Chapter 2

Mathematical Formulation of Real and Reactive Power Problems

Introduction

Theory and practice of power system operation show that the real power flow distribution in a network depends heavily on nodal voltage angular difference, while the reactive power flow distribution correspondingly depends on nodal voltage magnitudes. Guided by this philosophy, and computational efficiency, the original problem of optimal power flow can be decomposed into real and reactive power optimization sub-problems, thus also reflecting the way human operators deal with the real and reactive power flow control. In this chapter, the control devices influencing the flow of real and reactive power are briefly reviewed. With respect to active power generation cost, the input-output characteristics of thermal units is discussed. Finally, mathematical formulation of the two sub-problems is presented.

2.1 Real and Reactive Power Control Variables in Power Systems

The system control devices used to redistribute real power flow on the network are fewer than those for reactive flow. Among the system devices that can be remotely controlled by the dispatcher to influence the flows of active power directly in order to solve branch overloading are:

- Generator active power injections.
- Quadrature booster (phase shifting transformers) settings.
- Pumped storage input or output.
- Load shedding.
- MW interchange transactions between interconnected power companies.

Of these alternatives, load shedding which is a measure to reduce the amount of both real and reactive power demand in the system is undesirable because of curtailment of service to the consumers. This is mainly used under emergency conditions to maintain power balance when demand exceeds the generation capability. It could also help to boost the voltage of some system buses as a result of the reduction in the voltage drop along the line connecting these buses. Pumped storage is not used as a normal control, but more as a periodic control. For example, it may be used to reallocate the daily load cycle for economic reasons by means of pumping water up to a higher reservoir during the minimum load periods, then using the same device as a hydro generation plant during the peak-loads periods. The disadvantages inherent in the use of pumped storage facilities are that they are more expensive than conventional hydro generation, and that they are generally located at remote sites from the load centers. Power system dispatchers, however, rely primarily on shifting generator real power outputs to alter real power flow patterns, and in few places, upon phase shifting transformers.

Reactive power flow on the other hand is controlled by voltage-regulating devices which are strategically located in the network by the system planners in order to absorb or supply reactive power. These compensators are used to obtain desired reactive flows for power factor correction, desired voltage level, and reduction of real power losses. The most common devices for reactive compensation are as discussed below:

- **Synchronous Generators**

  The reactive power output of a generating unit can easily be regulated by adjusting its excitation level. The generator excitation characteristics are fixed by machine design. However, in a large interconnected network, reactive power generated has to be transmitted through the lines and transformers to the load centers. The size of the conductors used in the transmission lines, transformers and generator size required are larger than the actual ones. Therefore, a local reactive power control should be introduced to provide a desirable reactive power flow and a voltage profile from the economics point of view.

- **Synchronous Condensers**

  A synchronous condenser is a synchronous machine running without a prime mover or mechanical load. By controlling the field excitation within its design limits, it can either generate or absorb reactive power depending on whether it is over- or under-excited. With voltage regulator, it can automatically adjust the reactive power output to maintain constant terminal voltage. It draws a small amount of active power from the power system to supply losses [36]. Synchronous condensers found useful application in voltage and reactive power control at both transmission and sub-transmission levels and are usually connected to the tertiary windings of transformers.

Due to their high operating and purchasing cost, they have been largely superseded by static VAR compensators (see below). Their application has often been at High Voltage Direct Current (HVDC) converter stations. The high purchasing and operating costs of synchronous compensators is offset by the following advantages as compared with static compensators: Their reactive power production is unaffected by the system voltage. A synchronous condenser has a property of being able to withstand 10% to 20% overload capability for up to 30 minutes, and this can be decisive in some emergency conditions when the need for reactive power is crucial.

- **Solid State Electronically Controlled Reactance**

Static VAR Compensators (SVC) are shunt-connected static generators and/or absorbers whose outputs are varied so as to control certain parameters of the electric power system. A static VAR system is an aggregation of SVCs and mechanically switched capacitors or reactors whose outputs are coordinated. At the sub-transmission and distribution system level, they are used for balancing the three-phase of a system supplying unbalanced load. They also find application in critical bus locations to regulate a voltage that fluctuates due to a random load. Their typical installation would be near large time-varying industrial loads such as arc-furnaces used in steel making.

- **Tap-positions of Transformers**

Transformers equipped with tap-changing facilities constitute an important means of controlling voltage throughout the system at all voltage levels. Transformers used to change voltage from one subsystem to another are often furnished with on-load tap changer (OLTC) facilities. They may be controlled either manually or automatically. Usually there are many such transformers interconnecting transmission system of different voltage levels. The taps of these transformers provide a convenient means of controlling reactive power flow between sub-systems which in turn can be used to control the voltage profile, and minimize the active and reactive power losses. Changing the transformer tap varies the turn ratio, which also affects the impedance. Since the transformer impedance is mainly reactive, raising or lowering the tap has a significant effect on reactive power losses.

The control of a single transformer leads to a change in its terminal voltage and also influences the flow of reactive power through it. The resulting effect on the other buses will depend on the network configuration and load/generation distribution as some buses will experience increase in voltage and other bus voltages decrease. It is therefore required to coordinate the control of the tap changers of all transformers interconnecting the subsystems in order to change the general voltage level.

- **Reactive Shunt Elements**

This could be shunt capacitors or shunt reactors. Shunt capacitors supply the reactive power and boost the local voltage. They are used in the distribution systems for power factor correction and feeder voltage control. The objective of power factor correction is to provide reactive power close to the point where it is being consumed, rather than from the remote sources. Power factor correction is provided by means of fixed and switched shunt capacitors at various voltage levels throughout the distribution systems. In transmission systems, capacitors are used to compensate for the reactive power losses and maintaining satisfactory voltage profiles during heavy loading conditions.

Banks of capacitors of appropriate sizes are connected either directly to the tertiary winding of a transformer or to the high voltage bus, as shown in figure 2.1 (a and b). They are breaker-switched either automatically by a voltage relay or manually. Switching of capacitor banks can mitigate the total system losses, improve the voltage profile, and reduce the size (kVA capacities) of the apparatus in the system. The main advantage of shunt capacitors are their low cost and the flexibility of installation and operation.

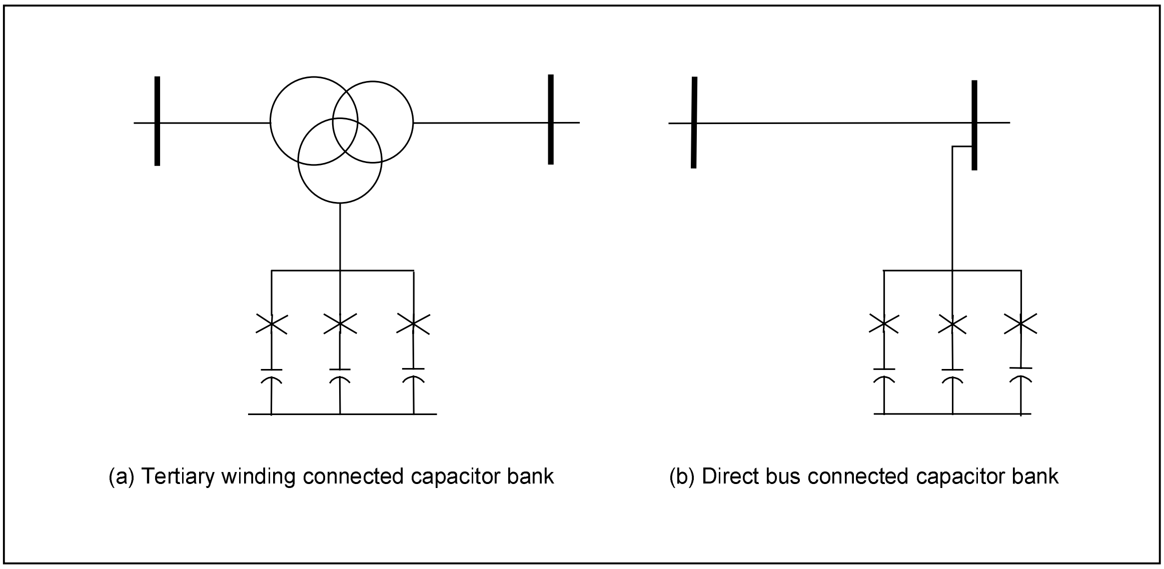

Figure 2.1 Capacitor banks connections

They are readily applied at various points in the network, thus contributing to efficiency of power transmission and distribution. The disadvantage is that their reactive power is proportional to the square of the voltage (constant impedance). At low voltage, the reactive power output is reduced when it is likely to be needed most.

Shunt reactors on the other hand, are used to compensate for the effects of line charging capacitance, especially during light load periods or on open circuit. When there is open circuit at the far end of a line, the capacitive line-charging current flowing through the source impedance (mostly inductive reactance) will cause a rise in sending end voltage. The "Ferranti" effect [36] will cause a further rise in the voltage at the receiving end. A shunt reactor may be connected to the tertiary winding of the adjacent transformer or to the High Voltage (HV) bus. During heavy loading conditions, there might be need for the reactors to be disconnected. In some applications, tapped reactors with on-voltage tap changer facilities have been used to allow variation of the reactance value. Shunt reactors are similar in construction with transformers

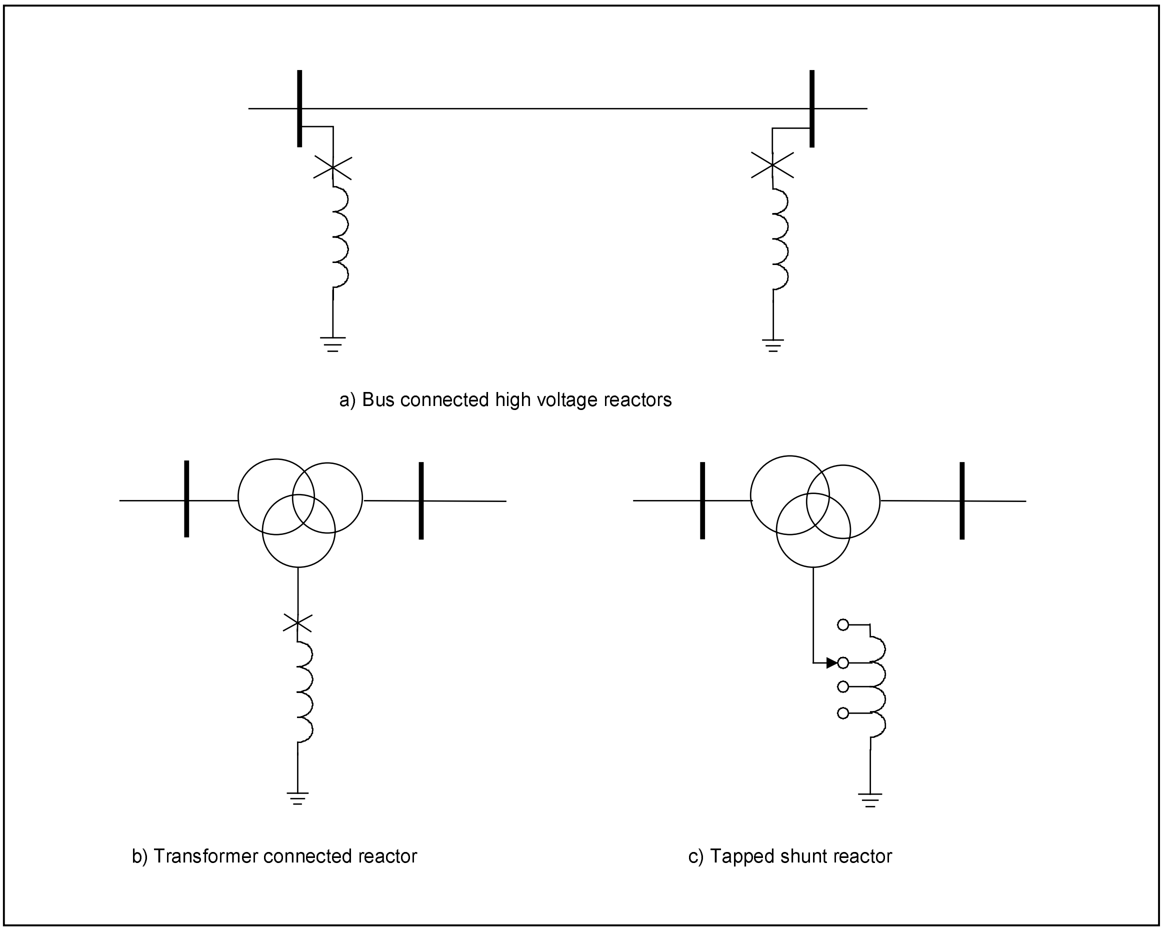

Figure 2.2 Shunt reactors connections

except that they have a single winding (per phase) on an iron core with air-gaps immersed in oil. They may be of either single- or three-phase construction. The various connection types are shown in figure 2.2 (a, b & c).

- **Switching of Transmission Line**

  Switching of long extra high voltage lines which have substantial charging capacitance also influence reactive power flows. The particular line may be switched off during the light load period to reduce the capacitive effect to correct voltage problem. However, during peaking or medium load periods, the use of line maneuver to correct voltage problem is applied as a last resort to avoid load shedding because of subsequent reduction of system reliability and is not so common.

These control devices described above contribute to the satisfactory operation of the power system by maintaining system voltages and frequency and other system variables within their
acceptable limits.

2.2 Formulation and Decomposition of the Problem

The purpose of an optimal power flow (OPF) program is to determine the optimal operation state of a power system by optimizing a particular objective while fulfilling certain operating and physical constraints. In general, OPF is expressed as nonlinear optimization problem as follows:

$$\mathrm{F}(\boldsymbol{x}_\mathrm{P}, \boldsymbol{x}_\mathrm{Q}) \rightarrow \min \tag{2.1}$$

subject to the following equality and inequality constraints

$$\begin{aligned} &\mathrm{P}_\mathrm{EC}(\boldsymbol{x}_\mathrm{P}, \boldsymbol{x}_\mathrm{Q}) = 0 \\ &\mathrm{P}_\mathrm{IC}(\boldsymbol{x}_\mathrm{P}, \boldsymbol{x}_\mathrm{Q}) \le 0 \\ &\mathrm{Q}_\mathrm{EC}(\boldsymbol{x}_\mathrm{P}, \boldsymbol{x}_\mathrm{Q}) = 0 \\ &\mathrm{Q}_\mathrm{IC}(\boldsymbol{x}_\mathrm{P}, \boldsymbol{x}_\mathrm{Q}) \le 0 \end{aligned} \tag{2.2}$$

where $\boldsymbol{x}_\mathrm{P}$ is a vector of real variables consisting of vectors for real power injection P_G and nodal voltage angles δ; likewise $\boldsymbol{x}_\mathrm{Q}$ is a reactive variable vector including respectively vectors for reactive power injection Q_G, nodal voltage magnitude $|\mathrm{V}|$, steps of on-load tap changing transformer T and switching states of shunt elements (capacitors and reactors); P_EC and P_IC are real power network equations and inequality constraints for P_G and branch flows; Q_{EC} and Q_{IC} are reactive power network equations and inequality constraints for Q_G, $|V|$, and T.

With the inherent weak coupling characteristics between real and reactive power flows in a power system [58], the original problem of equation (2.1) can be decomposed into two sub-problems, and different optimization techniques can be employed to solve them separately while taking the advantage of large scale application. The two sub-problems are:

a) Real Power Dispatch Problem

During the solution process, reactive power variables $\boldsymbol{x}_Q$ play a minor role and are treated as constant. The objective function here is stated as

$$F_P(\boldsymbol{x}_P) \rightarrow \min \tag{2.3}$$

subject to the following equality and inequality constraints

$$\begin{aligned} P_{EC}(\boldsymbol{x}_P) &= 0 \\ P_{IC}(\boldsymbol{x}_P) &\le 0 \end{aligned} \tag{2.4}$$

b) Reactive Power Dispatch Problem

By the same inference as above, the real power variables $\boldsymbol{x}_P$ can be deemed constant during the solution process. The objective function here is stated as

$$F_Q(\boldsymbol{x}_Q) \rightarrow \min \tag{2.5}$$

subject to the following equality and inequality constraints

$$\begin{aligned} Q_{EC}(\boldsymbol{x}_Q) &= 0 \\ Q_{IC}(\boldsymbol{x}_Q) &\le 0 \end{aligned} \tag{2.6}$$

2.3 Economic Load Dispatch Problem

The principal components of electric power cost to the consumer are the generation or production cost, the transmission cost, the distribution cost, administrative costs, plus some allowed rate of return or profit on the investment. In the case of predominant thermal generation - as found in many countries all over the world -, the production cost significantly contributes to the total cost of electric power to the consumers and is the main concern of power engineers responsible for economic comparison [23]. This present work therefore considers system with predominantly thermal generation.

Economic load dispatch is the determination of the amount of power that each generator in a system should produce so that the total cost of generation is minimized and the customers demands are met while satisfying system constraints. The important factors for economic operation of the system are operating efficiency of generating plants, fuel and operating cost, and transmission losses. It is also obvious that the most efficient generator in the system does not necessarily yield the least cost as it may be too far from the load center or situated in a region where the fuel cost is very high.

2.3.1 Modeling of Thermal Units Input-Output Characteristics

2.3.1.1 Simple Steam Unit

The steam unit characteristics describes the relationship between the gross input and the net output of a unit. The gross input is the total fuel power supplied to the unit and the net output is the net electrical active power output available to the utility system. Power utilities normally utilize a single heat rate curve with upper and lower limits on generation. A typical input-output characteristics of a steam unit is as shown in figure 2.3. The gross input, shown on the ordinate, is designated by **H**. This term **H** is declared in terms of heat energy requirements (Millions of British thermal units per hour **MBtu/h**). The output power shown on the abscissa, marked with $\mathbf{P_G}$, is the net electrical active power output of the steam unit given in **MW**. This idealized curve is usually approximated by a smooth quadratic polynomial function of the form

$$H_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad , \quad P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{2.7}$$

where α_i, β_i and γ_i are respectively the constant, linear and quadratic cost coefficients.

Minimum load limitations which may not necessarily be zero are generally caused by fuel combustion stability and inherent steam generator design constraints. For example, most supercritical units cannot operate below 30% of their design capability. A series of straight-line segments may also be used to represent the input-output characteristics.

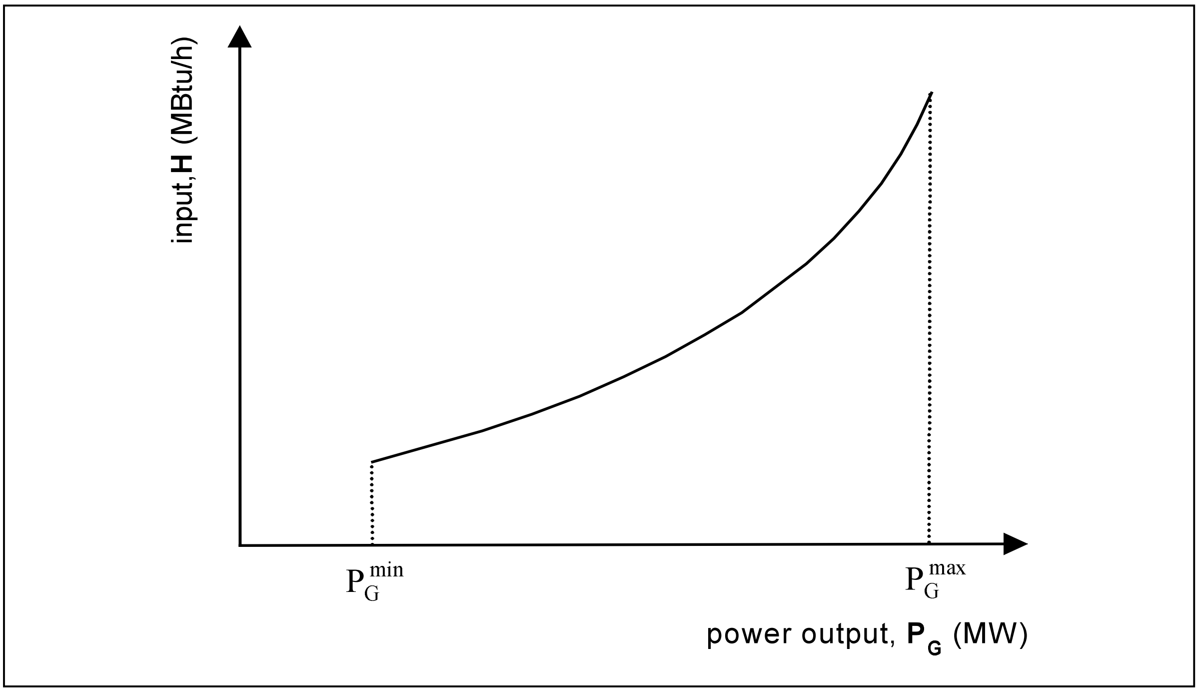

Figure 2.3 Input-output characteristics of a simple steam turbine generator

2.3.1.2 Steam Units with Several Steam Admission Valves

Large steam turbine generating units have a number of steam admission valves that are opened in sequence in order to obtain ever-increasing output of the unit [39]. When a valve is first open, the throttling losses increase rapidly and the incremental heat rate rises suddenly. Input-output curve for a steam turbine generating unit with four steam admission valves is as shown in figure 2.4. A purely polynomial approximation cannot fit this valve point effect satisfactorily. In order to incorporate the effects of valve point, a sinusoidal contribution is superimposed on the polynomial approximation [59]. This is modeled by the equation

$$H_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 + \left|\zeta_i \sin\left(\upsilon_i (P_{Gi} - P_{Gi}^{min})\right)\right| \quad , \; P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \qquad (2.8)$$

where coefficients ζ_i and ν_i are weight factors of associated components superimposed on the quadratic fuel cost function in order to simulate the valve point loading.

Another important characteristic of a steam unit is the unit heat rate characteristics which is **H/P** versus **P**. This characteristics shows the heat input per kilowatt hour of output versus the megawatt output of the unit. By knowing the fuel type and cost $ć_i$ in CU/MBtu, where CU is taken as a fictional currency unit, the input-output curve can be translated into a generation cost $F_i(P_{Gi})$ in CU/h by using

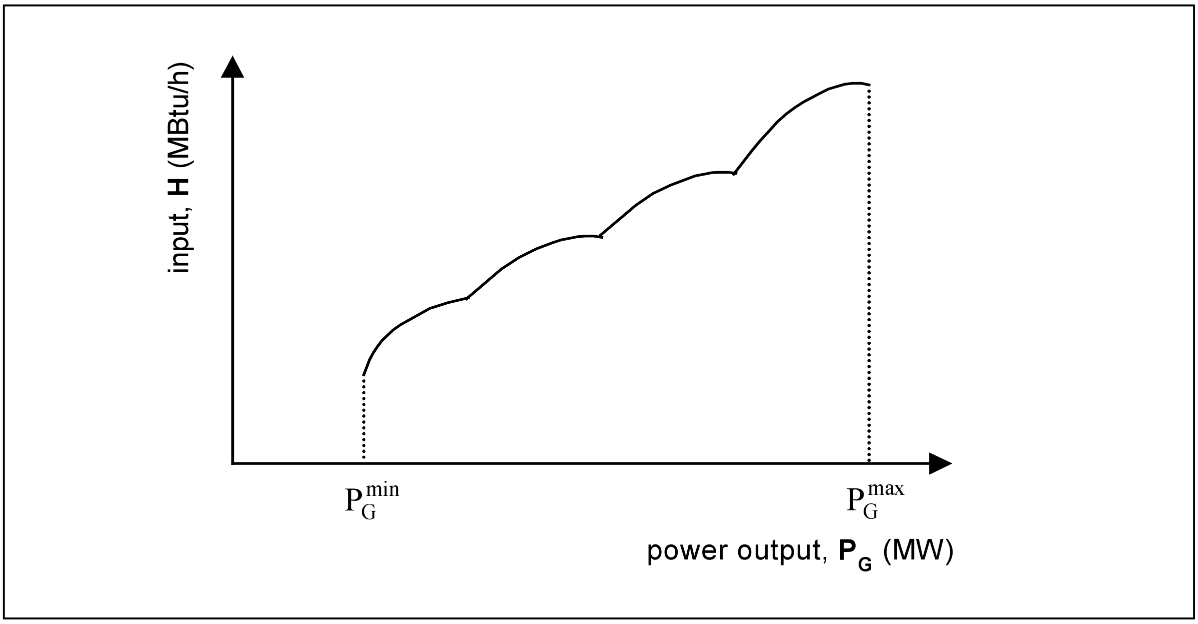

Figure 2.4 Input-output characteristics of a simple steam turbine generator with four steam admission valves

$$F_i(P_{Gi}) = H_i(P_{Gi})\cdot \acute{c}_i \tag{2.9}$$

2.3.2 Mathematical Formulation of Economic Load Dispatch

Consider a system consisting of n_G thermal generating units connected to the transmission network, the economic load dispatch seeks to minimize the total fuel cost of operating a power system subject to operation constraints. This objective function can be mathematically expressed by

$$F_T = \sum_{i=1}^{n_G} F_i(P_{Gi}) \quad \rightarrow \; \min \tag{2.10}$$

where

$F_i(P_{Gi})$ generation cost of unit i

P_{Gi} actual real power output of unit i

n_G total number of on-line thermal units

The minimization of this objective function is further subjected to the following set of constraints:

- Equality constraint of matching the sum of total load demand, sum of generating station auxiliary equipment power requirements, and total system losses with the overall on-line generating units power output given by

$$\sum_{i=1}^{n_G} P_{Gi} - P_T - \sum_{k=1}^{n_L} P_{Dk} - \sum_{i=1}^{n_G} P_{HDi} = \varphi \tag{2.11}$$

P_{Dk}	actual real power demand at node k
P_{HDi}	actual real house load of unit i
P_T	real power transmission losses
n_L	total number of loads

- Inequality constraints of each generating unit real power output ranging between its lower and upper limits to ensure stable operation as determined by its ratings given by

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \quad ; \; i = 1,2,\ldots n_G \tag{2.12}$$

In a closely meshed, very good constructed transmission network the above problem definition is adequate. Conversely, in a far extended weakly meshed network it is important to consider additional constraints of the network security such that the magnitude of network branches (transformers and lines) current I_l^{max} or apparent power flow S_l^{max} capacities as dictated by the thermal ratings are not violated hence security constraint economic load dispatch. This is given by

$$\begin{array}{lll} |I_l| \le I_l^{max} & \text{with} \quad I_l^{prot} \ge I_l^{max} & \\ & \quad \text{or} & l = 1,2,\ldots..n_{Br} \\ |S_l| \le S_l^{max} & \text{with} \quad S_l^{prot} \ge S_l^{max} & \end{array} \tag{2.13}$$

where n_{Br} is the total number of branches; I_l^{prot} and S_l^{prot} are respectively the magnitudes of current and apparent power flow for the branch protection.

2.4 Reactive Power Dispatch and Voltage Control

The reactive power and voltage control has a significant influence on security of the power system. For efficient and reliable operation of power systems, voltages at the terminal of all equipment in the system must be maintained within desired limits for power system stability enhancement. Conventionally, minimization of total transmission line losses has been considered to be the main objective in reactive power dispatch. Of recent the trend has been towards the elimination of security constraint violations. Proper redistribution of reactive power generations will offer the following benefits:

- Reduction in real power transmission losses caused by unnecessary reactive power flows which will consequently result in the lowest production cost.
- Increase in system security from augmented reactive power reserves for emergencies.

The reactive power dispatch objective thus seeks to minimize the active power losses in the network. Neglecting the shunt losses, the transmission losses P_T for the n_{Br} branches of the system under regard are given by equation (C.4) in appendix C

$$P_T = \sum_{n_{Br}} g_{km}\left(|V_k|^2 + |V_m|^2 - 2|V_k V_m|\cos\delta_{km}\right) \quad \rightarrow \quad \min \tag{2.14}$$

where

k and m	end nodes of branch
g_{km}	branch conductance
$\|V_k\|$ and $\|V_m\|$	voltage magnitudes at nodes k and m
δ_{km}	voltage angular difference

The minimization of this objective function is further subjected to a set of following operational constraints:

- equations of net real and reactive power flow into the network at a typical node k given by

$$P_k = |V_k| \sum_{m=1}^{n_B} |Y_{km} V_m| \cos(\delta_{km} - \theta_{km})$$

$$k = 1,2,\ldots.n_B \tag{2.15}$$

$$Q_k = |V_k| \sum_{m=1}^{n_B} |Y_{km} V_m| \sin(\delta_{km} - \theta_{km})$$

where

$Y_{km}=\|Y_{km}\|\cos(\theta_{km})+j\|Y_{km}\|\sin(\theta_{km})$	elements of bus admittance matrix
θ_{km}	admittance phase angle
n_B	total number of nodes

- nodal voltage limits including the load bus and generator voltages should be maintained within their permissible range to ensure proper and efficient power supply to consumers given by the inequality

$$V_k^{\min} \le |V_k| \le V_k^{\max} \qquad k = 1,2,........n_B \qquad (2.16)$$

- ranges and steps of transformer taps operating limits given by

$$T_j^{\min} \le T_j \le T_j^{\max} \qquad j = 1,2,......n_T \qquad (2.17)$$

where
n_T total number of tap changing transformers

- the reactive power output of the generators must be within the definite limits imposed by the source capabilities and reactive power restrictions of the units:

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \qquad i = 1,2, ...n_{PV} + sl \qquad (2.18)$$

where
sl slack node
n_{PV} total number of voltage-controlled or PV nodes