CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

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1. (a) $2z + \overline{z} = -5 + j$
   $2(x + jy) + (x - jy) = -5 + j$
   $3x + jy = -5 + j$
   $x = \frac{-5}{3}, y = 1$ so, $z = \frac{-5}{3} + j$
   $|z|^2 = z\overline{z}$
   $|z|^2 = (\frac{-5}{3} + j)(\frac{-5}{3} - j) = \frac{34}{9}$

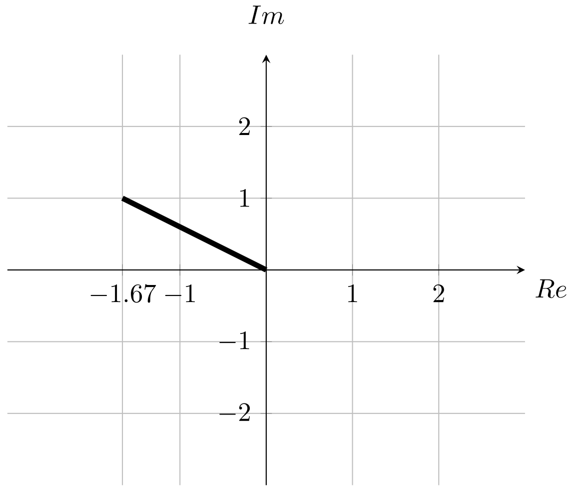


Figure 1: z

   (b) $r^5 e^{5j\theta} = 32j$
   $r^5 e^{5j\theta} = 2^5 e^{j\frac{\pi}{2}}$ $\quad r = 2,\ 5\theta = \frac{\pi}{2} + 2\pi k$
   $k = -2 \hookrightarrow \theta = \frac{-7\pi}{10}$ $\quad z_1 = 2e^{-j\frac{7\pi}{10}}$
   $k = -1 \hookrightarrow \theta = \frac{-3\pi}{10}$ $\quad z_2 = 2e^{-j\frac{3\pi}{10}}$
   $k = 0 \hookrightarrow \theta = \frac{\pi}{10}$ $\quad z_3 = 2e^{j\frac{\pi}{10}}$
   $k = 1 \hookrightarrow \theta = \frac{\pi}{2}$ $\quad z_4 = 2e^{j\frac{\pi}{2}}$
   $k = 2 \hookrightarrow \theta = \frac{9\pi}{10}$ $\quad z_5 = 2e^{j\frac{9\pi}{10}}$

   (c) $z_1 = (1 + j) = \sqrt{2}e^{\frac{j\pi}{4}}$, $\quad z_2 = (\frac{1}{2} + \frac{\sqrt{3}}{2}j) = e^{\frac{j\pi}{3}}$, $\quad z_3 = (j - 1) = \sqrt{2}e^{\frac{-j\pi}{4}}$
   $z = \frac{z_1 z_2}{z_3} = e^{\frac{j5\pi}{6}} \quad \longrightarrow |z| = 1, \quad \theta = \frac{5\pi}{6}\ or \frac{-\pi}{6}$

   (d) $e^{\frac{-j\pi}{2}} = -j$
   $z = j(-j) = 1 = e^{j2\pi}$

2.

3. (a)
   (b) $3\delta[n+7] - 4\delta[n+4] + 2\delta[n+2] - \delta[n+1] - \delta[n-1] + 4\delta[n-4]$

4. (a) $x(t + T) = 5sin(3t - (\frac{\pi}{4} + 3T)) = 2\pi m$
   $T = \frac{\frac{\pi}{4} - 2\pi m}{3}$ $\quad$ for m=0, $\quad T = \frac{\pi}{12}$
   The signal is periodic with the fundamental period $T_0 = \frac{\pi}{12}$

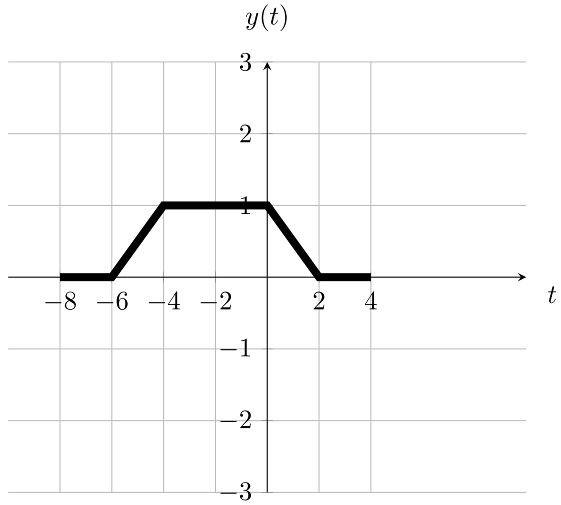


Figure 2: t $vs.$ $y(t)$

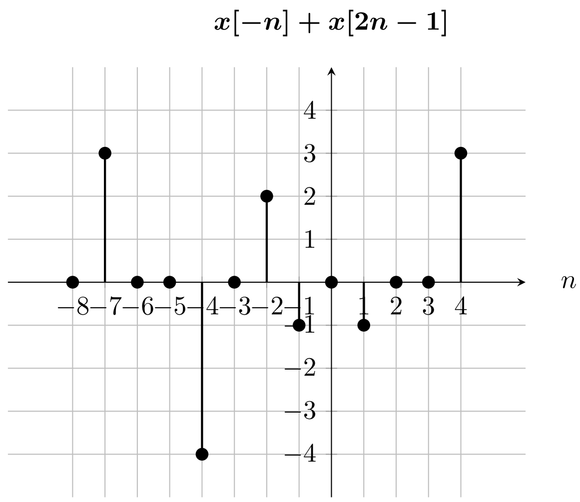


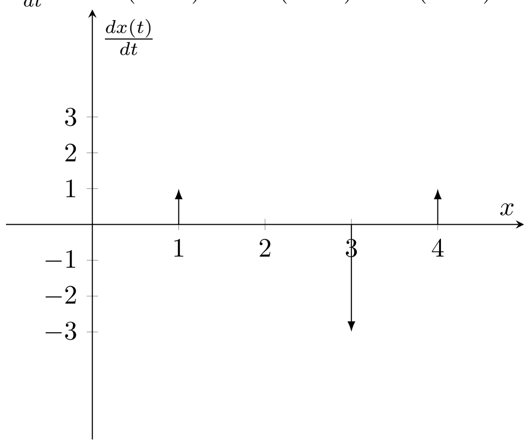
Figure 3: n vs. $x[-2n] + x[n-2]$

(b) for x_1 $\quad w_0 = \frac{13\pi}{10},\ N_0 = \frac{2\pi}{w_0}m = \frac{20m}{13},\quad m = 13 \longrightarrow N_0 = 20$
for x_2 $\quad w_0 = \frac{7\pi}{10},\ N_0 = \frac{2\pi}{w_0}m = \frac{20m}{7},\quad m = 7 \longrightarrow N_0 = 20$
LCM is 20, so the signal is periodic with the fundamental period $N_0 = 20$

(c) $7N_0 = 2\pi m,$ the smallest integer number m satisfy this equation is m=1.
for m=1 $\longrightarrow N_0 = \frac{2\pi}{7}$ Since N_0 is not an integer number, this signal is not periodic.

5. (a) $x(t) \;=\; u(t-1) \;-\; 3u(t-3) \;+\; u(t-4)$

(b) $\frac{dx(t)}{dt} \;=\; \delta(t-1) \;-\; 3\delta(t-3) \;+\; \delta(t-4)$



6. (a) $y(t) = tx(2t+3)$

i. **Memory** Has memory, $y(3) = 3x(9)$

ii. **Stability** Not stable, t is unbounded

iii. **Causality** Not causal, $y(3) = 3x(9)$ output depends on future input values

iv. **Linearity** Superposition property hold, therefore linear

v. **Invertibility** Not invertible, because $x(2t-3) = \frac{y(t)}{t}$ is not defined when $t = 0$

vi. **Time Invariance** Not time invariant $x_2(2t+3) = x_1(t-\tau), y_2(t) = tx_2(2t+3) = tx_1(2t-2\tau+3) \neq y_1(t-\tau)$

(b) $y[n] = \sum_{k=1}^{inf} x[n-k]$

i. **Memory** Has memory, accumulates past values of input to generate output

ii. **Stability** Not stable, take $x[n] = a^n$, then $y[n] = \sum_{k=1}^{\infty} a^{n-k} = a^n \sum_{k=1}^{\infty} a^{-k}, a^n$ is unbounded

iii. **Causality** Causal, $y[n] = x[n-1] + x[n-2] + x[n-3] + ...$,output depends on past input values

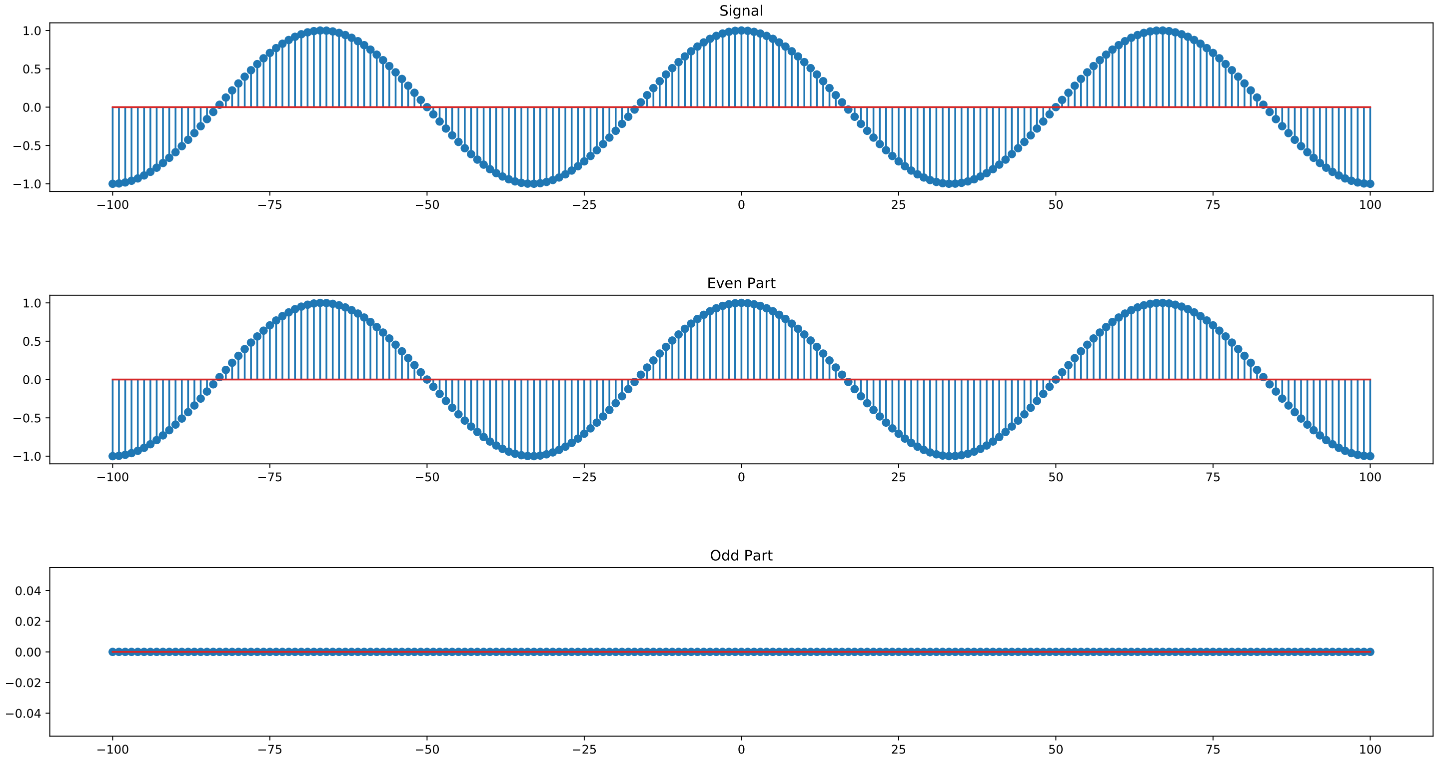
iv. **Linearity** Linear, Superposition property holds
$y[n] = \sum_{k=1}^{\infty}(a_1x_1[n-k] + a_2x_2[n-k]) = a_1\sum_{k=1}^{\infty} x_1[n-k] + a_2\sum_{k=1}^{\infty} x_2[n-k]) = a_1y_1[n] + a_2y_2[n]$

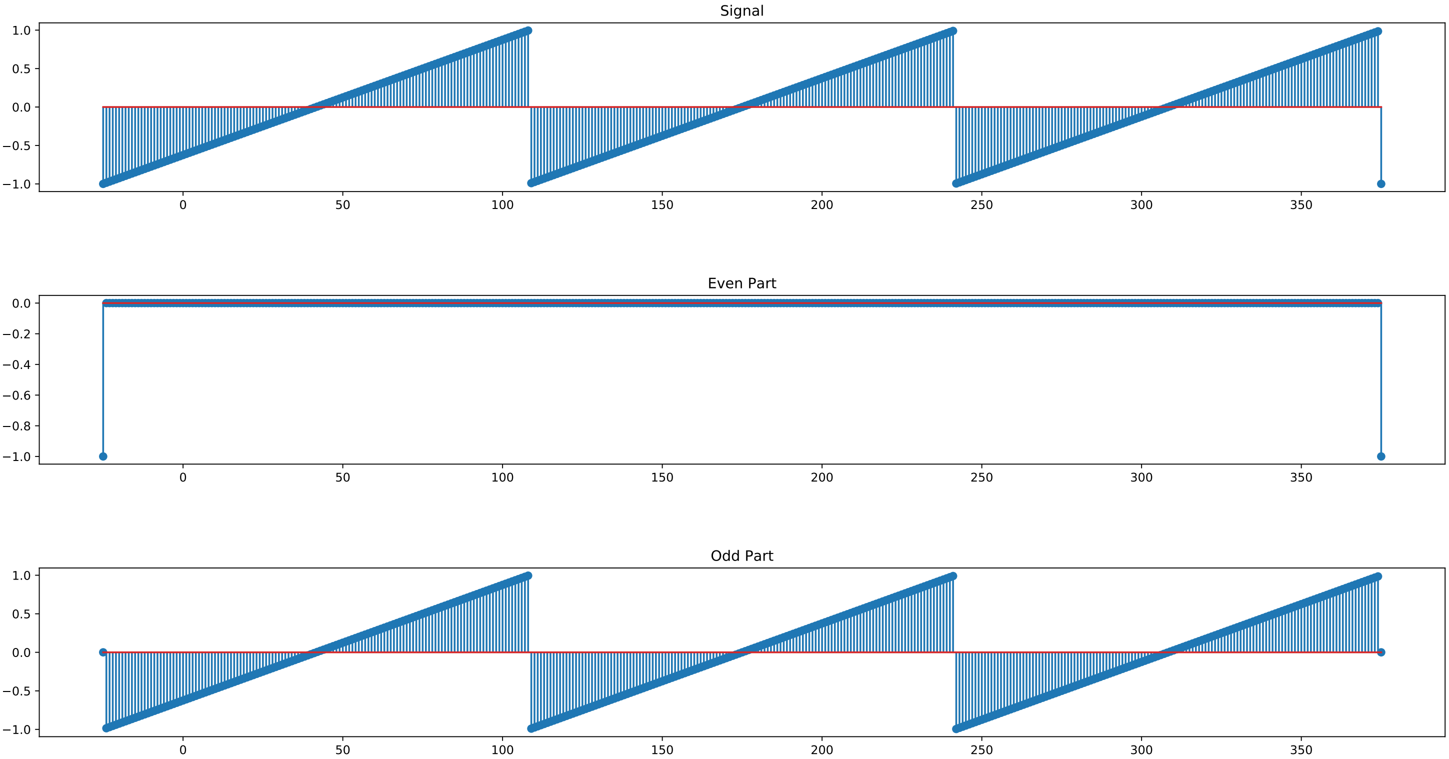
v. **Invertibility** Invertible, consider 2 different inputs generate same output.
Then we have $y[n] = \sum_{k=1}^{\infty} x[n-k] = \sum_{k=1}^{\infty} x'[n-k]$ and $x[n-k] \neq x'[n-k]$.
$\sum_{k=1}^{\infty} x[n-k] - x'[n-k] = 0$
Then, x[n-k]-x'[n-k]=0 must hold, but we assumed $x[n-k] \neq x'[n-k]$.

vi. **Time Invariance** Time invariant
$x[n-m] = x[(n-m)-1] + x[(n-m)-2] + x[(n-m)-3] + ...$
$y[n-m] = \sum_{k=1}^{\infty} x[(n-1)-k-m] = y[n-1-m]$

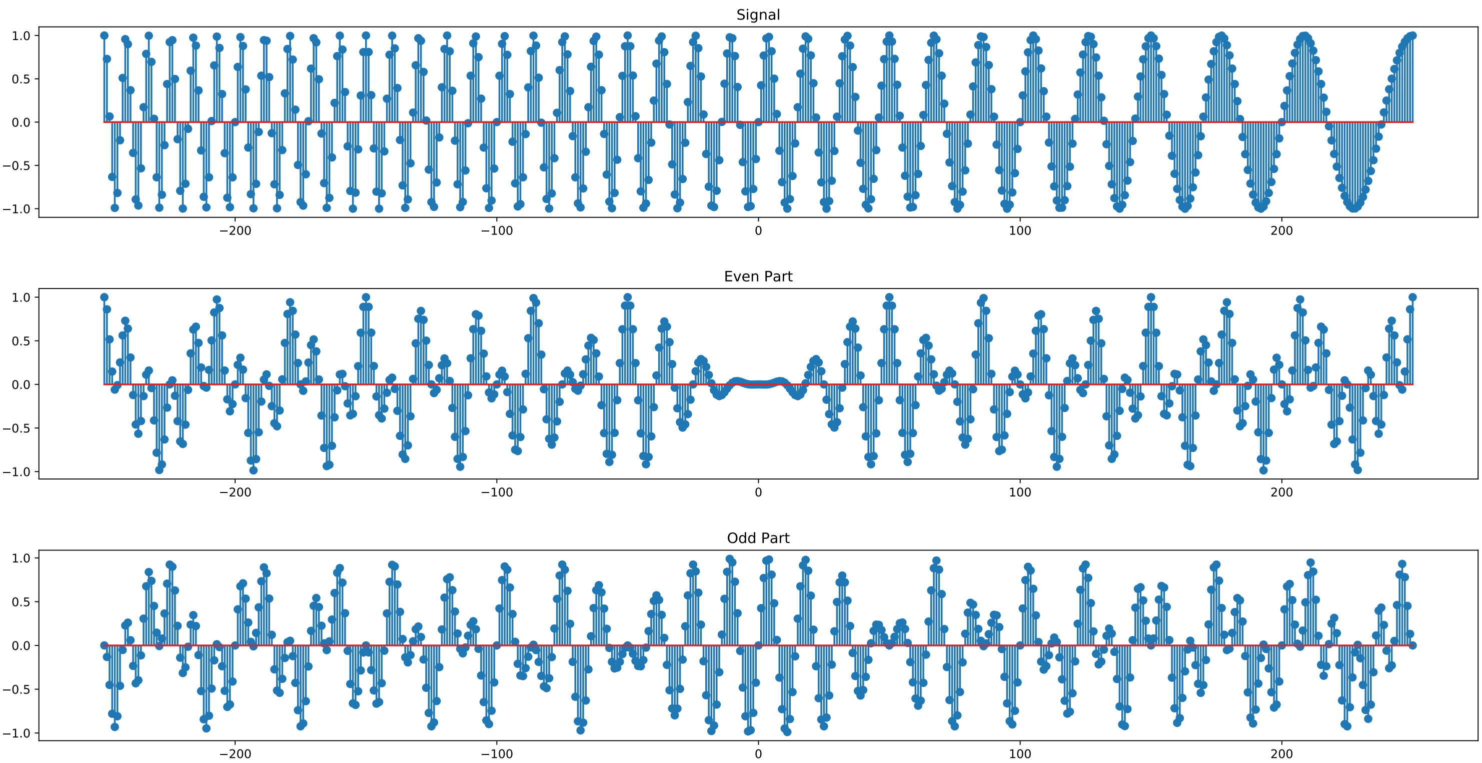
7. (a)

```
import matplotlib.pyplot as plt

def even_odd(x, s_i):
    x_odd = [0]*len(x)
    x_even = [0]*len(x)

    for n in range(s_i, len(x)+s_i):
        x_odd[n-s_i] = 0.5*(x[n-s_i] - x[-n+s_i-1])
        x_even[n-s_i] = 0.5*(x[n-s_i] + x[-n+s_i-1])

    plt.subplot(3,1,1)
    plt.stem(range(s_i, len(x)+s_i), x, use_line_collection=1)
    plt.title('Signal')

    plt.subplot(3,1,2)
    plt.stem(range(s_i, len(x)+s_i), x_even, use_line_collection=1)
    plt.title('Even Part')

    plt.subplot(3,1,3)
    plt.stem(range(s_i, len(x)+s_i), x_odd, use_line_collection=1)
    plt.title('Odd Part')

    plt.tight_layout()
    plt.show()


f = 'shifted_sawtooth_part_a.csv'
with open(f, 'r') as file:
    data = file.read().splitlines()
    s_i, x = data[0].split(',')[0], data[0].split(',')[1:]
x = [float(i) for i in x]

even_odd(x, int(s_i))
```


Signal
Even Part
Odd Part
1.0
0.5
0.0
−0.5
−1.0
0.04
0.02
0.00
−0.02
−0.04
−100
−75
−50
−25
0
25
50
75
100

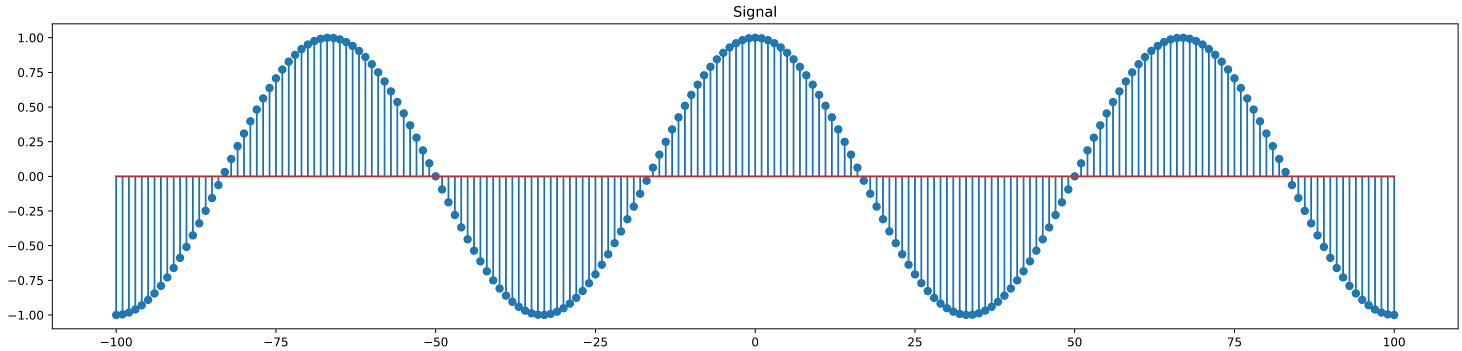


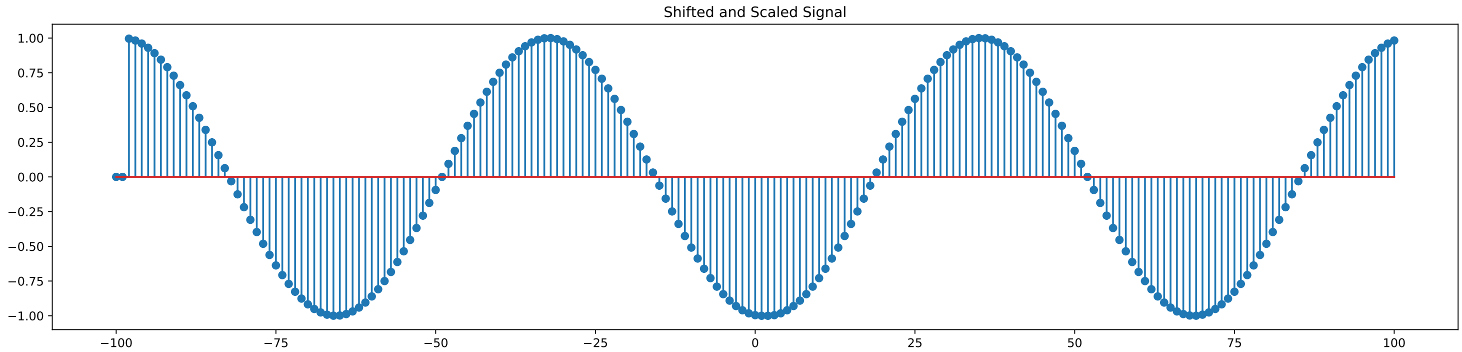
Signal
Even Part
Odd Part
1.0
0.5
0.0
−0.5
−1.0
−0.2
−0.4
−0.6
−0.8
0
50
100
150
200
250
300
350

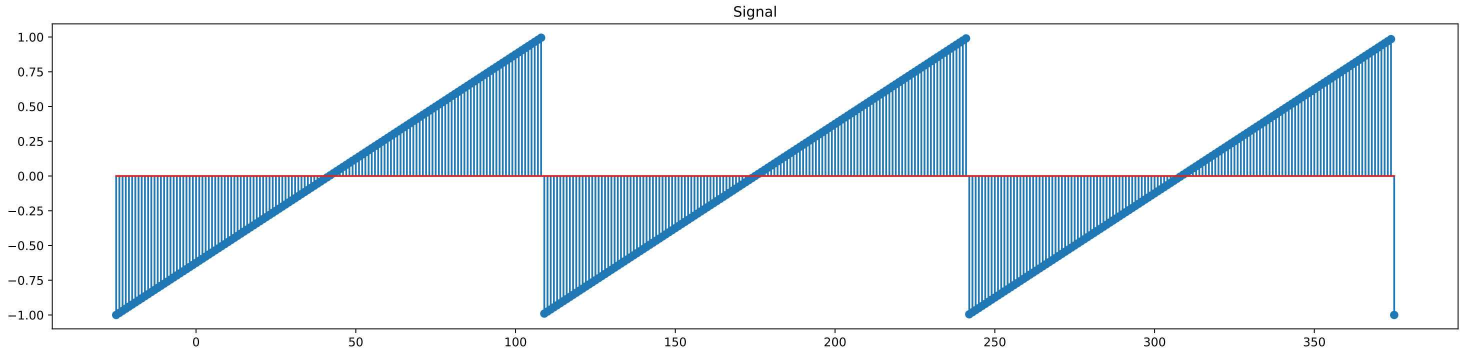

Signal
1.0
0.5
0.0
−0.5
−1.0
−200
−100
0
100
200
Even Part
Odd Part

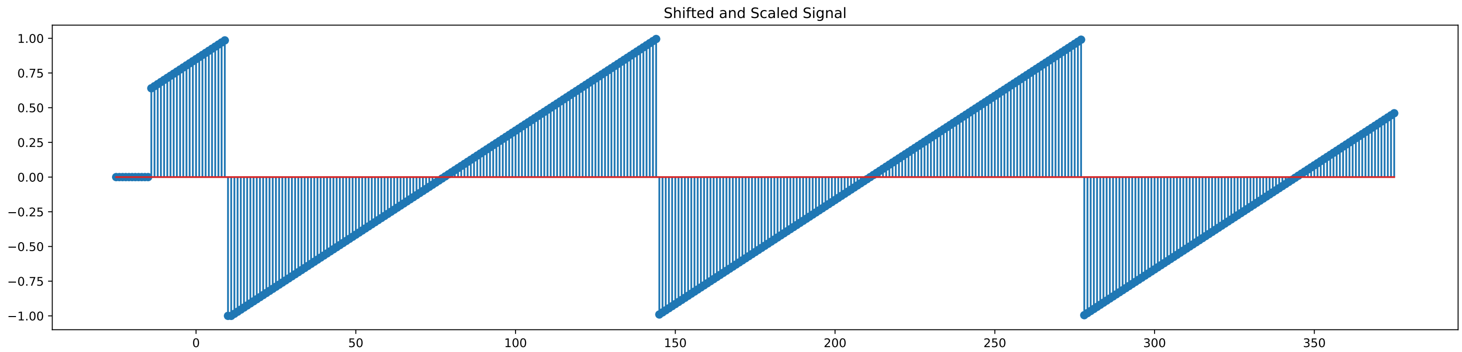
(b)

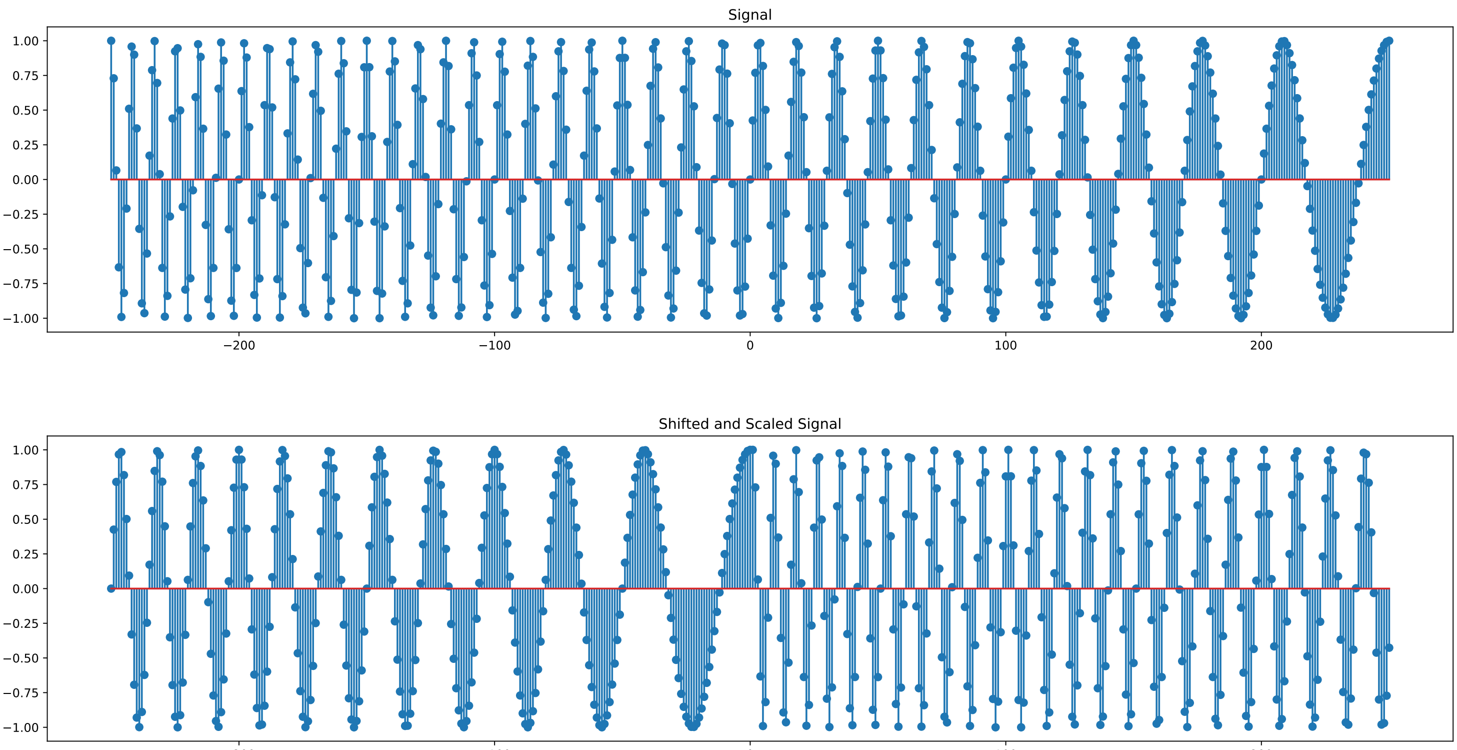
```
import matplotlib.pyplot as plt

def shifted_scaled_signal(x, s_i, a, b):
    xnew = [0]*len(x)
    for n in range(s_i, len(x)+s_i):
        if (n-a-b) >= s_i and (n-a-b) < s_i+len(x):
            xnew[n-s_i] = x[n-a-b]

    plt.subplot(2,1,1)
    plt.stem(range(s_i, len(x)+s_i), x, use_line_collection=1)
    plt.title('Signal')

    plt.subplot(2,1,2)
    plt.stem(range(s_i, len(x)+s_i), xnew, use_line_collection=1)
    plt.title('Shifted and Scaled Signal')

    plt.tight_layout()
    plt.show()


f = 'chirp_part_b.csv'
with open(f, 'r') as file:
    data = file.read().splitlines()
    s_i, a, b, x = data[0].split(',')[0], data[0].split(',')[1], data[0].split(',')[2], data[0].spli
x = [float(i) for i in x]

shifted_scaled_signal(x, int(s_i), int(a), int(b))
```


Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−100
−75
−50
−25
0
25
50
75
100


Shifted and Scaled Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−100
−75
−50
−25
0
25
50
75
100


Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
50
100
150
200
250
300
350


Shifted and Scaled Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
50
100
150
200
250
300
350


Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−200
−100
0
100
200
Shifted and Scaled Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−200
−100
0
100
200