THE HOLOMORPHIC EMBEDDING LOAD FLOW METHOD (HELM)

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Summary

- **WHAT:**

  A novel, general-purpose method for solving the load flow equations of power systems of any size.

- **WHY:**
  - NR shortcomings
  - HELM is *deterministic and complete*: it ensures that if there is a solution the method will find it and, conversely, if there is no solution (voltage collapse) the method will unequivocally signal such condition as well.

Summary

- **HOW:**

  The method is based on a holomorphic embedding procedure that extends the voltage variables into analytic functions in the complex plane. This provides a framework to obtain and study the solutions using the full power of complex analysis techniques:

  - The holomorphic embedding method provides a *non-iterative* procedure for constructing the complex power series of voltages at a well-defined reference point, where it is trivial to identify the correct branch of the multivalued problem
  - Then uses analytical continuation by means of algebraic approximants to reach the solution. It can be proven that the continuation propagates the chosen branch to the maximal (in logarithmic capacity) possible domain on the complex plane. This proves that the method is deterministic and non-equivocal.

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Summary

- **Relevance:**

1. **As an enabler of new real-time applications**:
   - HELM allows to *reliably* implement real-time applications that perform non-supervised *exploratory* load flows under all possible conditions
   - Examples: Contingency Analysis, Limit-Violations solvers, Restoration plan builders
2. **As an enabler of new powerful analysis tools:**
   - new insights into the load flow problem
   - new magnitudes with practical application (e.g. new measures of distance to voltage collapse)
   - comprehensive framework for computing the multiple solutions to the problem
   - exact treatment of any general (algebraic) constraints under the same methodology

Summary

- **Relevance:**

3. Final comments

- The mathematical foundations (complex analysis, geometric function theory) are advanced, but the numerical implementation is straightforward
- Performance characteristics that make it competitive and even superior to fast-decoupled load flow algorithms, thus making it a good general-purpose load flow method.
- The method has been implemented in industrial-strength EMS products now operating at several large utilities in Europe, Mexico, and the US, and has been granted two US Patents (7519506 and 7979239).

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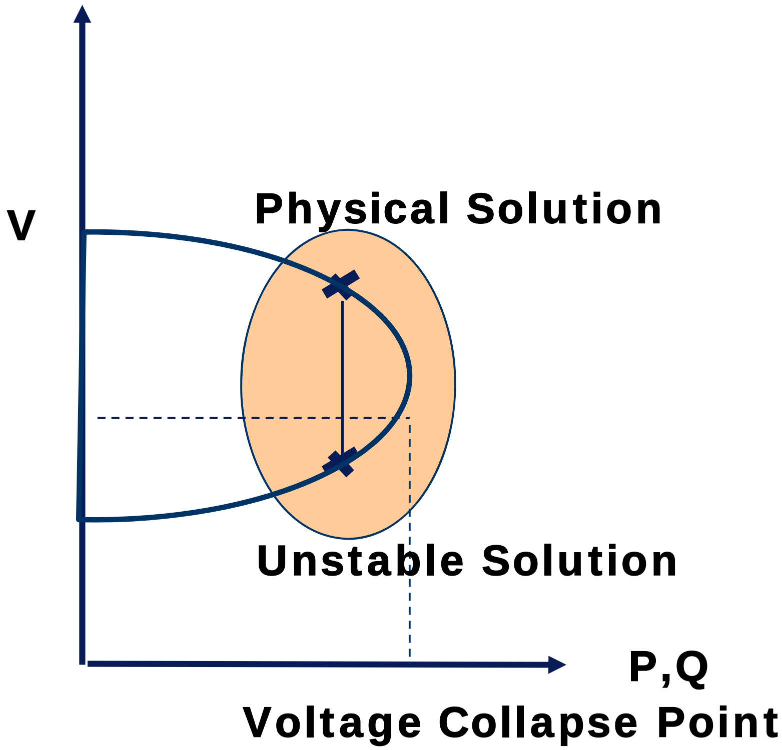
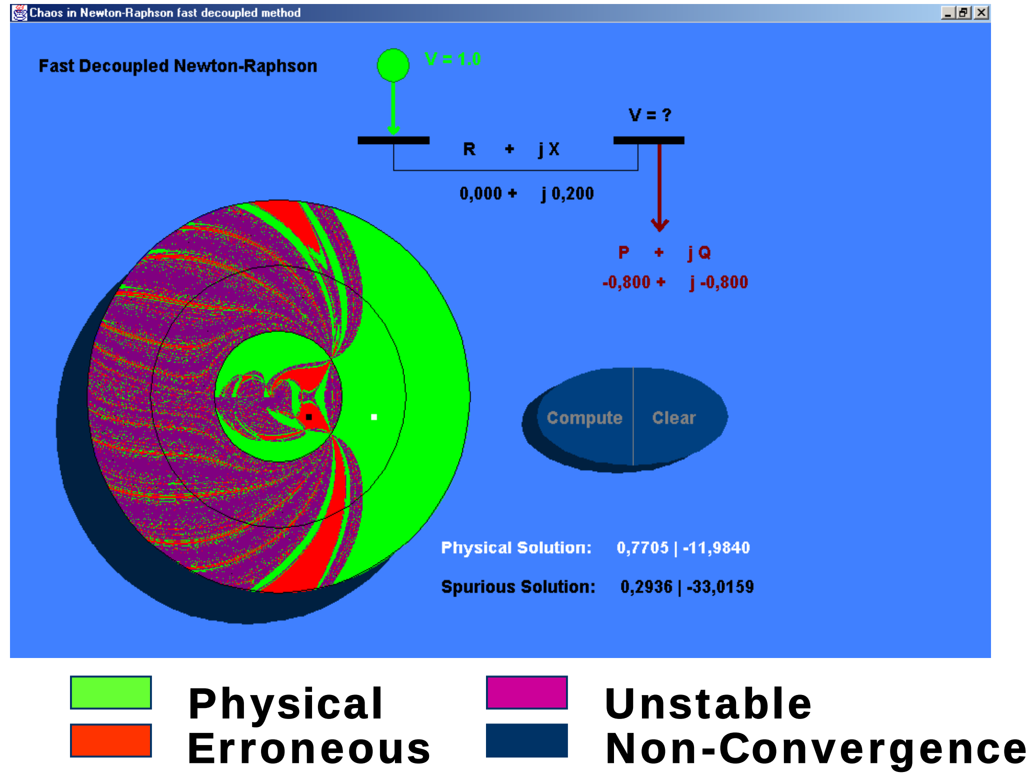
The problem with iterative methods

HELM



Newton-Raphson



Note: Every point within the circumference represents a possible starting point for Newton-Raphson calculation.

While HELM always finds the physical solution, N-R can converge to either the correct physical solution, an unstable solution, a wrong solution, or not converge at all.

The problem with iterative methods

For more information about the "fractality" problem:

-Bibliography:

- R. Klump and T.Overbye, *"A new method for finding low-voltage power flow solutions,"* in Power Engineering Society Summer Meeting, 2000. IEEE, vol. 1, 2000, pp. 593–597.
- J. Thorp and S. Naqavi, *"Load-flow fractals draw clues to erratic behaviour," IEEE Comput. Appl. Power,* vol. 10, no. 1, pp. 59–62, jan 1997.
- Gridquant, *"Convergence Issues with Newton-Raphson Method",* 2012. http://www.gridquant.com/technology/

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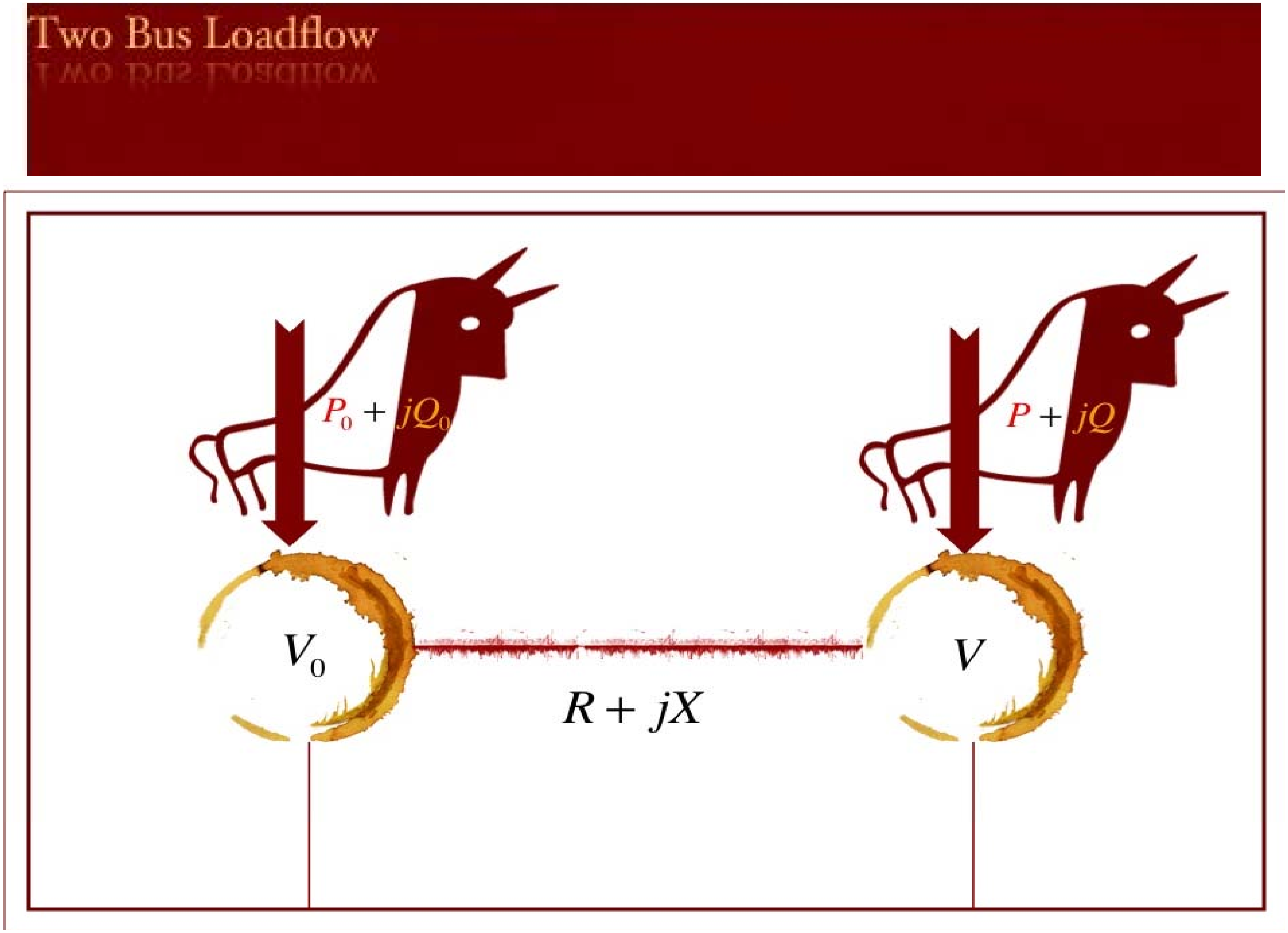
The HELM method



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The HELM method
Two bus system

The load flow equation for the two-bus system:

$$V = V_0 + Z\frac{S^*}{V^*} \quad ; \quad Z = R + jX \quad ; \quad S = P + jQ$$

Introducing dimensionless variables:

$$U \equiv \frac{V}{V_0}$$

$$\sigma \equiv \frac{ZS^*}{|V_0|^2} \quad ; \quad \sigma_R = \frac{XQ + RP}{|V_0|^2} \quad ; \quad \sigma_I = \frac{XP - RQ}{|V_0|^2}$$

The load flow equation in its most essential form:

$$\boxed{U = 1 + \frac{\sigma}{U^*}}$$

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The HELM method
Two bus system

Exact solution(s):

$$U_R = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \sigma_R - \sigma_I^2} \quad ; \quad U_I = \sigma_I$$

subject to the condition:

$$D \equiv \frac{1}{4} + \sigma_R - \sigma_I^2 \geq 0$$

Remember that the load flow problem is multi-valued in general. However, only one solution actually makes sense in the real system. All the other so-called "low-voltage" solutions would be unstable in any well-designed power system.

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Holomorphic Embedding

Holomorphic embedding:

$$\sigma \rightarrow s\sigma$$

Embedded load flow equations:

$$U(s) = 1 + \frac{s\sigma}{\bar{U}(s)} \qquad \textbf{with} \qquad \bar{U}(s) \equiv U^*(s^*)$$

$$\bar{U}(s) = 1 + \frac{s\sigma^*}{U(s)}$$

This is now a pair of functional equations involving two holomorphic functions *U*, *Û* of the complex variable *s*.
At the limit *s*=1, we recover the original load flow equation
At the limit *s*=0, the system is trivially solvable

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The method procedure

Once the equations are under the proposed embedding, this is how the method works:

- Since *U(s)* is **holomorphic**, consider the **power series expansion** about $s=0$. The embedded equations allow us to find all the coefficients of the power series as the solution to **a succession of linear systems** (1x1 in the two-bus case), order after order. The solution of each system yields the information to find the right-hand side for the next one.
- Using **analytic continuation** techniques (Padé Approximants), the solution at $s=1$ can be constructed. Stahl's theorem guarantees that the result is maximal: if there is a solution, the Padé Approximants will find it; if there is no solution, the Padé Approximants will signal this condition as well.

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The method procedure

Once the equations are under the proposed embedding, this is how the method works:

- Furthermore the solution is analytically continued from the well-defined reference solution at s=0:

  it is guaranteed to be the correct operative solution.

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References on HELM

For further reference:

-Bibliography

- “The Holomorphic Embedding Load Flow Method”, A. Trias. Accepted for publication in IEEE Trans. Power Syst., PES General Meeting, July 2012.
- “Two-bus load flow: exact developments”, A. Trias. Notes available at URL: http://www.gridquant.com/technology/

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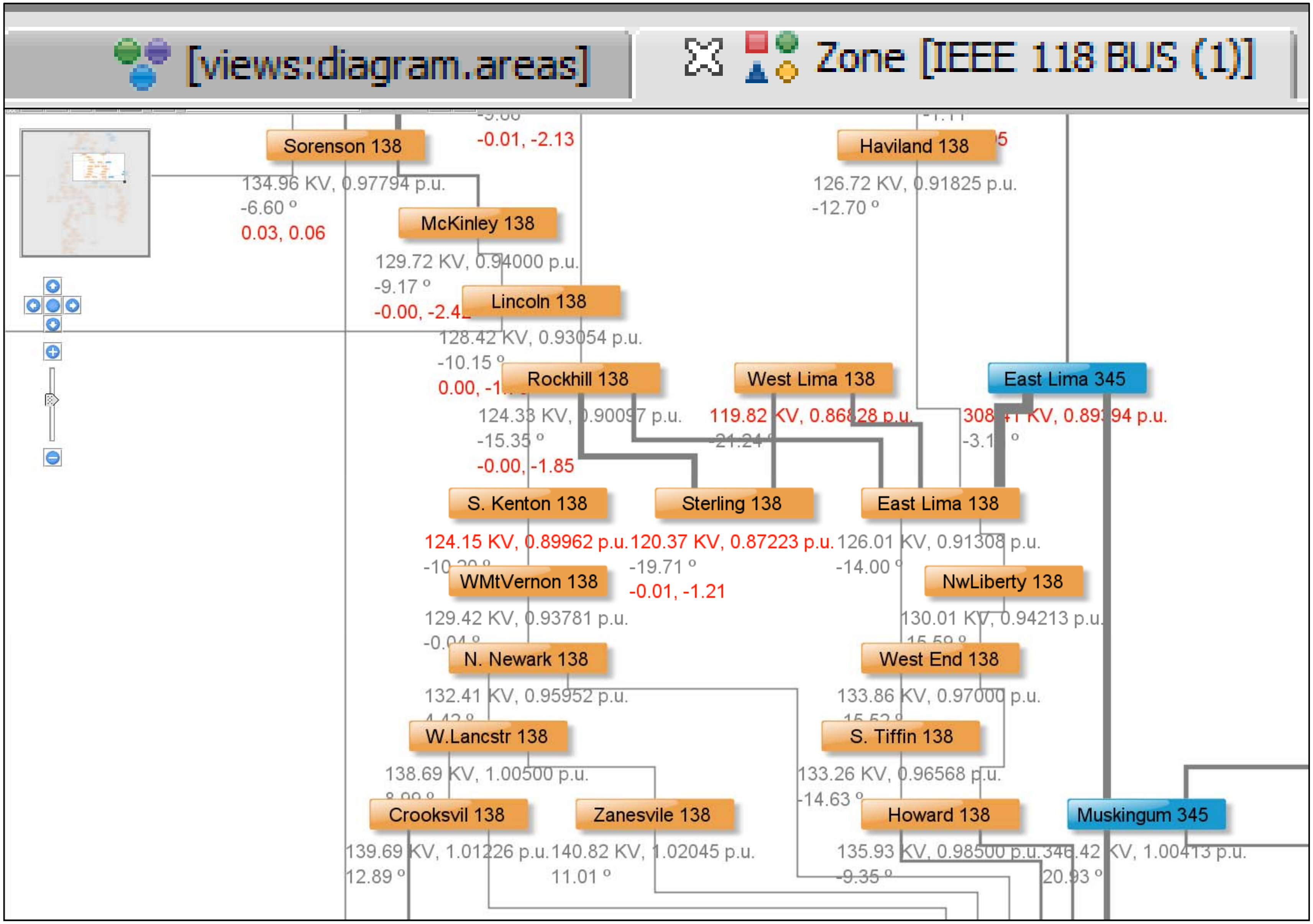
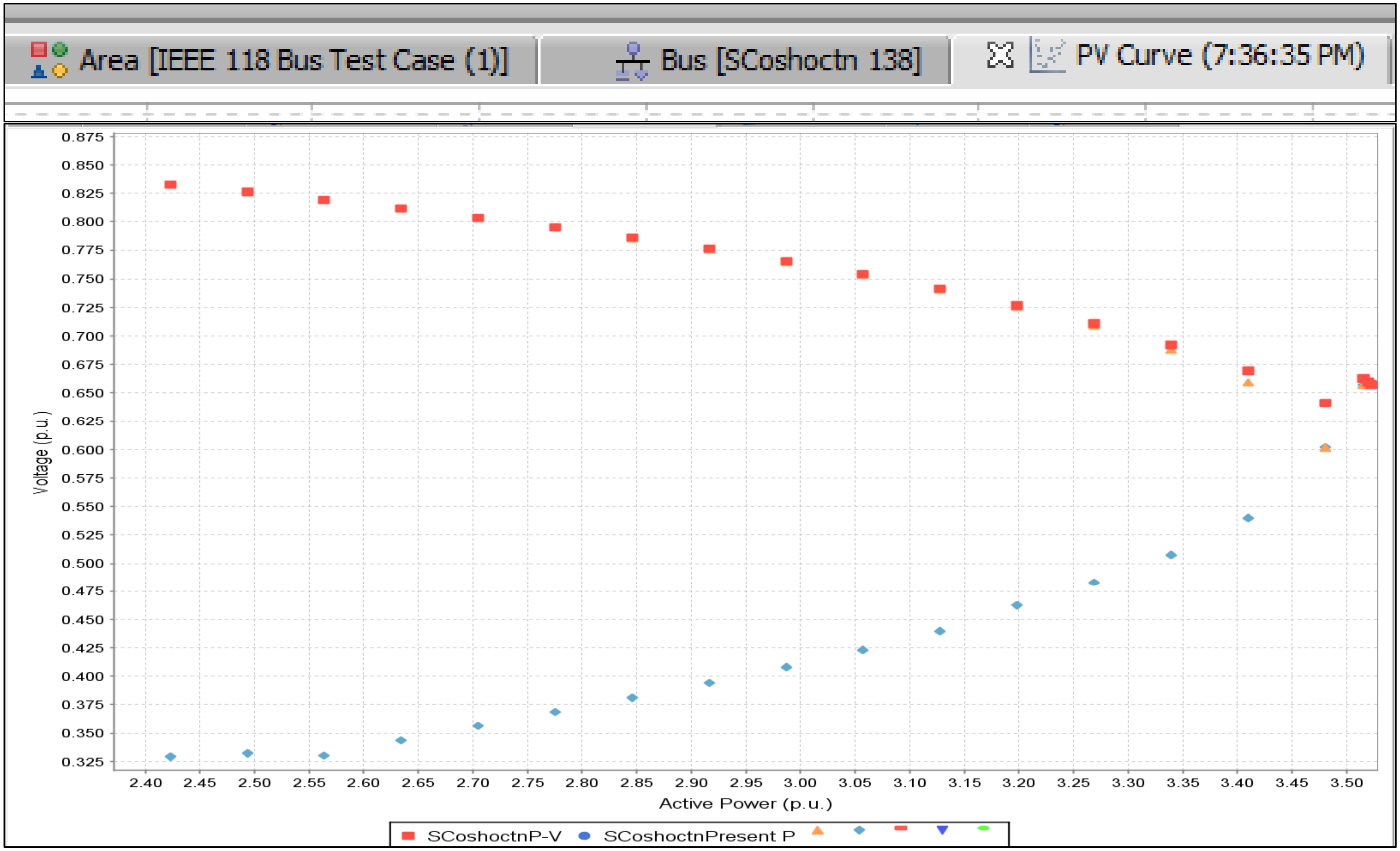
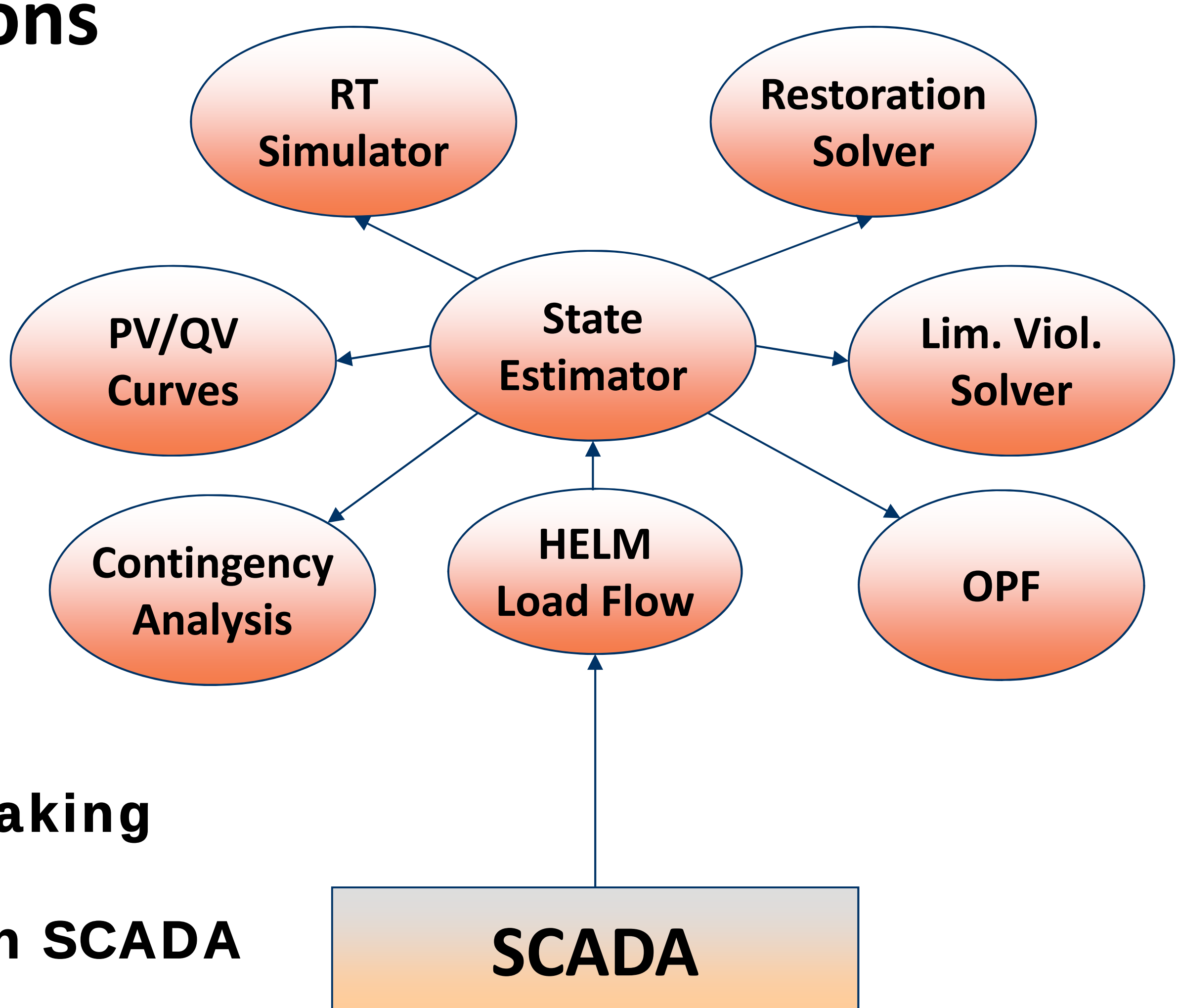
Examples (ALPS off-line Simulation)



Examples (ALPS off-line Simulation)



AGORA: A grid management system for Real-Time operations



- monitoring
- analysis
- optimal decision making
- full integration with SCADA

AGORA: Advanced Grid Observation Reliable Algorithms

A grid management system for Real-Time operation
"in Real-Time operation, we can't afford to have a convergence rate of less than 100%"

- AGORA applications are based on new, ground-breaking algorithmic advances
- Based on HELM, a new, *direct* method (US Patent No. 7 519 506, April 2009) for solving the load-flow equations has been developed
- **Several industry-first applications:**
  - Advanced Topological State Estimation
  - PV-QV Curves on any node in Real time
  - Limits Violations Solver
  - Restoration Solver

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AGORA: Advanced Grid Observation Reliable Algorithms

A grid management system for Real-Time operation
"in Real-Time operation, we can't afford to have a convergence rate of less than 100%"

- The solvers give answers in terms of real SCADA actions
- And they are fully checked via load-flow computation on the current estimated state

AGORA Restoration Solver

Search for the Optimal Path

- First system in the market that computes restoration plans in real time
- “Like a GPS navigator” for the Operator
- Monitors real-time network state, and re-computes when necessary
- Allows the user to simulate and evaluate the plan