A COMPARISON OF LOAD FLOW ANALYSIS USING DISTFLOW, GAUSS-SEIDEL, AND OPTIMAL LOAD FLOW ALGORITHMS

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Abstract – The state of a power system and the methods of calculating this state are extremely important in evaluating the operation of the power system, the control of this system, and the determination of future expansion for the power system. The state of the power system is determined through load flow analysis that calculates the power flowing in the lines of the system. There are several different methods to determine the load flow of a given system. For the purposes of this paper, only three methods of load flow algorithms will be evaluated: Gauss-Seidel, Optimal Load Flow, and the DistFlow method.

I. INTRODUCTION

Distribution automation (DA) oversees the control of power distribution systems, including connected loads, with the objectives, among others, of improving overall system efficiency in the use of both capital and energy, and increasing reliability of service to essential loads [1]. As distribution systems grow in size and complexity, and customers' demand ever-increasing power requirements, it is clear that DA becomes even more important.

One of the roles of DA is load reconfiguration, which involves remote control of switches and breakers to permit routine, daily or seasonal configuration of feeders or feeder segments for the purpose of taking advantage of load diversity among feeders [1]. Reconfiguration allows the system to effectively serve larger loads without requiring feeder reinforcement, or new construction, or to serve the same loads with lower energy losses in the system feeders. To permit a comparison of various configurations upon feeder losses, a load flow must be carried out for each configuration. For a system having N switches and breakers, there are 2^N possible configurations, and it is thus clear that the speed at which the load flow is carried out will be the limiting factor in any reconfiguration for loss minimization algorithm.

Traditionally, load flows are calculated using the Gauss-Seidel and Optimal Load Flow (or Newton-Raphson) methods. In [2], Wu *et al* proposed a new technique that took advantage of the radial structure of most power distribution systems. Wu *et al* called their method, *DistFlow*, and claimed it was computationally superior to the other two methods because of the simplified method of calculating the optimal load flow. The purpose of this research was to compare the accuracy and speed of computation of the following load flow algorithms: Gauss-Seidel, Optimal Load Flow, and the DistFlow method.

The first two load flow methods require the determination of an admittance matrix. This can pose a serious problem when the matrix is sparse and cannot be inverted, which is the case in many applications with the Optimal Load Flow. The DistFlow method is computationally superior to the other two methods because it does not require the admittance matrix calculation to optimize the distribution system. With this in mind, the paper will show the general development of each of the three load flow methods and their speed of optimization.

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II. GAUSS-SEIDEL METHOD

The Gauss-Seidel algorithm is an iterative numerical procedure which attempts to find a solution to the system of linear equations by repeatedly solving the linear system until the iteration solution is within a predetermined acceptable bound of error. It is a robust and reliable load flow method that provides convergence to extremely complex power systems. With the bus admittance matrix, and applying Kirchhoff's current law, the following equation is derived.

$$I = Y_{bus} V \tag{1}$$

where I is the current, V is the voltage and Y is the bus admittance matrix. The k_{th} (of N) nodal current is:

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n \tag{2}$$

which can be expanded into the following form:

$$I_k = Y_{kk} V_k + \sum_{n=1}^{N} Y_{kn} V_n \tag{3}$$

Re-arranging Equation (3) yields:

$$V_k = \frac{I_k}{V_{kk}} - \frac{1}{Y_{kk}} \sum_{n=1}^{N} Y_{kn} V_n \tag{4}$$

With the complex power at a given node, Equation (4) will yield the following result:

$$V_k = \frac{1}{Y_{kk}} \left(\frac{P_k - jQ_k}{V_k^*} - \sum_{n=1}^{N} Y_{kn} V_n^{(i)} \right) \tag{5}$$

Since the Gauss-Seidel method is an iterative procedure, Equation (5) yields the following:

$$V_k^{(i+1)} = \frac{1}{Y_{kk}} \left(\frac{P_k - jQ_k}{\left(V_k^i\right)^*} - \sum_{n=1}^{N} Y_{kn} V_n^{(i)} \right) \tag{6}$$

III. NEWTON-BASED OPTIMAL LOAD FLOW

Solving for Optimal Load Flow determines the optimal settings for the control variables in the power system. The objectives of this method are the following: active power cost optimization, active power loss minimization, minimum control-shift, and minimum number of controls rescheduled. The overall optimal load flow algorithm involves the solution of optimal load flows and the updates of Lagrange multipliers. The optimal load flow will converge to a solution that yields the minimal production costs for a utility by solving for the optimal system parameters.

It is understood that the primary goal of OPF is the minimization of the costs of meeting the load demand while maintaining the security of the system. To observe an extensive development of the algorithm, see Luenberger [5]. The solution for the load flow with this method requires the use of the following Lagrangian equation:

$$L(z) = f(x) + \mu^T h(x) + \lambda^T g(x) \tag{7}$$

where μ and λ are vectors of the Lagrange multipliers, and g (x) only includes the active inequality constraints. The inequality constraints allow for the Lagrangian multiplier to be adjusted depending on the situation. If, for example, the bus voltage is within a certain level, there is no requirement to activate the inequality constraint. A derivative of the algorithm to determine what constraint is to be implemented, and when it is to be used, is given in Sun *et al* [4]. The Gradient and Hessian of the Lagrangian can be defined by the following equations:

$$Gradient = \left[\frac{\partial L(z)}{\partial z_i} \right] \tag{8}$$

The gradient is simply the first partial derivative of the Lagrangian.

$$Hessian = \left[\frac{\partial^2 L(z)}{\partial z_i \partial z_j} \right] \tag{9}$$

The Hessian is a matrix of the second partial derivatives of the Lagrangian. The necessary conditions for optimization are derived from the following equation:

$$\overline{V_z} L(z^*) = 0 \tag{10}$$

IV. SIMPLIFIED DISTFLOW METHOD

The DistFlow method [2] uses a set of recursive equations called DistFlow branch equations that provide the optimal solution by estimating the power loss reduction due to a branch exchange. This recursive analysis of the distribution system is carried out until the load flow configuration is optimal. A series of branch exchanges ensures the final configuration minimizes line losses. It can also be used for radial distribution feeders, and in the capacitor placement problem.

The following equations represent the recursive set of power flow equations, where P_i, Q_i, V_i represent the real power, reactive power, and voltage magnitudes:

$$P_{i+1} = P_i - r_i\left(\frac{P_i + Q_i^2}{V_i^2}\right) - P_{Li+1} \tag{11}$$

$$Q_{i+1} = Q_i - x_i\left(\frac{P_i + Q_i^2}{V_i^2}\right) - Q_{Li+1} \tag{12}$$

$$V^2_{i+1} = V_i^2 + 2(r_i P_i + x_i Q_i) + (r_i^2 + x_i^2)\left(\frac{P_i + Q_i^2}{V_i^2}\right) \tag{13}$$

The above equations can be simplified to the following three:

$$P_{i+1} = \sum_{k=i+2}^{n} P_{Lk} \tag{14}$$

$$Q_{i+1} = \sum_{ki+2}^{n} Q_{Lk} \tag{15}$$

$$V_{i+1}^2 = V_i^2 - 2(r_i P_i + x_i Q_i) \tag{16}$$

It is necessary to calculate the power loss due to a branch exchange in order to determine if there are any candidates for a branch exchange. The following equation illustrates the power loss due to a branch exchange:

$$LP_i \approx r_i(P_i^2 + Q_i^2) \tag{17}$$

The following equation illustrates the total loss to the distribution system.

$$\overline{LP} = \sum_{l=0}^{n-1} r_i(P_i^2 + Q_i^2) \tag{18}$$

The power reduction due to a branch exchange must be determined. If this value is less than zero, the branch is considered a candidate for exchange. The branch with the lowest value is exchanged. This process is repeated until there are no longer any candidates for branch exchange. The following equation shows how this value of power loss is calculated:

$$\Delta L\overline{P_{bm}} = 2\Delta P_m + 2\Delta Q_m - \Delta\left(P_m^2 + Q_m^2\right) \tag{19}$$

V. RESULTS

Figure 1 illustrates the distribution system that was used for all three of the load flow methods, which was taken from Civanlar *et al* [3]. All three methods of load flow were implemented using Matlab. It was found that with the computing power, and the relatively simple distribution system, that all three methods converged to their respective solutions relatively quickly.

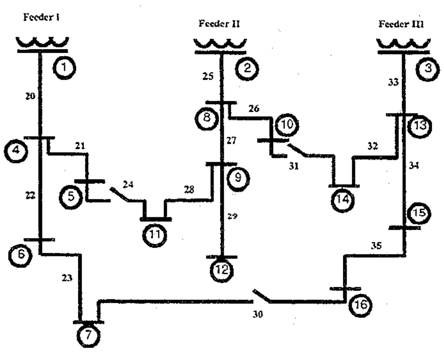


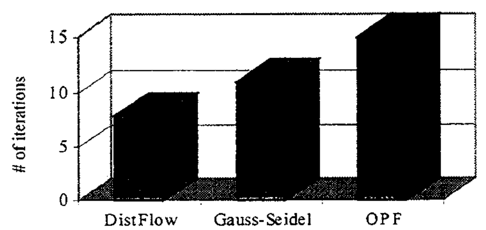
Figure 1: Test Distribution System

However, the DistFlow method had a greater speed of convergence and fewer iterations to obtain an optimal solution. Both The Gauss-Seidel and OPF methods converged slower, and they both had a significantly greater number of iterations than the DistFlow. Graph 1 and Graph 2 show the results of the load flow analysis on the distribution system shown in Figure 1.

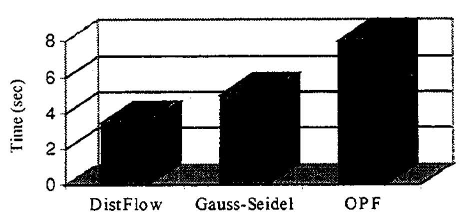
The results of this analysis show the primary reason why the electric power industry must always work towards perfecting the methodology of their load flow methods. The use of high speed computer programs, like Matlab, and the capability of supplying this information dynamically to the distribution system, makes it possible for the power industry to supply energy in the most cost effective manner.

The different load flow methods were not tested on a number of different distribution systems; therefore there is insufficient information to pick the most cost cost-effective load flow method. It is reasonable to expect different results from the load flow methods on different distribution systems.

Graph 1: Number of Iterations



Graph 2: Time for convergence



VI. CONCLUSIONS

The Gauss-Seidel and Newton- Raphson are general-purpose techniques that can solve most linear systems. The Newton-Raphson takes longer because of the need to recalculate the Jacobian. The DistFlow method has been optimized for radial distribution systems and does not work without reconfiguration in order to minimize feeder losses. The paper has shown that the DistFlow load method is a better method in the determination of load flows provided that the network can be reconfigured in order to achieve optimization. Future work will include the restructuring of this method in order to optimize the distribution system without requiring reconfiguration of the network.

VI. REFERENCES

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