TVINNFALLAGREINING I- FORMÚLUBLAÐ

Benedikt Steinar Magnússon <bsm@hi.is>, https://github.com/benediktmag/formulublad

1. Tvinntölur og tvinngild föll

$\cos(\theta) = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin(\theta) = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$(\cos(\theta) + \sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$

2. Fáguð föll

$$\overline{\frac{\partial f}{\partial z}} = \frac{\partial \overline{f}}{\partial \overline{z}} \qquad \overline{\frac{\partial f}{\partial \overline{z}}} = \frac{\partial \overline{f}}{\partial z}$$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \qquad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

Höfuðgrein hornsins: $\operatorname{Arg} z = 2\arctan\left(\dfrac{y}{|z| + x}\right)$, $\operatorname{Arg} : \mathbb{C} \setminus \mathbb{R}_- \to]-\pi, \pi[$.

Höfuðgrein lografallsins: $\operatorname{Log} z = \ln|z| + i\operatorname{Arg}(z)$, $\operatorname{Log} : \mathbb{C} \setminus \mathbb{R}_- \to \mathbb{C}$.

3. Cauchy-setning og Cauchy-formúla

$$\text{Vegheildi:} \quad \int_C f dz = \int_\gamma f dz = \int_\gamma f dx + i f dy = \int_a^b f(\gamma(t))\gamma'(t) dt$$

$$\int_C f d\bar{z} = \int_\gamma f d\bar{z} = \int_\gamma f dx - i f dy = \int_a^b f(\gamma(t))\overline{\gamma'(t)} dt$$

$$\left| \int_C f(z)\ dz \right| \leq \int_\gamma |f(z)|\,|dz| \leq \max_{z\in C} |f(z)| \int_\gamma |dz| = \max_{z\in C} |f(z)|\, L(C)$$

$$f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n, \qquad c_n = \frac{1}{2\pi i} \int_{\partial D(a,r)} \frac{f(\zeta)}{(\zeta - z)^{n+1}} \partial\zeta$$

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_{\partial\Omega} \frac{f(\zeta)}{(\zeta - z)^{n+1}} \partial\zeta$$

$$f(z) = \frac{1}{2\pi i} \int_{\partial\Omega} \frac{f(\zeta)}{\zeta - z}\, d\zeta - \frac{1}{\pi} \iint_\Omega \frac{\partial_{\bar{\zeta}} f(\zeta)}{\zeta - z}\, d\xi\, d\eta.$$

$$\int_{\partial\Omega} \frac{f(z)}{Q(z)} dz = 2\pi i \sum_{\alpha_j \in \Omega} \frac{f(\alpha_j)}{Q'(\alpha_j)}.$$

$$\int_{-\infty}^{+\infty} \frac{f(x)}{Q(x)}\, dx = 2\pi i \sum_{\alpha_j \in H_+} \frac{f(\alpha_j)}{Q'(\alpha_j)}$$

4. Leifareikningur

$$\int_{\partial\Omega} f(z)\,dz = 2\pi i \sum_{\alpha\in\omega\cap A} \operatorname{Res}(f,\alpha)$$

Leif í m-ta stigs skauti:

$$\operatorname{Res}(f,\alpha) = \frac{g^{(m-1)}(\alpha)}{(m-1)!}$$

Stofnbrotaliðun:

$$\frac{P(z)}{Q(z)} = \sum_{j=1}^{k} \frac{A_{j,0}}{(z-\alpha_j)^{m_j}} + \cdots + \frac{A_{j,m_j-1}}{(z-\alpha_j)}, \quad A_{j,\ell} = \frac{f_j^{(\ell)}(\alpha_j)}{\ell!}, \quad f_j(z) = \frac{(z-\alpha_j)^{m_j}P(z)}{Q(z)}$$

5. Almenn Cauchy-setning

$$I(\xi,z)f(z) = \frac{1}{2\pi i}\int_\xi \frac{f(\zeta)}{\zeta - z}\partial\zeta \qquad I(\xi,z)f^{(n)}(z) = \frac{n!}{2\pi i}\int_\xi \frac{f(\zeta)}{(\zeta-z)^{n+1}}\partial\zeta$$

$$\int_\xi f(\zeta)\,d\zeta = 2\pi i \sum_{\alpha\in I(\xi)\cap A} \operatorname{Res}(f,\alpha) I(\xi,\alpha)$$

$$\sum_{\alpha\in I(\xi)} \omega(f,\alpha)I(\xi,\alpha) = \frac{1}{2\pi i}\int_\xi \frac{f'(z)}{f(z)}dz = \frac{1}{2\pi i}\int_{f_*\xi}\frac{dw}{w} = I(f_*\xi,0)$$