and Day of month
daily_expense['Month'] = daily_expense['Date'].dt.month
daily_expense['Day'] = daily_expense['Date'].dt.day

# (d) Lag features (Will be used later)
# Create a lag feature (previous day's expense)
daily_expense['Lag1'] = daily_expense['DailyExpense'].shift(1)
# Fill missing values (e.g., first day) with the mean or zero
daily_expense['Lag1'].fillna(daily_expense['DailyExpense'].mean(), inplace=True)


# (e) Rolling 7-day average of expenses (Will be used later)
# This can help the model learn from recent spending trends
daily_expense['Rolling7'] = (
    daily_expense['DailyExpense']
    .rolling(window=7, min_periods=1)
    .mean()
)

# (f) Log transform the target to reduce the impact of large spikes
daily_expense['LogExpense'] = np.log1p(daily_expense['DailyExpense'])
```

Random Forest

Base Function

```python
In [27]: # Function to train model and evaluate RMSE for a given feature set
def evaluate_feature_set(X, y, tscv=None, use_tscv=True):
    # Split data into training and test sets
    X_train, X_test, y_train, y_test = train_test_split(
        X, y, test_size=0.2, random_state=42
    )

    # Set up a simple parameter grid (or use your existing grid)
    rf = RandomForestRegressor(random_state=42)
    param_grid = {
        'n_estimators': [100, 200, 300, 400, 500],
        'max_depth': [10, 20, 30, 40, 50],
        'min_samples_split': [2, 5, 10, 15, 20],
        'min_samples_leaf': [1, 2, 4, 8, 16],
        'max_features': ['sqrt', 'log2'],
        'bootstrap': [True, False]
    }

    if use_tscv:
        grid_search = GridSearchCV(
            rf, param_grid, cv=tscv, scoring='neg_mean_squared_error', n_jobs=-1
        )
    else:
        grid_search = GridSearchCV(
            rf, param_grid, cv=5, scoring='neg_mean_squared_error', n_jobs=-1
        )

    grid_search.fit(X_train, y_train)
```

```
    print("Best Params from Grid Search:", grid_search.best_params_)

    best_rf = grid_search.best_estimator_
    y_pred_log = best_rf.predict(X_test)
    y_pred = np.expm1(y_pred_log)
    y_test_exp = np.expm1(y_test)

    mse = mean_squared_error(y_test_exp, y_pred)
    rmse = np.sqrt(mse)
    return {'rmse': rmse, 'grid_search': grid_search, 'y_pred': y_pred, 'y_test': y_test_exp}
```

Evaluate

Without Lag1 and Rolling7

In [28]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_A = ['DayOfWeek', 'IsWeekend', 'Month', 'Day']  # without Lag1 and Rolling7

X = daily_expense[features_A]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_A, best_params_A = result['rmse'], result['grid_search'].best_params_
best_rf_A = result['grid_search'].best_estimator_
y_pred_A, y_test_A = result['y_pred'], result['y_test']
feature_importance_A = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set A (no Lag1, no Rolling7) RMSE:", rmse_A)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 20, 'max_features': 'sqrt', 'min_samples_leaf': 4, 'min_samples_split': 2,
'n_estimators': 100}
Feature Set A (no Lag1, no Rolling7) RMSE: 38.66275658792042
```

With Lag1

In [29]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_B = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Lag1']  # with Lag1 only

X = daily_expense[features_B]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_B, best_params_B = result['rmse'], result['grid_search'].best_params_
best_rf_B = result['grid_search'].best_estimator_
y_pred_B, y_test_B = result['y_pred'], result['y_test']
feature_importance_B = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set B (with Lag1, no Rolling7) RMSE:", rmse_B)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 'sqrt', 'min_samples_leaf': 8, 'min_samples_split': 2,
'n_estimators': 100}
Feature Set B (with Lag1, no Rolling7) RMSE: 39.3385956464684
```

With Rolling7

In [30]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_C = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7']  # with Rolling7 only

X = daily_expense[features_C]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_C, best_params_C = result['rmse'], result['grid_search'].best_params_
best_rf_C = result['grid_search'].best_estimator_
y_pred_C, y_test_C = result['y_pred'], result['y_test']
feature_importance_C = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set C (no Lag1, with Rolling7) RMSE:", rmse_C)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 'sqrt', 'min_samples_leaf': 4, 'min_samples_split': 2,
'n_estimators': 100}
Feature Set C (no Lag1, with Rolling7) RMSE: 37.275187253048614
```

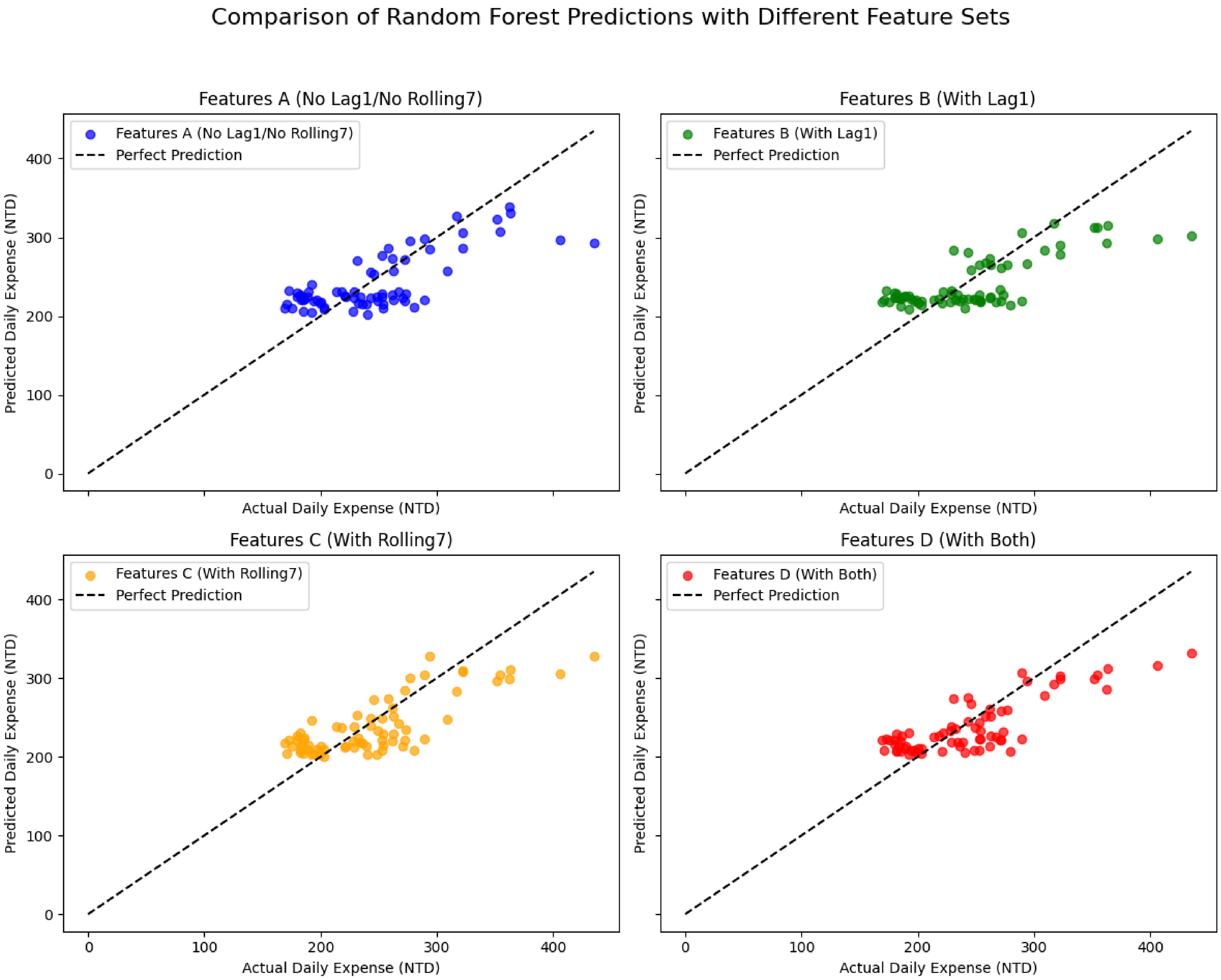
With Lag1 & Rolling7

In [31]:
```
tscv = TimeSeriesSplit(n_splits=5)

features_D = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Lag1', 'Rolling7']  # with both Lag1 and Rolling7

X = daily_expense[features_D]
y = daily_expense['LogExpense']

result = evaluate_feature_set(X, y, tscv, use_tscv=True)
rmse_D, best_params_D = result['rmse'], result['grid_search'].best_params_
best_rf_D = result['grid_search'].best_estimator_
y_pred_D, y_test_D = result['y_pred'], result['y_test']
feature_importance_D = result['grid_search'].best_estimator_.feature_importances_

print("Feature Set D (with Lag1, with Rolling7) RMSE:", rmse_D)
```

```
Best Params from Grid Search: {'bootstrap': True, 'max_depth': 10, 'max_features': 'sqrt', 'min_samples_leaf': 4, 'min_samples_split': 2,
'n_estimators': 200}
Feature Set D (with Lag1, with Rolling7) RMSE: 36.19773875656066
```

Result

In [32]:
```
# Create a 2x2 grid of subplots
fig, axes = plt.subplots(2, 2, figsize=(12, 10), sharex=True, sharey=True)
axes = axes.flatten()

# List of (true values, predictions, label, color) for each feature set
plot_data = [
    (y_test_A, y_pred_A, 'Features A (No Lag1/No Rolling7)', 'blue'),
    (y_test_B, y_pred_B, 'Features B (With Lag1)', 'green'),
    (y_test_C, y_pred_C, 'Features C (With Rolling7)', 'orange'),
    (y_test_D, y_pred_D, 'Features D (With Both)', 'red'),
]

for ax, (y_test, y_pred, title, color) in zip(axes, plot_data):
    ax.scatter(y_test, y_pred, color=color, alpha=0.7, label=title)
    # Determine maximum value for perfect prediction line in each subplot
    max_val = max(y_test.max(), y_pred.max())
    ax.plot([0, max_val], [0, max_val], 'k--', label='Perfect Prediction')
    ax.set_xlabel('Actual Daily Expense (NTD)')
    ax.set_ylabel('Predicted Daily Expense (NTD)')
    ax.set_title(title)
    ax.legend()

fig.suptitle('Comparison of Random Forest Predictions with Different Feature Sets', fontsize=16)
plt.savefig('data\\experiment_result\\random_forest_comparison.png')
plt.tight_layout(rect=[0, 0.03, 1, 0.95])
plt.show()
```



Feature Importance

In [33]:
```
# Feature Set A: Without Lag1 and Rolling7
feature_importance_df_A = pd.DataFrame({'Feature': features_A, 'Importance': feature_importance_A})
print("Feature Importances for Feature Set A (No Lag1/No Rolling7):")
print(feature_importance_df_A)
print("\n" + "="*50 + "\n")

# Feature Set B: With Lag1 only
feature_importance_df_B = pd.DataFrame({'Feature': features_B, 'Importance': feature_importance_B})
print("Feature Importances for Feature Set B (With Lag1):")
```

```
print(feature_importance_df_B)
print("\n" + "="*50 + "\n")

# Feature Set C: With Rolling7 only
feature_importance_df_C = pd.DataFrame({'Feature': features_C, 'Importance': feature_importance_C})
print("Feature Importances for Feature Set C (With Rolling7):")
print(feature_importance_df_C)
print("\n" + "="*50 + "\n")

# Feature Set D: With both Lag1 and Rolling7
feature_importance_df_D = pd.DataFrame({'Feature': features_D, 'Importance': feature_importance_D})
print("Feature Importances for Feature Set D (With Both Lag1 & Rolling7):")
print(feature_importance_df_D)
```

```
Feature Importances for Feature Set A (No Lag1/No Rolling7):
     Feature  Importance
0  DayOfWeek    0.406320
1  IsWeekend    0.282965
2      Month    0.144396
3        Day    0.166320

==================================================

Feature Importances for Feature Set B (With Lag1):
     Feature  Importance
0  DayOfWeek    0.406350
1  IsWeekend    0.314623
2      Month    0.064919
3        Day    0.073103
4       Lag1    0.141005

==================================================

Feature Importances for Feature Set C (With Rolling7):
     Feature  Importance
0  DayOfWeek    0.335043
1  IsWeekend    0.239466
2      Month    0.078241
3        Day    0.108969
4   Rolling7    0.238281

==================================================

Feature Importances for Feature Set D (With Both Lag1 & Rolling7):
     Feature  Importance
0  DayOfWeek    0.292754
1  IsWeekend    0.229379
2      Month    0.054489
3        Day    0.079247
4       Lag1    0.130432
5   Rolling7    0.213698
```

Cross Validation

In [34]:
```
# Function to perform cross-validation for a given feature set and best estimator
def cross_val_feature_set(features, best_rf):
    X_set = daily_expense[features]
    y_set = daily_expense['LogExpense']

    cv_scores = cross_val_score(
        best_rf,
        X_set,
        y_set,
        cv=5,
        scoring='neg_mean_squared_error',
        n_jobs=-1
    )
    mse_scores = -cv_scores
    rmse_scores = np.sqrt(mse_scores)
    return rmse_scores, rmse_scores.mean(), rmse_scores.std()
```

In [35]:
```
# Perform cross-validation on each feature set using the best estimator from grid search
rmse_scores_A, mean_rmse_A, std_rmse_A = cross_val_feature_set(features_A, best_rf_A)
rmse_scores_B, mean_rmse_B, std_rmse_B = cross_val_feature_set(features_B, best_rf_B)
rmse_scores_C, mean_rmse_C, std_rmse_C = cross_val_feature_set(features_C, best_rf_C)
rmse_scores_D, mean_rmse_D, std_rmse_D = cross_val_feature_set(features_D, best_rf_D)
```

In [36]:
```
print("Cross-Validation RMSE for Feature Set A (No Lag1/No Rolling7):")
print(rmse_scores_A)
print("Mean RMSE:", mean_rmse_A)
print("Standard Deviation of RMSE:", std_rmse_A)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set B (With Lag1):")
print(rmse_scores_B)
print("Mean RMSE:", mean_rmse_B)
print("Standard Deviation of RMSE:", std_rmse_B)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set C (With Rolling7):")
print(rmse_scores_C)
print("Mean RMSE:", mean_rmse_C)
print("Standard Deviation of RMSE:", std_rmse_C)
print("\n" + "="*50 + "\n")
print("Cross-Validation RMSE for Feature Set D (With Both Lag1 & Rolling7):")
print(rmse_scores_D)
print("Mean RMSE:", mean_rmse_D)
print("Standard Deviation of RMSE:", std_rmse_D)
```

```
Cross-Validation RMSE for Feature Set A (No Lag1/No Rolling7):
[0.14123133 0.18620571 0.15099621 0.19368341 0.16927219]
Mean RMSE: 0.16827776916692772
Standard Deviation of RMSE: 0.019990040177141778

==================================================

Cross-Validation RMSE for Feature Set B (With Lag1):
[0.13739528 0.17689063 0.15231579 0.19879546 0.16225005]
Mean RMSE: 0.16552944218569143
Standard Deviation of RMSE: 0.02103611466426487

==================================================

Cross-Validation RMSE for Feature Set C (With Rolling7):
[0.12797353 0.17395989 0.14638658 0.17412378 0.15168051]
Mean RMSE: 0.15482485750497763
Standard Deviation of RMSE: 0.017553818642204157

==================================================

Cross-Validation RMSE for Feature Set D (With Both Lag1 & Rolling7):
[0.12514251 0.16964064 0.14900217 0.17844627 0.1477058 ]
Mean RMSE: 0.1539874791836075
Standard Deviation of RMSE: 0.018652610616576897
```

In [37]:

```
result_df = pd.DataFrame({
    'feature_set': ['A', 'B', 'C', 'D'],
    'RMSE': [rmse_A, rmse_B, rmse_C, rmse_D],
    'best_params': [best_params_A, best_params_B, best_params_C, best_params_D],
    'best_rf': [best_rf_A, best_rf_B, best_rf_C, best_rf_D],
    'features': [features_A, features_B, features_C, features_D],
    'feature_importance': [feature_importance_A, feature_importance_B, feature_importance_C, feature_importance_D],
    'cross_validation_RMSE': [rmse_scores_A, rmse_scores_B, rmse_scores_C, rmse_scores_D],
    'cross_validation_RMSE_mean': [mean_rmse_A, mean_rmse_B, mean_rmse_C, mean_rmse_D],
    'cross_validation_RMSE_std': [std_rmse_A, std_rmse_B, std_rmse_C, std_rmse_D],
})

result_df.to_csv('data\\experiment_result\\random_forest_result.csv', index=False)
```

K-Means Clustering

In [38]:

```
from sklearn.cluster import KMeans
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import silhouette_score, davies_bouldin_score, calinski_harabasz_score
```

In [39]:

```
# Selecting features for clustering.
# We include some of the engineered features that capture temporal and trend information.
cluster_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7', 'Lag1']
```

In [40]:

```
# Create a subset of data for clustering.
X_cluster = daily_expense[cluster_features].copy()

# Standardize the features (recommended for K-Means).
scaler = StandardScaler()
X_cluster_scaled = scaler.fit_transform(X_cluster)
```

In [41]:

```
# Determine the optimal number of clusters using the Elbow Method.
sse = []  # Sum of Squared Errors for each k
sil_scores = []
db_scores = []
ch_scores = []

k_values = range(2, 31)
for k in k_values:
    # Note: In scikit-learn 1.4+ use n_init='auto'
    kmeans = KMeans(n_clusters=k, random_state=42, n_init='auto')
    labels = kmeans.fit_predict(X_cluster_scaled)

    sil = silhouette_score(X_cluster_scaled, labels)
    db = davies_bouldin_score(X_cluster_scaled, labels)
    ch = calinski_harabasz_score(X_cluster_scaled, labels)

    sse.append(kmeans.inertia_)
    sil_scores.append(sil)
    db_scores.append(db)
    ch_scores.append(ch)
```

In [42]:

```
# Get the corresponding SSE value for optimal_k_elbow:
optimal_k_elbow = 6
sse_optimal = sse[optimal_k_elbow - k_values[0]]  # adjust index if k_values does not start at 1

# Compute optimal k based on Silhouette Score and Davies-Bouldin Index
optimal_k_sil = k_values[sil_scores.index(max(sil_scores))]
sil_optimal = max(sil_scores)

optimal_k_db = k_values[db_scores.index(min(db_scores))]
db_optimal = min(db_scores)

optimal_k_ch = k_values[np.argmax(ch_scores)]
ch_optimal_value = max(ch_scores)
```

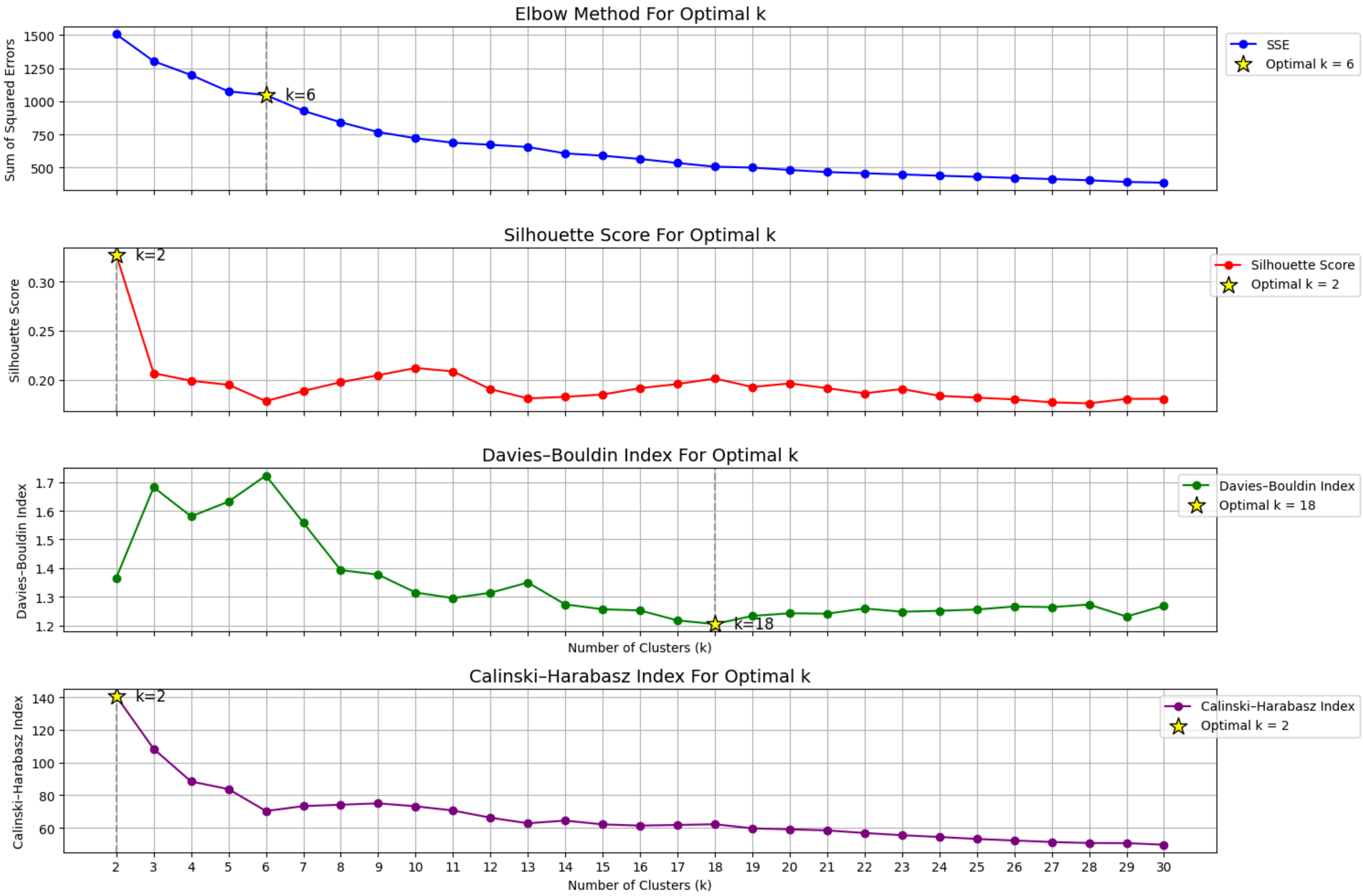
```python
print(f"Optimal k based on Elbow Method: {optimal_k_elbow} (SSE={sse_optimal:.2f})")
print(f"Optimal k based on Silhouette Score: {optimal_k_sil} (Score={sil_optimal:.2f})")
print(f"Optimal k based on Davies-Bouldin Index: {optimal_k_db} (Score={db_optimal:.2f})")
print(f"Optimal k based on Calinski-Harabasz Index: {optimal_k_ch} (Score={ch_optimal_value:.2f})")
```

```
Optimal k based on Elbow Method: 6 (SSE=1047.89)
Optimal k based on Silhouette Score: 2 (Score=0.33)
Optimal k based on Davies-Bouldin Index: 18 (Score=1.20)
Optimal k based on Calinski-Harabasz Index: 2 (Score=140.54)
```

In [43]:

```python
fig, axes = plt.subplots(4, 1, figsize=(15, 10), sharex=True)

# --- 1) Elbow Method (SSE) ---
axes[0].plot(k_values, sse, marker='o', color='b', label="SSE")
axes[0].scatter(
    optimal_k_elbow, sse_optimal,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_elbow}'
)
axes[0].axvline(optimal_k_elbow, color='gray', linestyle='--', alpha=0.7)
axes[0].text(
    optimal_k_elbow + 0.5, sse_optimal,
    f"k={optimal_k_elbow}",
    fontsize=12,
    verticalalignment='center'
)
axes[0].set_ylabel("Sum of Squared Errors")
axes[0].set_title("Elbow Method For Optimal k", fontsize=14)
axes[0].grid(True)
axes[0].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# --- 2) Silhouette Score ---
axes[1].plot(k_values, sil_scores, marker='o', color='r', label="Silhouette Score")
axes[1].scatter(
    optimal_k_sil, sil_optimal,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_sil}'
)
axes[1].axvline(optimal_k_sil, color='gray', linestyle='--', alpha=0.7)
axes[1].text(
    optimal_k_sil + 0.5, sil_optimal,
    f"k={optimal_k_sil}",
    fontsize=12,
    verticalalignment='center'
)
axes[1].set_ylabel("Silhouette Score")
axes[1].set_title("Silhouette Score For Optimal k", fontsize=14)
axes[1].grid(True)
axes[1].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# --- 3) Davies-Bouldin Index ---
axes[2].plot(k_values, db_scores, marker='o', color='g', label="Davies-Bouldin Index")
axes[2].scatter(
    optimal_k_db, db_optimal,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_db}'
)
axes[2].axvline(optimal_k_db, color='gray', linestyle='--', alpha=0.7)
axes[2].text(
    optimal_k_db + 0.5, db_optimal,
    f"k={optimal_k_db}",
    fontsize=12,
    verticalalignment='center'
)
axes[2].set_xlabel("Number of Clusters (k)")
axes[2].set_ylabel("Davies-Bouldin Index")
axes[2].set_title("Davies-Bouldin Index For Optimal k", fontsize=14)
axes[2].grid(True)
axes[2].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

axes[3].plot(k_values, ch_scores, marker='o', color='purple', label="Calinski-Harabasz Index")
axes[3].scatter(
    optimal_k_ch, ch_optimal_value,
    s=200, marker='*',
    color='yellow', edgecolors='black', linewidths=1,
    zorder=10, label=f'Optimal k = {optimal_k_ch}'
)
axes[3].axvline(optimal_k_ch, color='gray', linestyle='--', alpha=0.7)
axes[3].text(
    optimal_k_ch + 0.5, ch_optimal_value,
    f"k={optimal_k_ch}",
    fontsize=12,
    verticalalignment='center'
)

axes[3].set_xlabel("Number of Clusters (k)")
axes[3].set_ylabel("Calinski-Harabasz Index")
axes[3].set_title("Calinski-Harabasz Index For Optimal k", fontsize=14)
axes[3].grid(True)
axes[3].legend(loc='upper right', bbox_to_anchor=(1.13, 1.0))

# Set common x-ticks across subplots
axes[3].set_xticks(k_values)
```

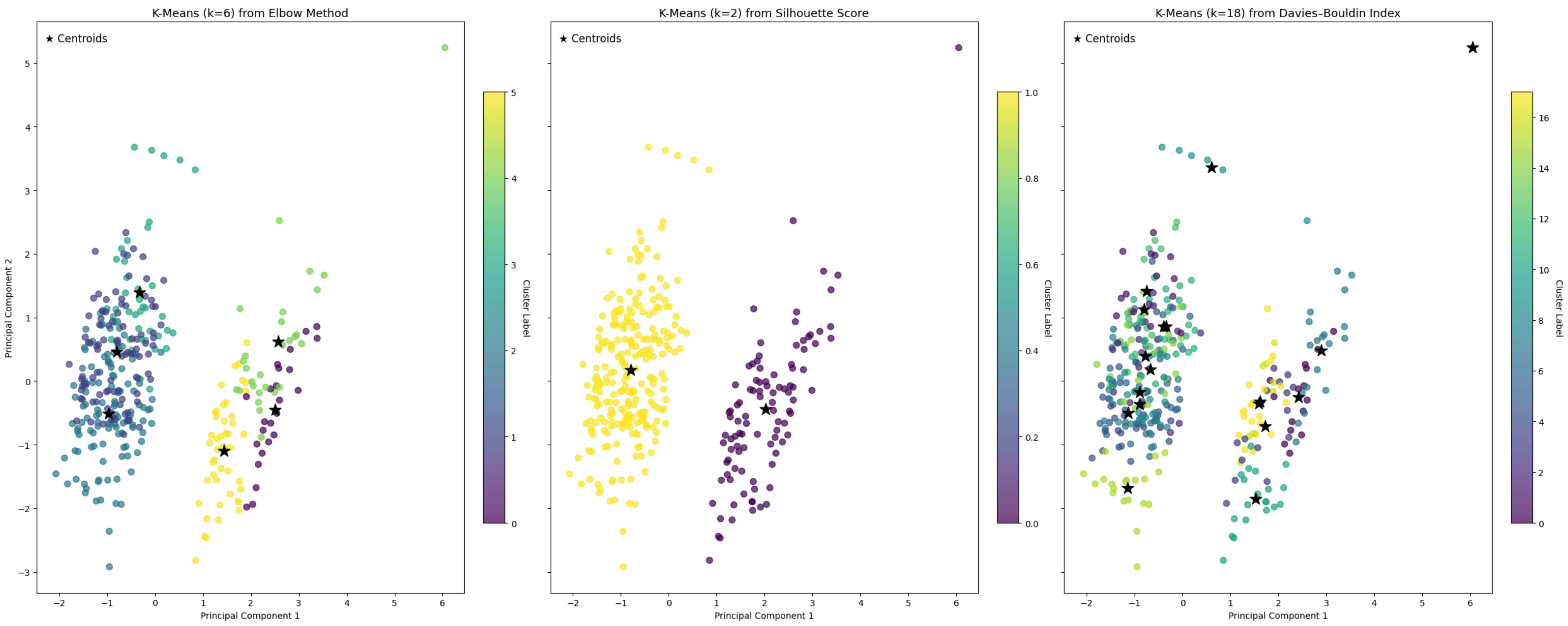
```
plt.tight_layout()
plt.savefig('data/experiment_result/kmeans_comparison_chart.png', dpi=150)
plt.show()
```



In [58]:

```
# Based on the elbow plot, choose an optimal number of clusters.
k_methods = [
    (optimal_k_elbow, "Elbow Method"),
    (optimal_k_sil, "Silhouette Score"),
    (optimal_k_db, "Davies-Bouldin Index")
]

# X_cluster_scaled is your scaled data for clustering
# Perform PCA once for consistent axes
pca = PCA(n_components=2, random_state=42)
X_pca_all = pca.fit_transform(X_cluster_scaled)

fig, axes = plt.subplots(1, 3, figsize=(25, 10), sharex=True, sharey=True)

for i, (k, method_name) in enumerate(k_methods):
    ax = axes[i]

    # Fit KMeans for the current k
    kmeans_model = KMeans(n_clusters=k, random_state=42, n_init='auto')
    clusters = kmeans_model.fit_predict(X_cluster_scaled)

    # Scatter plot of PCA-transformed data
    scatter = ax.scatter(
        X_pca_all[:, 0],
        X_pca_all[:, 1],
        c=clusters,
        cmap='viridis',
        alpha=0.7,
        s=50
    )

    # Plot cluster centroids in PCA space
    centers_2d = pca.transform(kmeans_model.cluster_centers_)
    ax.scatter(
        centers_2d[:, 0],
        centers_2d[:, 1],
        c='black',
        marker='*',
        s=200
    )

    # Title & labels
    ax.set_title(f"K-Means (k={k}) from {method_name}", fontsize=13)
    ax.set_xlabel("Principal Component 1")
    if i == 0:
        ax.set_ylabel("Principal Component 2")

    # Add a colorbar for cluster labels
    cbar = plt.colorbar(scatter, ax=ax, fraction=0.046, pad=0.04)
    cbar.set_label('Cluster Label', rotation=270, labelpad=15)

    # Place a small text label in the top-left corner indicating the black star is for centroids
```

```python
    # Using Unicode star (U+2605) for a small star symbol
    ax.text(
        0.02, 0.98,  # x,y in Axes fraction
        u"\u2605 Centroids",
        transform=ax.transAxes,
        color='black',
        fontsize=12,
        ha='left',
        va='top',
        bbox=dict(facecolor='white', alpha=0.5, edgecolor='none')  # optional background box
    )

plt.tight_layout()
plt.savefig('data\\experiment_result\\kmeans_pca_comparison.png', dpi=200, bbox_inches='tight')
plt.show()
```



In [45]:

```python
features_to_describe = ['DailyExpense', 'Rolling7', 'Lag1']

for k, model in k_methods:
    kmeans_model = KMeans(n_clusters=k, random_state=42, n_init='auto')
    clusters = kmeans_model.fit_predict(X_cluster_scaled)
    col_name = f'Cluster_{model}_{k}'

    print(f'\n{col_name}')
    # Print counts of samples per cluster
    print(pd.Series(clusters).value_counts())

    # Optionally add the cluster labels to your DataFrame for further analysis
    daily_expense[col_name] = clusters

    # Use .describe() to calculate count, mean, std, min, 25%, 50%, 75%, and max
    stats = daily_expense.groupby(col_name)[features_to_describe].describe().round(2)
    print("\nDescriptive statistics:")
    print(stats)

    # Alternatively, if you want to compute specific metrics using .agg():
    custom_stats = daily_expense.groupby(col_name)[features_to_describe].agg({
        'DailyExpense': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'median', lambda x: x.quantile(0.75)],
        'Rolling7': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'median', lambda x: x.quantile(0.75)],
        'Lag1': ['mean', 'std', 'min', 'max', lambda x: x.quantile(0.25), 'median', lambda x: x.quantile(0.75)]
    })
    # Rename the lambda columns for clarity
    custom_stats.columns = ['_'.join(col).strip() for col in custom_stats.columns.values]
    print("\nCustom statistics:")
    print(custom_stats)
```

```
Cluster_Elbow Method_6
2    115
1     96
5     46
3     42
4     28
0     25
Name: count, dtype: int64

Descriptive statistics:
                       DailyExpense                                         \
                              count    mean     std    min     25%    50%
Cluster_Elbow Method_6
0                              25.0  260.92   50.41  195.0  218.00  251.0
1                              96.0  220.35   34.43  168.0  190.75  221.0
2                             115.0  220.47   31.00  168.0  193.50  221.0
3                              42.0  231.38   30.98  170.0  210.00  239.5
4                              28.0  309.14  103.65  195.0  256.75  296.5
5                              46.0  320.24   59.16  191.0  283.00  316.0

                                       Rolling7          ...                \
                           75%    max    count    mean  ...     75%     max
Cluster_Elbow Method_6                                   ...
0                       298.00  396.0     25.0  245.36  ...  253.29  270.14
1                       242.25  304.0     96.0  239.99  ...  249.43  266.29
2                       240.50  303.0    115.0  237.07  ...  245.79  262.57
3                       249.75  289.0     42.0  270.13  ...  271.07  323.71
4                       322.75  759.0     28.0  254.42  ...  261.43  320.57
5                       354.00  451.0     46.0  242.07  ...  252.04  284.14

                         Lag1                                               \
                        count    mean     std    min     25%    50%     75%
Cluster_Elbow Method_6
0                        25.0  334.76   47.95  272.0  300.00  322.0  354.00
1                        96.0  224.66   37.49  168.0  194.00  219.0  240.50
2                       115.0  228.25   37.86  168.0  196.50  230.0  250.00
3                        42.0  266.19   45.20  185.0  242.25  253.5  282.25
4                        28.0  323.96  104.11  193.0  257.25  299.0  354.50
5                        46.0  210.54   28.72  169.0  187.25  200.5  239.00


                          max
Cluster_Elbow Method_6
0                       451.0
1                       369.0
2                       377.0
3                       403.0
4                       759.0
5                       268.0

[6 rows x 24 columns]

Custom statistics:
                        DailyExpense_mean  DailyExpense_std  DailyExpense_min  \
Cluster_Elbow Method_6
0                              260.920000         50.414052               195
1                              220.354167         34.434065               168
2                              220.469565         31.002072               168
3                              231.380952         30.976744               170
4                              309.142857        103.653166               195
5                              320.239130         59.159740               191

                        DailyExpense_max  DailyExpense_<lambda_0>  \
Cluster_Elbow Method_6
0                                    396                   218.00
1                                    304                   190.75
2                                    303                   193.50
3                                    289                   210.00
4                                    759                   256.75
5                                    451                   283.00

                        DailyExpense_median  DailyExpense_<lambda_1>  \
Cluster_Elbow Method_6
0                                     251.0                   298.00
1                                     221.0                   242.25
2                                     221.0                   240.50
3                                     239.5                   249.75
4                                     296.5                   322.75
5                                     316.0                   354.00

                        Rolling7_mean  Rolling7_std  Rolling7_min  ...  \
Cluster_Elbow Method_6                                             ...
0                          245.355714     12.410378    217.857143  ...
1                          239.988095     12.779745    215.000000  ...
2                          237.072588     12.794384    200.500000  ...
3                          270.129252     20.261182    241.714286  ...
4                          254.418367     23.361982    220.714286  ...
5                          242.065217     18.056241    202.428571  ...

                        Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Elbow Method_6
0                                239.428571       247.714286
1                                229.928571       240.142857
2                                228.285714       238.428571
3                                260.035714       265.000000
4                                238.071429       253.500000
5                                231.035714       237.500000
```

```
                        Rolling7_<lambda_1>   Lag1_mean    Lag1_std  Lag1_min  \
Cluster_Elbow Method_6
0                                253.285714  334.760000   47.946394     272.0
1                                249.428571  224.656250   37.492688     168.0
2                                245.785714  228.249605   37.860765     168.0
3                                271.071429  266.190476   45.203193     185.0
4                                261.428571  323.964286  104.108733     193.0
5                                252.035714  210.543478   28.721426     169.0

                        Lag1_max  Lag1_<lambda_0>  Lag1_median  \
Cluster_Elbow Method_6
0                          451.0           300.00        322.0
1                          369.0           194.00        219.0
2                          377.0           196.50        230.0
3                          403.0           242.25        253.5
4                          759.0           257.25        299.0
5                          268.0           187.25        200.5

                        Lag1_<lambda_1>
Cluster_Elbow Method_6
0                                354.00
1                                240.50
2                                250.00
3                                282.25
4                                354.50
5                                239.00

[6 rows x 21 columns]

Cluster_Silhouette Score_2
1    253
0     99
Name: count, dtype: int64

Descriptive statistics:
                           DailyExpense                                      \
                                  count    mean    std    min    25%    50%
Cluster_Silhouette Score_2
0                                  99.0  302.12  76.10  191.0  252.0  297.0
1                                 253.0  222.24  32.47  168.0  192.0  222.0

                                        Rolling7          ...          \
                              75%    max    count    mean  ...      75%
Cluster_Silhouette Score_2                                 ...
0                           326.0  759.0     99.0  246.39  ...  254.79
1                           244.0  304.0    253.0  243.67  ...  253.71

                                    Lag1                                     \
                              max  count    mean    std    min    25%    50%
Cluster_Silhouette Score_2
0                           320.57   99.0  273.99  86.45  169.0  202.0  253.0
1                           323.71  253.0  233.18  41.61  168.0  201.0  231.0


                              75%    max
Cluster_Silhouette Score_2
0                           319.5  759.0
1                           253.0  403.0

[2 rows x 24 columns]

Custom statistics:
                            DailyExpense_mean  DailyExpense_std  \
Cluster_Silhouette Score_2
0                                  302.121212         76.097316
1                                  222.237154         32.474885

                            DailyExpense_min  DailyExpense_max  \
Cluster_Silhouette Score_2
0                                        191               759
1                                        168               304

                            DailyExpense_<lambda_0>  DailyExpense_median  \
Cluster_Silhouette Score_2
0                                             252.0                297.0
1                                             192.0                222.0

                            DailyExpense_<lambda_1>  Rolling7_mean  \
Cluster_Silhouette Score_2
0                                             326.0     246.389971
1                                             244.0     243.666535

                            Rolling7_std  Rolling7_min  ...  \
Cluster_Silhouette Score_2                              ...
0                              19.111819    202.428571  ...
1                              18.550536    200.500000  ...

                            Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Silhouette Score_2
0                                    234.357143       245.571429
1                                    231.571429       242.714286

                            Rolling7_<lambda_1>   Lag1_mean   Lag1_std  \
Cluster_Silhouette Score_2
0                                    254.785714  273.989899  86.446928
1                                    253.714286  233.184603  41.607929
```

```
                              Lag1_min  Lag1_max  Lag1_<lambda_0>  Lag1_median  \
Cluster_Silhouette Score_2
0                                169.0     759.0            202.0        253.0
1                                168.0     403.0            201.0        231.0

                              Lag1_<lambda_1>
Cluster_Silhouette Score_2
0                                       319.5
1                                       253.0

[2 rows x 21 columns]

Cluster_Davies-Bouldin Index_18
8     41
6     34
4     32
11    26
14    24
15    20
12    20
10    20
7     19
17    18
1     18
2     18
0     15
3     15
5     15
16    10
9      6
13     1
Name: count, dtype: int64

Descriptive statistics:
                                DailyExpense                                 \
                                       count    mean     std    min     25%
Cluster_Davies-Bouldin Index_18
0                                       15.0  282.87   61.97  203.0  233.50
1                                       18.0  234.83   34.50  180.0  214.25
2                                       18.0  234.83   28.21  170.0  231.00
3                                       15.0  294.93   40.14  234.0  263.00
4                                       32.0  222.78   28.97  171.0  202.00
5                                       15.0  226.53   34.19  178.0  201.50
6                                       34.0  222.82   34.13  168.0  193.00
7                                       19.0  276.26   54.27  196.0  233.00
8                                       41.0  212.34   33.34  168.0  189.00
9                                        6.0  333.50  208.53  243.0  244.50
10                                      20.0  281.75   49.56  195.0  250.50
11                                      26.0  231.31   32.14  169.0  206.50
12                                      20.0  214.50   30.09  179.0  190.00
13                                       1.0  253.00     NaN  253.0  253.00
14                                      24.0  221.29   34.76  173.0  187.25
15                                      20.0  205.30   26.35  175.0  187.25
16                                      10.0  366.20   55.09  290.0  325.25
17                                      18.0  315.83   69.13  191.0  278.00

                                                      Rolling7          ...  \
                                   50%     75%    max    count    mean  ...
Cluster_Davies-Bouldin Index_18                                         ...
0                                280.0  320.50  400.0     15.0  248.71  ...
1                                236.5  253.50  297.0     18.0  249.15  ...
2                                234.5  248.75  278.0     18.0  257.21  ...
3                                297.0  319.50  362.0     15.0  235.21  ...
4                                230.0  241.50  280.0     32.0  236.32  ...
5                                224.0  244.00  289.0     15.0  262.09  ...
6                                226.0  243.25  303.0     34.0  240.13  ...
7                                277.0  318.50  403.0     19.0  254.11  ...
8                                201.0  223.00  304.0     41.0  233.30  ...
9                                248.0  256.75  759.0      6.0  321.07  ...
10                               279.0  312.50  377.0     20.0  231.76  ...
11                               236.0  253.50  276.0     26.0  258.28  ...
12                               211.0  227.75  289.0     20.0  247.88  ...
13                               253.0  253.00  253.0      1.0  311.29  ...
14                               225.0  244.00  288.0     24.0  236.31  ...
15                               199.0  215.50  268.0     20.0  218.00  ...
16                               370.0  405.00  436.0     10.0  267.54  ...
17                               316.5  365.25  451.0     18.0  242.40  ...

                                                 Lag1                        \
                                    75%     max count    mean    std    min
Cluster_Davies-Bouldin Index_18
0                                253.93  269.14  15.0  318.27  41.56  236.0
1                                255.75  261.86  18.0  255.28  31.78  209.0
2                                268.32  278.00  18.0  256.17  31.02  218.0
3                                240.79  261.14  15.0  242.53  47.92  174.0
4                                242.18  250.86  32.0  238.34  27.86  175.0
5                                266.00  272.86  15.0  254.93  39.15  185.0
6                                248.86  262.57  34.0  211.32  27.78  168.0
7                                261.43  283.57  19.0  368.11  45.77  301.0
8                                239.43  253.29  41.0  209.32  27.94  168.0
9                                322.54  323.71   6.0  251.17   5.64  244.0
10                               240.42  249.86  20.0  235.55  39.09  185.0
11                               264.04  267.86  26.0  234.27  27.93  187.0
12                               252.64  271.14  20.0  306.50  46.95  245.0
13                               311.29  311.29   1.0  759.00    NaN  759.0
14                               243.36  253.71  24.0  213.46  30.76  173.0
```

```
15                              222.25  233.14  20.0  198.79  22.20  171.0
16                              275.82  284.14  10.0  205.10  26.90  170.0
17                              250.46  274.29  18.0  219.06  37.03  169.0


                                   25%    50%     75%    max
Cluster_Davies-Bouldin Index_18
0                               295.00  318.0  342.00  400.0
1                               235.25  249.0  279.75  322.0
2                               232.50  247.5  267.00  324.0
3                               196.00  245.0  282.50  317.0
4                               229.75  240.0  253.00  309.0
5                               231.00  248.0  286.00  318.0
6                               190.00  210.0  229.25  271.0
7                               328.00  362.0  400.00  451.0
8                               189.00  203.0  223.00  314.0
9                               247.00  251.5  254.50  259.0
10                              200.00  233.5  256.75  314.0
11                              213.25  238.0  258.50  276.0
12                              279.25  295.0  324.25  403.0
13                              759.00  759.0  759.00  759.0
14                              190.25  211.5  228.25  288.0
15                              181.50  196.0  206.75  248.0
16                              191.00  193.5  225.00  250.0
17                              186.50  219.0  247.25  296.0

[18 rows x 24 columns]

Custom statistics:
                                DailyExpense_mean  DailyExpense_std  \
Cluster_Davies-Bouldin Index_18
0                                      282.866667         61.974496
1                                      234.833333         34.496803
2                                      234.833333         28.213993
3                                      294.933333         40.143433
4                                      222.781250         28.974666
5                                      226.533333         34.188274
6                                      222.823529         34.129407
7                                      276.263158         54.267693
8                                      212.341463         33.336624
9                                      333.500000        208.532731
10                                     281.750000         49.558976
11                                     231.307692         32.138163
12                                     214.500000         30.091964
13                                     253.000000               NaN
14                                     221.291667         34.758145
15                                     205.300000         26.352070
16                                     366.200000         55.087405
17                                     315.833333         69.130099

                                DailyExpense_min  DailyExpense_max  \
Cluster_Davies-Bouldin Index_18
0                                            203               400
1                                            180               297
2                                            170               278
3                                            234               362
4                                            171               280
5                                            178               289
6                                            168               303
7                                            196               403
8                                            168               304
9                                            243               759
10                                           195               377
11                                           169               276
12                                           179               289
13                                           253               253
14                                           173               288
15                                           175               268
16                                           290               436
17                                           191               451

                                DailyExpense_<lambda_0>  DailyExpense_median  \
Cluster_Davies-Bouldin Index_18
0                                                233.50                280.0
1                                                214.25                236.5
2                                                231.00                234.5
3                                                263.00                297.0
4                                                202.00                230.0
5                                                201.50                224.0
6                                                193.00                226.0
7                                                233.00                277.0
8                                                189.00                201.0
9                                                244.50                248.0
10                                               250.50                279.0
11                                               206.50                236.0
12                                               190.00                211.0
13                                               253.00                253.0
14                                               187.25                225.0
15                                               187.25                199.0
16                                               325.25                370.0
17                                               278.00                316.5

                                DailyExpense_<lambda_1>  Rolling7_mean  \
Cluster_Davies-Bouldin Index_18
0                                                320.50     248.714286
1                                                253.50     249.150794
2                                                248.75     257.214286
```

```
3                                  319.50     235.209524
4                                  241.50     236.316667
5                                  244.00     262.085714
6                                  243.25     240.126050
7                                  318.50     254.112782
8                                  223.00     233.299652
9                                  256.75     321.071429
10                                 312.50     231.758929
11                                 253.50     258.280220
12                                 227.75     247.878571
13                                 253.00     311.285714
14                                 244.00     236.309524
15                                 215.50     217.996429
16                                 405.00     267.542857
17                                 365.25     242.396825

                            Rolling7_std  Rolling7_min  ...  \
Cluster_Davies-Bouldin Index_18                          ...
0                              10.785748    230.714286  ...
1                               9.507370    224.428571  ...
2                              11.907532    242.714286  ...
3                              11.739007    213.428571  ...
4                               7.830816    223.000000  ...
5                               6.108542    251.142857  ...
6                              10.610945    217.857143  ...
7                              14.499791    222.428571  ...
8                               9.881636    215.000000  ...
9                               2.110711    318.285714  ...
10                             12.252890    202.428571  ...
11                              6.475383    242.857143  ...
12                             11.412703    229.571429  ...
13                                   NaN    311.285714  ...
14                              9.853647    216.428571  ...
15                              7.373439    200.500000  ...
16                             10.467269    252.285714  ...
17                             12.760943    221.142857  ...

                            Rolling7_<lambda_0>  Rolling7_median  \
Cluster_Davies-Bouldin Index_18
0                                    240.642857       247.857143
1                                    246.107143       248.785714
2                                    247.642857       253.500000
3                                    230.357143       235.571429
4                                    229.250000       237.285714
5                                    258.857143       262.714286
6                                    232.678571       239.857143
7                                    247.071429       254.000000
8                                    224.857143       234.714286
9                                    319.500000       321.285714
10                                   223.464286       231.928571
11                                   254.642857       258.642857
12                                   240.750000       244.714286
13                                   311.285714       311.285714
14                                   228.857143       235.500000
15                                   214.535714       219.000000
16                                   259.964286       267.071429
17                                   234.892857       240.214286

                            Rolling7_<lambda_1>   Lag1_mean   Lag1_std  \
Cluster_Davies-Bouldin Index_18
0                                    253.928571  318.266667  41.561429
1                                    255.750000  255.277778  31.783839
2                                    268.321429  256.166667  31.020392
3                                    240.785714  242.533333  47.917886
4                                    242.178571  238.343750  27.862114
5                                    266.000000  254.933333  39.147097
6                                    248.857143  211.323529  27.781311
7                                    261.428571  368.105263  45.771042
8                                    239.428571  209.317073  27.938718
9                                    322.535714  251.166667   5.636193
10                                   240.419643  235.550000  39.088934
11                                   264.035714  234.269231  27.930711
12                                   252.642857  306.500000  46.948460
13                                   311.285714  759.000000        NaN
14                                   243.357143  213.458333  30.764151
15                                   222.250000  198.785227  22.202398
16                                   275.821429  205.100000  26.904977
17                                   250.464286  219.055556  37.025384

                            Lag1_min  Lag1_max  Lag1_<lambda_0>  \
Cluster_Davies-Bouldin Index_18
0                              236.0     400.0           295.00
1                              209.0     322.0           235.25
2                              218.0     324.0           232.50
3                              174.0     317.0           196.00
4                              175.0     309.0           229.75
5                              185.0     318.0           231.00
6                              168.0     271.0           190.00
7                              301.0     451.0           328.00
8                              168.0     314.0           189.00
9                              244.0     259.0           247.00
10                             185.0     314.0           200.00
11                             187.0     276.0           213.25
12                             245.0     403.0           279.25
13                             759.0     759.0           759.00
14                             173.0     288.0           190.25
15                             171.0     248.0           181.50
```

```
16                               170.0     250.0          191.00
17                               169.0     296.0          186.50

                             Lag1_median  Lag1_<lambda_1>
Cluster_Davies-Bouldin Index_18
0                                  318.0          342.00
1                                  249.0          279.75
2                                  247.5          267.00
3                                  245.0          282.50
4                                  240.0          253.00
5                                  248.0          286.00
6                                  210.0          229.25
7                                  362.0          400.00
8                                  203.0          223.00
9                                  251.5          254.50
10                                 233.5          256.75
11                                 238.0          258.50
12                                 295.0          324.25
13                                 759.0          759.00
14                                 211.5          228.25
15                                 196.0          206.75
16                                 193.5          225.00
17                                 219.0          247.25

[18 rows x 21 columns]
```

Comparing How Cluster Assignments Change from k to k+1

In [46]:

```
assignment_changes = []
previous_labels = None
previous_k = None

for k in k_values:
    kmeans = KMeans(n_clusters=k, random_state=42, n_init='auto')
    labels = kmeans.fit_predict(X_cluster_scaled)

    if previous_labels is not None:
        # Count how many labels differ
        changes = np.sum(labels != previous_labels)
        fraction_changed = changes / len(labels)
        assignment_changes.append((previous_k, k, fraction_changed))

    previous_labels = labels
    previous_k = k

# Convert to a DataFrame for easier reading
changes_df = pd.DataFrame(assignment_changes, columns=["k_old", "k_new", "fraction_changed"])
print("Fraction of points changing clusters when going from k to k+1:")
print(changes_df)
```

```
Fraction of points changing clusters when going from k to k+1:
    k_old  k_new  fraction_changed
0       2      3          0.352273
1       3      4          0.116477
2       4      5          0.116477
3       5      6          0.221591
4       6      7          0.301136
5       7      8          0.735795
6       8      9          0.187500
7       9     10          0.090909
8      10     11          0.116477
9      11     12          0.076705
10     12     13          0.150568
11     13     14          0.025568
12     14     15          0.142045
13     15     16          0.110795
14     16     17          0.036932
15     17     18          0.073864
16     18     19          0.079545
17     19     20          0.056818
18     20     21          0.792614
19     21     22          0.068182
20     22     23          0.076705
21     23     24          0.085227
22     24     25          0.105114
23     25     26          0.059659
24     26     27          0.062500
25     27     28          0.088068
26     28     29          0.065341
27     29     30          0.076705
```

In [47]:

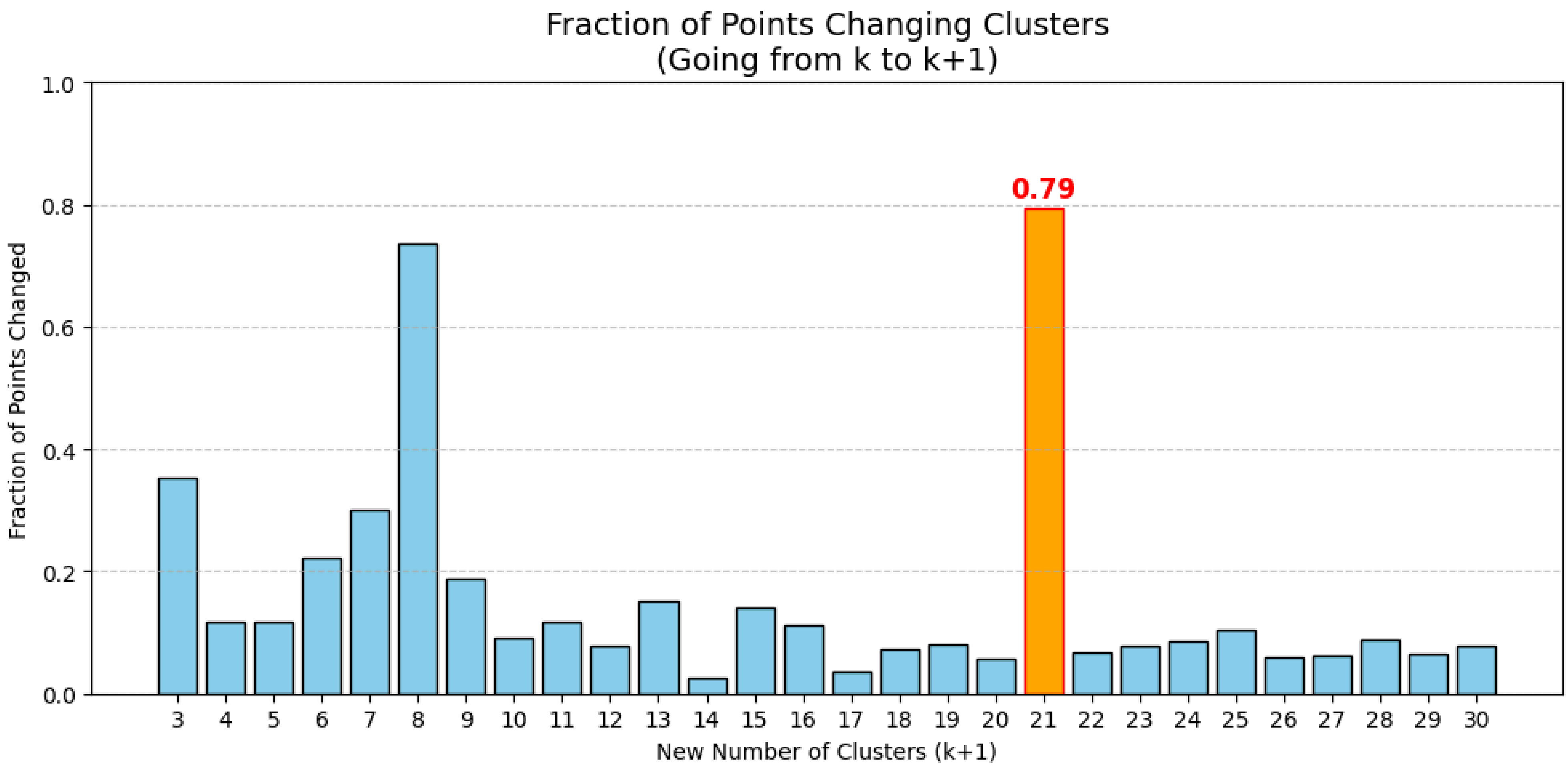
```
# Identify the row with the maximum fraction_changed
max_idx = changes_df['fraction_changed'].idxmax()
max_k = changes_df.loc[max_idx, 'k_new']
max_val = changes_df.loc[max_idx, 'fraction_changed']
print(f"Maximum change in cluster assignments occurs when going from k={max_k-1} to k={max_k} ({max_val:.2%} change)")
```

```
Maximum change in cluster assignments occurs when going from k=20 to k=21 (79.26% change)
```

In [48]:

```
plt.figure(figsize=(10, 5))

# Create a bar chart
bars = plt.bar(
    changes_df['k_new'],
    changes_df['fraction_changed'],
    color='skyblue',
```

```
    edgecolor='black'
)

# Highlight the bar with the maximum fraction changed
for bar in bars:
    bar_center = bar.get_x() + bar.get_width()/2
    if np.isclose(bar_center, max_k, atol=0.1):
        bar.set_color('orange')
        bar.set_edgecolor('red')

# Annotate the highest bar
plt.text(
    max_k,
    max_val + 0.02,  # Slightly above the bar
    f"{max_val:.2f}",
    ha='center',
    color='red',
    fontsize=12,
    fontweight='bold'
)

plt.title("Fraction of Points Changing Clusters\n(Going from k to k+1)", fontsize=14)
plt.xlabel("New Number of Clusters (k+1)")
plt.ylabel("Fraction of Points Changed")
plt.ylim(0, 1)  # Fractions range from 0 to 1
plt.xticks(changes_df['k_new'])
plt.grid(axis='y', linestyle='--', alpha=0.7)
plt.tight_layout()
plt.savefig('data/experiment_result/cluster_assignment_changes.png', dpi=150)
plt.show()
```



Cluster on First X Month, Then See How The Data Fits

In [49]:

```
cutoff_date = pd.to_datetime("2025-06-01")
train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

# Scale them separately or together, depending on your approach
scaler = StandardScaler()
train_features = train_data[cluster_features]
test_features = test_data[cluster_features]

X_train_scaled = scaler.fit_transform(train_features)
X_test_scaled = scaler.transform(test_features)

k = 6  # or your chosen k
kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
kmeans_split.fit(X_train_scaled)

# Assign clusters to training data
train_clusters = kmeans_split.predict(X_train_scaled)
train_data['Cluster'] = train_clusters

# Assign clusters to test data
test_clusters = kmeans_split.predict(X_test_scaled)
test_data['Cluster'] = test_clusters

print("Training cluster distribution:")
print(train_data['Cluster'].value_counts())

print("\nTest cluster distribution:")
print(test_data['Cluster'].value_counts())
```

```
Training cluster distribution:
Cluster
0    31
5    26
3    26
1    24
4    22
2    16
Name: count, dtype: int64

Test cluster distribution:
Cluster
4    115
3     47
1     23
2     22
Name: count, dtype: int64
```

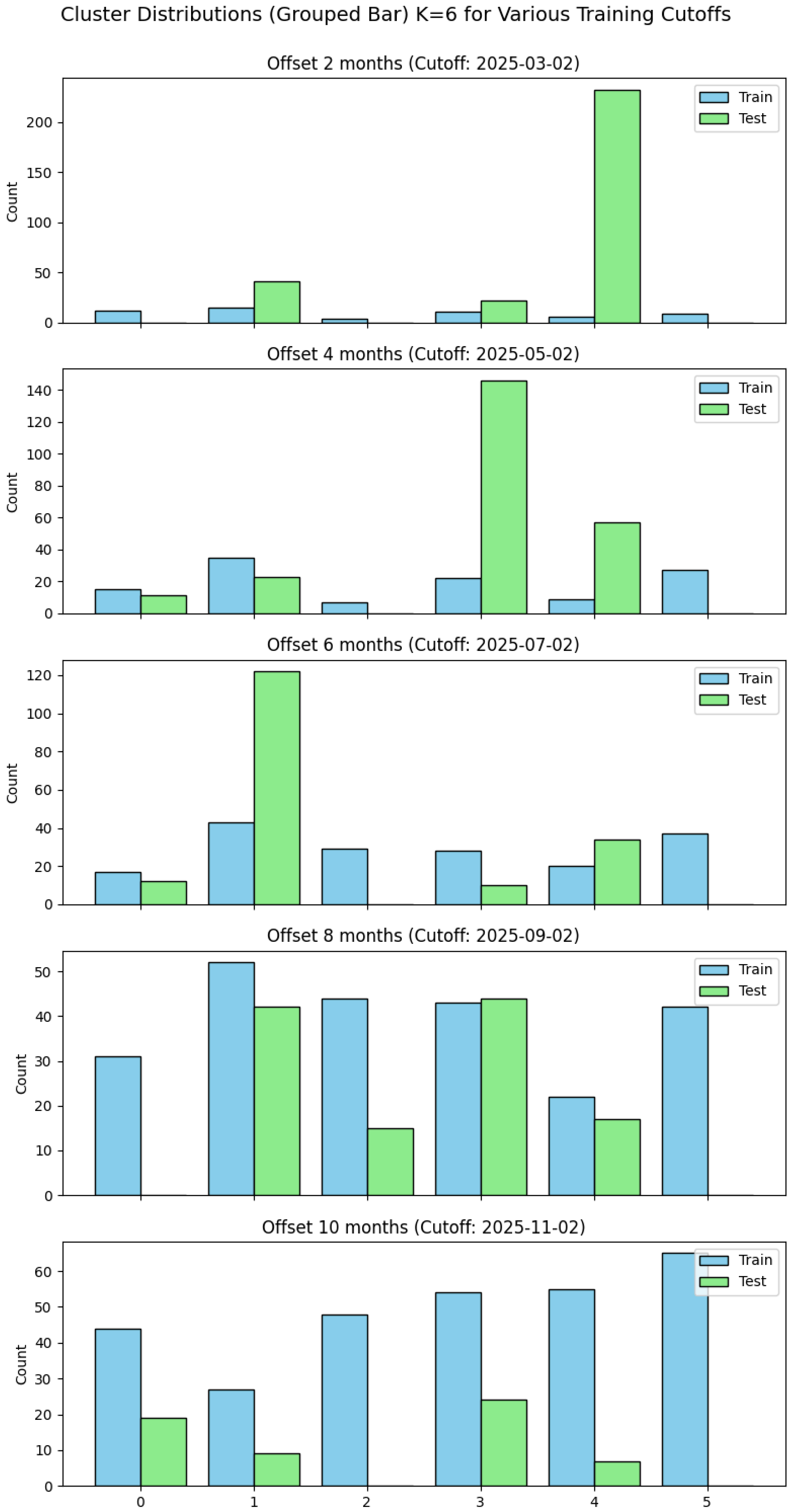
Check for 2, 4, 6, 8, 10 Months

In [50]:
```
offsets = [2, 4, 6, 8, 10]

start_date = daily_expense['Date'].min()
split_results = {}
```

In [51]:
```
# Loop over each offset to compute cluster assignments and count samples per cluster
for offset in offsets:
    cutoff_date = start_date + pd.DateOffset(months=offset)

    # Split the data
    train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
    test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

    # Scale the features (fit scaler on train and transform test)
    scaler = StandardScaler()
    train_features = train_data[cluster_features]
    test_features = test_data[cluster_features]

    X_train_scaled = scaler.fit_transform(train_features)
    X_test_scaled = scaler.transform(test_features)

    # Fit K-Means on training data with chosen k (e.g., 6)
    k = 6
    kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
    kmeans_split.fit(X_train_scaled)

    # Predict clusters for training and test data
    train_clusters = kmeans_split.predict(X_train_scaled)
    test_clusters = kmeans_split.predict(X_test_scaled)

    train_data['Cluster'] = train_clusters
    test_data['Cluster'] = test_clusters

    # Count samples per cluster
    train_counts = train_data['Cluster'].value_counts().sort_index()
    test_counts = test_data['Cluster'].value_counts().sort_index()

    # Store results for this offset
    split_results[offset] = {
        'cutoff_date': cutoff_date,
        'train_counts': train_counts,
        'test_counts': test_counts
    }
```

In [52]:
```
fig, axes = plt.subplots(nrows=len(offsets), ncols=1, figsize=(8, 3 * len(offsets)), sharex=True)

x_positions = np.arange(k)  # k is the number of clusters, e.g. 6
bar_width = 0.4

for i, offset in enumerate(offsets):
    result = split_results[offset]
    cutoff_date = result['cutoff_date']
    train_counts = result['train_counts']
    test_counts = result['test_counts']

    # Ensure both Series cover all clusters [0..k-1], fill missing with 0
    train_counts = train_counts.reindex(range(k), fill_value=0)
    test_counts = test_counts.reindex(range(k), fill_value=0)

    ax = axes[i] if len(offsets) > 1 else axes  # handle single-subplot case

    # Plot grouped bars
    ax.bar(x_positions - bar_width/2, train_counts.values, width=bar_width,
           color='skyblue', edgecolor='black', label='Train')
    ax.bar(x_positions + bar_width/2, test_counts.values, width=bar_width,
           color='lightgreen', edgecolor='black', label='Test')

    ax.set_xticks(x_positions)
    ax.set_xticklabels([str(c) for c in range(k)])
    ax.set_ylabel("Count")
    ax.set_title(f"Offset {offset} months (Cutoff: {cutoff_date.date()})")
    ax.legend(loc='upper right')

plt.suptitle("Cluster Distributions (Grouped Bar) K=6 for Various Training Cutoffs", fontsize=14, y=1)
plt.savefig('data/experiment_result/train_test_cluster_distribution_k_6.png', dpi=150, bbox_inches='tight')
```

```
plt.tight_layout()
plt.show()
```



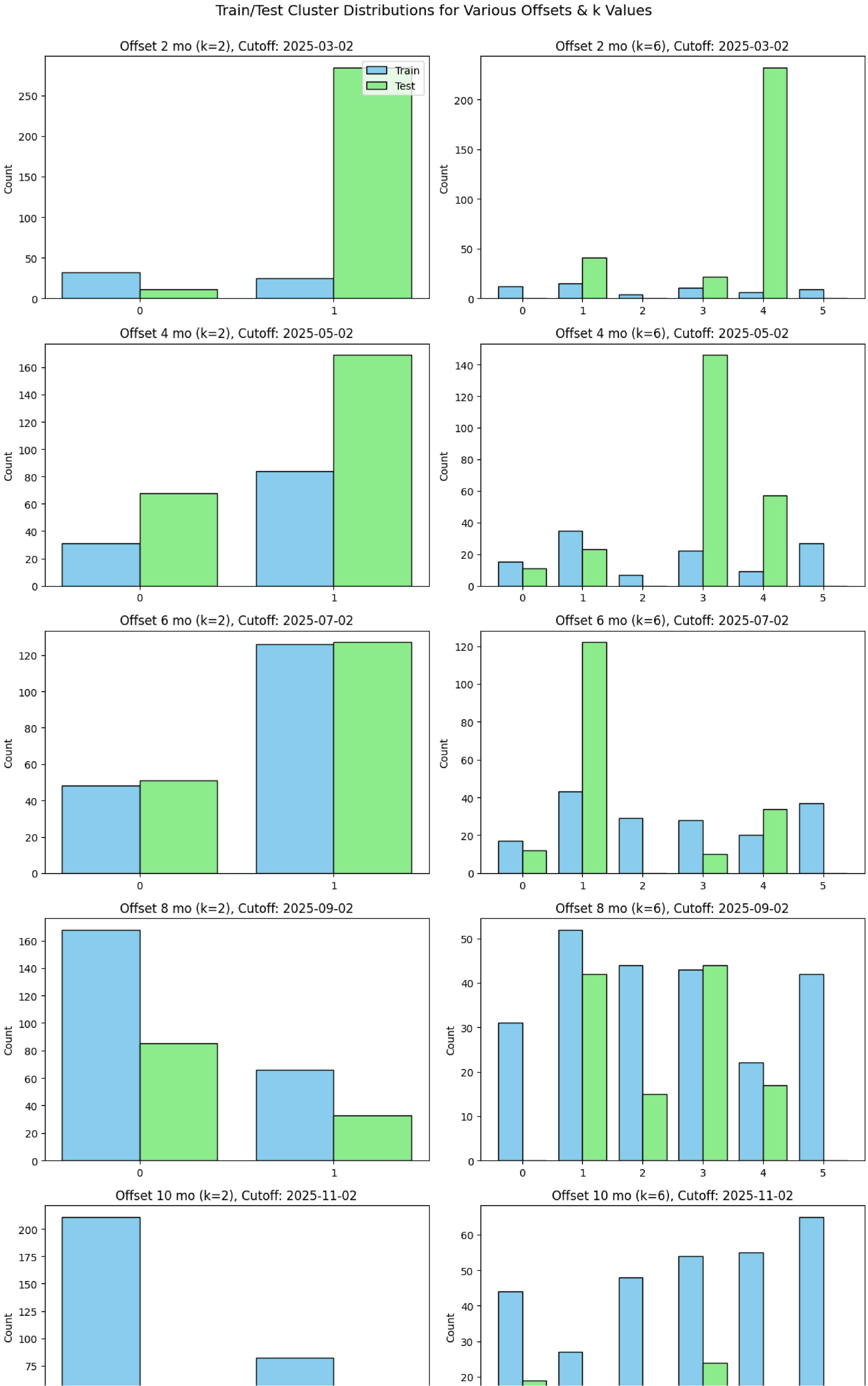
Check for 2, 4, 6, 8, 10 Months Using K=6 and K=2

In [54]:
```
offsets = [2, 4, 6, 8, 10]
k_vals = [2, 6]
start_date = daily_expense['Date'].min()

split_results = {offset: {} for offset in offsets}
```

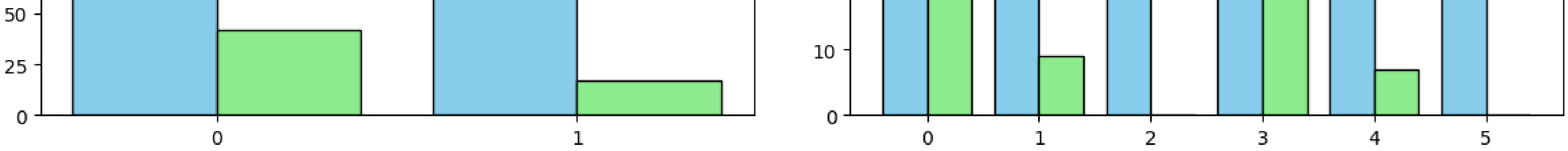
In [55]:
```
for offset in offsets:
    cutoff_date = start_date + pd.DateOffset(months=offset)

    # Split the data
    train_data = daily_expense[daily_expense['Date'] < cutoff_date].copy()
    test_data = daily_expense[daily_expense['Date'] >= cutoff_date].copy()

    # Scale the features (fit scaler on train, then transform test)
    scaler = StandardScaler()
    train_features = train_data[cluster_features]
    test_features = test_data[cluster_features]

    X_train_scaled = scaler.fit_transform(train_features)
    X_test_scaled = scaler.transform(test_features)

    for k in k_vals:
        # Fit K-Means on training data
        kmeans_split = KMeans(n_clusters=k, random_state=42, n_init='auto')
        kmeans_split.fit(X_train_scaled)

        # Predict clusters for training and test data
        train_clusters = kmeans_split.predict(X_train_scaled)
        test_clusters = kmeans_split.predict(X_test_scaled)

        # Count samples per cluster
        train_counts = pd.Series(train_clusters).value_counts().sort_index()
        test_counts = pd.Series(test_clusters).value_counts().sort_index()

        # Store the results
        split_results[offset][k] = {
            'cutoff_date': cutoff_date,
            'train_counts': train_counts,
            'test_counts': test_counts
        }
```

In [56]:
```
n_offsets = len(offsets)
n_kvals = len(k_vals)
fig, axes = plt.subplots(
    nrows=n_offsets, ncols=n_kvals,
    figsize=(12, 4 * n_offsets),
    sharex=False, sharey=False
)

bar_width = 0.4

for i, offset in enumerate(offsets):
    for j, k in enumerate(k_vals):
        ax = axes[i, j] if n_offsets > 1 else axes[j]  # handle single-row case
        results = split_results[offset][k]

        cutoff_date = results['cutoff_date']
        train_counts = results['train_counts']
        test_counts = results['test_counts']

        # Ensure we have a count for each cluster index [0..k-1]
        train_counts = train_counts.reindex(range(k), fill_value=0)
        test_counts = test_counts.reindex(range(k), fill_value=0)

        x_positions = np.arange(k)

        # Grouped bar chart: train vs test
        ax.bar(
            x_positions - bar_width/2,
            train_counts.values,
            width=bar_width,
            color='skyblue',
            edgecolor='black',
            label='Train'
        )
        ax.bar(
            x_positions + bar_width/2,
            test_counts.values,
            width=bar_width,
            color='lightgreen',
            edgecolor='black',
            label='Test'
        )

        ax.set_xticks(x_positions)
        ax.set_xticklabels([str(c) for c in range(k)])
        ax.set_ylabel("Count")
        ax.set_title(f"Offset {offset} mo (k={k}), Cutoff: {cutoff_date.date()}")

        if i == 0 and j == 0:
            ax.legend(loc='upper right')

plt.suptitle("Train/Test Cluster Distributions for Various Offsets & k Values", fontsize=14, y=1)
```

```
plt.savefig('data/experiment_result/train_test_cluster_distribution_multiple_offsets_and_k_vals.png', dpi=150, bbox_inches='tight')
plt.tight_layout()
plt.show()
```



Train/Test Cluster Distributions for Various Offsets & k Values
Offset 2 mo (k=2), Cutoff: 2025-03-02
Offset 2 mo (k=6), Cutoff: 2025-03-02
Offset 4 mo (k=2), Cutoff: 2025-05-02
Offset 4 mo (k=6), Cutoff: 2025-05-02
Offset 6 mo (k=2), Cutoff: 2025-07-02
Offset 6 mo (k=6), Cutoff: 2025-07-02
Offset 8 mo (k=2), Cutoff: 2025-09-02
Offset 8 mo (k=6), Cutoff: 2025-09-02
Offset 10 mo (k=2), Cutoff: 2025-11-02
Offset 10 mo (k=6), Cutoff: 2025-11-02
Train
Test
Count



GMM (Gaussian Mixture Model)

In [ ]:
```
from sklearn.mixture import GaussianMixture
from sklearn.preprocessing import StandardScaler
import seaborn as sns
```

In [ ]:
```
cluster_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day', 'Rolling7', 'Lag1']
```

In [ ]:
```
scaler = StandardScaler()
X_cluster_scaled = scaler.fit_transform(daily_expense[cluster_features])
```

In [ ]:
```
n_components = 6
```

In [ ]:
```
gmm = GaussianMixture(n_components=n_components, random_state=42)
gmm.fit(X_cluster_scaled)
```

Out[ ]:
```
▾ GaussianMixture
GaussianMixture(n_components=6, random_state=42)
```

In [ ]:
```
# Predict cluster labels and get the membership probabilities
gmm_labels = gmm.predict(X_cluster_scaled)
gmm_probabilities = gmm.predict_proba(X_cluster_scaled)

# Add the GMM cluster assignments and probabilities to your DataFrame
daily_expense['GMM_Cluster'] = gmm_labels
# For example, you might store the maximum probability (confidence) for each point
daily_expense['GMM_Confidence'] = gmm_probabilities.max(axis=1)

print("GMM Cluster distribution:")
print(daily_expense['GMM_Cluster'].value_counts())

# Optionally, display the first few rows with cluster labels and confidence
print(daily_expense[['Date', 'DailyExpense', 'GMM_Cluster', 'GMM_Confidence']].head())
```

```
GMM Cluster distribution:
GMM_Cluster
2    132
1     99
0     49
5     47
3     22
4      3
Name: count, dtype: int64
        Date  DailyExpense  GMM_Cluster  GMM_Confidence
1 2025-01-02           206            2        0.999993
2 2025-01-03           195            2        0.999993
3 2025-01-04           295            5        1.000000
4 2025-01-05           263            0        1.000000
5 2025-01-06           230            2        0.999994
```

Descriptive Statistics

In [ ]:
```
features_numeric = ['DailyExpense', 'Rolling7', 'Lag1']
print("=== Numeric Descriptive Statistics by Cluster ===")
profile_stats = daily_expense.groupby('GMM_Cluster')[features_numeric].describe().round(2)
print(profile_stats)
```

```
=== Numeric Descriptive Statistics by Cluster ===
            DailyExpense                                                \
                   count    mean     std    min     25%    50%     75%    max
GMM_Cluster
0                   49.0  268.43   54.05  191.0  222.00  258.0  309.00  403.0
1                   99.0  220.91   34.25  169.0  191.00  221.0  243.00  304.0
2                  132.0  223.13   31.51  168.0  194.75  227.0  246.25  303.0
3                   22.0  222.86   31.15  168.0  200.50  217.5  245.75  289.0
4                    3.0  442.00  276.22  253.0  283.50  314.0  536.50  759.0
5                   47.0  328.32   52.77  245.0  292.50  321.0  358.00  451.0

            Rolling7         ...                  Lag1                 \
               count    mean ...     75%     max count    mean     std
GMM_Cluster                  ...
0               49.0  244.03 ...  254.00  283.57  49.0  322.84   57.82
1               99.0  242.64 ...  254.57  268.57  99.0  220.87   29.11
2              132.0  240.41 ...  249.50  278.00 132.0  230.63   36.74
3               22.0  267.84 ...  272.43  323.71  22.0  303.95   48.94
4                3.0  297.67 ...  315.93  320.57   3.0  419.33  294.21
5               47.0  245.58 ...  254.64  284.14  47.0  213.79   32.34


               min     25%    50%     75%    max
GMM_Cluster
0            185.0  289.00  318.0  354.00  451.0
1            168.0  194.50  220.0  240.00  288.0
2            168.0  201.25  231.5  253.00  324.0
3            244.0  254.50  300.5  321.25  403.0
4            244.0  249.50  255.0  507.00  759.0
5            169.0  191.00  201.0  243.50  288.0

[6 rows x 24 columns]
```

In [ ]:

```python
# For categorical features, calculate percentages (e.g., DayOfWeek, IsWeekend)
categorical_features = ['DayOfWeek', 'IsWeekend', 'Month', 'Day']
print("\n=== Categorical Distribution by Cluster (in %): ===")
cat_profile = daily_expense.groupby('GMM_Cluster')[categorical_features].agg(
    lambda x: x.value_counts(normalize=True).mul(100).round(2).to_dict()
)
print(cat_profile)
```

```
=== Categorical Distribution by Cluster (in %): ===
                                                     DayOfWeek   IsWeekend  \
GMM_Cluster
0                                                 {6: 100.0}  {1: 100.0}
1            {4: 24.24, 3: 23.23, 1: 21.21, 2: 20.2, 0: 11.11}  {0: 100.0}
2              {3: 20.45, 2: 20.45, 4: 19.7, 0: 19.7, 1: 19.7}  {0: 100.0}
3             {0: 59.09, 2: 18.18, 1: 13.64, 3: 4.55, 4: 4.55}  {0: 100.0}
4                                       {6: 66.67, 5: 33.33}  {1: 100.0}
5                                                 {5: 100.0}  {1: 100.0}

                                                         Month  \
GMM_Cluster
0            {3: 10.2, 8: 10.2, 1: 8.16, 2: 8.16, 4: 8.16, ...
1            {11: 19.19, 12: 19.19, 9: 18.18, 8: 17.17, 10:...
2            {1: 16.67, 3: 15.91, 4: 15.91, 2: 15.15, 5: 15...
3            {10: 36.36, 8: 13.64, 9: 13.64, 12: 13.64, 7: ...
4                                      {10: 66.67, 11: 33.33}
5            {5: 10.64, 8: 10.64, 1: 8.51, 4: 8.51, 6: 8.51...

                                                           Day
GMM_Cluster
0            {9: 6.12, 16: 6.12, 23: 6.12, 5: 4.08, 13: 4.0...
1            {4: 5.05, 10: 5.05, 24: 5.05, 25: 5.05, 30: 4....
2            {3: 4.55, 21: 4.55, 14: 4.55, 23: 3.79, 17: 3....
3            {22: 9.09, 2: 9.09, 13: 9.09, 3: 9.09, 16: 9.0...
4                              {11: 33.33, 12: 33.33, 2: 33.33}
5            {8: 6.38, 15: 6.38, 4: 4.26, 12: 4.26, 20: 4.2...
```

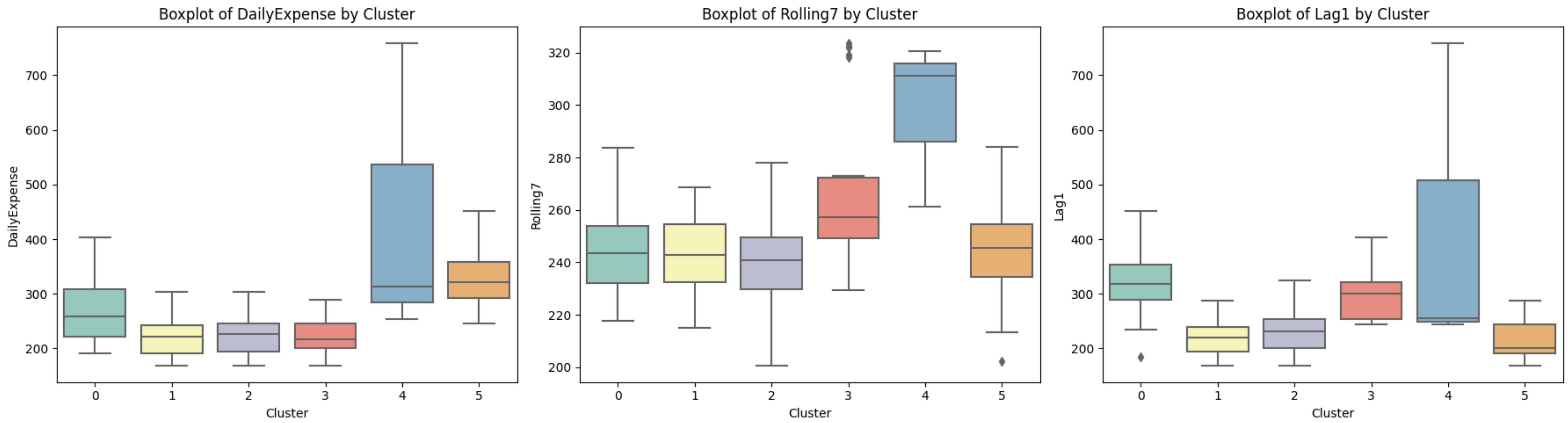
In [ ]:

```python
# Investigate a small cluster (e.g., Cluster 4)
print("\n=== Detailed Profile for Cluster 4 ===")
cluster_4 = daily_expense[daily_expense['GMM_Cluster'] == 4]
print(cluster_4.describe())
```

```
=== Detailed Profile for Cluster 4 ===
                      Date  DailyExpense  DayOfWeek  IsWeekend      Month  \
count                    3      3.000000   3.000000        3.0   3.000000
mean   2025-10-18 16:00:00    442.000000   5.666667        1.0  10.333333
min    2025-10-11 00:00:00    253.000000   5.000000        1.0  10.000000
25%    2025-10-11 12:00:00    283.500000   5.500000        1.0  10.000000
50%    2025-10-12 00:00:00    314.000000   6.000000        1.0  10.000000
75%    2025-10-22 12:00:00    536.500000   6.000000        1.0  10.500000
max    2025-11-02 00:00:00    759.000000   6.000000        1.0  11.000000
std                    NaN    276.219116   0.577350        0.0   0.577350

             Day        Lag1    Rolling7  LogExpense  GMM_Cluster  \
count   3.000000    3.000000    3.000000    3.000000          3.0
mean    8.333333  419.333333  297.666667    5.974408          4.0
min     2.000000  244.000000  261.142857    5.537334          4.0
25%     6.500000  249.500000  286.214286    5.644953          4.0
50%    11.000000  255.000000  311.285714    5.752573          4.0
75%    11.500000  507.000000  315.928571    6.192946          4.0
max    12.000000  759.000000  320.571429    6.633318          4.0
std     5.507571  294.211375   31.969479    0.580692          0.0

       GMM_Confidence
count    3.000000e+00
mean     9.999999e-01
min      9.999997e-01
25%      9.999998e-01
50%      1.000000e+00
75%      1.000000e+00
max      1.000000e+00
std      1.893129e-07
```

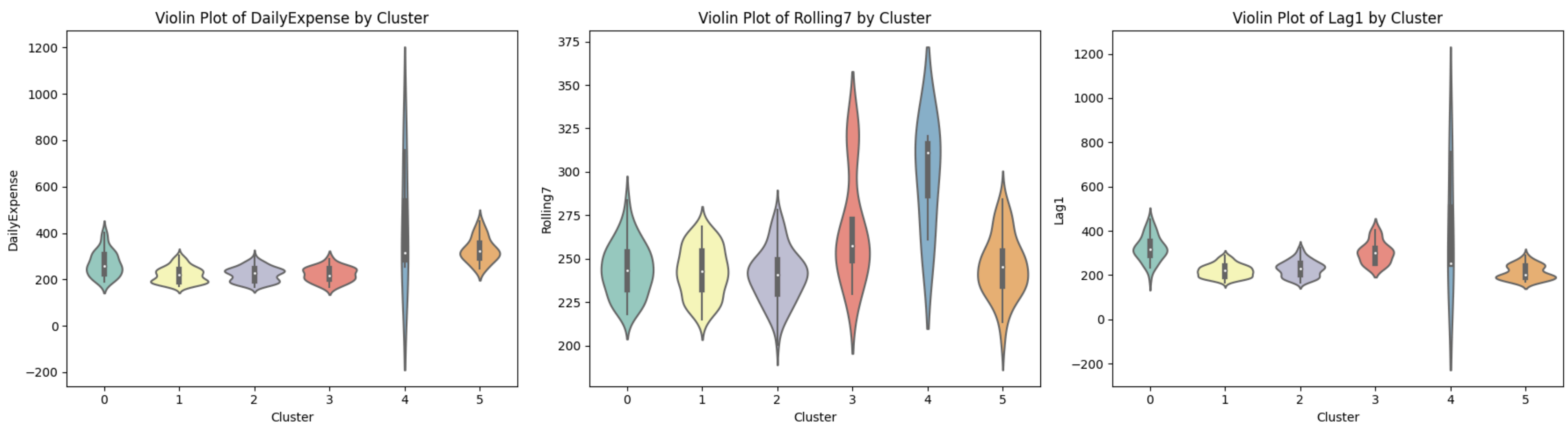
In [ ]:

```
plt.figure(figsize=(18, 5))
for i, feat in enumerate(features_numeric):
    plt.subplot(1, 3, i+1)
    sns.boxplot(x='GMM_Cluster', y=feat, data=daily_expense, palette='Set3')
    plt.title(f"Boxplot of {feat} by Cluster")
    plt.xlabel("Cluster")
    plt.ylabel(feat)
plt.tight_layout()
plt.show()
```



In [ ]:

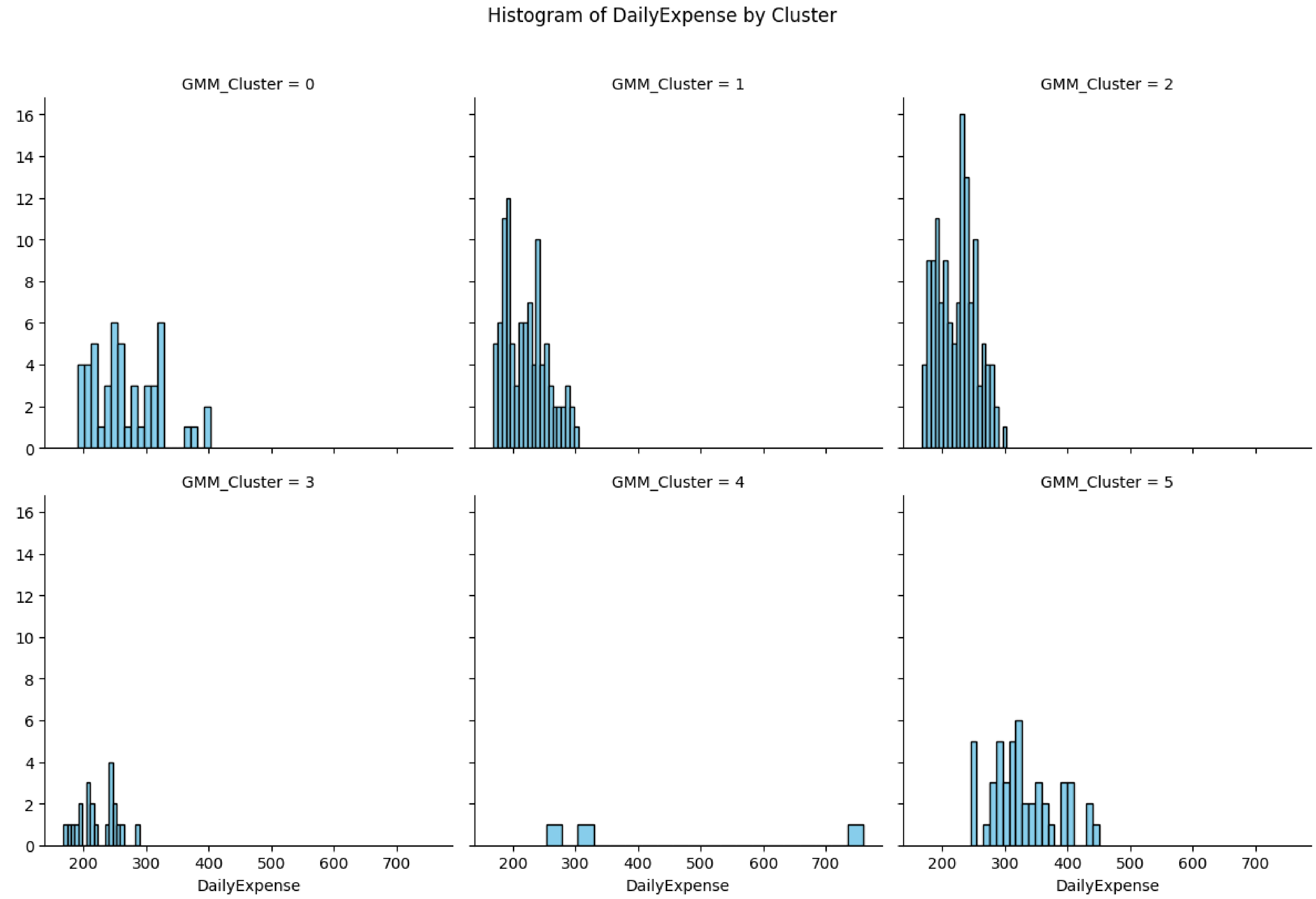
```
plt.figure(figsize=(18, 5))
for i, feat in enumerate(features_numeric):
    plt.subplot(1, 3, i+1)
    sns.violinplot(x='GMM_Cluster', y=feat, data=daily_expense, palette='Set3')
    plt.title(f"Violin Plot of {feat} by Cluster")
    plt.xlabel("Cluster")
    plt.ylabel(feat)
plt.tight_layout()
plt.show()
```



In [ ]:

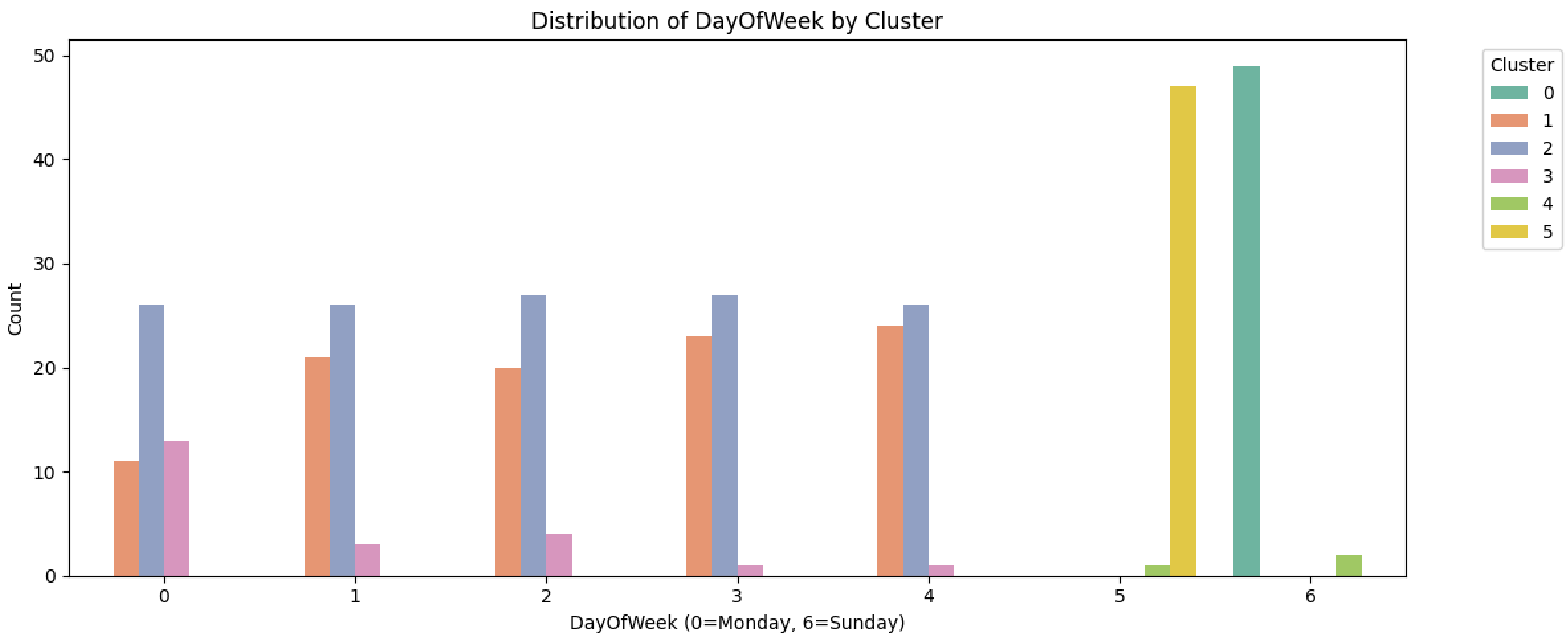
```
g = sns.FacetGrid(daily_expense, col="GMM_Cluster", col_wrap=3, height=4)
g.map(plt.hist, "DailyExpense", bins=20, color='skyblue', edgecolor='black')
g.fig.suptitle("Histogram of DailyExpense by Cluster", y=1.02)
plt.tight_layout()
plt.show()
```

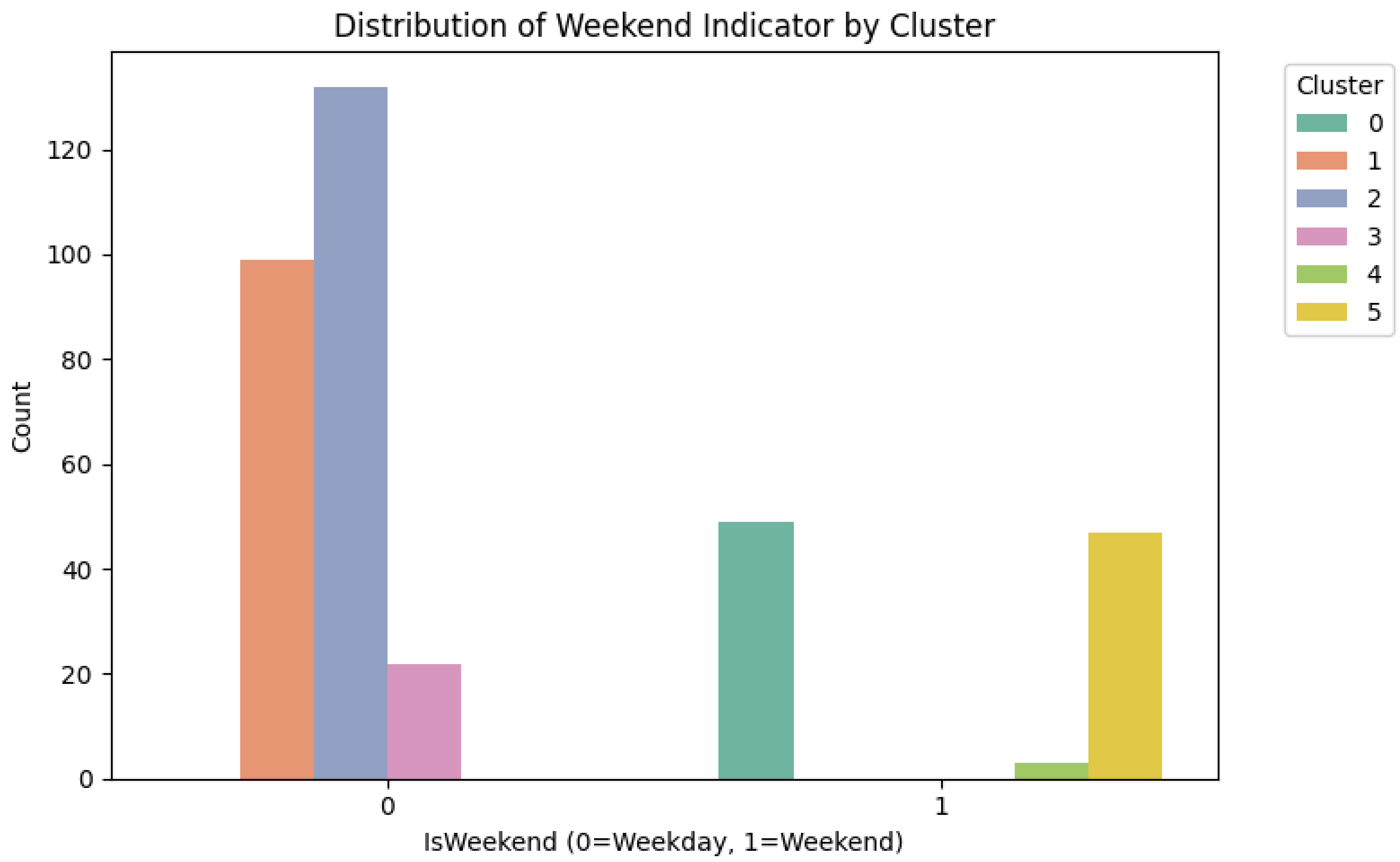
```
C:\Users\Davon\AppData\Local\Packages\PythonSoftwareFoundation.Python.3.11_qbz5n2kfra8p0\LocalCache\local-packages\Python311\site-packages
\seaborn\axisgrid.py:118: UserWarning: The figure layout has changed to tight
  self._figure.tight_layout(*args, **kwargs)
C:\Users\Davon\AppData\Local\Temp\ipykernel_17252\3102834032.py:4: UserWarning: The figure layout has changed to tight
  plt.tight_layout()
```



In [ ]:

```python
plt.figure(figsize=(12, 5))
sns.countplot(x='DayOfWeek', hue='GMM_Cluster', data=daily_expense, palette='Set2')
plt.title("Distribution of DayOfWeek by Cluster")
plt.xlabel("DayOfWeek (0=Monday, 6=Sunday)")
plt.ylabel("Count")
plt.legend(title="Cluster", bbox_to_anchor=(1.05, 1), loc='upper left')
plt.tight_layout()
plt.show()

plt.figure(figsize=(8, 5))
sns.countplot(x='IsWeekend', hue='GMM_Cluster', data=daily_expense, palette='Set2')
plt.title("Distribution of Weekend Indicator by Cluster")
plt.xlabel("IsWeekend (0=Weekday, 1=Weekend)")
plt.ylabel("Count")
plt.legend(title="Cluster", bbox_to_anchor=(1.05, 1), loc='upper left')
plt.tight_layout()
plt.show()
```





Radar Chart for Cluster Mean Profiles

In [ ]:
```
features_radar = ['DailyExpense', 'Rolling7', 'Lag1']
cluster_means = daily_expense.groupby('GMM_Cluster')[features_radar].mean().reset_index()

# Normalize each feature (min-max normalization) for fair comparison
def normalize(series):
    return (series - series.min()) / (series.max() - series.min())

for feat in features_radar:
    cluster_means[feat] = normalize(cluster_means[feat])

# Prepare the angles for the radar chart
num_vars = len(features_radar)
angles = np.linspace(0, 2 * np.pi, num_vars, endpoint=False).tolist()
angles += angles[:1]

# Plot radar chart
fig, ax = plt.subplots(figsize=(6, 6), subplot_kw=dict(polar=True))
for i, row in cluster_means.iterrows():
    values = row[features_radar].tolist()
    values += values[:1]  # close the circle
    ax.plot(angles, values, label=f"Cluster {int(row['GMM_Cluster'])}")
    ax.fill(angles, values, alpha=0.25)
ax.set_thetagrids(np.degrees(angles[:-1]), features_radar)
ax.set_title("Radar Chart of Cluster Mean Profiles (Normalized)", fontsize=14)
ax.legend(loc='upper right', bbox_to_anchor=(1.3, 1.1))
plt.tight_layout()
plt.show()
```