

STÆRÐFRÆÐIGREINING I/IA & II/IIA - FORMÚLUBLAÐ

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Stærðfræðigreining I/IA:

I.1. Tölur og föll

$$\begin{array}{lll} \sin^2(x) + \cos^2(x) = 1 & \sin(-x) = -\sin(x) & \cos(-x) = \cos(x) \\ \sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) & \sin(\pi - x) = \sin(x) & \cos(\pi - x) = -\cos(x) \\ \cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) & \sin(\pi/2 - x) = \cos(x) & \cos(\pi/2 - x) = \sin(x) \\ \tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)} & \sin^2(x) = \frac{1 - \cos(2x)}{2} & \cos^2(x) = \frac{1 + \cos(2x)}{2} \\ & \sin(2x) = 2\sin(x)\cos(x) & \cos(2x) = 2\cos^2(x) - 1 \end{array}$$

I.3. Afleiður

Taylormargliða:

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Skekkjumat: $E_n(x) = \frac{f^{(n+1)}(X)}{(n+1)!}(x-a)^{n+1}.$

I.4. Torræð föll

Andhverfur hornafalla

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \arccos(x) = \frac{-1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

Breiðbogaföll

$$\begin{array}{lll} \sinh(x) = \frac{e^x - e^{-x}}{2} & \cosh(x) = \frac{e^x + e^{-x}}{2} & \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \\ \operatorname{arsinh}(x) = \ln(x + \sqrt{x^2 + 1}) & \operatorname{arcosh}(x) = \ln(x + \sqrt{x^2 - 1}) & \operatorname{artanh}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \end{array}$$

I.6. Heildun

Innsetning: $\int f(g(x))g'(x)dx = \int f(u)du$ Hluth.: $\int u'(x)v(x)dx = u(x)v(x) - \int u(x)v'(x)dx$

I.7. Rúmmál, massi og massamiðja

Kúla: $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$.

Sívalningur: $V = \pi r^2 h$, $S = 2\pi r h$.

Keila með grunnflöt A : $V = \frac{1}{3}Ah$.

Snúðar

Snúðið um x -ás:

$$V = \pi \int_a^b f(x)^2 dx, \quad S = 2\pi \int_a^b |f(x)|\sqrt{1+(f'(x))^2} dx.$$

Snúðið um y -ás:

$$V = 2\pi \int_a^b x f(x) dx, \quad S = 2\pi \int_a^b |x|\sqrt{1+(f'(x))^2} dx.$$

Lengd grafs: $s = \int_a^b \sqrt{1+(f'(x))^2} dx.$

Massamiðja plötu

$$\bar{x} = \frac{M_{x=0}}{m} = \frac{\int_a^b x \delta(x) f(x) dx}{\int_a^b \delta(x) f(x) dx} \quad \bar{y} = \frac{M_{y=0}}{m} = \frac{\frac{1}{2} \int_a^b \delta(x) f(x)^2 dx}{\int_a^b \delta(x) f(x) dx}.$$

I.8. Diffurjöfnur

Línulegar fyrsta stigs diffurjöfnur, $y' + p(x)y = q(x)$

$$y(x) = e^{-\mu(x)} \int e^{\mu(x)} q(x) dx, \quad \mu(x) = \int p(x) dx.$$

Línuleg annars stigs diffurjafna með fastastuðla, $ay'' + by' + cy = 0$

Tilvik I: $y(x) = Ae^{r_1 x} + Be^{r_2 x}$ ef kennijafnan hefur tvær ólíkar rauntölulausnir r_1 og r_2 .

Tilvik II: $y(x) = Ae^{kx} + Bxe^{kx}$ ef kennijafnan hefur eina tvöfalda rauntölulausn $k = -\frac{b}{2a}$.

Tilvik III: $y(x) = Ae^{kx} \cos(\omega x) + Be^{kx} \sin(\omega x)$ ef kennijafnan hefur engar rauntölulausnir, bara tvinntölulausnir $r_1 = k + i\omega$ og $r_2 = k - i\omega$ þar sem $k = -\frac{b}{2a}$ og $\omega = \frac{\sqrt{4ac-b^2}}{2a}$.

I.9. Runur og raðir

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

I.10. Veldaraðir

$$\text{Kvótapróf: } \frac{1}{R} = L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \quad \text{Rótarpróf: } \frac{1}{R} = L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots \quad \text{fyrir öll } x$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad \text{fyrir öll } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad \text{fyrir öll } x$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \quad \text{fyrir } -1 < x < 1$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \dots \quad \text{fyrir } -1 < x < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \text{fyrir } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad \text{fyrir } -1 \leq x \leq 1$$

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \quad \text{fyrir öll } x$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \quad \text{fyrir öll } x$$