# STÆRÐFRÆÐIGREINING IIIA - FORMÚLUBLAÐ

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## 1. Linear Ordinary Differential Equations

### Fird Order Linear Equations

$y' + p(x)y = g(x)$:

$$y(x) = e^{-M(x)} \left( C + \int g(x) e^{M(x)} \, dx \right) \qquad \text{where } M(x) = \int p(x) dx.$$

### Reduction of order

$y'' + p(x)y' + q(x)y = 0$:

$$u_2(x) = u_1(x) \int \frac{e^{-P(t)}}{u_1^2(t)} \, dt, \qquad \text{where } P(x) = \int p(x) \, dx.$$

## 2. Separation of Variables

## 3. Series Solutions of Linear Equations

### Recursive formula for the coefficients at an ordinary point

$$(k+2)(k+1)c_{k+2} + \sum_{j=0}^{k} (k-j+1) a_j c_{k-j+1} + \sum_{j=0}^{k} b_j c_{k-j} = 0.$$

### Recursive formula for the coefficients at a regular singular point

$$P(k+\gamma)c_k + \sum_{j=0}^{k-1} ((j+\gamma)a_{k-j} + b_{k-j})c_j = 0, \qquad k \in \mathbb{N},$$

with the understanding that for $k = 0$ the above sum is empty, and the indicial polynomial $P$ is given by

$$P(X) = X(X-1) + a_0 X + b_0.$$

## 4. Existence Theory

### Picard iteration

$$\phi_{m+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_m(t)) \, dt.$$

## 5. The Exponential of a Matrix

## 7. Sturm-Liouville Theory

### Sturm-Liouville form

$$Ly = \frac{d}{dx} \left( p(x) \frac{dy}{dx} \right) + q(x)y = g(x)$$

**Inner product and norm**

$$\langle u, v\rangle = \int_a^b u(x)\overline{v(x)}\,dx \qquad\qquad \|u\|_2 = \sqrt{\langle u, u\rangle}$$

## -. Fourier series

**Fourier series $f$ on $[-L, L]$: Exponential form:**

$$\begin{aligned} f(x) &\sim \frac{c_0}{2} + \sum_{n=-\infty}^{\infty} c_n e^{\frac{inx\pi}{L}}, \\ c_n &= \hat{f}(n) = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{n\pi x}{L}}\,dx, & n \in \mathbb{Z}. \end{aligned}$$

**Fourier series $f$ on $[-L, L]$: Trigonometric form**

$$\begin{aligned} f(x) &\sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \\ a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx, & n \geq 0 \\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx, & n \geq 1 \end{aligned}$$

**Solving Ordinary Differential Equations**

If $P(D)u = f$, then $P(in)\hat{u}(n) = \hat{f}(n)$ and

$$u(x) = \sum_{n=-\infty}^{\infty} \frac{\hat{f}(n)}{P(in)} e^{inx}.$$