

Vacation Sheet 2

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Submit neat answers to all the questions you attempt (include all the working you wish to be marked, e.g. calculations, diagrams). Partial answers should also be handed in: these may be awarded partial credit.

1. Again we test understanding of mathematical language. Decide which of the following statements are true, and which are false. Justify your answers.

- (a) $b^2 - 4ac \geq 0$ implies that $a^2 + bx + c = 0$ has a root
- (b) a and b are odd $\Rightarrow a \times b$ is odd
- (c) $a^2 + bx + c = 0$ has a root implies that $b^2 - 4ac > 0$

2. We give calculus question in the examination style.

- (a) Let $y = (x + 3)(x - 5)$

Write down the roots of y and the derivative $\frac{dy}{dx}$ of y

Find the gradient of the tangent lines to y at the roots, and hence the equations of both tangent lines.

Do these lines meet? If so, where?

Convey the above information in a brief sketch.

- (b) y is a quadratic which passes through the point $(1, 18)$ and has derivative $\frac{dy}{dx} = 2x - 11$

Determine y using integration.

- (c) Differentiate and integrate the following functions. (*Hint: π is nothing more than a constant, like any other number*)

- i. $y = x^{100}$
- ii. $y = 0$
- iii. $y = 1 + x + x^2$
- iv. $y = 1 + x + \dots + x^{100}$
- v. $y = x^\pi$

3. This question is about finding solutions to systems of equations. Recall that a **solution** to a list of equations is a value that makes every separate equation true; graphically, a solution is a point where the graphs of every equation intersect. Thus a particular system may have none, one, two, etc. solutions. Some even have infinitely many! Try to find all solutions of the following systems algebraically. Draw graphs if you can.

(a)

$$7x + 8y = 10$$

$$2x + 3y = 5$$

(b)

$$y = x^2$$

$$2y = x + 6$$

4. Sometimes having quadratic equations or other polynomials in the denominator can be a problem (you will see this when you begin to apply calculus to such expressions). We use the method of **partial fractions** to *decompose* the more complicated expression into the sum of simple ones. Use the method to simplify the following.

(a)

$$\frac{5x + 19}{(x + 3)(x + 5)}$$

(b)

$$\frac{1}{(x + 5)(x + 7)}$$

(c)

$$\frac{x}{x^2 - 100}$$