KTH Stockholm
EECS :: CST
Visualization, Autumn 2019, DD2257
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Homework assignment No. 1

Due September 5, 2019

Task 1.1: Curvilinear Grid around a Cylinder — 6 P

Consider a cylinder in 3D with a radius of $r = 5$ and a length of $\ell = 20$. It is oriented along the z-axis and one of its ends is centered around the origin.

Create a curvilinear grid around this cylinder with the following properties:

- The grid has $n_\phi = 20$ grid points in angular direction (along a circle in the xy-plane).
- The grid has $n_r = 5$ grid points in radial direction (from the cylinder outwards).
- The grid has $n_z = 8$ grid points in z-direction.
- The grid has a thickness of $b = 5$ around the cylinder.

Describe the curvilinear grid using coordinate functions for the grid vertices such as $x(i, j, k)$, $y(i, j, k)$, $z(i, j, k)$, where i, j, k are the indices of the vertices!

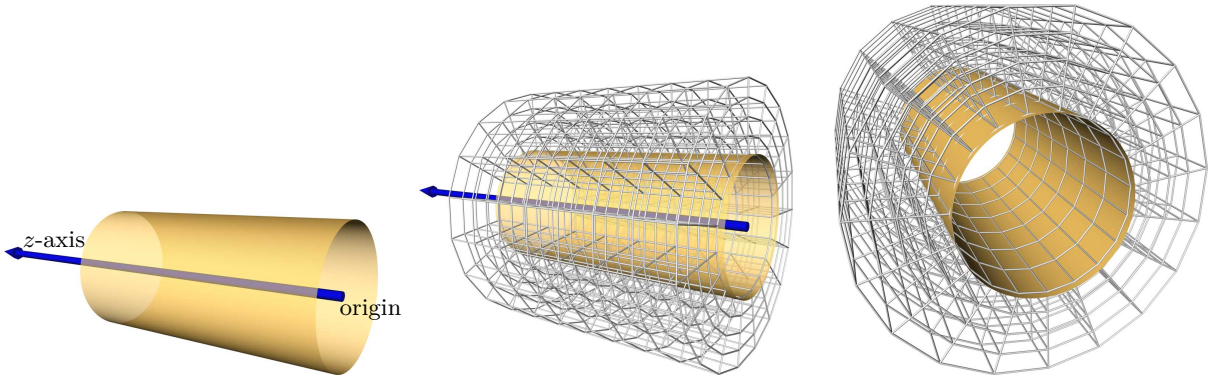

Figure 1: Curvilinear mesh around a cylinder.

Task 1.2: Order Independence of Bi-linear Interpolation — 3 P

Let f_{00}, f_{10}, f_{01}, and f_{11} be four coefficients at the corners of a uniform square and let $f(x, y)$ be the function that performs a bi-linear interpolation of these coefficients.

Bi-linear interpolation can be expressed as the consecutive application of linear interpolation along each dimension.

Show that the result of bi-linear interpolation is *independent* of the order of linear interpolations, i.e., that linear interpolation can be performed in x-direction first followed by the y-direction or **vice versa**.

Task 1.3: Bilinear Interpolation in the Unit Square **1+2+1 P**

Given is the bilinear function $f(x, y)$ with $f(0, 0) = 3$, $f(1, 0) = 2$, $f(0, 1) = 2$, $f(1, 1) = 3$ as shown in Figure 2.

(a) Compute $f(0.5, 0.5)$.

(b) Determine the formula for the gradient of the given f.

(c) Compute the gradient at $(0.5, 0.5)$.

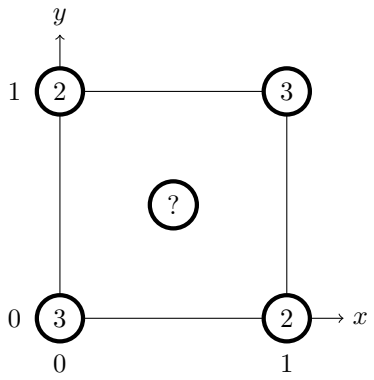

Figure 2: A bilinear cell of a 2D scalar field.

Task 1.4: Bilinear Interpolation **1 P**

Given is the bilinear function $f(x, y)$ with $f(0, 0) = 1$, $f(2, 0) = 3$, $f(0, 1) = 5$, $f(2, 1) = 7$ as shown in Figure 3. Compute $f(1, 0.5)$.

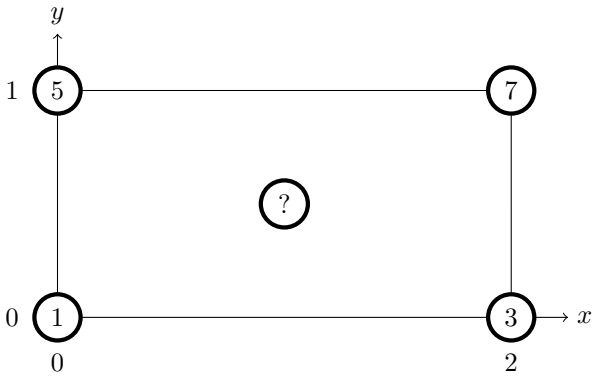

Figure 3: A bilinear cell of a 2D scalar field.

Task 1.5: Interpolation **2+2+2 P**

Given are 3 points $\mathbf{p}_0 = (0, 0), \mathbf{p}_1 = (1, 0), \mathbf{p}_2 = (0, 1)$ equipped with the scalar values $s_0 = 1, s_1 = 2, s_2 = 3$. Determine the interpolating function $s(x, y)$ (i.e., write down the formula) for

(a) linear interpolation,

(b) Shepard interpolation.

(c) For both types of interpolation, compute the values of $s(1/2, 0)$, $s(0, 1/2)$, $s(1/2, 1/2)$, $s(2/3, 2/3)$. Use $p = 2$ for Shepard interpolation.