STÆRÐFRÆÐIGREINING I/IA & II/IIA - FORMÚLUBLAÐ

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Stærðfræðigreining I/IA:

I.1. Tölur og föll

$\sin^2(x) + \cos^2(x) = 1$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$

$\sin(-x) = -\sin(x)$
$\sin(\pi - x) = \sin(x)$
$\sin(\pi/2 - x) = \cos(x)$
$\sin^2(x) = \frac{1-\cos(2x)}{2}$
$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(-x) = \cos(x)$
$\cos(\pi - x) = -\cos(x)$
$\cos(\pi/2 - x) = \sin(x)$
$\cos^2(x) = \frac{1+\cos(2x)}{2}$
$\cos(2x) = 2\cos^2(x) - 1$

I.3. Afleiður

Taylormargliða:

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

Skekkjumat: $E_n(x) = \frac{f^{(n+1)}(X)}{(n+1)!}(x-a)^{n+1}$.

I.4. Torræð föll

Andhverfur hornafalla

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

Breiðbogaföll

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\mathrm{arsinh(x)} = \ln\left(x + \sqrt{x^2+1}\right) \qquad \mathrm{arcosh(x)} = \ln\left(x + \sqrt{x^2-1}\right) \qquad \mathrm{artanh(x)} = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

I.6. Heildun

Innsetning: $\int f(g(x))g'(x)dx = \int f(u)du$ Hluth.: $\int u'(x)v(x)dx = u(x)v(x) - \int u(x)v'(x)dx$

I.7. Rúmmál, massi og massamiðja

Kúla: $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$.
Sívalningur: $V = \pi r^2 h$, $S = 2\pi r h$.
Keila með grunnflöt A: $V = \frac{1}{3}Ah$.

Snúðar

Snúið um x-ás:

$$V = \pi \int_a^b f(x)^2\, dx, \qquad S = 2\pi \int_a^b |f(x)|\sqrt{1 + (f'(x))^2}\, dx.$$

Snúið um y-ás:

$$V = 2\pi \int_a^b x f(x)\, dx, \qquad S = 2\pi \int_a^n |x|\sqrt{1 + (f'(x))^2}\, dx.$$

Lengd grafs: $s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$.

Massamiðja plötu

$$\overline{x} = \frac{M_{x=0}}{m} = \frac{\int_a^b x\delta(x)f(x)\,dx}{\int_a^b \delta(x)f(x)\,dx} \qquad \overline{y} = \frac{M_{y=0}}{m} = \frac{\frac{1}{2}\int_a^b \delta(x)f(x)^2\,dx}{\int_a^b \delta(x)f(x)\,dx}.$$

I.8. Diffurjöfnur

Línulegar fyrsta stigs diffurjöfnur, $y' + p(x)y = q(x)$

$$y(x) = e^{-\mu(x)}\int e^{\mu(x)}q(x)\,dx, \qquad \mu(x) = \int p(x)\,dx.$$

Línuleg annars stigs diffurjafna með fastastuðla, $ay'' + by' + cy = 0$

Tilvik I: $y(x) = Ae^{r_1x} + Be^{r_2x}$ ef kennijafnan hefur tvær ólíkar rauntölulausnir r_1 og r_2.
Tilvik II: $y(x) = Ae^{kx} + Bxe^{kx}$ ef kennijafnan hefur eina tvöfalda rauntölulausn $k = -\frac{b}{2a}$.
Tilvik III: $y(x) = Ae^{kx}\cos(\omega x) + Be^{kx}\sin(\omega x)$ ef kennijafnan hefur engar rauntölulausnir, bara tvinntölulausnir $r_1 = k + i\omega$ og $r_2 = k - i\omega$ þar sem $k = -\frac{b}{2a}$ og $\omega = \frac{\sqrt{4ac-b^2}}{2a}$.

I.9. Runur og raðir

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = 1$$

I.10. Veldaraðir

$$\text{Kvótapróf:} \quad \frac{1}{R} = L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \qquad \text{Rótarpróf:} \quad \frac{1}{R} = L = \lim_{n\to\infty}\sqrt[n]{|a_n|}$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots \qquad \text{fyrir öll } x$$

$$\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \text{fyrir öll } x$$

$$\cos x = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad \text{fyrir öll } x$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty}x^n = 1 + x + x^2 + x^3 + \cdots \qquad \text{fyrir } -1 < x < 1$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty}nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots \qquad \text{fyrir } -1 < x < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad \text{fyrir } -1 < x \leq 1$$

$$\tan^{-1}x = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad \text{fyrir } -1 \leq x \leq 1$$

$$\sinh x = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots \qquad \text{fyrir öll } x$$

$$\cosh x = \sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots \qquad \text{fyrir öll } x$$