CHAPTER – 3

OPTIMAL POWER FLOW PROBLEM & SOLUTION METHODOLOGIES

3.0 INTRODUCTION

This chapter covers existing methodologies for solution of Optimal Power Flow (OPF) problem. They include formulation of OPF problem, objective function, constraints, applications and in-depth coverage of various popular OPF methods.

The OPF methods are broadly grouped as Conventional and Intelligent. The conventional methodologies include the well known techniques like Gradient method, Newton method, Quadratic Programming method, Linear Programming method and Interior point method. Intelligent methodologies include the recently developed and popular methods like Genetic Algorithm, Particle swarm optimization. Solution methodologies for optimum power flow problem are extensively covered in this chapter.

3.1 OPTIMAL POWER FLOW PROBLEM

In an OPF, the values of some or all of the control variables need to be found so as to optimise (minimise or maximize) a predefined objective. It is also important that the proper problem definition with clearly stated objectives be given at the onset. The quality of the solution depends on the accuracy of the model studied. Objectives must be modeled and its practicality with possible solutions.

Objective function takes various forms such as fuel cost, transmission losses and reactive source allocation. Usually the objective function of interest is the minimisation of total production cost of scheduled generating units. This is most used as it reflects current economic dispatch practice and importantly cost related aspect is always ranked high among operational requirements in Power Systems.

OPF aims to optimise a certain objective, subject to the network power flow equations and system and equipment operating limits. The optimal condition is attained by adjusting the available controls to minimise an objective function subject to specified operating and security requirements.

Some well-known objectives can be identified as below:

Active power objectives

1. Economic dispatch (minimum cost, losses, MW generation or transmission losses)
2. Environmental dispatch
3. Maximum power transfer

Reactive power objectives

MW and MVAr loss minimization

General goals

1. Minimum deviation from a target schedule
2. Minimum control shifts to alleviate Violations
3. Least absolute shift approximation of control shift

Among the above the following objectives are most commonly used:

(a) Fuel or active power cost optimisation

(b) Active power loss minimisation

(c) VAr planning to minimise the cost of reactive power support

The mathematical description of the OPF problem is presented below:

3.1.1 OPF Objective Function for Fuel Cost Minimization

The OPF problem can be formulated as an optimization problem [2, 5, 6, 18] and is as follows:

Total Generation cost function is expressed as:

$$F(P_G) = \sum_{i=1}^{N_G} \left(\alpha_i + \beta_i P_{G_i} + \gamma_i p_{G_i}^2 \right) \tag{3.1}$$

The objective function is expressed as:

$$Min\, F(P_G) = f(x,u) \tag{3.2}$$

Subject to satisfaction of Non linear Equality Constraints:

$$g(x,u) = 0 \tag{3.3}$$

and Non linear Inequality Constraints:

$$h(x,u) \le 0 \tag{3.4}$$

$$u^{\min} \le u \le u^{\max} \tag{3.5}$$

$$x^{\min} \le x \le x^{\max} \tag{3.6}$$

$F(P_G)$ is total cost function $f(x, u)$ is the scalar objective, $g(x, u)$ represents nonlinear equality constraints (power flow equations), and $h(x, u)$ is the nonlinear inequality constraint of vector arguments x, u.

The vector x contains dependent variables consisting of:

- Bus voltage magnitudes and phase angles
- MVAr output of generators designated for bus voltage control
- Fixed parameters such as the reference bus angle
- Non controlled generator MW and MVAr outputs

- Non controlled MW and MVAr loads
- Fixed bus voltages, line parameters

The vector u consists of control variables including:

- Real and reactive power generation
- Phase – shifter angles
- Net interchange
- Load MW and MVAr (load shedding)
- DC transmission line flows
- Control voltage settings
- LTC transformer tap settings

The equality and inequality constraints are:

- Limits on all control variables
- Power flow equations
- Generation / load balance
- Branch flow limits (MW, MVAr, MVA)
- Bus voltage limits
- Active / reactive reserve limits
- Generator MVAr limits
- Corridor (transmission interface) limits

3.1.2 Constraints for Objective Function of Fuel Cost Minimization

Consider Fig 3.1 representing a standard IEEE 14 Bus single line diagram. 5 Generators are connected to 5 buses. For a given system load, total system generation cost should be minimum.

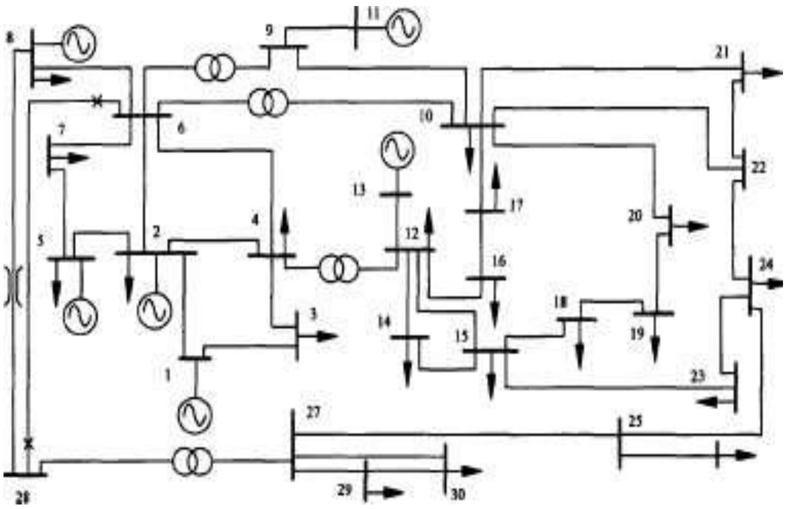

Fig: 3.1 IEEE 14 – Bus Test System

The network equality constraints are represented by the load flow equations [18]:

$$P_i(V,\delta) - P_{Gi} + P_{Di} = 0 \tag{3.7}$$

$$Q_i(V,\delta) - Q_{Gi} + Q_{Di} = 0 \tag{3.8}$$

where:

$$P_i(V,\delta) = |V_i| \sum_{i=1}^{N} |V_i|\,|Y_{ij}| \cos(\delta_i - \delta_j - \Phi_{ij}) \tag{3.9}$$

$$Q_i(V,\delta) = |V_i| \sum_{i=1}^{N} |V_i|\,|Y_{ij}| \sin(\delta_i - \delta_j - \Phi_{ij}) \tag{3.10}$$

$$Y_{ij} = |Y_{ij}| \angle \Phi_{ij} \tag{3.11}$$

and Load balance equation.

$$\sum_{i=1}^{N_G}((P_{Gi}) - \sum_{i=1}^{N_D}(P_{Di}) - P_L = 0 \tag{3.12}$$

The Inequality constraints representing the limits on all variables, line flow constraints,

$$V_{i\min} \le V_i \le V_{i\max}\,, \quad i = 1,\ldots,N, \tag{3.13}$$

$$P_{G_i\min} \le P_{G_i} \le P_{G_i\max}\,, \quad i = 1,\ldots,N_G \tag{3.14}$$

$$Q_{G_i \min} \le Q_{G_i} \le Q_{G_i \max}\,, \quad i = 1, \ldots, N_{G_q,} \tag{3.15}$$

$$-k_{vi}\, I_{l\max} \le V_i - V_j \le K_{vj}\, I_{l\max}, \quad l = 1, \ldots, N_l \tag{3.16}$$

i, j are the nodes of line *l*.

$$-k_{\delta_i} I_{l\max} \le \delta_i - \delta_j \le K_{\delta j} I_{l\max}\,, \quad l = 1, \ldots, N_l \tag{3.17}$$

i, j are the nodes of line *l*.

$$S_{l_i} \le S_{l_{i\max}} \quad i = 1, \ldots, N_l \tag{3.18}$$

$$T_{k\min} \le T_k \le T_{k\max} \quad i = 1, \ldots, N_l \tag{3.19}$$

$$\delta_{i\min} \le \delta_i \le \delta_{i\max} \tag{3.20}$$

3.1.3 OPF Objective Function for Power Loss Minimization

The objective functions to be minimized are given by the sum of line losses

$$P_L = \sum_{k=1}^{N_l} P_{l_k} \tag{3.21}$$

Individual line losses P_{1_k} can be expressed in terms of voltages and phase angles as

$$P_{l_k} = g_k \left[V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j) \right] \tag{3.22}$$

The objective function can now be written as

$$Min\ P_L = \sum_{i=1}^{N_l} g_k \left[(V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j) \right] \tag{3.23}$$

This is a quadratic form and is suitable for implementation using the quadratic interior point method.

The constraints are equivalent to those specified in Section 3.1.1 for cost minimization, with voltage and phase angle expressed in rectangular form.

3.1.4 Constraints for Objective Function of Power Loss Minimization

The controllable system quantities are generator MW, controlled voltage magnitude, reactive power injection from reactive power sources and transformer tapping. The objective use herein is to minimize the power transmission loss function by optimizing the control variables within their limits. Therefore, no violation on other quantities (e.g. MVA flow of transmission lines, load bus voltage magnitude, generator MVAR) occurs in normal system operating conditions. These are system constraints to be formed as equality and inequality constraints as shown below.

The Equality constraints are given by Eqns. (3.7) – (3.12)

The Inequality constraints are given by Eqns. (3.13) – (3.20)

3.1.5 Objectives of Optimal Power Flow

Present commercial OPF programs can solve very large and complex power systems optimization problems in a relatively less time. Many different solution methods have been suggested to solve OPF problems.

In a conventional power flow, the values of the control variables are predetermined. In an OPF, the values of some or all of the control variables need to be known so as to optimize (minimize or maximize) a predefined objective. The OPF calculation has many applications in power systems, real-time control, operational planning, and planning [19–24]. OPF is used in many modern energy management systems (EMSs).

OPF continues to be significant due to the growth in power system size and complex interconnections [25 – 29]. For example, OPF should support deregulation transactions or furnish information on what reinforcement is required. OPF studies can decide the tradeoffs between reinforcements and control options as per the results obtained from carrying out OPF studies. It is clarified when a control option enhances utilization of an existing asset (e.g., generation or transmission), or when a control option is an inexpensive alternative to installing new facilities. Issues of priority of transmission access and VAr pricing or auxiliary costing to afford price and purchases can be done by OPF [2, 3, 28].

The main goal of a generic OPF is to reduce the costs of meeting the load demand for a power system while up keeping the security of the system. From the viewpoint of an OPF, the maintenance of system security requires keeping each device in the power system within its desired operation range at steady-state. This will include maximum and minimum outputs for generators, maximum MVA flows on transmission lines and transformers, as well as keeping system bus voltages within specified ranges.

The secondary goal of an OPF is the determination of system marginal cost data. This marginal cost data can aid in the pricing of MW transactions as well as the pricing auxiliary services such as voltage support through MVAR support. The OPF is capable of performing all of the control functions necessary for the power system. While the economic dispatch of a power system does control generator MW output, the OPF controls transformer tap ratios and phase shift angles as well. The OPF also is able to monitor system security issues including line overloads and low or high voltage problems. If any security problems occur, the OPF will modify its controls to fix them, i.e., remove a transmission line overload.

The quality of the solution depends on the accuracy of the model used. It is essential to define problem properly with clearly stated objectives be given at the onset. No two-power system utilities have the same type of devices and operating requirements. The model form presented here allows OPF development to easily customize its solution to different cases under study [32–38].

OPF, to a large extent depends on static optimization method for minimizing a scalar optimization function (e.g., cost). It was first introduced in the 1960s by Tinney and Dommel [29]. It employs first-order gradient algorithm for minimization objective function subject to equality and inequality constraints. Solution methods were not popular as they are computationally intensive than traditional power flow. The next generation OPF has been greater as power systems operation or planning need to know the limit, the cost of power, incentive for adding units, and building transmission systems a particular load entity.

3.1.6 Optimal Power Flow Challenges

The demand for an OPF tool has been increasing to assess the state and recommended control actions both for off line and online studies, since the first OPF paper was presented in 60's. The thrust for OPF to solve problems of today's deregulated industry and the unsolved problem in the vertically integrated industry has posed further challenges to OPF to evaluate the capabilities of existing OPF in terms of its potential and abilities [30].

Many challenges are before OPF remain to be answered. They can be listed as given below.

1. Because of the consideration of large number of variety of constraints and due to non linearity of mathematical models OPF poses a big challenge for the mathematicians as well as for engineers in obtaining optimum solutions.
2. The deregulated electricity market seeks answer from OPF, to address a variety of different types of market participants, data model requirements and real time processing and selection of appropriate costing for each unbundled service evaluation.
3. To cope up with response time requirements, modeling of externalities (loop flow, environmental and simultaneous transfers), practicality and sensitivity for on line use.
4. How well the future OPF provide local or global control measures to support the impact of critical contingencies, which threaten system voltage and angle stability simulated.

5. Future OPF has to address the gamut of operation and planning environment in providing new generation facilities, unbundled transmission services and other resources allocations.

   Finally it has to be simple to use and portable and fast enough.

After brief overview of the applications of Optimal Power Flow as mentioned above, detailed explanation of the most common applications is given below.

3.2 OPF SOLUTION METHODOLOGIES

A first comprehensive survey regarding optimal power dispatch was given by H.H.Happ [31] and subsequently an IEEE working group [32] presented bibliography survey of major economic-security functions in 1981. Thereafter in 1985, J. Carpentier presented a survey [33] and classified the OPF algorithms based on their solution methodology. In 1990, B. H. Chowdhury et *al* [34] did a survey on economic dispatch methods. In 1999, J. A. Momoh et *al* [3] presented a review of some selected OPF techniques.

The solution methodologies can be broadly grouped in to two namely:

1. Conventional (classical) methods
2. Intelligent methods.

The further sub classification of each methodology is given below as per the Tree diagram.

O P F Solution Methodologies

O P F Methods

- Conventional Methods
  - Gradient Methods
    - Generalised Reduced
    - Reduced Gradient
    - Conjugate Gradient
  - Hessian – based
  - Newton – based
  - Linear Programming
  - Quadratic Programming
  - Interior Point
- Intelligent Methods
  - Artificial Neural Networks
  - Fuzzy Logic
  - Evolutionary Programming
  - Ant Colony
  - Particle Swarm Optimisation

Fig: 3.2 Tree diagram indicating OPF Methodologies

3.2.1 Conventional Methods

Traditionally, conventional methods are used to effectively solve OPF. The application of these methods had been an area of active research in the recent past. The ***conventional methods*** are based on mathematical programming approaches and used to solve different size of OPF problems. To meet the requirements of different objective functions, types of application and nature of constraints, the popular conventional methods is further sub divided into the following [2, 3]**:**

(a) Gradient Method [2, 5, 6, 29]

(b) Newton Method [35]

(c) Linear Programming Method [2, 5, 6, 36]

(d) Quadratic Programming Method [5]

(e) Interior Point Method [2, 5, 6, 37]

Even though, excellent advancements have been made in classical methods, they suffer with the following disadvantages: In most cases, mathematical formulations have to be simplified to get the solutions because of the extremely limited capability to solve real-world large-scale power system problems. They are weak in handling qualitative constraints. They have poor convergence, may get stuck at local optimum, they can find only a single optimized solution in a single simulation run, they become too slow if number of variables are large and they are computationally expensive for solution of a large system.

3.2.2 Intelligent Methods

To overcome the limitations and deficiencies in analytical methods, *Intelligent methods* based on *Artificial Intelligence* (AI) techniques have been developed in the recent past. These methods can be classified or divided into the following,

a) Artificial Neural Networks (ANN) [38]

b) Genetic Algorithms (GA) [5, 6, 8, 14]

c) Particle Swarm Optimization (PSO) [5, 6, 11, 15, 16]

d) Ant Colony Algorithm [39]

The major advantage of the Intelligent methods is that they are relatively versatile for handling various qualitative constraints. These methods can find multiple optimal solutions in single simulation run. So they are quite suitable in solving multi objective optimization problems. In most cases, they can find the global optimum solution. The main advantages of Intelligent methods are: Possesses learning ability, fast, appropriate for non-linear modeling, etc. whereas, large dimensionality and the choice of training methodology are some disadvantages of Intelligent methods.

Detailed description on important aspects like Problem formulation, Solution algorithm, Merits & Demerits and Researchers' contribution on each of the methodology as referred above is presented in the coming sections.

The contribution by Researchers in each of the methodology has been covered with a lucid presentation in Tabular form. This helps the reader to quickly get to know the significant contributions and salient features of the contribution made by Researchers as per the Ref. No. mentioned in the list of References.

3.3 CONVENTIONAL METHODOLOGIES

The list of OPF Methodologies is presented in the Tree diagram Fig. 3.1. It starts with Gradient Method.

3.3.1 Gradient Method

The Generalised Reduced Gradient is applied to the OPF problem [29] with the main motivation being the existence of the concept of the state and control variables, with load flow equations providing a nodal basis for the elimination of state variables. With the availability of good load flow packages, the sensitivity information needed is provided. This in turn helps in obtaining a reduced problem in the space of the control variables with the load flow equations and the associated state variables eliminated.

3.3.1.1 OPF Problem Formulation

The objective function considered is total cost of generation. The objective function to be minimized is

$$F(P_G) = \sum_{all\ gen} F_i\,(P_{Gi}) \tag{3.24}$$

Where the sum extended to all generation on the power system including the generator at reference bus.

The unknown or state vector x is defined as,

$$x = \begin{bmatrix} \delta_i \\ |V_i| \\ \delta_i \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{on each PO bus} \\ \text{on each PV bus} \end{matrix} \tag{3.25}$$

another vector of independent variables, y is defined as [2]:

$$y = \begin{bmatrix} \delta_k \\ |V_k| \\ P_k^{net} \\ Q_k^{net} \\ P_k^{net} \\ |V_k|^{sch} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{on the Slack bus / reference} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{on each PO bus} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{on each PV} \end{matrix} \tag{3.26}$$

The vector y represents all the parameters known that must be specified. Some of these parameters can be adjusted (for example the generator output, P_k^{net} and the generator bus voltage). While some of the parameters are fixed, such as P and Q at each load bus in respect of OPF calculations. This can be understood by dividing the vector y into two parts, u and p.

$$y = \begin{bmatrix} u \\ p \end{bmatrix} \tag{3.27}$$

where u represents the vector of control or adjustable variables and p represents the fixed or constant variables.

Now with this we can define a set of m equations that govern the power flow [2]:

$$g(x,y)=\begin{bmatrix} P_{Gi}\left(|V|,\delta\right)-P_{Gi}^{net} \\ Q_{Gi}\left(|V|,\delta\right)-Q_{Gi}^{net} \\ P_{Gk}\left(|V|,\delta\right)-P_{Gk}^{net} \end{bmatrix} \begin{array}{l} \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{for each } PQ \text{ (load) bus} \\ \left.\begin{array}{l} \\ \end{array}\right\} \begin{array}{l}\text{for each } PV \text{ (generator) bus} \\ \text{not including reference bus}\end{array} \end{array} \tag{3.28}$$

These equations are the bus equations usually referred in Newton Power Flow. It may be noted that the reference bus power generation is not an independent variable. In other words the reference bus generation always changes to balance the power flow, which cannot be specified at the beginning of the calculations.

The cost function / objective function can be expressed as a function of the control variables and state variables. For this, cost function is divided in to the following.

$$F(P_G)=\sum_{gen} F_i\,(P_{Gi})+F_{ref}\,(P_{G\,ref}) \tag{3.29}$$

Where F_{ref} is the cost function of reference bus.

And the first summation does not include the reference bus. The P_{Gi} s are all independent, controlled variables, where as P_{Gref} is a function of the network voltages and angles.

i.e. $P_{Gref}=P_{ref}\,(|v|,\,\delta)$ (3.30)

the cost function becomes

$$= \sum_{gen} F_i(P_{Gi}) + F_{ref}(P_{ref}(v,\ \delta)) \quad = \quad f(x,\ u) \tag{3.31}$$

To solve the optimization problem, we can define Lagrangian function as

$$\Lambda\ (x,\ u,\ p) = f(x,\ u) + \lambda^T g(x,\ u,\ p) \tag{3.32}$$

This can be further written as,

$$\Lambda\ \ (x,u,p) = \sum_{gen} F_i(P_{Gi}) + F_{ref}\,[P_{ref}\,(|v|,\delta)] + [\lambda_1,\lambda_2,..,\lambda_N]\begin{bmatrix} P_i(v,\delta) - P_i^{net} \\ Q_i(v,\delta) - Q_i^{net} \end{bmatrix} \tag{3.33}$$

Thus we have a Lagrange function that has a single objective function and N Lagrange multipliers one for each of the N power flow equations.

3.3.1.2 Solution Algorithm

To minimize the cost function, subject to the constraints, the gradient of Lagrange function is set to zero [2]:

$$\nabla\Lambda\ = 0 \tag{3.34}$$

To do this, the gradient vector is separated in to three parts corresponding to the variables x, u and λ.

It is represented as

$$\nabla\mathcal{L}_x = \frac{\partial\mathcal{L}}{\partial x} = \frac{\partial f}{\partial x} + \left(\frac{\partial g}{\partial x}\right)^T \lambda = 0 \tag{3.35}$$

$$\nabla\mathcal{L}_u = \frac{\partial\mathcal{L}}{\partial u} = \frac{\partial f}{\partial u} + \left(\frac{\partial g}{\partial u}\right)^T \lambda = 0 \tag{3.36}$$

$$\nabla\mathcal{L}_\lambda = \frac{\partial\mathcal{L}}{\partial\lambda} = g(x,\ u,\ p) = 0 \tag{3.37}$$

Eq. (3.35) consists of a vector of derivation of the objective function w.r.t the state variables x. Since the objective function itself is not a function of the state variable except for the reference bus, this becomes:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial}{\partial P_{ref}} F_{ref}(P_{ref}) \frac{\partial P_{ref}}{\partial \delta_1} \\ \frac{\partial}{\partial P_{ref}} F_{ref}(P_{ref}) \frac{\partial P_{ref}}{\partial |V_1|} \\ \vdots \\ \end{bmatrix} \tag{3.38}$$

The $\frac{\partial g}{\partial x}$ term in equation (3.35) is actually the Jacobian matrix for the Newton Power flow which is already known. That is:

$$\left[\frac{\partial g}{\partial x}\right] = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial |V_1|} & \frac{\partial P_1}{\partial \delta_2} & \frac{\partial P_1}{\partial |V_2|} & & \cdots \\ \frac{\partial Q_1}{\partial \delta_1} & \frac{\partial Q_1}{\partial |V_1|} & \frac{\partial Q_1}{\partial \delta_2} & \frac{\partial Q_1}{\partial |V_2|} & & \cdots \\ \frac{\partial P_2}{\partial \delta_1} & \frac{\partial P_2}{\partial |V_1|} & & & \cdots & \\ \frac{\partial Q_2}{\partial \delta_1} & \frac{\partial Q_2}{\partial |V_1|} & & & \cdots & \\ \vdots & & & & \vdots & \\ \end{bmatrix} \tag{3.39}$$

This matrix has to be transposed for use in Eq. (3.35). Eq. (3.36) is the gradient of the Lagrange function w.r.t the control variables. Here the vector $\frac{\partial f}{\partial u}$ is vector of the derivatives of the objective function w.r.t the control variables.

$$\frac{\partial f}{\partial u} = \begin{bmatrix} \frac{\partial}{\partial P_1} \; F_1(P_1) \\ \frac{\partial}{\partial P_2} \; F_2(P_2) \\ \vdots \end{bmatrix} \tag{3.40}$$

The other term in Eq. (3.36), $\left(\frac{\partial g}{\partial u}\right)$ actually consists of a matrix of all zeroes with some -1 terms on the diagonals, which correspond to Eq. in $g(x, u, p)$ where a control variable is present. Finally Eq. (3.37) consists of the power flow equation themselves.

Algorithm for Gradient Method

The solution steps of the gradient method of OPF are as follows.

Step 1: Given a set of fixed parameters *p*, assume a starting set of control variables '*u*'.

Step 2: Solve for Power flow. This guarantees Eq. (3.33) is satisfied.

Step 3: Solve Eq. (3.32) for λ

$$\lambda = -\left[\left(\frac{\partial g}{\partial x}\right)\right]^{T^{-1}} \frac{\partial f}{\partial x} \tag{3.41}$$

Step 4: Substitute λ from Eq. (3.41) into Eq. (3.36) and compute the gradient.

$$\nabla \mathcal{L} = \frac{\partial \mathcal{L}}{\partial u} = \frac{\partial f}{\partial u} + \left[\frac{\partial g}{\partial u}\right]^T \cdot \lambda \tag{3.42}$$

Step 5: If $\nabla \mathcal{L}$ equals zero within the prescribed tolerance, the minimum has been reached other wise:

Step 6: Find a new set of control variables.

$$u^{new} = u^{old} + \Delta u \tag{3.43}$$

where $\Delta u = -\alpha \nabla \Lambda$

Here Δu is a step in negative direction of the gradient. The step size is adjusted by the positive scalar α.

In this algorithm, the choice of α is very critical. Too small a value of α guarantees the convergence, but slows down the process of convergence; too a high a value of α causes oscillations around the minimum. Several methods are available for optimum choice of step size.

3.3.1.3 OPF Solution by Gradient Method — Researchers' Contribution

The Significant Contributions/Salient Features of Presentations made by Researchers are furnished below:

Sl.No.	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions/Salient Features
1	Dommel H.W. and Tinney W.F [29]	Optimal power flow solutions	*IEEE Transactions on Power Apparatus and Systems,* PAS- 87, pp. 1866–1876, October 1968.	➢ Using penalty function optimization approach, developed nonlinear programming (NLP) method for minimization of fuel cost and active power losses. ➢ Verification of boundary, using Lagrange multiplier approach, is achieved. ➢ Capable of solving large size power system problems up to 500 buses. ➢ Its drawback is in the modeling of components such as transformer taps that are accounted in the load flow but not in the optimization routine.

2	C. M. Shen and M.A. Laughton [40]	Determina-tion of Optimum Power System Operating Conditions	Proceedings of IEEE, vol. 116, No. 2, pp. 225-239, 1969.	➢ Provided solutions for power system problems by an iterative indirect approach based on Lagrange-Kuhn-Tucker conditions of optimality. ➢ A sample 135 kV British system of 270 buses was validated by this method and applied to solve the economic dispatch objective function with constraints. ➢ Constraints include voltage levels, generator loading, reactive-source loading, transformer-tap limits, transmission-line loading. ➢ This method shown less computation time, with a tolerance of 0.001, when compared to other penalty function techniques.
3	O. Alasc and B Stott [41]	Optimum Load Flow with steady state security	*IEEE Transactions on Power Apparatus and Systems,* PAS- 93, pp.745–754, 1974.	➢ Developed a non linear programming approach based on reduced gradient method utilizing the Lagrange multiplier and penalty- function technique. ➢ This method minimises the cost of total active power generation. ➢ Steady state security and insecurity constraints are incorporated to make the optimum power flow calculation a powerful and practical tool for system operation and design. . ➢ Validated on the 30- bus IEEE test system and solved in 14.3 seconds. ➢ The correct choice of gradient step sizes is crucial to the success of the algorithm.

3.3. 1. 4 Merits and Demerits of Gradient Method

The Merits and Demerits of Gradient Method are summarized and given below.

Merits

1) With the Gradient method, the Optimal Power Flow solution usually requires 10 to 20 computations of the Jacobian matrix formed in the Newton method.
2) The Gradient procedure is used to find the optimal power flow solution that is feasible with respect to all relevant inequality constraints. It handles functional inequality constraints by making use of penalty functions.
3) Gradient methods are better fitted to highly constrained problems.
4) Gradient methods can accommodate non linearities easily compared to Quadratic method.
5) Compact explicit gradient methods are very efficient, reliable, accurate and fast.

This is true when the optimal step in the gradient direction is computed automatically through quadratic developments.

Demerits

1) The higher the dimension of the gradient, the higher the accuracy of the OPF solution. However consideration of equality and inequality constraints and penalty factors make the relevant matrices less sparse and hence it complicates the procedure and increases computational time.

2) Gradient method suffers from the difficulty of handling all the inequality constraints usually encountered in optimum power flow.

3) During the problem solving process, the direction of the Gradient has to be changed often and this leads to a very slow convergences. This is predominant, especially during the enforcement of penalty function; the selection of degree of penalty has bearing on the convergence.

4) Gradient methods basically exhibit slow convergence characteristics near the optimal solution.

5) These methods are difficult to solve in the presence of inequality constraints.

3.3.2 Newton Method

In the area of Power systems, Newton's method is well known for solution of Power Flow. It has been the standard solution algorithm for the power flow problem for a long time The Newton approach [42] is a flexible formulation that can be adopted to develop different OPF algorithms suited to the requirements of different applications. Although the Newton approach exists as a concept entirely apart from any specific method of implementation, it would not be possible to develop practical OPF programs without employing special sparsity techniques. The concept and the techniques together comprise the given approach. Other Newton-based approaches are possible.

Newton's method [2, 35] is a very powerful solution algorithm because of its rapid convergence near the solution. This property is especially useful for power system applications because an initial guess near the solution is easily attained. System voltages will be near rated system values, generator outputs can be estimated from historical data, and transformer tap ratios will be near 1.0 p.u.

3.3.2.1 OPF Problem Formulation

Eqns. (3.1) – (3.6) describe the OPF Problem and constraints.

3.3.2.2 Solution Algorithm

The solution for the Optimal Power Flow by Newton's method requires the creation of the Lagrangian as shown below [35, 42]:

$$L(z) = f(x) + \mu^T h(x) + \lambda^T g(x) \tag{3.44}$$

where $z = \begin{bmatrix} x & \mu & \lambda \end{bmatrix}^T$, μ and λ are vectors of the Lagrange multipliers, and *g(x)* only includes the active (or binding) inequality constraints.

A gradient and Hessian of the Lagrangian is then defined as

Gradient = $\nabla L(z) = \left[\dfrac{\partial L(z)}{\partial z_i} \right]$ = a vector of the first partial derivatives of the Lagrangian (3.45)

$$\text{Hessian} = \nabla^2 L(z) = H = \left[\frac{\partial^2 L(z)}{\partial z_i \partial z_j} \right] = \begin{bmatrix} \dfrac{\partial^2 L(z)}{\partial x_i \partial x_j} & \dfrac{\partial^2 L(z)}{\partial x_i \partial \mu_j} & \dfrac{\partial^2 L(z)}{\partial x_i \partial \lambda_j} \\ \dfrac{\partial^2 L(z)}{\partial \mu_i \partial x_j} & 0 & 0 \\ \dfrac{\partial^2 L(z)}{\partial \lambda_i \partial x_j} & 0 & 0 \end{bmatrix} \leftarrow \text{a matrix of the second partial derivatives of the Lagrangian}$$

(3. 46)

It can be observed that the structure of the Hessian matrix shown above is extremely sparse. This sparsity is exploited in the solution algorithm.

According to optimization theory, the Kuhn-Tucker necessary conditions of optimality can be mentioned as given under,

Let $Z^* = [x^*, \lambda^*, \mu^*]$, is the optimal solution.

$$\nabla_x L(z^*) = \nabla_x L([x^*, \lambda^*, \mu^*]) = 0 \qquad (3.47)$$

$$\nabla_\lambda L(z^*) = \nabla_\lambda L([x^*, \lambda^*, \mu^*]) = 0 \qquad (3.48)$$

$$\nabla_\mu L(z^*) = \nabla_\mu L([x^*, \lambda^*, \mu^*]) = 0 \qquad (3.49)$$

$\lambda_i^* \geq 0$ if $h(x^*) = 0$ (i.e., the inequality constraint is active) (3.50)

$\lambda_i^* = 0$ if $h(x^*) \leq 0$ (i.e., the inequality constraint is not active) (3.51)

$\mu_i^* = 0$ Real (3.52)

By solving the equation $\nabla_z L(z^*) = 0$, the solution for the optimal problem can be obtained.

It may be noted that special attention must be paid to the inequality constraints of this problem. As noted, the Lagrangian only includes those inequalities that are being enforced. For example, if a bus voltage is within the desired operating range, then there is no need to activate the inequality constraint associated with that bus voltage. For this Newton's method formulation, the inequality constraints have to be handled by separating them into two sets: active and inactive. For efficient algorithms, the determination of those inequality constraints that are active is of utmost importance. While an inequality constraint is being enforced, the sign of its associated Lagrange multiplier at solution determines whether continued enforcement of the constraint is necessary. Essentially the Lagrange multiplier is the negative of the derivative of the function that is being minimized with respect to the enforced constraint. Therefore, if the multiplier is positive, continued enforcement will result in a decrease of the function, and enforcement is thus maintained. If it is negative, then enforcement will result in an increase of the function, and enforcement is thus stopped. The outer loop of the flow chart in Fig. 3.2 performs this search for the binding or active constraints.

Considering the issues discussed above, the solution of the minimization problem can be found by applying Newton's method.

Algorithm for Newton method

Once an understanding of the calculation of the Hessian and Gradient is attained, the solution of the OPF can be achieved by using the Newton's method algorithm.

Step 1: Initialize the OPF solution.

a) Initial guess at which inequalities are violated.
b) Initial guess z vector (bus voltages and angles, generator output power, transformer tap ratios and phase shifts, all Lagrange multipliers).

Step 2: Evaluate those inequalities that have to be added or removed using the information from Lagrange multipliers for hard constraints and direct evaluation for soft constraints.

Step 3: Determine viability of the OPF solution. Presently this ensures that at least one generator is not at a limit.

Step 4: Calculate the Gradient (Eq. (3.51)) and Hessian (Eq. (3.52)) of the Lagrangian.

Step 5: Solve the Eq. $[H]\Delta z = \nabla L(z)$.

Step 6: Update solution $z_{new} = z_{old} - \Delta z$.

Step 7: Check whether $\| \Delta z \| < \varepsilon$. If not, go to Step 4, otherwise continue.

Step 8: Check whether correct inequalities have been enforced. If not go to Step 2. If so, problem solved.

3.3.2.3 OPF Solution by Newton Method — Researchers Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions
1	A. M. H. Rashed and D. H. Kelly [43]	Optimal Load Flow Solution Using Lagrangian Multipliers and the Hessian Matrix	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-93, pp. 1292-1297, 1974.	➢ While using Lagrange multiplier and Newton's method, the method also introduced an acceleration factor to compute the update controls ➢ As an extension of Tinney's work, it employs a nonlinear programming methodology based on the homotopy continuation algorithm for minimizing loss and cost objective functions ➢ Validation of voltage magnitude was done on 179-bus system and results are comparable to augmented MINOS schemes.
2	H. H. Happ. [44]	Optimal Power Dispatch	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-93, no. 3, pp. 820-830 May/June, 1974.	➢ Application of Lagrange multipliers to an economic dispatch objective function was presented. ➢ Obtained solution for incremental losses, using the Jacobian matrix attained from Newton –Raphson load flow. ➢ Results obtained on a 118 bus test system are good for both on-line and off-line operations. ➢ Comparable with the B-matrix method in terms of optimum production cost for total generation, losses, and load.

3	David I. Sun, Bruce Ashley, Brian Brewer, Art Hughes, William F. Tinney [35]	Optimal Power Flow by Newton Approach	*IEEE Transactions on Power Apparatus and Systems,* vol.PAS-103, no. 10, pp. 2864-2879, Oct 1984.	➢ Network sparsity techniques and Lagrange multiplier approach was used ➢ Solution for reactive power optimization based on Newton method was presented. ➢ Quadratic approximation of the Lagrangian was solved at each iteration and also validated on an actual 912-bus system. ➢ Approach is suitable for practical large systems due to super-linear convergence to Kuhn-Tucker condition makes.
4	Maria, G. A. and Findlay, J. A., A [45]	Newton optimal power flow program for Ontario hydro EMS	*IEEE Transactions on Power Systems,* vol. PWRS-2, pp. 576-584, Aug. 1987.	➢ Initially, the augmented Lagrangian is formed. ➢ Set of Non linear equations as, first partial derivatives of the augmented objective with respect to the control variables are obtained. ➢ All the Non linear equations are solved simultaneously by the NR method unlike the Dommel and Tinney method, where only part of these equations is solved by the NR method.
5	M. V. F. Pereira, L. M. V. G. Pinto, S. Granville and A. Monticelli [46]	A Decomposition Approach to Security Constrained Optimal Power Flow with Post Contingency Corrective Rescheduling	9th PSCC Conference, pp. 585-591, 1987.	➢ Solution for Economic dispatch problem with security constraints using Bender's decomposition approach is obtained. ➢ In addition, solution is also obtained for dispatch problems like : *the pure economic dispatch problem, the security-constrained dispatch problem and the security-constrained dispatch with re-scheduling problem* ➢ This method linearises AC/DC power flows and performs sensitivity analysis of load variations ➢ Practical testing of the method has shown encouraging results.
6	C. W. Sanders and C. A. Monroe [47]	An Algorithm for Real-Time Security Constrain	*IEEE Transactions on Power Systems,* vol. PWRS-2, no. 4,	➢ For security constrained dispatch calculations provided an algorithm ➢ The method was validated on a 1200 bus 1500 line practical power system.

		ed Dispatch	pp. 175-182 November 1987.	➢ Designed constrained economic dispatch calculation (CEDC) in order to achieve following goals: a) Establish economic base points to load frequency control (LFC); b) Enhance dependability of service by considering network transmission limitations, c)Furnish constrained participation factors, d) Adaptable to current control computer systems. ➢ CEDC is efficient compared to benchmark OPF algorithm and adapts the basic Lagrange multiplier technique for OPF. It is assumed to be in the standard cubic polynomial form algorithm. ➢ The objective of CEDC was optimized subject to area constraints, line constraints, and the line-group constraints. Computation of constraint-sensitivity factors was done to linearise security constraints. ➢ The sensitivity factors can be decided from telemetry-based values of the fractional system load internal to the bounded area. ➢ Load flow was adapted to stimulate the periodic incremental system losses but not as a constraint.
7	A. Monticelli M. V. F. Pereira, and S. Granville [48]	Security Constrained Dispatch	*IEEE Transactions on Power Systems,* vol. PWRS-2, no. 4, pp. 175-182, November 1987.	➢ An algorithm based on mathematical programming decomposition, for solving an economic dispatch problem with security constraints is presented. ➢ Separate contingency analysis with generation rescheduling can be done to estimate constraint violation. ➢ Preventive control actions are built in and an automatic way of adjusting the controls are included ➢ Using Monticelli's method, the specific dispatch problem with rescheduling was tested on the

				IEEE 118-bus test system. ➢ Detecting infeasibility is also included in this method. employed
8	Monticelli and Wen-Hsiung E. Liu [49]	Adaptive Movement Penalty Method for the Newton's Optimal Power Flow	*IEEE Transactions on Power Systems,* vol 7, no. 1, pp. 334-342, 1992.	➢ This method introduces adaptive movement penalties to ensure positive definitiveness and convergence is attained without any negative affect. ➢ Handling of penalties is automatic and tuning is not required. ➢ Results are encouraging when tested on the critical 1650 –bus system.
9	S. D. Chen and J. F. Chen [50]	A new algorithm based on the Newton-Raphson approach for real-time emission dispatch	*Electric Power System Research,* vol. 40 pp. 137-141, 1997.	➢ An algorithm based on Newton-Raphson (NR) method covering sensitivity factors to solve emission dispatch in real-time is proposed. ➢ Development of Jacobian matrix and the B-coefficients is done in terms of the generalized generation shift distribution factor. ➢ Computation of penalty factor and incremental losses is simplified with fast Execution time.
10	K. L. Lo and Z. J. Meng [51]	Newton-like method for line outage simulation	*IEE Proceedings-Generations, Transmission, Distribution,* vol. 151, no. 2, pp. 225-231, March 2004.	➢ Fixed Newton method and the modification of the right hand side vector method are presented for simulation of line outage ➢ Above methods have better convergence characteristics than Fast decoupled load flow method and Newton based full AC load flow method.
11	X. Tong and M. Lin [52]	Semi smooth Newton-type algorithms for solving optimal power flow problems	*Proceedings of IEEE/PES Transmission and Distribution Conference,* Dalian, China, pp. 1-7, 2005.	➢Semi smooth Newton- type algorithm is presented where in General inequality constraints and bounded constraints are tackled separately. ➢The KKT system of the OPF is altered to a system of non smooth bounded constrained equations with inclusion of diagonal matrix and the non linear complementary function. ➢Number of variables is less with low computing cost.

3.3.2.4 Merits and Demerits of Newton Method

The Merits and Demerits of Newton Method are summarized and given below.

Merits

1) The method has the ability to converge fast.
2) It can handle inequality constraints very well.
3) In this method, binding inequality constraints are to be identified, which helps in fast convergence.
4) For any given set of binding constraints, the process converges to the Kuhn-Tucker conditions in fewer iterations.
5) The Newton approach is a flexible formulation that can be used to develop different OPF algorithms to the requirements of different applications.
6) With this method efficient and robust solutions can be obtained for problems of any practical size.
7) Solution time varies approximately in proportion to network size and is relatively independent of the number of controls or inequality constraints.
8) There is no need of user supplied tuning and scaling factors for the optimisation process.

Demerits

1) The penalty near the limit is very small by which the optimal solution will tend to the variable to float over the limit
2) It is not possible to develop practical OPF programs without employing sparsity techniques.

3) Newton based techniques have a drawback of the convergence characteristics that are sensitive to the initial conditions and they may even fail to converge due to inappropriate initial conditions.

3.3.3 Linear Programming Method

Linear Programming (L.P) method [2, 5] treats problems having constraints and objective functions formulated in linear form with non negative variables. Basically the simplex method is well known to be very effective for solving LP problems.

The Linear Programming approach has been advocated [53] on the grounds that

(a) The L.P solution process is completely reliable.

(b) The L.P solutions can be very fast.

(c) The accuracy and scope of linearised model is adequate for most engineering purposes.

It may be noted that point (a) is certainly true while point (b) depends on the specific algorithms and problem formulations. The observation (c) is frequently valid since the transmission network is quasi linear, but it needs to be checked out for any given system and application.

3.3.3.1 OPF Problem Formulation

The L.P based algorithm solves the OPF problems as a succession of linear approximations.

The objective function can be written in the following form [3]:

Minimise $F(x^0 + \Delta x,\ u^0 + \Delta u)$ (3.53)

Subject to $g'(x^0 + \Delta x,\ u^0 + \Delta u) = 0$ (3.54)

$$h'\ (x^0 + \Delta x,\ u^0 + \Delta u) \leq 0 \tag{3.55}$$

where x^0, u^0 are the initial values of x and u.

$\Delta x, \Delta u$ are the shifts about the initial points.

g', h' are the linear approximations to the original are the non linear constraints.

3.3.3.2 Solution Algorithm

The basic steps required in the L.P based OPF algorithm is as follows [53]:

Step 1: Solve the power flow problem for nominal operating conditions.

Step 2: Linearise the OPF problem (express it in terms of changes about the current exact system operating point) by,

a) Treating the limits of the monitored constraints as changes with respect to the values of these quantities, accurately calculated from the power flow.

b) Treating the incremental control variables Δu as changes about the current control variables (affected by shifting the cost curves).

Step 3: Linearise the incremental network model by,

a) Constructing and factorising the network admittance matrix (unless it has not changed since last time performed)

b) Expressing the increamental limits obtained in step 2 (b) in terms of incremental control variables Δu .

Step 4: Solve the linearly constrained OPF problem by a special dual piece wise linear relaxation L.P algorithm computing the increamental control variables.

Step 5: Update the control variables $u = u + \Delta u$ and solve the exact non linear power flow problem.

Step 6: If the changes in the control variables in step 4 are below user defined tolerances the solution has not been reached. If not go to step 4 and continue the cycle.

It may be observed that step 4 is the key step since it determines the computational efficiency of the algorithm. The algorithm solves the network and test operating limits in sparse form while performing minimisation in the non – sparse part. For steps 1 and 5, solving the exact non linear power flow problem $g(x,u) = 0$ is required to provide an accurate operating x^0. With this the optimisation process, can be initiated as a starting point or at a new operating point following the rescheduling of control variables. However the power flow solution may be performed using either the Newton- Raphson (NR) power flow method or Fast Decoupled power flow (FDFF) method.

As can be seen from Eq. (3.29), the optimisation problem solved at each iteration is a linear approximation of the actual optimisation problem. Steps 2 and 3 in the LP based OPF algorithm correspond to forming the linear network model and it can be expressed in terms of changes about the operating point. The Linearised network constraints models can be derived using either a Jacobian – based coupled formulation given by

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \quad or \quad \Delta u \; PQ = J \, \Delta x \qquad (3.56)$$

or a decoupled formulation based on the modified Fast decoupled power flow equations given as under

$$B' \Delta\delta = \Delta P \qquad (3.57)$$

$$B'' \Delta V = \Delta Q \qquad (3.58)$$

In most applications of L.P based OPF, the later model is used.

3.3.3.3 OPF Solution by LP Method — Researchers Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions/ Salient Features
1	D. W. Wells [54]	Method for Economic Secure Loading of a Power System	*Proceedings of IEEE*, vol. 115, no. 8, pp. 606-614, 1968.	➢ A linear programming method to formulate an economical schedule, consistent with network security requirements for loading plants in power system, was developed. ➢ Simplex method was used to solve cost objective and its constraints. Further a scheme was adopted for selecting and updating variables at the buses. ➢ It is a decomposition approach based on Dantzig and Wolfe's algorithm. ➢ The drawbacks are: (a) optimum results may not be obtained for an infeasible situation and (2) Digital computers may create rounding errors by which constraints may be overloaded.

2	C. M. Shen and M. A. Laughton [55]	Power System Load Scheduling with Security Constraints using Dual Linear Programming	*Proceedings of IEEE,* vol. 117, no. 1, pp. 2117-2127, 1970.	➢ A dual linear programming technique was tested on a 23-bus theoretical power system. ➢ The problem formulation has taken in to account single-line outages. ➢ Revised simplex method was adopted to obtain solutions for both primal and dual problems. ➢ Changes in system networks were made based on variational studies of system dispatch load. ➢ The method has shown encouraging results.
3	B. Stott and E. Hobson [56]	Power System Security Control Calculation using Linear Programming, Parts I & II	*IEEE Transactions Power Apparatus and Systems,* vol. PAS-97 no.5, pp. 1713-1731, Sept. 1978.	➢ The method provided control actions to relieve network overloads during emergency conditions. ➢ A linear programming iterative technique was used for network sparsity selection of binding constraints and the implementation of a dual formation. ➢ The computational burden was reduced, due to larger number of buses of the LP method. ➢ The method has six prioritized objective functions. The method is capable of handling load shedding, high voltage taps, large sized systems. Further, it also handles infeasibility using heuristics. ➢ It is proved to be very efficient at various load levels. The sensitivity is robust at different generator and line outages, but it is restricted to linear objective functions.
4	B. Stott and J. L. Marinho [57]	Linear Programming for Power System Network Security Applications	*IEEE Transactions Power Apparatus and Systems,* vol. PAS-98, No.3,	➢ A modified revised simplex technique was used for calculations of security dispatch and emergency control, on large power systems. ➢ It has used multi-segment generator cost curves and sparse matrix techniques.

			pp. 837-848 June 1979.	➢ A generalized linear programming code was followed instead of classical linear programming approach. ➢ Solutions were obtained by Linearization of the objective functions which were quadratic cost curves and the weighted least square approach. ➢ Practical components such as transformer tap setting were included and the results were fast and efficient.
5	W. O. Stadlin and D.L. Fletcher [58]	Voltage versus Reactive Current Model for Dispatch and Control	*IEEE Transactions Power Apparatus and Systems,* vol. PAS-101, pp. 3751-3758, October 1982.	➢ A network modeling technique showing the effect of reactive control of voltage, by using a current model for voltage/reactive dispatch and control is described. ➢ The method allows the typical load flow equation to be decomposed in to reactive power and voltage magnitude. ➢ Sensitivity coefficients are worked out from voltage and VAR coefficients further, other devices such as current models, transformer taps, incremental losses, and sensitivity of different models can also be modeled. ➢ Efficiency of the voltage/VAR model is dependent on the estimation of load characteristics and modeling of equivalent external network. ➢ The method was verified on a 30-bus IEEE test system.
6	M. R. Irving and M. J. H. Sterling [59]	Economic dispatch of Active Power with Constraints Relaxation	IEE Proceedings C, Vol. 130, No. 4, 1983.	➢ Using an AC power flow, the problem of economic dispatch of active power with constraints relaxation was solved by the LP method. ➢ It is able to solve up to 50-generation and 30-node systems.
7	E. Houses and G.	Real and Reactive Power	*IEEE Transaction s on Power*	➢ Quasi-Newton linear programming using a variable weights method with multiple objective functions is proposed.

	Irisarri [60]	System Security Dispatch Using a Variable Weights Optimisation Method	*Apparatus and Systems,* vol. PAS-102, pp. 1260-1268, 1983.	➢ Hessian matrix was improved by sparsity coding in place of full Hessian. ➢ Set of penalty functions with variable weights coefficients are represented as linearised constraints. ➢ Feasibility retention and optimum power flow solution is obtained with the use of "Guiding function". ➢ The method is verified on 14 and 118-bus systems and performance is comparable to o methods on small-size systems.
8	S. A. Farghal, M. A. Tantawy, M. S. Abou-Hussein, S. A. Hassan and A. A. Abou-Slela [61]	A Fast Technique for Power System Security Assessment Using Sensitivity Parameters of Linear Programming	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-103, No. 5, pp. 946-953 May 1984.	➢ A method is developed for real-time control of power system under emergency conditions. ➢ Insecure system operating conditions are corrected by using sensitivity parameters. It is achieved through set of control actions based on the optimal re-dispatch function. ➢ Transmission line overload problems are taken care of. ➢ Classical dispatch fast decoupled load flow and ramp rate constraints were used in this method. ➢ It was validated on a 30-bus system for various loads and is appropriate for on-line operation.
9	R. Mota-Palomino and V. H. Quintana [62]	A Penalty Function-Linear Programming Method for Solving Power System Constrained Economic Operation Problems	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-103, pp. 1414-1442, June 1984.	➢ Solved constrained Economic Operation Problem using a non-conventional linear programming technique involving a piece-wise differentiable penalty function approach. ➢ Objectives of contingency constrained economic dispatch (CED) with linear constraints were achieved by employing this method. ➢ Optimal solution was attained, independent of a feasible starting point and it was verified on a 10-, 23-, and 118-bus systems.

				➢ At a certain point pseudo gradient of the penalty function is a linear combination of the column of the active set matrix or not, decides the descent direction. The method's optimal step-size was decided by choosing the direction so that the active constraints remain active or feasible and hence, only inactive constraints were considered to determine a step-size. ➢ In all the cases analysed, the approach requires few iterations to obtain an optimal solution compared to standard primal simplex methods. ➢ CPU time is reduced by lesser number of iterations due to equality constraints formed as a result of entry of artificial variables linked with constraints. ➢ It is used in both dual linear programming formulations and quadratic programming problems.
10	R. Mota-Palomino and V. H. Quintana [63]	Sparse Reactive Power Scheduling by a Penalty-Function-Linear Programming Technique	*IEEE Transactions on Power Systems,* vol. PWRS-1, pp. 31-39, 1986.	➢ Solution for Reactive power dispatch problems is provided using an algorithm based on penalty-function linear programming. ➢ A sparse reactive power sensitivity matrix was modeled as a by this method. It is a powerful constraint relaxation approach to handle linearised reactive dispatch problems. ➢ Many constraint violations are permitted and infeasibility is overcome by selecting a point closer to a feasible point. ➢ Sensitivity matrix (bipartite graph) takes care of large size systems and helps to decide which constraints are binding.

				➢ The reactive power dispatch problem includes various vector functions: A vector of costs coupled with changes in (a) Generated voltages at voltage-controlled nodes. (b) Shunt susceptance connected to the nodes of the system. (c) Transformer turns ratios. ➢ Boundaries are set, on the variations in the variables and the reactive current generations, by the inequality constraints. The problem formulation covers both hard and soft constraints. ➢ Sensitivity graph was employed to define a subset of constraints for reducing the computational burden. This subset was built in for the formulation of linear programming problem ➢ It is an efficient method due to sparsity technique in its formulation and performance was good when tested on a 256-node, 58-voltage-controlled-node interconnected.
11	M. Santos-Neito and V. H. Quintana [64]	Linear Reactive Power Studies for Longitudinal Power Systems	9^{th} PSCC Conference, pp. 783-787, 1987.	➢ Linear reactive power flow problems were solved by a penalty function linear programming algorithm ➢ A scheme for handling infeasibility is included. ➢ Analysis was made on three objectives i.e., real power losses, load voltage deviation, and feasibility enforcement of violated constraints. ➢ The method was verified on a 253-bus Mexican test system.
12	T. S. Chung and Ge Shaoyun	A recursive LP-based approach for optimal	*Electric Power System Research,*	➢ Achieved optimal capacitor allocation and reduction of line losses in a distribution system, using recursive linear programming.

	[65]	capacitor allocation with cost-benefit consideration	vol. 39, pp. 129-136, 1997.	➢ Computational time and memory space are reduced as this method does not require any matrix inversion.
13	E. Lobato, L. Rouco, M. I. Navarrete, R. Casanova and G. Lopez [66]	An LP-based optimal power flow for transmission losses and generator reactive margins minimization	*Procedure of IEEE porto power tech conference,* Portugal, Sept. 2001.	➢ LP based OPF was applied for reduction of transmission losses and Generator reactive margins of the system. ➢ Integer variables represented the discrete nature of shunt reactors and capacitors ➢ Objective function and the constraints are linearised in each iteration, to yield better results.
14	F. G. M, Lima, F. D. Galiana, I. Kockar and J. Munoz [67]	Phase shifter placement in large scale systems via mixed integer linear programming	*IEEE Transactions Power Systems,* vol. 18, no. 3, pp. 1029-1034, Aug. 2003.	➢ Design analysis was made on the combinatorial optimal placement of Thyristor Controlled Phase Shifter Transformer (TCPST) in large scale power systems, using Mixed Integer Linear Programming. ➢ The number, network location and settings of phase shifters to enhance system load ability are determined under the DC load flow model. ➢ Restrictions on the installation investment or total number of TCPSTs are satisfied ➢ Execution time is considerably reduced compared to other available similar cases.

3.3.3.4 Merits and Demerits of Linear Programming Method

The Merits and Demerits of Linear Programming Method are summarized and given below.

Merits

1) The LP method easily handles Non linearity constraints

2) It is efficient in handling of inequalities.

3) Deals effectively with local constraints.

4) It has ability for incorporation of contingency constraints.

5) The latest LP methods have over come the difficulties of solving the non separable loss minimisation problem, limitations on the modeling of generator cost curves.

6) There is no requirement to start from a feasible point .The process is entered with a solved or unsolved power flow. If a reactive balance is not initially achievable, the first power flow solution switches in or out the necessary amount of controlled VAR compensation

7) The LP solution is completely reliable

8) It has the ability to detect infeasible solution

9) The LP solution can be very fast.

10) The advantages of LP approach ,such as, complete computational reliability and very high speed enables it , suitable for real time or steady mode purposes

Demerits

1) It suffers lack of accuracy.

2) Although LP methods are fast and reliable, but they have some disadvantages associated with the piecewise linear cost approximations.

3.3.4 Quadratic Programming Method

Quadratic Programming (QP) is a special form of NLP. The objective function of QP optimisation model is quadratic and the constraints are in linear form. Quadratic Programming has higher accuracy than LP – based approaches. Especially the most often used objective function is a quadratic.

The NLP having the objective function and constraints described in Quadratic form is having lot of practical importance and is referred to as quadratic optimisation. The special case of NLP where the objective function is quadratic (i.e. is involving the square, cross product of one or more variables) and constraints described in linear form is known as quadratic programming. Derivation of the sensitivity method is aimed at solving the NLP on the computer. Apart from being a common form for many important problems, Quadratic Programming is also very important because many of the problems are often solved as a series of QP or Sequential Quadratic Programming (SQP) problems [18, 68].

Quadratic Programming based optimisation is involved in power systems [69] for maintaining a desired voltage profile, maximising power flow and minimizing generation cost. These quantities are generally controlled by complex power generation which is usually having two limits. Here minimisation is considered as maximisation can be determined by changing the sign of the objective function. Further, the quadratic functions are characterized by the matrices and vectors.

3.3.4.1 OPF Problem Formulation

The objective and constraint functions may be expressed, respectively, as [5]:

$$f(x) = \frac{1}{2} x^T R x + a^T x \qquad (3.59)$$

$$f_i(x) = \frac{1}{2} x^T H_i x + b_i^T x \qquad i = 1, 2, \ldots m \qquad (3.60)$$

R together with *H* are *n* square and symmetrical matrices, and *x*, *a* together with *b* are *n* vectors. As per the definition, the constraints are bounded by,

$C_i \leq f_i(x) \leq D_i$, for all $i = 1, 2, \ldots m$ (*m* number of constraints (3.61)

Among these *i*, the first *p* are equalities $(C_i = D_i$, for $i \leq p)$.

Let $$A = \begin{bmatrix} S & F_x^T \\ F_x & 0 \end{bmatrix}, \; y = \begin{bmatrix} \Delta x \\ \Delta Z \end{bmatrix}, \; b = \begin{bmatrix} -U \\ \Delta K \end{bmatrix} \qquad (3.62)$$

$$Ay = b \qquad (3.63)$$

The matrix *A* and *n*-vector *U* in Eq. (3.63) can be found by using:

$$F_x^T = \left[H_1 x + b_1, H_2 x + b_2, \ldots, H_m x + b_m \right] \qquad (3.64)$$

$$S = R + \sum_{i=1}^{m} \lambda_i H_i \, , w = a + \sum_{i=1}^{m} \lambda_i b_i \qquad (3.65)$$

and $U = Sx + w$ (3.66)

Consider an *m*-vector J with component $J_i = (1, 2, \ldots, m)$, defined by

$$T = \min\left[\lambda_i, (D_i - K_i)\right] \qquad (3.67)$$

and

$$J_i = \max\left[T, (C_i - K_i)\right] \qquad (3.68)$$

for all i=1, 2... m. It can be concluded that a set of EKT (Extended Kuhn Tucker conditions) is satisfied if and only if $U=0$ and $J=0$.

To Compute ΔK :

Consider a change Δx about a known x. The second-order approximation of the objective function can be written as

$$f(x+\Delta x) = f(x) + f_x \Delta x + \frac{1}{2}\Delta x^T f_{xx} \Delta x \qquad (3.69)$$

Where the partial derivatives are evaluated at x. This is done to reduce Eq. (3.69) by a proper adjustment of ΔK. To this effect, it is assumed that $\Delta K = qJ$, where J is computed from Eqns. (3.67) and (3.68). The increments Δx and Δz caused by ΔK satisfy Eq. (3.63):

$$A\begin{bmatrix} \Delta x \\ \Delta z \end{bmatrix} = \begin{bmatrix} -U \\ qJ \end{bmatrix}, \qquad (3.70)$$

which consists of

$$S\Delta x + F_x^T \Delta x = -U$$

and $F_x \Delta_x = \Delta k = qJ$.

Vectors u and v are defined in such a way that $\Delta x = qu$ and $F_x^T = (\Delta z - qv) = (q-1)U$ where the last equation is satisfied in the sense of LSMN (Least Square Solution with Minimum Norm). Then, by eliminating Δx and Δz, from Eq. (3.70) the following is obtained

$$A\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -u \\ J \end{bmatrix} \qquad (3.71)$$

Then substitution of $\Delta x = qu$ into Eq. (3.69) gives

$$f(x+\Delta x)=f(x)+qf_x u+\frac{1}{2}q^2 N$$

where $N=u^T f_{xx}u$ (3.72)

The minimum of $f(x+\Delta x)$ for positive q occurs at

$$q=-\frac{f_x u}{N} \tag{3.73}$$

If $f_x u<0$ and $N>0$. q is chosen as given by Eq. (3.73) only when $f_x u<0$ and N > 0 and $\Delta K=J$ or q = 1 is chosen otherwise. For q not equal to one, ΔK_i is revised for all i, by

$$T=\min\left[qJ_i,(D_i-K_i)\right] \tag{3.74}$$

$$\Delta K_i=\max\left[T,(C_i-K_i)\right] \tag{3.75}$$

Using the ΔK, Δx and Δz are solved from Eq. (3.63) and then x and z are updated.

3.3.4.2 Solution Algorithm

The algorithm to be implemented in a computer program is mentioned below.

1. Input Data

   (a) n, m, p and $\in$(to replace zero, usually lies between 10^{-3} and 10^{-5})

   (b) R, a, H_i, C_i, and D_i for all i=1, 2,..., m

2. Initialization

   Set x_i = 0 and Z_i = 0 ($\lambda_i=0$) for all i or use any other preference.

3. Testing EKT Conditions

(a) Calculate K_i and U_i (the i th component of U) and then J_i from Eqs. (3.67) and (3.68)

(b) A set of EKT Conditions is reached if $|U_i| < \varepsilon$ and $|J_i| < \varepsilon$ for all i. Otherwise go to step 4.

4. Solving for u and v.

   (a) Solve u and v from Eq. (3.71) by using LSMN.

   (b) Calculate N by Eq. (3.72) and then go to step 5. If $N > 0$ and $f_x u < 0$. Go to part

   (c) other wise.

   (d) Update x by $x+u$ and z by $z+v$, and then go to step 3.

5. Determining ΔK

   (a) Calculate q by Eq. (3.73) and then find ΔK_i from Eq. (3.74) & (3.75) for all i.

   (b) Solve Δx and Δz from Eq. (3.63) by using LSMN.

   (c) Update x by $x + \Delta x$ and z by $z + \Delta z$, and then go to step 3.

In using the algorithm, it should be understood that several set of EKT conditions are to be explored. This is achieved by varying the initial values. However, at times, intuitive judgment is helpful in deciding the smallest one is the solution of the problem.

3.3.4.3 OPF Solution by Quadratic Programming Method—Researchers Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions / Salient Features
1	G. F. Reid and L. Hasdorf [70]	Economic Dispatch Using Quadratic Programming	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-92, pp. 2015-2023, 1973.	➢ Quadratic programming method based on Wolf's algorithm specialized to solve the economic dispatch problem is implemented. ➢ Penalty factors or the selection of the gradient step size are not essential. ➢ The method was developed purely for research purposes; therefore, the model used is limited and employs the classical economic dispatch with voltage, real, and reactive power as constraints. ➢ The CPU time is less as convergence is very fast. It increases with system size. ➢ Validated on 5-, 14-, 30-, 57- and 118-bus systems.
2	B. F. Wollenberg and W. O. Stadlin. [71]	A Real Time Optimizer for Security Dispatch	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-93, pp. 1640-1649, 1974.	➢ Real-time solutions with dependable & satisfactory results are achieved by employing structured, sparsity programmed matrix solution techniques ➢ Contingency constrained economic dispatch requirements are met by the decomposition algorithm which is one of the original works for economic dispatch. ➢ Two methods, derived from the Dantzig-Wolfe algorithm and quadratic formulations to solve the

				economic dispatch problem, are compared. ➢ The method is able to deal with practical components of a power system and the optimization schedule is included in the power flow with no area interchange. ➢ Easily applicable to other optimization schedules and was validated on a practical 247-bus system.
3	T. C.Giras N. Sarosh and S. N. Talukdar [72]	A Fast and Robust Variable Metric Method for Optimum Power Flows	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-96, No. 3, pp. 741-757, May/June 1977.	➢ Solved an OPF problem, having an infeasible initial starting point, by Quasi-Newton technique using the Han-Powell algorithm. ➢ A decomposition technique using the Berna, Locke, Westberg (BLW) decomposition is adopted. ➢ Due to excellent linear convergence qualities of power flow, the execution is fast and was validated on small synthetic systems. ➢ The method can be of production grade quality subject to its performance in more rigorous tests.
4	R.C. Burchett , H.H. Happ, D.R. Vierath, K.A. Wirgau [25]	Developments in Optimal Power Flow	*IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-101, No. 2, pp. 406 - 414, Feb 1982.	➢ Four objective functions namely, fuel cost, active and reactive losses, and new shunt capacitors are solved by Quadratic Programming (QP) method. ➢ Run time and the robustness of QP method are superior to an augmented Lagrangian method. This is evident from: a) QP method required an execution time of five minutes to solve up to 2000 buses on large mainframe computers.

				b) A feasible solution from an infeasible starting point was obtained by formation of a sequence of quadratic programs that converge to the optimal solution of the original nonlinear problem. ➢ OPF solutions based on the above methods, for four different systems with a range of 350-,1100-,1600- and 1900- buses, are evaluated and the observations are: ➢ The QP method employs the exact second derivatives, while second method adopts an augmented Lagrangian to solve a sequence of sub-problems with a changed objective. The later is based wholly on the first derivative information. ➢ By this method a viable solution can be obtained in the presence of power flow divergence. MINOS was employed as the optimization method. ➢ It can obtain different VARs and can avoid voltage collapse, but has the drawback to decide which constraints to be included and which not to be included in the active set. ➢ Development of the economic dispatch OPF problem by this method is much more complex than the classical economic dispatch problem.

5	K. Aoki and T. Satoh [73]	Economic Dispatch with Network Security Constraints Using Parametric Quadratic Programming	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-101, No.9, pp. 3502-3512, September 1982.	➢ Provided solution, for the economic dispatch problem with DC load flow type network security constraints by an efficient method, which is treated as a research grade tool. ➢ A parametric quadratic programming (PQP) method based on simplex approach is employed, to surmount problems associated with transmission losses as a quadratic form of generator outputs. ➢ The method, using an upper bounding and relaxation of constraints technique, compares well with AC load flow algorithms. It is applicable to large systems as computational effort is reduced by using DC the load flow. ➢ The constraints included are generation limits, an approximation of the DC load flow, branch flow limits and transmission line losses. A pointer is used to limit the number of variables to the number of generators. ➢ CPU time of 0.2-0.4seconds was obtained for all cases studied and tested against a number of other recognized methods.
6	G. C. Contaxis , B. C. Papadis, and C. Delkis [74]	Decoupled Power System Security Dispatch	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-102, pp. 3049-3056, September	➢ Solution is provided to the optimal power flow problem by decomposing it in to a real and a reactive sub problem ➢ The economic dispatch–cost function is solved as, real sub problem and the cost function with respect to the slack bus is solved as, the reactive sub problem. The economic dispatch objective with

			1983.	constraints is solved as, the two sub problems combined. ➢ The OPF problem is treated as a non linear constrained optimisation problem, identifying system losses, operating limits on the generators and security limits on lines. ➢ Beale's optimisation technique is used for solving Quadratic programming with linear constraints. ➢ Efficiency of this method is assured by using the solution of real sub problem as input to the other sub problem until solution for the full problem attained. ➢ The performance of the system was verified on a 27-bus system by computing system losses using bus impedance matrix which in turn is utilized, to determine the B-matrix by increasing the speed of computation.
7	S. N. Talukdar, T. C. Giras and V. K. Kalyan [75]	Decomposition for Optimal Power Flows	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-102, No. 12, pp. 3877-3884, Dec. 1983.	➢ A quadratic programming method based on the Han-Powell algorithm, which employs Berna, Locke and Westerberg (BLW) technique was used to solve practical size hypothetical systems of 550 & 1110 buses. It can be applied to solve systems of 2000 buses or greater. ➢ By this method, the problem is reduced to a quadratic programming form, but the selection of step-size is not completely accomplished. ➢ An optimal solution was obtained with diverse initial starting forms

				and the algorithm can be easily extended to solve constrained economic dispatch problem.
8	R. C. Burchett, H. H. Happ and D. R. Vierath [68]	Quadratically Convergent Optimal Power Flow	*IEEE Transactions on Power Apparatus and Systems,* vol. PAS-103, pp. 3267-3275, Nov. 1984.	➢ The observations given earlier under Ref. No 25 hold good here since present document is an extension of "Developments in Optimal Power Flow" mentioned in *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-101, No. 2, p.p 406 - 414, Feb 1982. ➢ The new points focused are: a)Sparsity techniques are used and the method results in quadratic convergence b) The non convergent power flow constraint is overcome by adding capacitor bank.
9	M. A. El-Kady, B. D. Bell, V. F. Carvalho, R. C. Burdhett, H. H. Happ, and D. R. Vierath [76]	Assessment of Real-Time Optimal Voltage Control	*IEEE Transactions on Power Systems,* vol. PWRS-1, No. 2, pp. 99-107, May 1986.	➢ Solution for the OPF problem for voltage control is provided by applying a Quadratic programming algorithm. ➢ The method was adapted to the Ontario Hydro Power System, considering variation of the total system load over a 24-hour period. ➢ A General Electric version24 OPF package based on a sequence of quadratic OPF sub-problems was implemented using a VAX 11/780 computer. ➢ This method was validated on a 380-bus, 65-generator, 550-line, and 85-transformer system for developing and maintaining the voltage below a specified upper limit.

				➢ Some of the constraints are tap changers, real and reactive generation, transformer taps. Expected run time for larger machines was obtained by verifying the method on a 1079-bus system on an IBM 3081 mainframe computer. ➢ The execution time, for 1079 bus system, is reduced to two minutes and 16 seconds from seven minutes and five seconds by adopting direct method instead of Quasi-Newton method.
10	K. Aoki, A. Nishikori and R. T. Yokoyana [77]	Constrained Load Flow Using Recursive Quadratic Programming	*IEEE Transactions on Power Systems,* vol. PWRS-2, No. 1, pp. 8-16, Feb. 1987.	➢ It is a capable, realistic and perfect algorithm for handling constrained load flow (CLF) problems. ➢ Control variable adjustment is achieved and the CFL problem is considered as a set of nonlinear programming problems. ➢ MINOS method is adapted, to deal with the nonlinear constraints and Quasi-Quadratic programming problem formulation is used. ➢ Reactive power, voltage magnitude, and transformer tap ratios are the constraints in order of priority of this method and adapts a prearranged order of priority for entering the Q's as they influence the objective function. ➢ Every stage of algorithm is verified to make certain that controls are properly adjusted in the order of preference. To deal with the control load flow problem, additional controls are required.

				➢ Capability aspect of this approach with other algorithms was evaluated by analyzing the sensitivities of their controls. ➢ If proper adjustment of controls is not done divergence may occur. Step-size method is followed to assure convergence but obtaining proper step-size is a problem. ➢ Lagrange multiplier path decides modification pattern of the approach. ➢ Encouraging results were obtained when verified on a 135-bus real-scale system of the Chouguku Electric Power Company.
11	A. D. Papalexopoulos, C. F. Imparato and F. F. Wu [78]	Large Scale Optimal Power Flow: Effects of Initialization Decoupling and Discretization	*IEEE Transactions on Power Systems,* vol. PWRS-4, No. 2, pp. 748-759, May 1989.	➢ The requirements namely (1) properly implemented second order OPF solution methods are robust with respect to different starting points and (2) the decoupled OPF solution is expected to be almost as accurate as the full OPF solution, are demonstrated by this method. ➢ The sensitivity of OPF results, the main goal of this paper, was demonstrated by evaluation of the performance of a 1500-bus Pacific Gas and Electric System, after making exhaustive studies for the model formulation, using second-order OPF solution method, developed by Burchett and Happ. ➢ Because of the non-convex nature of the problem, the robustness of the OPF was tested at different initial starting points. The goal of the paper was to demonstrate the sensitivity of OPF results.

				➢ It was observed that the decoupled problem is good for large systems and the method improves computation time by three to four folds. ➢ The load modeling does not have a great effect on the final results and a constraint relaxation technique is employed in the method. ➢ The application of state estimation is observed be key action and a selection criterion to attain a large mis-match was implemented to get OPF results. ➢ Decoupling of the problem reduces the computation burden for large problems and permits to utilise different optimisation cycles for the sub problems. ➢ The method was validated on a practical 1549-bus system, 20% of which were PV buses where summer peak, partial peak and off-peak and winter peak cases were studied. ➢ Loss and cost minimisation studies were conducted for three issues namely, sensitivity of OPF Solutions with respect to starting points employed in the solutions, accuracy of active / reactive decoupled approach to OPF solution and outcome of discretization of transformer taps on the OPF solution
12	J. A. Momoh [79]	A generalized quadratic-based model for optimal	CH2809-2/89/0000-0261 $1.00 © IEEE, pp.	➢ A generalised quadratic-based model for OPF, as an extension of basic Kuhun-Tucker conditions is provided. ➢ The OPF algorithm covers conditions for feasibility, convergence and optimality.

		power flow	261-267, 1989.	➢ Multiple objective functions and selectable constraints can be solved by using hierarchical structures.. ➢ The generalised algorithm using sensitivity of objective functions with optimal adjustments in the constraints in it's a global optimal solution. ➢ Computational memory and execution time required have been reduced.
13	N. Grudinin [80]	Reactive power optimization using successive quadratic programming method	*IEEE Transactions on Power Systems,* vol. 13, No. 4, pp. 1219-1225, November 1998.	➢ Reactive power optimisation is achieved by employing successive quadratic programming method. ➢ Economical and security objective functions are solved by using bicriterion reactive power optimisation model. ➢ Newton type quadratic programming method is employed for solving Quadratic programming. ➢ An efficient algorithm for approximation of initial problem by quadratic programming is explained. ➢ Developed a new modified successive quadratic programming method. It employs search of the best optimal point between two solutions on sequential approximating programming procedure. This is regarded as change of objective function in this interval and contravention of inequality constraints.
14	G. P. Granelli and M. Montagna [81]	Security constrained economic dispatch using dual	*Electric Power System Research,* vol. 56, pp.	➢ Security-constrained economic dispatch is solved by using Dual sequential quadratic programming ➢ By using relaxing transmission limit, a dual feasible starting point could be obtained and by adapting

		quadratic programming	71-80, 2000.	the dual quadratic algorithm, the constraint violations are enforced. ➢ The method has reduced computation time and provided good accuracy It is comparable with SQP method of NAG routine.
15	X. Lin, A. K. David and C. W. Yu [82]	Reactive power optimization with voltage stability consideration in power market systems	*IEE Proc,-- Generation Transmission Distribution,* vol. 150, no. 3, pp. 305-310, May 2003.	➢ An OPF for competitive market was created, by employing a method based on integrated cost analysis and voltage stability analysis. ➢ Solution was obtained by using sequential quadratic programming. ➢ Optimum reactive power dispatch was attained under different voltage stability margin requirements in normal and outage conditions when verified on IEEE 14-bus test system.
16	A. Berizzi, M. Delfanti, P. Marannino, M. S. Pasquadibisceglie and A. Silvestri [83]	Enhanced security-constrained OPF with FACTS devices	*IEEE Transactions on Power Systems,* vol. 20, no.3, pp. 1597-1605, August. 2005.	➢ Fixed the optimal setting and operation mode of UPFC and TCPAR by employing Security Constraint Optimal Power Flow (SCOPF) ➢ Solved the enhanced security-constrained OPF with FACTS devices using HP (Han Powel) algorithm. ➢ It is a proven method to solve non-linear problems with non-linear constraints, by using the solution of successive quadratic problems with linear constraints. It was implemented to CIGRE 63-bus system and Italian EHV network. Further, a global solution could be achieved at different starting points.

3.3.4.4 Merits and Demerits of Quadratic Programming Method

The Merits and Demerits of Quadratic Programming Method are summarized and given below.

Merits

1) The method is suited to infeasible or divergent starting points.
2) Optimum Power Flow in ill conditioned and divergent systems can be solved in most cases.
3) The Quadratic Programming method does not require the use of penalty factors or the determination of gradient step size which can cause convergence difficulties. In this way convergence is very fast.
4) The method can solve both the load flow and economic dispatch problems.
5) During the optimisation phase all intermediate results feasible and the algorithm indicates whether or not a feasible solution is possible.
6) The accuracy of QP method is much higher compared to other established methods.

Demerits

1) The main problem of using the Quadratic Programming in Reactive Power Optimisation are:
    a) Convergence of approximating programming cycle (successive solution of quadratic programming and load flow problems).
    b) Difficulties in obtaining solution of quadratic programming in large dimension of approximating QP problems.

c) Complexity and reliability of quadratic programming algorithms.

2) QP based techniques have some disadvantages associated with the piecewise quadratic cost approximations.

3.3.5 Interior Point Method

It has been found that, the projective scaling algorithm for linear programming proposed by N. Karmarkar is characterized by significant speed advantages for large problems reported to be as much as 12:1 when compared to the simplex method [12]. Further, this method has a polynomial bound on worst-case running time that is better than the ellipsoid algorithms. Karmarkar's algorithm is significantly different from Dantzig's simplex method. Karmarkar's interior point rarely visits too many extreme points before an optimal point is found. In addition, the IP method stays in the interior of the polytope and tries to position a current solution as the "center of the universe" in finding a better direction for the next move. By properly choosing the step lengths, an optimal solution is achieved after a number of iterations. Although this IP approach requires more computational time in finding a moving direction than the traditional simplex method, better moving direction is achieved resulting in less iterations. In this way, the IP approach has become a major rival of the simplex method and has attracted attention in the optimization community. Several variants of interior points have been proposed and successfully applied to optimal power flow [37, 84, and 85].

The Interior Point Method [2, 5, and 6] is one of the most efficient algorithms. The IP method classification is a relatively new optimization approach that was applied to solve power system optimization problems in the late 1980s and early 1990s and as can be seen from the list of references [69, 86 – 98].

The Interior Point Method (IPM) can solve a large scale linear programming problem by moving through the interior, rather than the boundary as in the simplex method, of the feasible reason to find an optimal solution. The IP method was originally proposed to solve linear programming problems; however later it was implemented to efficiently handle quadratic programming problems.

It is known as an interior method, since it finds improved search directions strictly in the interior of the feasible space as already shown in Fig 3.3.

The basic ideas involved in the iteration process of Interior Point Method as proposed by N. K.Karmarkar [12], are given below.

In order to have a comprehensive idea of the optimisation process, the difference between the simplex and interior point methods is described geometrically.

Consider an interior path, described by x_i, as shown in Fig 3.3. In the simplex method the solution goes from corner point to corner point, as indicated by x_i. The steepest descent direction is represented by c. The main features of the IPM as shown in Fig 3.3 are:

1) Starting from an interior point, the method constructs a path that reaches the optimal solution after few iterations (less than the simplex method).
2) The IPM leads to a “good assesment” of the optimal solution after the first few iterations. This feature is very important, because for each linearization of the original formulation an exact result of Quadratic Programming problem is not imperative. Normally it is enough to obtain a point near the optimal solution because each QP sub problem is already an approximation of the original problem.

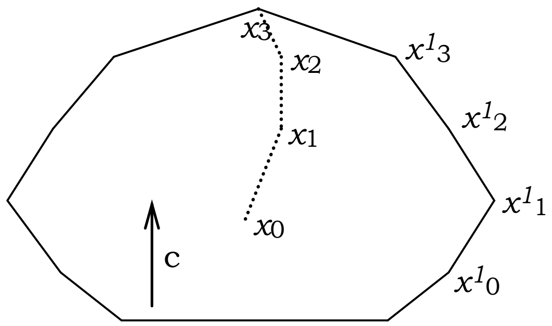

Fig: 3.3 Polytope of a two – dimension feasible region.

The interior point method starts by determining an initial solution using Mehrotra’s algorithm, to locate a feasible or near-feasible solution. There are two procedures to be performed in an iterative manner until the optimal solution has been found. The formal is the determination of a search direction for each variable in the search space by a Newton’s method. The lateral is the determination of a step length normally assigned a value as close to unity as possible to accelerate solution convergence while strictly maintaining primal and dual feasibility. A calculated solution in each iteration is be checked for optimality by the Karush – Kuhn – Tucker (KKT) conditions, which consist of primal feasibility, dual feasibility and complementary slackness.

3.3.5.1 OPF Problem Formulation by Primal — Dual Interior Point Method

As has been mentioned, the objective function considered in this project is to minimize the total production cost of scheduled generating units. OPF formulation consists of three main components: objective function, equality constraints, and inequality constraints.

An OPF problem is generally formulated as per Eq. (3.1) – (3.6).

Objective Function

The objective function is given by Eq. (3.2) and is reproduced below.

$$F_T = F(P_G) = \sum_{i=1}^{N_G} F_i\,(P_{G_i}) \;=\; \sum_{i=1}^{N_G} (\alpha_i + \beta_i\, P_{G_i} + \gamma_i\, P_{G_i}^2)$$

Equality Constraints

The equality constraints are active/reactive power flow equations as per Eq. (3.7) – (3.12).

Eq. (3.9) and (3.10) are nonlinear and can be linearized by the Taylor's expansion using

$$\begin{pmatrix} \Delta P\,(V,\delta) \\ \Delta Q\,(V,\delta) \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta V \end{pmatrix} \tag{3.76}$$

where

$\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$ is the Jacobian matrix

Transmission loss (P_L) given in the Eq. (3.12) can be directly calculated from the power flow.

Inequality Constraints

The inequality constraints consist of generator active/reactive power limits, voltage magnitude limits, and transformer tap position limits, are represented by Eq. (3.13) – (3.20).

3.3.5.2 Solution Algorithm

The PDIPM method is started by arranging a primal quadratic programming problem into a standard form as [5.6]:

Minimize $\frac{1}{2}\, x^T Q x + c^T x$ (3.77)

Subject to $\boldsymbol{Ax} = \boldsymbol{b},\ \boldsymbol{x} \geq \boldsymbol{0}$ (3.78)

Eq. (3.77) can be transformed into the corresponding dual problem having the form.

Maximize $-\frac{1}{2}\, \boldsymbol{x}^T \boldsymbol{Qx} + \boldsymbol{b}^T \boldsymbol{W}$ (3.79)

Subject to $-\boldsymbol{Qx} + \boldsymbol{A}^T \boldsymbol{w} + \boldsymbol{s} = \boldsymbol{c},\ \boldsymbol{s} \geq \boldsymbol{0}$ (3.80)

Stopping criteria for the algorithm is based on three conditions of Karush-Kuhn-Tucker (KKT): primal feasibility, dual feasibility, and complementary slackness; namely, these three conditions have to be satisfied and are given in Eq. (3.81), (3.82) and (3.83).

$$\boldsymbol{Ax} = \boldsymbol{b},\ \boldsymbol{x} \geq \boldsymbol{0} \qquad \text{(Primal feasibility)} \tag{3.81}$$

$$-Qx + A^T w + s = c,\ s \geq 0 \qquad \text{(Dual feasibility} \tag{3.82}$$

$$\boldsymbol{X\ Se} = \boldsymbol{\mu\ e} \qquad \text{(Complementary slackness)} \tag{3.83}$$

where

$$\boldsymbol{\mu}^k = \frac{(X^k)^T S^k}{n} \tag{3.84}$$

$$\boldsymbol{X} = diag\ (x_1, x_2, \ldots\ldots x_n) \tag{3.85}$$

$$\boldsymbol{S} = diag\ (S_1, S_2, \ldots\ldots S_n) \tag{3.86}$$

From the KKT conditions, the directions of translation are calculated using the Newton's method which yields the following system Eq.

$$\begin{bmatrix} A & \mathbf{0} & \mathbf{0} \\ -Q & A^T & I \\ S^k & \mathbf{0} & X^k \end{bmatrix} \begin{bmatrix} d_x^k \\ d_w^k \\ d_z^k \end{bmatrix} = \begin{bmatrix} Ax^k - b \\ -Qx^k + A^T w^k + s^k - c \\ X^k S^k e - \boldsymbol{\mu}^k e \end{bmatrix} \tag{3.87}$$

The right hand side of Eq. (3.87) is so-called slackness vectors and can be assigned to new variables as

$$t^k = b - Ax^k \tag{3.88}$$

$$u^k = Qx^k + c - A^T w^k - s^k \tag{3.89}$$

$$v^k = \boldsymbol{\mu}^k e - X^k S^k e \tag{3.90}$$

From Eq. (3.87) – (3.90), we have

$$Ad_x^k = t^k \tag{3.91}$$

$$-Qd_x^k + A^T d_w^k + d_s^k = u^k \tag{3.92}$$

$$S_k d_x^k + X_k d_s^k = v^k \tag{3.93}$$

Combining and rearranging Eq. (3.92) and Eq. (3.93) gives

$$-d_x^k + (S_k + X_k Q)^{-1} X_k A^T d_w^k = (S_k + X_k Q)^{-1} (X_k u^k - v^k) \tag{3.94}$$

With Eq. (3.91) and Eq. (3.94), a dual search direction can be derived a

$$d_w^k = \left[A(S_k + X_k Q)^{-1} X_k A^T\right]^{-1} - \left[A(S_k + X_k Q)^{-1}(X_k u^k - \upsilon^k) + t^k\right] \qquad (3.95)$$

The equation for a primal search direction can be derived from Eq. (3.94).

$$d_x^k = (S_k + X_k Q)^{-1}\left[X_k\,(A^T d_w^k - u^k) + \upsilon^k\right] \qquad (3.96)$$

With the primal search direction and Eq. (3.93), a slack search direction can be obtained by

$$d_x^k = X_k^{-1}(\upsilon^k - S_k\, d_x^k) \qquad (3.97)$$

To find appropriate step lengths while keeping the primal and dual problem feasible, Eq. (3.98) – Eq. (3.101) are used.

$$\alpha_P^k = \min\left(-x_j^k / d_{x_j}^k\right) \mid d_{x_j}^k < 0 \qquad (3.98)$$

$$\alpha_D^k = \min\left(-s_j^k / d_{s_j}^k\right) \mid d_{s_j}^k < 0 \qquad (3.99)$$

$$\alpha_{max}^k = \min\left(\alpha_P^k\ ,\ \alpha_D^k\right) \qquad (3.100)$$

$$\alpha^k = 0.99\,\alpha_{\max}^k \qquad (3.101)$$

An updated solution can be computed by Eq. (3.102) – (3.104).

$$x^{k+1} = x^k + \alpha^k d_x^k \qquad (3.102)$$

$$w^{k+1} = w^k + \alpha^k d_w^k \qquad (3.103)$$

$$s^{k+1} = s^k + \alpha^k d_s^k \qquad (3.104)$$

ALOGORITHM FOR PDIPM

The PDIPM algorithm applied to the OPF problem is summarized step-by-step as follows.

Step 1: Read relevant input data.

Step 2: Perform a base case power flow by a power flow subroutine.

Step 3: Establish an OPF model.

Step 4: Compute Eq. (3.88) – (3.90).

Step 5: Calculate search directions with Eq. (3.95) – (3.97).

Step 6: Compute primal, dual and actual step-lengths with Eq. (3.98) – (3.101).

Step 7: Update the solution vectors with Eq. (3.102) – (3.104).

Step 8: Check if the optimality conditions are satisfied by Eq. (3.81) – (3.83) and if $\mu \le \varepsilon$ (ε =0.001 is chosen).

If yes, go to the next step. Otherwise go to step 4.

Step 9: Perform the power flow subroutine.

Step 10: Check if there are any violations in Eq. (3.15) and Eq. (3.19). If no, go to the next step; otherwise, go to step 4.

Step 11: Check if a change in the objective function is less than or equal to the prespecified tolerance. If yes, go to the next step; otherwise, go to step 4.

Step 12: Print and display an optimal power flow solution.

3.3.5.3 Interior Point Method — Researchers Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No.	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions / Salient Features
1	Clements, K. A., Davis, P. W., and Frey, K. D. [88]	An Interior Point Algorithm for Weighted Least Absolute value Power System State Estimation	*IEEE/PES Winter Meeting,* 1991.	➢ Solved power system state estimation problems by employing a nonlinear programming interior point technique. It also helps in detection and identification of unwanted data. ➢ A logarithmic barrier function interior point method was employed, to accommodate inequality constraints. The Karush-Kuhn-Tucker (KKT) equations were solved by .Newton's method. ➢ Solved the problem in fewer iterations as compared to linear programming techniques, where the number of iterations depends on the size of the system. ➢ Encouraging results were obtained when validation of the method was done on up to a 118-bus system including 6-30-, 40-, and 55-bus systems. The selection of the initial starting points was a constraint. The CPU time was reduced by using Choleski-factorization technique.
2	Ponnambal-am K., Quintana, V. H., and Vannelli, A [89]	A Fast Algorithm for Power System Optimizati on Problems Using an Interior	*IEEE/PES Winter Meeting,* 1991.	➢ Solved the hydro-scheduling problem, using a newly developed dual affine (DA) algorithm (a variant of Karmarkar's interior point method). Equality and inequality constraints were included in the linear programming problem.

		Point Method		➢ Irrespective of the size of the problem, 20-60 iterations were required to attain the solution ➢ Using this algorithm, both linear and nonlinear optimization problems with large numbers of constraints, were solved. ➢ The algorithm was employed to solve, a large problem comprising 880 variables and 3680 constraints, and the sparsity of the constraint matrix was taken in to account. ➢ Preconditioned conjugate gradient method was employed to solve the normal equation in every iteration. ➢ This method was validated on up to 118 buses with 3680 constraints and it was realized that the dual affine algorithm is only suitable for a problem with inequality constraints. ➢ With the problem modified to a primal problem with only inequality constraints, Adler's method was employed to get initial feasible points and the method solved the 118-bus system over nine times quicker than an efficient simplex (MINOS) code. ➢ The advantage of the DA method over the simplex method for staircase-structured seasonal hydro-scheduling was recognized.
3	Momoh, J. A., Austin, R. A., and Adapa, R [90, 91]	a) Application of Interior Point Method to Economic Dispatch	*A) IEEE International Conference on Systems Man & Cybernetics,* 1992.	➢ Solved optimal power flow problems, economic dispatch, and VAR planning, by adapting a Quadratic Interior Point (QIP) method and also it also provides solution to linear and quadratic objective functions including linear constraints.

		b) Feasibility of Interior Point Method for VAR Planning	*B) Proceedings of North American Power Symposium,* Reno, Nevada, 1992.	➢ Solved economic dispatch objective in two phases: (1) the Interior Point algorithm gets the optimal generations and (2) violations are found by using the above generations in the load flow analysis. ➢ This method was verified on the IEEE 14-bus test, but objectives like security constrained economic dispatch or VAR planning were not considered. ➢ QIP was eight times faster than MINOS 5.0, and also the results attained were encouraging. ➢ Momoh's method has the ability to handle new variables and constraints and can be used on other computer platforms. ➢ Variation or sensitivity studies of load and generations were not conducted.
4	Luis S. Vargas, Victor H. Quintana, Anthony Vannelli [37]	A Tutorial Description Of An Interior Point Method And Its Applications To Security Constrained Economic Dispatch	*IEEE Transactions on Power Systems,* vol. 8, no. 3, pp. 1315 – 1324, Aug. 1993.	➢ Solved the power system security-constrained economic dispatch (SCED) problems using a successive linear programming (SLP) approach. ➢ The method adapted a new dual affine interior point algorithm solve the traditional OPF problem with power flow constraints, flows, real and reactive generation, transformer tap ratios, and voltage magnitude. ➢ The SCED problem was bifurcated into two steps and the load flow was solved independently for the optimization schedule. ➢ The active power was related to generation factors by using a distribution factor and was validated on the EEE 30- and 118-bus systems.

				➢ In interior point approach, the optimal solution was achieved in less number of iterations in comparison to MINOS 5.0 and proved to be faster than it by a speed factor of 36:1. ➢ Sensitivity analysis on the deviation of generation for the 30 and 118-bus systems was also conducted.
5	C. N. Lu, and M. R. Unum [92]	Network Constrained Security Control Using an Interior Point Algorithm	*IEEE Transactions on Power Systems,* Vol. No. 3, pp. 1068-1076, 1993.	➢ Solved different sizes of network constrained security control linear programming problems by employing IP method. ➢ It was used in the relief of network overloads by adapting active power controls and other controls such as the generation shifting, phase-shifter control HVDC link control, and load shedding. ➢ In this method, an initial feasible solution is attained using the linear programming technique and the original problem was resolved with the primal interior point algorithm. ➢ The initial starting point requires more work and the simplex method is employed as a post-processor. ➢ The technique requires less CPU time when compared with MINOS5.0, while convergence in the last few iterations of the process, may be time-consuming ➢ The method was tested on the IEEE 6-, 30-, and 118-bus system to show speed advantage over MINOS (simplex algorithm). ➢ The analysis indicated that interior point algorithm is suitable to practical power systems models.

				➢ Solved the contingency-constrained problem with the primal non-decomposed approach and complexity analysis was performed on the method.
6	Momoh, J. A., Guo S. X., Ogbuobiri C. E., and Adapa, R[93]	The Quadratic Interior Point Method for Solving Power System Optimization Problems	*IEEE Transactions on Power System,* Vol. 9, 1994.	➢ solved linear programming problems using an approach based on Karmarkar's interior point method for ➢ Extended quadratic interior point (EQIP) method based on improvement of initial conditions was employed to solve both linear and quadratic programming problems. ➢ This method is an addition of the dual affine algorithm and is capable of solving economic dispatch and VAR planning problems covered under power system optimization problems ➢ The method is able to accommodate the nonlinearity in objectives and constraints .and was verified on 118-bus system. ➢ Discrete control variables and contingency constrained problems were not addressed in the formulation of the method. ➢ Capability to start with a better initial starting point enhances efficiency of this method and the optimality criteria are well described. ➢ This EQIP method is faster by a factor of 5:1 in comparison to MINOS 5.0.
7	Granvilles [94]	Optimal Reactive	*IEEE Transactions*	➢ Solved the VAR planning objective function of installation cost and

		Dispatch through Interior Point Methods	*on Power System,* Vol. 9, pp. 136-146, Feb. 1994.	losses using an Interior Point (IP) method and ω was introduced as a trade factor. ➢ Primal-dual variant of IP was adapted in this research and the problem was a non-convex, nonlinear programming with nonlinear constraints ➢ VAR planning problem with losses was appropriately handled by primal-dual algorithm. A W-matrix technique is used in this method. ➢ Better computational performance was observed when primal-dual logarithmic barrier method was used in linear and quadratic programming problems. ➢ For obtaining acceptable results for loss minimization and reactive injection costs, appropriate weights must be indicated in order, for the algorithm. ➢ The method was validated on huge practical 1862- and 3462- bus systems and the method resolves infeasibility by regularly adjusting limits to hold load flow limits. ➢ Bender's decomposition algorithm was combined with primal-dual algorithm to get better efficiency of the technique.
8	Yan, X. Quintana, V.H. [95]	An efficient predictor-corrector interior point algorithm for	*IEEE Transactions on Power System,* Vol. 9, pp. 136-146, Feb. 1994	➢ Solved security-constrained economic dispatch (SCED) problem by an advanced interior point approach using successive linear programming. ➢ Predictor-corrector interior point method employed to solve nonlinear SCED problem after linearization.

		security-constrained economic dispatch		➢ Identified several significant issues in addition to explaining the fundamental algorithm.. They are detrimental to its capable accomplishment, including the tuning of barrier parameter, the selection of initial point, and so on. ➢ For minimizing the number of iterations necessary by the algorithm, analysis is done to assess impact of the vital variants on the performance of the algorithm. Few ideas like, adapting the feasibility condition to tune the method of computing barrier parameter μ and selecting initial point by using a relative small threshold, are suggested. ➢ Test results on power systems of 236 to 2124 buses, indicate suggested actions have improved performance of the algorithm by a factor of 2.The predictor-corrector method has shown advantage over a pure primal-dual interior point method.
9	Wei, H., Sasaki, H. and Yokoyama, R [96]	An application of interior point quadratic programming algorithm to power system optimization problems	*IEEE Transactions on Power System,* Vol. 11, pp. 260-266, 1996.	➢ Solved power system optimization problems with considerably reduced calculation time using, a new interior point quadratic programming algorithm. The algorithm has two special features: ➢ The search direction is the Newton direction, as it depends on the path-following interior point algorithm and hence the algorithm has quadratic convergence. ➢ A symmetric indefinite system is solved directly and hence the algorithm prevents the creation of

				$[AD^{-1}A^{T}]$ and accordingly generates lesser fill-ins compared to the case of factorizing the positive definite system matrix for big systems, resulting in an intense speed-up. ➢ The algorithm can begin from either a feasible (interior point) or an infeasible point (non interior point), because the rule of the interior point approach has been simplified. ➢ Performance on the IEEE test systems and a Japanese 344 bus system, proved robustness of the algorithm and considerable reduction in execution time than interior point method.
10	Granville, S. Mello, J. C. O. and Melo, A. C. G. [97]	Application of Interior Point Methods to Power Flow unsolvability	*IEEE Transactions on Power System*, Vol. 11, pp. 1096-1103, 1996.	➢ IP A direct interior point (IP) method was used to restore system solvability by application of an optimal power flow ➢ With the P-Q load representation, for a given set of active and reactive bus injections, power flow unsolvability happens when the power flow equations have no real solution. ➢ Rescheduling of active power of generators, adjustments on terminal voltage of generators, tap changes on LTC transformers, , lowest load shedding are treated as the set of control actions in the algorithm ➢ Surveillance of the effect of every control optimization, in system solvability, is possible with IP formulation. ➢ Computation of probabilistic indicators of solvability problems by

				a framework, considering the probability of contingencies, is explained. ➢ The task of control optimization is described in a real 11-bus power system and the probabilistic method is adapted to a 1600-bus power system obtained from the Brazilian South/Southeast/Central West system.
12	D. Xia-oying, W. Xifan, S. Yonghua and G. Jian [99]	The interior point branch and cut method for optimal flow	0-7803-7459-2/02/$17.00 © IEEE, pp. 651-655, 2002.	➢ Solved decoupled OPF problem using an Interior Point Branch and Cut Method (IPBCM). ➢ Solved Active Power Suboptimal Problem (APSOP) by employing Modern Interior Point Algorithm (MIPA) and adapted IPBCM to iteratively resolve linearizations of Reactive Power Suboptimal Problem (RPSOP). ➢ The RPSOP has fewer variables and limitations than original OPF problem, resulting in improving pace of computation.
13	Wei Yan, Y. Yu, D. C. Yu and K. Bhattarai [100]	A new optimal reactive power flow model in rectangular form and its solution by predictor corrector primal dual interior point method	*IEEE Transactions on Power System,* Vol. 21, no. 1, pp. 61-67, Feb. 2006.	➢ Predictor Corrector Primal Dual Interior Point Method (PCPDIPM) was employed or resolving the problem of the Optimal Reactive Power Flow (ORPF). ➢ Presented a new optimal reactive power flow model in rectangular form. In the complete optimal process, the Hessian matrices are constants and require evaluation only once. The computation time for this method is always less than conventional model in seven test cases.

3.3.5.4 Merits and Demerits of Interior Point Method

Merits

1. The Interior Point Method is one of the most efficient algorithms. Maintains good accuracy while achieving great advantages in speed of convergence of as much as 12:1 in some cases when compared with other known linear programming techniques.
2. The Interior Point Method can solve a large scale linear programming problem by moving through the interior, rather than the boundary as in the simplex method, of the feasible region to find an optimal solution.
3. The Interior Point Method is preferably adapted to OPF due to its reliability, speed and accuracy.
4. Automatic objective selection (Economic Dispatch, VAR planning and Loss Minimization options) based on system analysis.
5. IP provides user interaction in the selection of constraints.

Demerits

1. Limitation due to starting and terminating conditions
2. Infeasible solution if step size is chosen improperly.

3.4 INTELLIGENT METHODOLOGIES

Intelligent methods include Genetic Algorithm and Particle Swarm Optimization methods.

3.4.1 Binary Coded Genetic Algorithm Method

The drawbacks of conventional methods were presented in Section 1.3. All of them can be summarized as three major problems:

- Firstly, they may not be able to provide optimal solution and usually getting stuck at a local optimal.
- Secondly, all these methods are based on assumption of continuity and differentiability of objective function which is not actually allowed in a practical system.
- Finally, all these methods cannot be applied with discrete variables, which are transformer taps.

It is observed that Genetic Algorithm (GA) is an appropriate method to solve this problem, which eliminates the above drawbacks. GAs differs from other optimization and search procedures in four ways [8]:

- GAs work with a coding of the parameter set, not the parameters themselves. Therefore GAs can easily handle the integer or discrete variables.
- GAs search within a population of points, not a single point. Therefore GAs can provide a globally optimal solution.
- GAs use only objective function information, not derivatives or other auxiliary knowledge. Therefore GAs can deal with non-smooth, non-continuous and non-differentiable functions which are actually exist in a practical optimization problem.
- GAs use probabilistic transition rules, not deterministic rules.

We use GA because the features of GA are different from other search techniques in several aspects, such as:

- First, the algorithm is a multipath that searches many peaks in parallel and hence reducing the possibility of local minimum trapping.
- Secondly, GA works with a coding of parameters instead of the parameters themselves. The coding of parameter will help the genetic operator to evolve the current state into the next state with minimum computations.
- Thirdly, GA evaluates the fitness of each string to guide its search instead of the optimization function.

3.4.1.1 OPF Problem Formulation

The OPF problem is to minimize the fuel cost, set as an objective function, while satisfying several equality and inequality constraints. Equations (3.1) to (3.6) describe the OPF problem and constraints.

Genetic Algorithm Approach

The general purpose GA has the following steps: [108]

Step-1*: *Formation of Chromosome – Coding and Decoding:

GA operates on the encoded *binary* string of the problem parameters rather than the actual parameters of the system. Each string can be thought of as a chromosome that completely describes one candidate solution to the problem. Once the encoded structure of chromosome is formed, a population is then generated randomly which consists of certain number of chromosomes. With binary coding method, the active power generation of a particular generator P_{gi} would be coded as a binary string of '0's and '1's with length 4 digits.

As an example, for a 57-Bus power system with 7-Generators, the active power generations P_{Gi} for i=1, 2,..7 with a length of 4-Digits (can be different) is shown in Table-3.4.1.

Table 3.4.1 Coding of Active Power Generation [111, 113].

	P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	P_{G6}	P_{G7}	**code**
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0000
1	38.392	6.66	9.33	6.66	36.66	6.66	27.33	0001
2	76.784	13.32	18.66	13.32	73.32	13.32	54.66	0010
3	115.176	19.98	27.99	19.98	109.98	19.98	81.99	0011
4	153.568	26.64	37.32	26.64	146.64	26.64	109.32	0100
5	191.96	33.3	46.65	33.3	183.3	33.3	136.65	0101
6	230.352	39.96	55.98	39.96	219.96	39.96	163.98	0110
7	268.744	46.62	65.31	46.62	256.62	46.62	191.31	0111
8	307.136	53.28	74.64	53.28	293.28	53.28	218.64	1000
9	345.528	59.94	83.97	59.94	329.94	59.94	245.97	1001
10	383.92	66.6	93.3	66.6	366.6	66.6	273.3	1010
11	422.312	73.26	102.63	73.26	403.26	73.26	300.63	1011
12	460.704	79.92	111.96	79.92	439.92	79.92	327.96	1100
13	499.096	86.58	121.29	86.58	476.58	86.58	355.29	1101
14	537.488	93.24	130.62	93.24	513.24	93.24	382.62	1110
15	575.88	100.00	140.00	100.00	550.00	100.00	410.00	1111

Each of P_{Gi} is bounded with in P_{Gi}^{max} and P_{Gi}^{min}. The choice of string length depends on resolution required. The bit length L_i and corresponding resolution R_i of any P_{Gi} can be determined by the following equation:

$$R_i = \{(\beta_i - \gamma_i) / (2^{Li} - 1)\} \quad \text{for} \quad i = 1, 2, \ldots, 7 \tag{3.105}$$

and

$$P_{Gi} = \gamma_i + \text{decimal.(string)}.\ R_i \quad \text{for } i = 1, 2, .., 7 \tag{3.106}$$

Let the resolution R_i for i =1,2,..,7 is specified as (38.392, 6.66, 9.33, 6.66, 9.33, 6.66, 36.66, 6.66, 27.33 MW) and with corresponding bit lengths L_i for i =1,2,..,7. The parameter domain of P_{Gi} for i=1, 2,..,7 is presented in Table 3.4.1.

If the candidate parameter set is (575.88, 93.24, 121.29, 79.92, 403.26, 66.6, 245.97) then the chromosome is a binary string of:

[1111 1110 1101 1100 1011 1010 1001]

Decoding is the reverse procedure of coding.

The first step of any genetic algorithm is to create an initial population of GA by randomly generating a set of feasible solutions. A binary string of length L is associated to each member (individual) of the population. The string is usually known as a chromosome and represents a solution of the problem. A sampling of this initial population creates an intermediate population. Thus some operators (reproduction, crossover and mutation) are applied to an intermediate population in order to obtain a new one, this process is called Genetic Operation.

The process, that starts from the present population and leads to the new population, is called a generation process (Table 3.4.2).

Table 3.4.2 First generation of GA process for 57 bus example [111, 113].

Initial Population	P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	P_{G6}	P_{G7}	$f(P_{Gi})$	$f_{max} - f(P_{Gi})$
0010001100101100110111010000	76.784	19.98	18.66	79.92	476.58	86.58	0.00	1067300	293200
0111100010111111000010110111	268.744	53.28	102.63	100.00	0.00	73.26	19.31	915730	444770
1010011111111010000100100000	383.92	46.62	140.00	66.6	36.66	13.32	0.00	1360500	0.00
0110010101100100110000111110	230.352	33.3	55.98	26.64	439.92	19.98	382.62	30509.80	1329991
Max									1360500
Min									30509.80

***Step-2**: Genetic Operation-Crossover:*

Crossover is the primary genetic operator, which promotes the exploration of new regions in the search space. For a pair of parents selected from the population the recombination operation divides two strings of bits into segments by setting a crossover point at random locus,

Table 3.4.3 Single point crossovers [111, 113].

Locus = 3			
Parent1	0110010101100100110000111110	Child1	0111100010111111000010110111
Parent2	0111100010111111000010110111	Child2	0110010101100100110000111110

Table 3.4.4 The result after crossover and mutation of the first population [111, 113].

Chromosome	The result after crossover of the first population	The result after mutation of the first population
1	011001010110010011**0**000111110	011001010110010011**1**000111110
2	011110001**0**111111000010110111	011110001**1**111111000010110111
3	0110010101**10**0100110**0**011111**0**	0110010101**01**0100110**1**011111**1**
4	0111100010111111**000**010110111	0111100010111111**111**010110111

i.e. Single Point Crossover (Table 3.4.3). The segments of bits from the parents behind the crossover point are exchanged with each other to generate their off-spring. The mixture is performed by choosing a point of the strings randomly and switching the left segments of this point. The new strings belong to the next generation of possible solutions (Table 4.4). The strings to be crossed are selected according to their scores using the roulette wheel [8]. Thus, the strings with larger scores have more chances to be mixed with other strings because all the copies in the roulette have the same probability to be selected.

Step-3**: **Genetic Operation-Mutation:

Mutation is a secondary operator; it prevents the premature stopping of the algorithm in a local solution. This operator is defined by a random bit value change in a chosen string with a low probability. The mutation adds a random search character to the genetic algorithm (Table 3.4.4).

All strings and bits have the same probability of mutation. For example, in this string 011001010110010011**0**000111110, if the mutation affects bit number six, the string obtained is 011001010110010011**1**000111110 and the value of P_{G5} change from 439.92 to 513.24.

$$\underbrace{0110}_{P_{G1}}\underbrace{0101}_{P_{G2}}\underbrace{0110}_{P_{G3}}\underbrace{0100}_{P_{G4}}\underbrace{11\mathbf{0}0}_{P_{G5}}\underbrace{0011}_{P_{G6}}\underbrace{1110}_{P_{G7}} \xrightarrow[\text{mutation}]{\text{After}} \underbrace{0110}_{P_{G1}}\underbrace{0101}_{P_{G2}}\underbrace{0110}_{P_{G3}}\underbrace{0100}_{P_{G4}}\underbrace{11\mathbf{1}0}_{P_{G5}}\underbrace{0011}_{P_{G6}}\underbrace{1110}_{P_{G7}}$$

Step-4**: **Genetic Operation-Reproduction:

Reproduction is based on the principle of better fitness survival. It is an operator that obtains a fixed copies number of solutions according to their fitness value. If the score increases, the number of copies increases too. A score value is associated to a solution relying on its distance from the optimal solution (closer distances to the optimal solution mean higher scores).

Table 3.4.5 Second generation of GA process for 57 bus example [111, 113].

Second generation	P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	P_{G6}	P_{G7}	$f(P_{Gi})$	$f_{max} - f(P_{Gi})$
0110010101100100111000111110	230.35	33.3	55.98	26.6	513.2	19.98	382.62	5503.1	882116.9
0111100011111111000010110111	268.74	53.28	140.0	100.00	0.00	73.26	163.98	887620	0.00
0110010101010100110010111111	230.35	33.3	46.65	26.64	439.92	73.26	410.00	4985.56	882634.44
0111100010111111111010110111	268.74	53.28	102.63	100.00	513.24	73.26	191.31	8015.6	879604.4
Max									887620
Min									4985.56

Step-5: Evaluation-Candidate solutions fitness and cost function:

The cost function is defined as per Eq. (3.1) and is reproduced below for convenience

$$F(P_G) = \sum_{i=1}^{N_G} \left(\alpha_i + \beta_i P_{G_i} + \gamma_i p_{G_i}^2 \right) \qquad P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}$$

The objective is to search (P_{G1}, P_{G2}, P_{G3}, P_{G4}, P_{G5}, P_{G6}, P_{G7}) in their admissible limits to achieve the optimisation problem of OPF. The cost function f (P_{Gi}) takes a chromosome (a possible (P_{G1}, P_{G2}, P_{G3}, P_{G4}, P_{G5}, P_{G6}, P_{G7})) and returns a value. The value of the cost is then mapped into a fitness value *fit* (P_{G1}, P_{G2}, P_{G3}, P_{G4}, P_{G5}, P_{G6}, P_{G7}) so as to fit in the genetic algorithm.

To minimise f (P_{Gi}) is equivalent to getting a maximum fitness value in the searching process, a chromosome that has lower cost function should be assigned a larger fitness value. The objective of OPF should change to the maximisation of fitness used in the simulated roulette wheel as follows:

$$fitness_i = \begin{cases} f\max - fi(P_{Gi}), & \text{if } f\max \ge fi(P_{Gi});\ i = 1, NG, \\ 0, & \text{otherwise.} \end{cases} \qquad (3.107)$$

It should be given by the slack generator with considering different reactive constraints. Examples of reactive constraints are the min and the max reactive rate of the generators buses and the min and the max of the voltage levels of all buses. All these require a fast and robust load flow program with best convergence properties. The developed load flow process is based upon the full Newton-Raphson algorithm using the optimal multiplier technique.

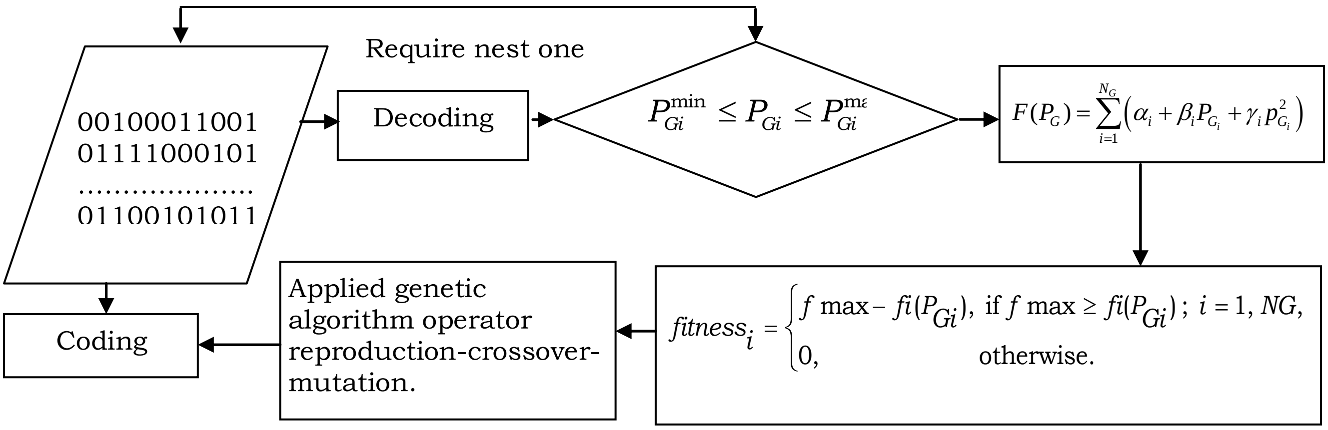

Fig: 3.4 A Simple flow chart of the GAOPF [113]

Step-6: Termination of the GA:

Since GA is a stochastic search method, it is difficult to formally specify convergence criteria. As the fitness of a population may remain static for a number of generations before a superior individual is found, the application of convergence termination criteria becomes problematic, a common practice is to terminate GA after a prespecified number of generations (in our case the number of generations is 300) and then test the fitness of best members in the last population. If no acceptable solutions are found, the GA may be restarted or fresh search initiated.

3.4.1.3 OPF Solution by Genetic Algorithm — Researchers' Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions / Salient Features
1	A. Bakritzs, V. Perirtridis and S. Kazarlis [101]	Genetic Algorithm Solution to the Economic Dispatch Problem	*IEE Proc,-Generation Transmission Distribution,* vol. 141, no. 4, pp. 377-382, July 1994.	➢ Solved Economic dispatch problems (two) using Genetic Algorithm method. Its merits are, the non restriction of any convexity limitations on the generator cost function and effective coding of GAs to work on parallel machines. ➢ GA is superior to Dynamic programming, as per the performance observed in Economic dispatch problem. ➢ The run time of the second GA solution (EGA method) proportionately increases with size of the system.
2	Po-Hung Chen and Hong-Chan Chang [14]	Large-Scale Economic Dispatch by Genetic Algorithm Discover	*IEEE Transactions on Power Systems,* Vol. 10, no. 4, pp. 1919 – 1926, Nov. 1995.	➢ Solved Large Scale Economic Dispatch problem by Genetic Algorithm. ➢ Designed new encoding technique where in, the chromosome has only an encoding normalized increamental cost. ➢ There is no correlation between total number of bits in the chromosome and number of units. ➢ The unique characteristic of Genetic Approach is significant in big and intricate systems which other approaches fails to accomplish. ➢ Dispatch is made more practical by

				flexibility in GA, due to consideration of network losses, ramp rate limits and prohibited zone's avoidance. ➢ This method takes lesser time compared to Lambda –iteration method in big systems.
3	L. L. Lai and J. T. Maimply [102]	Improved Genetic Algorithms for Optimal Power Flow under both normal contingent operation states	*Electrical power and Energy systems, Vol.19, No.*5, *pp.* 287-292, 1997.	➢ Provided solution by employing Improved Genetic Algorithm for optimal power flow in regular and contingent conditions. ➢ Contingent condition implies circuit outage simulation in one branch resulting in crossing limits of power flow in the other branch. ➢ The approach gives good performance and discards operational and insecure violations. ➢ The dynamical hierarchy of the coding procedure designed in this approach, enables to code numerous control variables in a practical system within a suitable string length. ➢ This method is therefore able to regulate the active power outputs of Generation, bus voltages, shunt capacitors / reactors and transformer tap settings to minimize the fuel costs. ➢ IGA obtains better optimal fuel cost of the normal case and global optimal point compared to gradient based conventional method.
4	Anastasios G. Bakirtzis and Pandel N. Biskas, Christoforos and	Optimal power flow by Enhanced Genetic Algorithm	*IEEE Transactions on Power Systems,* Vol.17*,* No.2, pp. 229-	➢ Solved Optimal Power Flow (OPF) with both continuous and discrete control variables, by Enhanced Genetic Algorithm (EGA), superior to Simple Genetic Algorithm (SGA). ➢ Unit active power outputs and generator bus voltage magnitudes

	Vasilios Petridis [103]		236, May 2002.	are considered as continuous control variables, while transformer-tap settings and switchable shunt devices are treated as discrete control variables. ➢ Branch flow limits, load bus voltage magnitude limits and generator reactive capabilities are incorporated as penalties in the GA fitness function (FF). ➢ Algorithm's effectiveness and accuracy are improved by using advanced and problem-specific operators. ➢ EGA-OPF solution and execution cost and time are superior compared to SGA,
5	Tarek Bouktir, Linda Slimani and M.Belkacemi [104]	A Genetic algorithm for solving the Optimal Power Flow problem	*Leonardo Journal of Sciences,* Issue 4, pp.44-58. June 2004.	➢ Provided solution to optimal power flow problem of large distribution system using simple genetic algorithm. ➢ The objective includes fuel cost minimisation and retaining the power outputs of generators, bus voltages, shunt capacitors / reactors and transformers tap-setting in their safe limits. ➢ Constraints are bifurcated in to active and passive to reduce the CPU time. ➢ Active constraints are incorporated in Genetic Algorithm to derive the optimal solution, as they only have direct access to the cost function. ➢ Conventional load flow program is employed to modify passive constraints, one time after the convergence on the Genetic Algorithm OPF (GAOPF) i.e., attaining the optimal solution.

				➢ Using simple genetic operations namely, proportionate reproduction, simple mutation and one point cross over in binary codes, results indicate that a simple GA will give good result. ➢ With more number of constraints typical to a large scale system, GA takes longer CPU time to converge.
6	Liladhur G. Sewtohul, Robert T.F. Ah King and Harry C.S. Rughooputh [105]	Genetic Algorithms for Economic Dispatch with valve point effect	*Proceedings of the 2004 IEEE International Conference on Networking, Sensing & Control,* Taipei, Taiwan, pp.1358-1363, March 21-24, 2004.	➢ Provided solution for Economic Dispatch with valve point effect using Genetic Algorithm ➢ In this method, four Genetic Algorithms namely, Simple Genetic Algorithm (SGA), SGA with generation – apart elitism, SGA with atavism and Atavistic Genetic Algorithm (AGA) are employed to get solution on three test systems: 3 – generator system, 13 – generator system and the standard IEEE 30-bus test system. ➢ On comparison of results, it is observed that all GA methods mentioned above are better than Lagrangian method with no valve effect. ➢ With valve point effect and ramping characteristics of Generators, AGA is superior to other GAs and the Tabu search. Further, the AGA alone circumvents entrapment in local solution. It is attributed to equilibrium in selective pressure and population diversity.

7	Chao-Lung Chiang [106]	Improved Genetic Algorithm for Power Economic Dispatch of Units with valve point effects and multiple fuels	*IEEE Transactions on Power Systems, Vol.20,* No.4, pp.1690-1699, Nov 2005.	➢ Improved Genetic Algorithm integrated with Multiplier Updating (IGA – MU) is employed to solve complicated problem of Power Economic dispatch of units having valve point effects and multiple fuels. ➢ An effective search to actively explore solutions is achieved by IGA coupled with an improved evolutionary direction operator. The MU is used to deal the equality and inequality constraints of the Power Economic Dispatch (PED) problem. ➢ The method has several important advantages namely, easy concept; simple implementation, more useful than earlier approaches, better performance, compared to CGA – MU (Conventional Genetic Algorithm with Multiplier Updating), robustness of logarithm, adaptable to large scale systems; automatic tuning of the randomly assigned penalty to a proper value, and the condition for only a small population in the accurate and practical PED problem.
8	Ashish Saini, Devendra K. Chaturvedi and A.K.Saxena [107]	Optimal Power Flow Solution: A GA-Fuzzy System approach	*International Journal of Emerging Electric Power System, Vol.5, Issue* 2, 2006.	➢ A GA – Fuzzy system was employed to solve the complex problem of OPF. ➢ Probabilities of GA operations such as cross over and mutation are decided by Fuzzy rule base. Algorithms for GA-OPF and are created and analysed. ➢ Results show that the GA-OPF has quicker convergence and smaller generation costs in comparison to other methods.

				➢ The GA-Fuzzy (GAF) OPF demonstrated better performance in respect of convergence, consistency in different runs and lower cost of generation in comparison to simple GA and other methods. ➢ The merits are due to the alterations in crossover and mutation probabilities value as directed by a set of Fuzzy rule base, though they are stochastic in nature.
9	M.Younes, M. Rahli and L. Abdelhakeem-Koridak [108]	Optimal Power based on Hybrid Genetic Algorithm	*Journal of Information Science and Engineering*, vol. 23, pp.1801-1816, Jan 2007.	➢ Hybrid Genetic Algorithm (combination of GA and Mat power) was used to solve OPF including active and reactive power dispatches. ➢ The method uses the Genetic Algorithm (GA) to get a close to global solution and the package of Mat lab – m files for solving power flow and optimal power flow problem (mat power) to decide the optimal global solution. ➢ Mat power is employed to adjust the control variables to attain the global solution. ➢ The method was validated on the modified IEEE 57 – bus system and the results show that the hybrid approach provides a good solution as compared to GA or Mat power alone.

3.4.1.4 Merits and Demerits of Genetic Algorithm

The Merits and Demerits of Genetic Algorithm are summarized and given below.

Merits

1. GAs can handle the Integer or discrete variables.
2. GAs can provide a globally optimum solution as it can avoid the trap of local optima.
3. GAs can deal with the non-smooth, non continuous, non-convex and non differentiable functions which actually exist in practical optimisation problems.
4. GAs has the potential to find solutions in many different areas of the search space simultaneously, there by multiple objectives can be achieved in single run.
5. GAs are adaptable to change, ability to generate large number of solutions and rapid convergence.
6. GAs can be easily coded to work on parallel computers.

De Merits

1. GAs are stochastic algorithms and the solution they provide to the OPF problem is not guaranteed to be optimum.
2. The execution time and the quality of the solution, deteriorate with the increase of the chromosome length, i.e., the OPF problem size.

   If the size of the power system is increasing, the GA approach can produce more in feasible off springs which may lead to wastage of computational efforts.

3.4.2 Particle Swarm Optimisation Method

Particle swarm optimization (PSO) is a population based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling [15, 16 and 17].

In PSO, the search for an optimal solution is conducted using a population of particles, each of which represents a candidate solution to the optimization problem. Particles change their position by flying round a multidimensional space by following current optimal particles until a relatively unchanged position has been achieved or until computational limitations are exceeded. Each particle adjusts its trajectory towards its own previous best position and towards the global best position attained till then. PSO is easy to implement and provides fast convergence for many optimization problems and has gained lot of attention in power system applications recently.

The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles .In PSO, each particle makes it s decision using its own experience together with its neighbor's experience.

3.4.2.1 OPF Problem Formulation

The OPF problem is to optimize the steady state performance of a power system in terms of an objective function while satisfying several equality and inequality constraints. Mathematically, the OPF problem can be represented by Eq. (3.1) – (3.6).

$$Min\ F(P_G) = f(x,u) = J(x,u)$$

Objective Function

The objective function is given by Eq. (3.1) and is reproduced below with an addition of function *J*.

$$J = F_T = F(P_G) = \sum_{i=1}^{N_G} F_i(P_{G_i}) = \sum_{i=1}^{N_G} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2)$$

Subject to: $g(x,u) = 0$

$$h(x,u) \le 0$$

$$x^T = [P_{G_1}, V_{L_1} \ldots . V_{L_{N_D}}, \ Q_{G_1} \ldots . Q_{G_{N_G}}, \ S_{l_1} \ldots S_{l_{N_l}}] \qquad (3.108)$$

where x is the vector of dependent variables consisting of slack bus power P_{G_1}, load bus voltages V_L, generator reactive power outputs Q_G, and transmission line loadings S_l. And hence, x represented as above. *NL*, *NG* and *nl* are number of load buses, number of generators, and number of transmission lines, respectively.

u is the vector of independent variables consisting of generator voltages V_G, generator real power outputs P_G except at the slack bus P_{G_1}, transformer tap settings *T*, and shunt VAR compensations Q_C. Hence, u can be expressed as

$$u^T = [V_{G_1} \dots V_{G_{N_G}} \ , \ P_{G_2} \dots . P_{G_{N_G}} \ , \ T_1 \dots T_{NT} \ , \ Q_{c_1} \dots . Q_{c_{NC}}] \tag{3.109}$$

Where *NT* and *NC* are the number of the regulating transformers and shunt compensators, respectively. *J* is the objective function to be minimized, *g* is the equality constraints representing typical load flow equations, *h* is the equality constraint representing system constraints as given below.

(a) *Generation constraints:* Generator voltages, real power outputs, and reactive power outputs are restricted by their lower and upper limits, are represented by Eq. (3.12) – (3.20).

(b) *Shunt VAR constraints:* Shunt VAR compensations are restricted by their limits as follows:

$$Q_{c_{i\min}} \le Q_{c_i} \le Q_{c_{i\max}} \ , \quad i = 1, \dots, NC \tag{3.110}$$

(c) *Security constraints:* These include the constraints of voltages at load buses and transmission line loadings as follows:

$$V_{L_{i\min}} \le V_{L_i} \le V_{L_{i\max}} \ , \quad i = 1, \dots, N_D \tag{3.111}$$

$$S_{l_i} \le S_{l_{i\max}} \ , \quad i = 1, \dots, N_l \tag{3.112}$$

It is the worth mentioning that the control variables are self constrained. The hard inequalities of P_{G_1}, V_L, Q_G and S_l can be incorporated in the objective function as quadratic penalty terms. Therefore, the objective function can be augmented as follows:

$$J_{aug} = J + \lambda_P (P_{G_1} - P_{G_{1\lim}})^2 + \lambda_V \sum_{i=1}^{N_D} (V_{L_i} - V_{G_{i\lim}})^2$$

$$+ \lambda_Q \sum_{i=1}^{N_G} (Q_{G_i} - Q_{G_{i\lim}})^2 + \lambda_S \sum_{i=1}^{N_l} (S_{l_i} - S_{l_{i\max}})^2 \qquad (3.113)$$

Where $\lambda_P, \lambda_V, \lambda_Q$ and λ_S are penalty factors and $x_{\lim}$ is the limit value of the dependent variable x given as

$$x_{\lim} = \begin{cases} x_{\max} & x > x_{\max} \\ x_{\min} & x < x_{\min} \end{cases} \qquad (3.114)$$

3.4.2.2 Solution Algorithm

Description of basic elements required for the development of Solution Algorithm is given below.

- Particle, $X(t)$: It is a candidate solution represented by an m -dimensional vector, where m is the number of optimised parameters. At time t, the jth particle $X_j(t)$ can be described as $X_j(t)$ = $[x_{j,1}(t)$,…… ……$x_{j,\,\mathrm{m}}\ (t)]$, where xs are the optimised parameters and $x_{j,\,k}(t)$ is the position of the jth particle with respect to the kth dimension, i.e. the value of the kth optimised parameter in the jth candidate solution.
- Population, pop (t): It is a set of n particles at time t, i.e. pop (t)= $[X_i(t)$,…. $X_{\mathrm{n}}(t)^T$.
- Swarm: It is an apparently disorganized population of moving particles that tend to cluster together, while each particle seems to be moving in a random direction.
- Particle velocity, $V(t)$ **:** It is the velocity of the moving particles represented by an m dimensional vector. At time t, the jth particle

velocity $V_j(t)$ can be described as $V_j(t) = [v_{j,1}(t), \ldots\ldots\ldots v_{j,m}(t)]$, where $v_{j,k}(t)$ is the velocity component of the jth particle with respect to k^{th} dimension.

- Inertia weight, $w(t)$: It is a control parameter to control the impact of the previous velocities on the present velocity. Thus it influences the trade off, between the global and local exploration abilities of the particles, large inertia weight to enhance the global exploration, is recommended at the initial stages where as for final stages, the inertia weight is reduced for better local exploration.
- Individual best $X^*(t)$: During the search process, the particle compares its fitness value at the current position, to the best fitness value it has ever attained at any time up to the current time. The best position that is associated with the best fitness encountered so far is called the individual best, $X^*(t)$. In this way, the best position $X^*(t)$.for each particle in the swarm, can be determined and updated during the search. For example, in a minimisation problem with objective function J, the individual best of the jth particle $X^*_j(t)$ is determined such that $J(X^*_j(t)) \leq J(X^*_j(\tau))$, $\tau \leq t$. For simplicity it is assumed that $J_j^* = J(X^*_j(t))$.For the jth particle, individual best can be expressed as $X^*_j(t) = [x^*_{j,1}(t) \ldots\ldots\ldots\ldots x^*_{j,m}(t)]$.
- Global best $X^{**}(t)$: It is the best position among all individual best positions (i.e. the best of all) achieved so far .Therefore ,the global best can be determined as such that $J(X^{**}_j(t)) \leq J(X^*_j(\tau))$, $j=1,\ldots\ldots.n$. For simplicity, assume that $J^{**}= J(X^{**}(t))$.

- Stopping criteria: the conditions under which the search process will terminate. In the present case, the search will terminate if one of the following conditions is met.,

a) The number of iterations since, the last change of the best solution is greater than a prespecified number.

or

b) The number of iterations reaches the maximum allowable number.

With the description of basic elements as above, the Solution algorithm is developed as given below.

- In order to make uniform search in the initial stages and very local search in later stages, an annealing procedure is followed. A decrement function for decreasing the inertia weight given as $w(t) = \alpha\, w(t\text{-}1)$, α is a decrement constant smaller than but close to 1, is considered here.
- Feasibility checks, for imposition of procedure of the particle positions, after the position updating to prevent the particles from flying outside the feasible search space.
- The particle velocity in the *k*th dimension is limited by some maximum value, $v_k^{\ max}$. With this limit, enhancement of local exploration space is achieved and it realistically simulates the incremental changes of human learning. In order to ensure uniform velocity through all dimensions, the maximum velocity in the *k*th dimension is given as :

$$v_{k\ \max} = (x_{k\ \max} - x_{k\ \min}) \,/\, N \qquad (3.115)$$

In PSO algorithm, the population has n particles and each particle is an m – dimensional vector, where m is the number of optimized parameters. Incorporating the above modifications, the computational flow of PSO technique can be described in the following steps.

Step 1 *(Initialization)*

- Set the time counter $t=0$ and generate randomly n particles, $[X_j(0),\ j=1,\ldots n]$, where $X_j(0)=[x_{j,1}(0),\ldots,x_{j,m}(0)]$.
- $x_{j,k}(0)$ is generated by randomly selecting a value with uniform probability over the kth optimized parameter search space $[x_{k\,\min}, x_{k\,\max}]$.
- Similarly, generate randomly initial velocities of all particles,$[V_j(0),\ j=1,\ldots n]$, where $V_j(0)=[v_{j,1}(0),\ldots,v_{j,m}(0)]$.
- $v_{j,k}(0)$ is generated by randomly selecting a value with uniform probability over the kth dimension $[-v_{k\,\max}, v_{k\,\max}]$.
- Each particle in the initial population is evaluated using the objective function J.
- For each particle, set $X_j^*(0)=X_j(0)$ and $J_j^*=J_j,\ j=1,\ldots,n$. Search for the best value of the objective function J_{best}.
- Set the particle associated with J_{best} as the global best, $X^{**}(0)$, with an objective function of J^{**}.
- Set the initial value of the inertia weight $w(0)$.

Step 2 *(Time updating)*

Update the time counter $t = t + 1$.

Step 3 *(Weight updating)*

Update the inertia weight $w(t) = \alpha\, w(t-1)$.

Step 4 *(Velocity updating)*

Using the global best and individual best of each particle, the *j*th particle velocity in the *k*th dimension is updated according to the following equation:

$$v_{j,k}(t) = w(t)\, v_{j,k}(t-1) + c_1\, r_1 (x^{*}_{j,k}(t-1) - x_{j,k}(t-1))$$
$$+\, c_2\, r_2 (x^{**}_{j,k}(t-1) - x_{j,k}(t-1)) \qquad (3.116)$$

Where c_1 and c_2 are positive constants and r_1 and r_2 are uniformly distributed random numbers in [0, 1]. It is worth mentioning that the second term represents the cognitive part of PSO where the particle changes its velocity based on its own thinking and memory. The third term represents the social part of PSO where the particle changes its velocity based on the social-psychological adaptation of knowledge. If a particle violates the velocity limits, set its velocity equal to the limit.

Step 5 *(Position updating)*

Based on the updated velocities, each particle changes its position according to the following equation:

$$x_{j,k}(t) = v_{j,k}(t) + x_{j,k}(t-1) \qquad (3.117)$$

If a particle violates its position limits in any dimension, set its position at proper limit.

Step 6 *(Individual best updating)*

Each particle is evaluated according to its updated position. If $J_j < J^{*}_j$, $j = 1, \ldots, n$, then update individual best as $X^{*}_j(t) = X_j(t)$ and $J^{*}_j = J_j$ and go to step 7; else go to step 7.

Step 7 *(Global best updating)*

Search for the minimum value $J_{\min}$ among J_j^* , where min is the index of the particle with minimum objective function, i.e. $\min \in \{j; j = 1,\ldots,n\}$. If $J_{\min} < J^{**}$, then update global best as $X^{**}(t) = X_{\min}(t)$ and $J^{**} = J_{\min}$ and go to step 8 ; else go to step 8.

Step 8 *(Stopping criteria)*

If one of the stopping criteria is satisfied then stop; else go to step 2.

3.4.2.3 PSO Method — Researches Contribution

The Significant Contributions/Salient Features of Researchers are furnished below:

Sl.No	Author [Ref. No]	Title of Topic	Journal / Publication Details	Significant Contributions / Salient Features
1	Hirotaka Yoshida, Kenichi Kawata, Yoshikazu Fukuyama [16]	A Particle Swarm Optimization for Reactive Power and Voltage Control Considering Voltage Security Assessment	*IEEE Transactions on Power Systems,* vol. 15, no. 4, pp. 1232 – 1239, Nov. 2000.	➢ Reactive power and voltage control (VVC), is handled by Particle Swarm Optimisation, while taking into account voltage security assesment (VSA). ➢ The method treats , VVC as a mixed integer nonlinear optimization problem (MINLP) and decides a control approach with continuous and independent control variables such as AVR operating values, OLTC tap positions, and the number of reactive power compensation equipment. ➢ Voltage security is taken care by adapting a continuation power flow (CPFLOW) and a voltage contingency analysis method.

				➢ The viability of the proposed method for VVC is confirmed on practical power systems with encouraging results.
2	M.A. Abido [17]	Optimal Power Flow using Particle Swarm Optimization	Electrical Power and Energy Systems 24, pp. 563 – 571. 2002	➢ Provided capable and dependable evolutionary based method, the Particle swarm optimization (PSO), to solve Optimal Power Flow problem. ➢ For optimal position of OPF problem control variables, PSO algorithm is used. ➢ Presumptions forced on the optimized objective functions are considerably removed by this optimisation technique in solving OPF problem, ➢ Validation was done for various objective functions such as fuel cost minimisation, enhancement of voltage profile and voltage stability. ➢ Observations prove that this method is better than the conventional methods and Genetic Algorithms in respect of efficacy and robustness.
3	Cui-Ru Wang, He-Jin Yuan, Zhi-Qiang Huang, Jiang-Wei Zhang and Chen-Jun Sun [18]	A Modified Particle Swarm Optimization Algorithm and its OPF Problem	*Proceedings of the Fourth International Conference on Machine Learning and Cybernetics,* Guangzhou, pp.2885-2889, Aug 2005.	➢ Solved OPF problem in a power system by employing modified particle swarm optimization (MPSO) algorithm. ➢ MPSO using swarm intelligence provides a new thinking for solution of nonlinear, non-differential and multi-modal problem. ➢ Particle understands from itself and the best one as well as from other particles in this algorithm ➢ Possibility to discover the global optimum is improved and the affect of starting position of the particles is reduced by enriched knowledge.

4	John G. Vlachogiannis and kwang Y. Lee [109]	A Comparative Study on Particle Swarm Optimisation for Optimal Steady- state Performance of Power Systems	*IEEE Transactions Power Systems,* vol.21, No 4, pp 1718-1728, Nov 2006.	➢ Three types of PSO algorithms were used to make a relative study on optimal steady – state performance of power systems. The Algorithms comprise enhanced GPAC, LPAC with constriction factor approach based on the passive congregation operator and the CA based on the coordinated aggregation operator. ➢ Above referred PSO algorithms were compared with the recent PSO and the usual interior-point OPF-based algorithm with reference to the solutions of optimization problems of reactive power and voltage control. ➢ The observations on IEEE 30-bus system and IEEE 118-bus systems show better performance of LPAC and a superb performance of CA. ➢ The CA attains a global optimum solution and shows improved convergence characteristics, adapting the least random parameters than others. However execution time is its major disadvantage.
5	Jong-Bae Park, Yun-Won Jeong, Joong-Rin Shin and kwang Y. Lee [110]	An Improved particle Swarm Optimisation for Nonconvex Economic Dispatch Problems	*IEEE Transactions Power Systems,* vol.25, No 1, pp 156-166, Feb 2010.	➢ Solved the nonconvex economic dispatch problems using an improved particle swarm optimization. ➢ Improved the performance of the conventional PSO by adopting this approach which uses the chaotic sequences and the crossover operation. ➢ The global searching ability and

				getaway from local minimum is enhanced by uniting, the chaotic sequences with the linearly decreasing inertia weights. ➢ Further, the diversity of the population is enlarged by adding the crossover operation. ➢ The global searching capability as well as preventing the solution from entrapment in local optima, by the above approaches.

3.4.2.4 Merits and Demerits of PSO Method

Merits

1. PSO is one of the modern heuristic algorithms capable to solve large-scale non convex optimisation problems like OPF.
2. The main advantages of PSO algorithms are: simple concept, easy implementation, relative robustness to control parameters and computational efficiency.
3. The prominent merit of PSO is its fast convergence speed.
4. PSO algorithm can be realized simply for less parameter adjusting.
5. PSO can easily deal with non differentiable and non convex objective functions.
6. PSO has the flexibility to control the balance between the global and local exploration of the search space.

Demerits

1. The candidate solutions in PSO are coded as a set of real numbers. But, most of the control variables such as transformer taps settings and switchable shunt capacitors change in discrete

manner. Real coding of these variables represents a limitation of PSO methods as simple round-off calculations may lead to significant errors.

2. Slow convergence in refined search stage (weak local search ability).

3.4 NEED FOR ALTERNATIVE METHDOLOGIES FOR OPTIMAL POWER FLOW SOLUTION

An exhaustive literature survey is carried out for the existing OPF methodologies and observations presented as above. With the knowledge gained, now, need for alternative OPF methodologies are discussed below. *The objective is to explore the necessity for alternative approaches for OPF solution that can overcome the disadvantages and retain advantages of the existing methodologies.*

3.4.1 Limitations of Mathematical Methods

For the sake of continuity, the limitations in mathematical methods presented in Section 3.2 of Chapter 3 are reproduced below:

- Limited capabilities in handling large-scale power system problems. They become too slow if the variables are large in number.
- They are not guaranteed to converge to global optimum of the general non convex problems like OPF.
- The methods may satisfy necessary conditions but not all the sufficient conditions. Also they are weak in handling qualitative constraints.
- Inconsistency in the final results due to approximations made

while linearising some of the nonlinear objective functions and constraints.

- Consideration of certain equality or inequality constraints makes difficulty in obtaining the solution.
- The process may converge slowly due to the requirement for the satisfaction of large number of constraints.
- Some mathematical models are too complex to deal with.
- These methods are difficult to apply for the problems with discrete variables such as transformer taps.

In addition, W.F. Tinney et.al. [29] have presented some more deficiencies in OPF. The salient deficiencies in OPF that influence the mathematical methods are: consideration of discrete variables in place of continuous variables and too large number of control actions.

3.4.2 Limitations of Genetic Algorithm Approach

The following *limitations* may be observed in GA approach:

- The solution deteriorates with the increase of chromosome length. Hence to limit its size, limitations are imposed in consideration of number of control variables.
- GA method tends to fail with the more difficult problems and needs good problem knowledge to be tuned.
- Careless representation in any of the schemes that are used in the formation of chromosomes shall nullify the effectiveness of mutation and crossover operators.
- The use is restricted for small problems such as those handling less variables, constraints etc.

- GA is a stochastic approach where the solution is not guaranteed to be the optimum.
- Higher computational time.
- Conventional methods rather than GA method are suited for finding a best solution of well behaved convex optimization problems of only few variables.

3.4.3 Improvements in Genetic Algorithm Approach

To overcome difficulties in conventional GA approaches, Anastasios G. Bakirtzis et.al [103] have proposed Enhanced Genetic Algorithm (EGA) for the solution of OPF problem. The EGA method has following features:

- The method considers control variables and constraints used in the OPF and penalty method treatment of the functional operating constraints. Control device parameters are treated as discrete control variables.
- Variable binary string length is used for better resolution to each control variable.
- The method avoids the unnecessary increase in the size of GA chromosome.
- Problem-specific operators incorporated in the EGA method makes the method suitable for solving larger OPF.

The test results presented in [103] are quite attractive. However the authors in their conclusions have presented the following *limitations of EGA* method:

- The method is claimed as stochastic and also said the solution to OPF is not guaranteed to be optimum.

- Execution time is high.
- The quality of solution is found to be deteriorating with the increase in length of chromosome i.e. the OPF problem size.
- If the size of power system is growing, the EGA approach can produce more infeasible strings which may lead to wastage of computational time, memory etc.

3.4.4 Objectives of alternative Methodologies

Because of the above, one has to think for alternative methodologies that can avoid all the difficulties in the various approaches and provide a better OPF solution. The proposed methodologies must aim the following objectives that will improve genetic algorithm for OPF solution.

- Need for large improvements in Speed.
- Need of Good accurate solution.
- Need for consideration of large varieties of constraints.
- Need for avoiding the blind search, encountering with infeasible strings, and wastage of computational effort.
- Need for consideration of System nonlinearities.
- Need for reduction in population size, number of populations in order to make the computational effort simple and effective.
- Need for testing other types of Genetic Algorithm methods instead of conventional GA that uses binary coded chromosomes.
- Need for thinking population is finite in contrast to assume it to be as infinite.
- Need for incorporating a local search method within a genetic

algorithm that can overcome most of the obstacles that arise as a result of finite population size.

- Need for a suitable local search method that can achieve a right balance between global exploration and local exploitation capabilities. These algorithms can produce solutions with high accuracy.
- Need for identification and selection of proper control parameters that influence exploitation of chromosomes and extraction of global optimum solution.
- Need for search of a local method that enhances overall search capability. The enhancement can be in terms of solution quality and efficiency.
- Need for the proper genetic operators that will resolve some of the problems that face genetic search.
- Need for reducing time for searching for a global optimum solution and memory needed to process the population.
- Need for improvements in coding and decoding of Chromosome that minimizes the population size.
- Need for undertaking *multi-objective OPF problem*. By integrating objective functions, other than cost objective function, it can be said economical conditions can be studied together with system security constraints and other system requirements.

Well designed GAs have shown the capability of handling highly multimodal functions that are hard to attack by other optimization methods. However, because of the high dimensionality of optimization space, caused by number of system parameters, different variety of objective functions and large number of system security constraints, the problem of OPF is still challenging and computationally expensive.

Incorporating a local search method can introduce new genes which can help to combat the genetic drift problem caused by the accumulation of stochastic errors due to finite populations. It can also accelerate the search engine towards the global optimum which in turn can guarantee that the convergence rate is large enough to obstruct any genetic drift. Due to its limited population size, a genetic algorithm may also sample bad representatives of good search regions and good representatives of bad regions. A local search method can ensure fair representation of the different search areas by sampling their local optima which in turn can reduce the possibility of premature convergence.

Conventional Genetic Algorithms can rapidly locate the regions in which the global optimum exists. However they take a relatively long time to locate it. A combination of a genetic and a local search method can speed up the search to locate the exact global optimum solution. In addition, applying a local search in conjunction with genetic algorithm can accelerate convergence to global optimum at a minimum time.

In real world problems, function evaluations are most time consuming. A local search algorithm's ability to locate local optima with high accuracy complements the ability of genetic algorithms to capture a global solution quickly and effectively.

Population size is crucial in a genetic algorithm. It determines the memory size and convergence speed and affects the search speed for a global solution.

Many researchers have contributed in the area of OPF by GA. All of them have used binary coded chromosome GAs, for optimizing variables. Use of continuous (real-valued) GAs is yet to be developed. Continuous GAs for solving problems with continuous search spaces, could overcome issues involved in the coding and decoding of binary GAs, such as 'deception', that results in premature convergence to a suboptimal solution [10], and "Hamming Cliffs", that makes gradual search over continuous space difficult [112]. The other benefit that arises from the use of continuous GAs as function optimizers is in achieving high precision for representing candidate solutions without increasing the computational burden.

Following sections describe the advantages of continuous GAs and Multi-objective Genetic algorithm (MOGA).

3.4.5 Advantages of continuous genetic algorithms over binary genetic algorithms

Following are the benefits of Continuous Genetic Algorithms (CGAs) in OPF problems:

- No need of using extra large Chromosomes as in the case of conventional binary Genetic algorithms (BGAs) which increases computational complexities.
- In CGAs the possibility of avoiding infeasible strings exists.
- In CGAs, there is no need for coding of chromosomes from Decimal

to binary while generating population and decoding of chromosomes back to Decimal while evaluating the objective function. This leads to increase in efficiency of GAs.

3.4.6 Advantages of Multi-Objective Genetic Algorithms

By integrating objective functions, other than cost objective function, it can be said economical conditions can be studied together with system security constraints and other system requirements.

The selected problem can be designated as a multi-criteria and multi-objective optimization problem which requires simultaneous optimization of two objectives with different individual optima. Objectives are such that none of them can be improved without degradation of another. Hence instead of a unique optimal solution, there exists a set of optimal tradeoffs between the objectives, the so called pareto-optimal solutions. In multi-objective optimization, the solutions are compared with each other based on non dominance property. For this class of problems GA based Multi-Objective Algorithm [115, 116] is more suitable.

3. 5 CONCLUSIONS

In this Chapter we have presented various popular techniques in Optimum Power Flow, covering both Conventional as well as Intelligent methodologies. To begin with, the Mathematical representation of optimal power flow problem is described by explaining the objective function along with non linear equality and non linear inequality constraints. The objective function is taken as minimisation of total production cost of scheduled generating units, as it reflects current economic dispatch practice and importantly cost related aspect is always ranked high among operational requirements in Power Systems.

The objectives of OPF have been mentioned, which include reduction of the costs of meeting the load demand for a power system while up keeping the security of the system and the determination of system marginal cost data to aid in the pricing of MW transactions as well as the pricing auxiliary services such as voltage support through MVAR support. In addition other applications of OPF are described and they include *Voltage Instability, Reactive power compensation and Economic dispatch.*

In addition, the challenges before OPF which remain to be answered are explained. It is to be mentioned, in the present research work, attempt is made to meet the challenge of coping up with response time requirements, for on line use.

For each of the Conventional and Intelligent methodology, detailed description is provided on important aspects like Problem formulation, Solution algorithm, Contribution of Researches and Merits & Demerits. The contribution by Researchers in each of the methodology has been covered with a lucid presentation in Tabular form. This helps the reader to quickly get to know the significant contributions and salient features of the contribution made by Researchers as per the Ref. No. mentioned in the list of References.

The conventional methods include Gradient method, Newton method, Linear Programming method, Quadratic Programming method and Interior Point method. Among these methods, the Interior Point method (IP) is found to be the most efficient algorithm. It maintains good accuracy while achieving the speed of convergence of as much as 12:1 in some cases when compared to other known linear programming methods. The IP method can solve large scale linear programming provided user interaction in the selection of constraints.

The Intelligent methods covered are PSO method and GA method. These methods are suitable in solving multiple objective problems as they are versatile in handling qualitative constraints. The advantages of the intelligent methods include learning ability, fast convergence and their suitability for non linear modeling. Among these two methods, GA method has better advantages such as handling both integer or discrete variables, providing globally optimum solutions dealing with non smooth, non continuous, non convex and non differentiable functions normally found in practical optimisation problems. Further GAs are adoptable to change, have ability to generate large number of solutions and provide rapid convergence.

Because of the nature of the problem, in recent times Genetic Algorithm approroach found to be more attracting the researchers to mitigate the OPF problem. This chapter explores the advantages and disadvantages in evolutionary algorithms like Genetic algorithms, continuous Genetic algorithms and multi-objective Genetic algorithms. With reference to OPF, this chapter provides basic up gradations required for OPF solution methodologies.