1 Problem statement

1.1 Version 1

Input:

- Set $\mathcal{R} = (R_1, R_2, ..., R_m)$ representing available resources, where each resource $R_k \in \mathcal{R}$ has capacity c_k.
- Value g, representing GPU capacity.
- Set $\mathcal{T} = (T_1, T_2, ..., T_n)$ representing tasks to be scheduled. Each task $T_i \in \mathcal{T}$ is characterized by processing time p_i, resource assignment function $a_r : \mathcal{T} \to \mathcal{R}$ and GPU offloading function $a_g : \mathcal{T} \to \{0, 1\}$.
- Value h, representing main frame length.

Output: Let $\mathcal{W} = (W_1, W_2, ..., W_\ell)$ be a set of isolation windows, where each window $W_j \in \mathcal{W}$ has length l_j. The goal is to find a window assignment function $a_w : \mathcal{T} \to \mathcal{W}$ assigning each task $T_i \in \mathcal{T}$ to an isolation window $W_j \in \mathcal{W}$ such that:

$$l_j \geq \frac{\max_{T_i \in \mathcal{T}: a_w(T_i) = W_j}(p_i)}{0.6}, \forall W_j \in \mathcal{W} \tag{1}$$

$$\sum_{W_j \in \mathcal{W}} l_j \leq h \tag{2}$$

$$\sum_{T_i \in \mathcal{T}: a_w(T_i) = W_j} \mathbb{1}_{[a_r(T_i) = R_k]} \leq c_k, \forall R_k \in \mathcal{R}, \forall W_j \in \mathcal{W} \tag{3}$$

$$\sum_{T_i \in \mathcal{T}: a_w(T_i) = W_j} a_g(T_i) \leq g, \forall W_j \in \mathcal{W} \tag{4}$$

ILP Model:

$$min \sum_{W_j \in \mathcal{W}} l_j \quad \text{subject to:} \tag{5}$$

$$\sum_{W_j \in \mathcal{W}} l_j \leq h \tag{6}$$

$$l_j \geq \frac{p_i \cdot w_{i,j}}{0.6} \quad \forall T_i \in \mathcal{T}, \forall W_j \in \mathcal{W} \tag{7}$$

$$l_j \leq l_{j-1} \quad \forall W_j \in \mathcal{W} \tag{8}$$

$$\sum_{W_j \in \mathcal{W}} w_{i,j} = 1 \quad \forall T_i \in \mathcal{T} \tag{9}$$

$$\sum_{T_i \in \mathcal{T}} w_{i,j} \cdot \mathbb{1}_{[a_r(T_i)=R_k]} \leq c_k \quad \forall W_j \in \mathcal{W}, \forall R_k \in \mathcal{R} \tag{10}$$

$$\sum_{T_i \in \mathcal{T}} w_{i,j} \cdot a_g(T_i) \leq g \quad \forall W_j \in \mathcal{W} \tag{11}$$

$$w_{i,j} \in \{0,1\} \quad \forall T_i \in \mathcal{T}, \forall W_j \in \mathcal{W} \tag{12}$$