CENG 384 - Signals and Systems for Computer Engineers
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Homework 2

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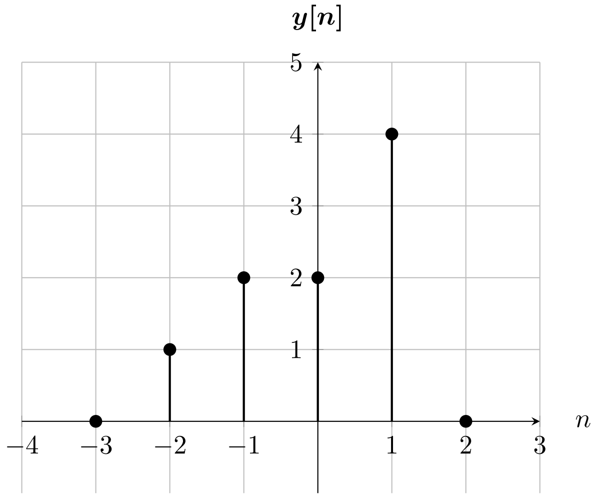
1. (a) $y'(t) + 5y(t) = x(t)$

   (b) homogenous solution for $x(t) = 0$ has the following from:
   $y_h(t) = Ce^{st}$
   $Cse^{st} + 5Ce^{st} = 0$, $s = -5$
   $y_h(t) = Ce^{-5t}$
   since this equation is linear,
   $y_p(t) = (Ae^{-t} + Be^{-3t})u(t)$
   $y_p'(t) = (-Ae^{-t} - 3Be^{-3t})u(t)$
   substituting this into equation, we get: $-Ae^{-t} - 3Be^{-3t} + 5Ae^{-t} + 5Be^{-3t} = e^{-t} + e^{-3t}$
   $A = \frac{1}{4}$, $B = \frac{1}{2}$
   general solution is the superposition of particular and homogenous solutions;
   $y(t) = Ce^{-5t} + \frac{1}{4}e^{-t} + \frac{1}{2}e^{-3t}$
   At t=0, the system is initially at rest, which means $y(0) = 0$
   $y(0) = C + \frac{1}{4} + \frac{1}{2} = 0$, $C = \frac{-3}{4}$
   overall solution is
   $y(t) = [\frac{-3}{4}e^{-5t} + \frac{1}{4}e^{-t} + \frac{1}{2}e^{-3t}]u(t)$

2. (a) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ where
   $y[-3] = 0$, $y[-2] = 1$, $y[-1] = 2$, $y[0] = 2$, $y[1] = 4$, $y[2] = 0$,



   (b) $\frac{d(x(t))}{dt} = \delta(t-1) + \delta(t+1)$
   $y(t) = \int \frac{d(x(t))}{dt} h(t-\tau)d\tau$
   $y(t) = \int [\delta(\tau - 1) + \delta(\tau + 1)]e^{-(t-\tau)}sin(t-\tau)u(\tau)dt$
   $y(t) = e^{-t}sin(t-1)u(t-1) + e^{-t}sin(t+1)u(t+1)$

3. (a) To find $y(t) = x(t) * h(t)$, we need to find:
   $y(t) = \int x(\tau)h(t-\tau)d\tau$
   $y(t) = \int e^{-\tau}u(\tau)e^{(-2(t-\tau))}u(t-\tau)d\tau$
   $y(t) = \int e^{(-2t+\tau)}u(\tau)u(t-\tau)d\tau$
   This integral nonzero only for $\tau \geq 0$ and $t - \tau \geq 0$
   $y(t) = \int_0^t e^{(-2t+\tau)}d\tau$
   $y(t) = \int_0^t e^{-2t}e^{\tau}d\tau$

$y(t) = e^{-2t}\int_0^t e^{\tau} d\tau$
$y(t) = e^{-2t}(e^t - 1)$

(b) To find $y(t) = x(t) * h(t)$, we need to find:
$y(t) = \int x(\tau)h(t-\tau)d\tau$
$x(\tau)$ is nonzero only for $0 \geq \tau \geq 1$, the integral becomes:
$y(t) = \int e^{3(t-\tau)} d\tau$
$y(t) = e^{3t}\int e^{-3\tau} d\tau$, since $\int_0^1 e^{-3\tau} d\tau = \frac{e^3-1}{3e^3} = \frac{1}{3} - \frac{1}{3}e^{-3}$, we can substitute this into equation for $y(t)$
$y(t) = e^{3t}[\frac{1}{3} - \frac{1}{3}e^{-3}]$ for $t \geq 0$.

4. (a) Leave y[n] alone in the left hand side of the equation;
$y[n] = y[n-1] + y[n-2]$, using initial conditions and input, evaluate the values of $y[n]$ for all n, as follows;
$y[2] = y[1] + y[0] = 1 + 1 = 2$
$y[3] = y[2] + y[1] = 2 + 1 = 3$
$y[4] = y[3] + y[2] = 3 + 2 = 5$
$y[5] = y[4] + y[3] = 5 + 3 = 8$
$y[6] = y[5] + y[4] = 8 + 5 = 13$
... Series of Fibonacci numbers, limit of the sum of the first n Fibonacci numbers does not exist as n goes to infinity. The series diverge.

(b) $y_h(t) = Ce^{st}$
$y'_h(t) = Cse^{st}$
$y''_h(t) = Cs^2e^{st}$
$y'''_h(t) = Cs^3e^{st}$
$Cs^3e^{st} - 6Cs^2e^{st} + 13Cse^{st} - 10Ce^{st} = 0$
$s^3 - 6s^2 + 13s - 10 = (s-2)(s^2 - 4s + 5) = 0$
$s_1 = 2$, $s_2 = 2 + 1$, $s_3 = 2 - i$
We get, $y(t) = C_1e^{(2t)} + C_2e^{(2+i)t} + C_3e^{(2-i)t}$
$y(0) = C_1 + C_2 + C_3 = 1$
$y'(0) = 2C_1 + (2+i)C_2 + (2-i)C_3 = \frac{3}{2}$
$y''(0) = 4C_1 + (3+4i)C_2 + (3-4i)C_3 = 3$
$C_1 = 2$, $C_2 = \frac{-2+i}{4}$, $C_3 = \frac{-2-i}{4}$
$y(t)$ becomes:
$y(t) = 2e^{(2t)} + \frac{-2+i}{4}e^{(2+i)t} + \frac{-2-i}{4}e^{(2-i)t}$

5. (a) $y_p(t) = Acos(5t) + Bsin(5t)$
$y'_p(t) = -5Asin(5t) + 5Bcos(5t)$
$y''_p(t) = -25cos(5t) - 25sin(5t)$
substituting this into equation, we get:
$-25cos(5t) - 25sin(5t) - 25Asin(5t) + 25Bcos(5t) + 6Acos(5t) + 6Bsin(5t) = cos(5t)$
$19A + 25B = 1$, $-25A - 19B = 0$
$A = \frac{-19}{986}$, $B = \frac{25}{986}$
$y_p(t) = \frac{-19}{986}cos(5t) + \frac{25}{986}sin(5t)$

(b) $y_h(t) = Ce^{st}$
substituting this into equation, we get:
$Cs^2e^{st} + 5Cse^{st} + 6Ce^{st} = 0$
$s^2 + 5s + 6 = 0$, $s = -2, s = -3$
$y_h(t) = C_1e^{-2t} + C_2e^{-3t}$

(c) $y(t) = C_1e^{-2t} + C_2e^{-3t} + \frac{-19}{986}cos(5t) + \frac{25}{986}sin(5t)$
$y(0) = C_1 + C_2 - \frac{-19}{986} = 0$
$y'(0) = -2C_1 - 3C_2 + \frac{125}{986} = 0$
$C1 = \frac{-68}{986}$, $C2 = \frac{87}{986}$

6. (a) We will take z transform of $w[n] - \frac{1}{2}w[n-1] = x[n]$,
$w[z] - \frac{1}{2}z^{-1}w[z] = x[z]$
$\frac{w[z]}{x[z]} = \frac{1}{1-\frac{1}{2}z^{-1}} = h_0(z)$ If we take inverse
$h_0[n] = (\frac{1}{2})^n u(n)$

(b) $h_z = h_0[z] * h_0[z] = \frac{1}{(1-\frac{1}{2}z^{-1})^2}$
$h_z = \frac{A}{(1-\frac{1}{2}z^{-1})^2} + \frac{B}{1-\frac{1}{2}z^{-1}}$
$A(1 - \frac{1}{2}z^{-1}) + B = 1$
$A + B = 1$, $\frac{-A}{2} = 0$
$A = 0, B = 1$

$h_z = \frac{1}{(1-\frac{1}{2}z^{-1})^2}$
take inverse transform $h[n] = (n+1)(1/2)^n u[n]$

(c) $h_z = \frac{1}{(1-\frac{1}{2}z^{-1})^2}$
$\frac{y[z]}{x[z]} = \frac{1}{1-z^{-1}+\frac{1}{4}z^{-2}}$
$y[z][1 - z^{-1} + \frac{1}{4}z^{-2}] = x[z]$
$y[z] - y[z]z^{-1} + \frac{1}{4}y[z] = x[z]$
$y[n] - y[n-1] + \frac{1}{4}y[n-2] = x[n]$

7. (a)

(b)