Vacation Sheet 8

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Submit neat answers to all the questions you attempt (include all the working you wish to be marked, e.g. calculations, diagrams). Partial answers should also be handed in: these may be awarded partial credit. Try to complete the questions at an examination pace; time yourself!

1. Given that $y = 3x^7 - 1 + x^{\frac{1}{2}}$ for $x > 0$
   Find $\dfrac{dy}{dx}$

2. (a) Express $\sqrt{147}$ in the "surd form" $a\sqrt{3}$, where a is an integer.
   (b) Express $(7 - \sqrt{5})^2$ in the "surd form" $a + b\sqrt{3}$, where a and b are integers.
   (c) Express $36 - 16\sqrt{5}$ in the form $(a - b\sqrt{5})^2$, where a and b are integers.

3. Let $f(x) = \dfrac{1}{x}$
   (a) Sketch $4f(x) + 3$, marking all intersections and asymptotes.
   (b) Give the equations of all asymptotes.
   (c) Find both $\dfrac{df}{dx}$ and $\dfrac{d}{dx}(4f(x) + 3)$

4. Consider the two straight lines defined by:

$$3x + 5y = 12$$

$$2x + 10y = 7$$

   (a) Sketch them both and convince yourself that they intersect (i.e. are not parallel)
   (b) Solve the system of equations algebraically.
   (c) What is the area of the triangle bounded by the two lines and the y-axis?

5. The quadratic $3x^2 - 5x + (k + 2)$ has no real roots.

(a) Give the range of possible values for k.

(b) Sketch the graph, giving the value of the y-intercept (in terms of k).

6. Find
$$\int (2 - 7\sqrt{2})^2 dx$$

7. Given that a curve has equation $y = f(x)$, passes through the point $P = (2, 7)$, and satisfies
$$f'(x) = 3x^2 - 4$$
Find and sketch $f(x)$.