

TVINNFALLAGREINING I - FORMÚLUBLAÐ

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1. Tvinntölur og tvinngild föll

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$$

2. Fáguð föll

$$\overline{\frac{\partial f}{\partial z}} = \frac{\partial \bar{f}}{\partial \bar{z}}$$
$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}$$
$$\overline{\frac{\partial f}{\partial \bar{z}}} = \frac{\partial \bar{f}}{\partial z}$$
$$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

Höfuðgrein hornsins: $\text{Arg } z = 2 \arctan \left(\frac{y}{|z| + x} \right)$, $\text{Arg} : \mathbb{C} \setminus \mathbb{R}_- \rightarrow]-\pi, \pi[$.

Höfuðgrein lografallsins: $\text{Log } z = \ln |z| + i\text{Arg}(z)$, $\text{Log} : \mathbb{C} \setminus \mathbb{R}_- \rightarrow \mathbb{C}$.

3. Cauchy-setning og Cauchy-formúla

Vegheildi:

$$\int_C f dz = \int_{\gamma} f dz = \int_{\gamma} f dx + i f dy = \int_a^b f(\gamma(t)) \gamma'(t) dt$$
$$\int_C f d\bar{z} = \int_{\gamma} f d\bar{z} = \int_{\gamma} f dx - i f dy = \int_a^b f(\gamma(t)) \overline{\gamma'(t)} dt$$

$$\left| \int_C f(z) dz \right| \leq \int_{\gamma} |f(z)| |dz| \leq \max_{z \in C} |f(z)| \int_{\gamma} |dz| = \max_{z \in C} |f(z)| L(C)$$

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n, \quad c_n = \frac{1}{2\pi i} \int_{\partial D(a,r)} \frac{f(\zeta)}{(\zeta-z)^{n+1}} \partial \zeta$$

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_{\partial \Omega} \frac{f(\zeta)}{(\zeta-z)^{n+1}} \partial \zeta$$

$$f(z) = \frac{1}{2\pi i} \int_{\partial \Omega} \frac{f(\zeta)}{\zeta-z} d\zeta - \frac{1}{\pi} \iint_{\Omega} \frac{\partial_{\bar{\zeta}} f(\zeta)}{\zeta-z} d\xi d\eta.$$

$$\int_{\partial \Omega} \frac{f(z)}{Q(z)} dz = 2\pi i \sum_{\alpha_j \in \Omega} \frac{f(\alpha_j)}{Q'(\alpha_j)}.$$

$$\int_{-\infty}^{+\infty} \frac{f(x)}{Q(x)} dx = 2\pi i \sum_{\alpha_j \in H_+} \frac{f(\alpha_j)}{Q'(\alpha_j)}$$

4. Leifareikningur

$$\int_{\partial\Omega} f(z) dz = 2\pi i \sum_{\alpha \in \omega \cap A} \text{Res}(f, \alpha)$$

Leif í m-ta stigs skauti:

$$\text{Res}(f, \alpha) = \frac{g^{(m-1)}(\alpha)}{(m-1)!}$$

Stofnbrotaliðun:

$$\frac{P(z)}{Q(z)} = \sum_{j=1}^k \frac{A_{j,0}}{(z - \alpha_j)^{m_j}} + \cdots + \frac{A_{j,m_j-1}}{(z - \alpha_j)}, \quad A_{j,\ell} = \frac{f_j^{(\ell)}(\alpha_j)}{\ell!}, \quad f_j(z) = \frac{(z - \alpha_j)^{m_j} P(z)}{Q(z)}$$

5. Almenn Cauchy-setning

$$I(\xi, z)f(z) = \frac{1}{2\pi i} \int_{\xi} \frac{f(\zeta)}{\zeta - z} \partial\zeta \quad I(\xi, z)f^{(n)}(z) = \frac{n!}{2\pi i} \int_{\xi} \frac{f(\zeta)}{(\zeta - z)^{n+1}} \partial\zeta$$

$$\int_{\xi} f(\zeta) d\zeta = 2\pi i \sum_{\alpha \in I(\xi) \cap A} \text{Res}(f, \alpha) I(\xi, \alpha)$$

$$\sum_{\alpha \in I(\xi)} \omega(f, \alpha) I(\xi, \alpha) = \frac{1}{2\pi i} \int_{\xi} \frac{f'(z)}{f(z)} dz = \frac{1}{2\pi i} \int_{f_* \xi} \frac{dw}{w} = I(f_* \xi, 0)$$

7. Einfaldlega samanhangandi svæði

Schwarz–Pick: Ef $f \in \mathcal{O}(\mathbb{D})$ og $f(\mathbb{D}) \subset \overline{\mathbb{D}}$, þá er

$$\left| \frac{f(z_1) - f(z_2)}{1 - \overline{f(z_1)} f(z_2)} \right| \leq \left| \frac{z_1 - z_2}{1 - \overline{z_1} z_2} \right|, \quad z_1, z_2 \in \mathbb{D},$$

og

$$|f'(z)| \leq \frac{1 - |f(z)|^2}{1 - |z|^2}, \quad z \in \mathbb{D}.$$

Jafnaðarmerki gildir í báðum ójöfnum ef f er sjálfsmótun á \mathbb{D} .