Vacation Sheet 4

Ben Eills

December 17, 2011

Submit neat answers to all the questions you attempt (include all the working you wish to be marked, e.g. calculations, diagrams). Partial answers should also be handed in: these may be awarded partial credit.

1. We test knowledge of transformations.

   (a) Graph the following transformations for a constant a.

      i. $f(x) \longrightarrow f(x+a)$

      ii. $f(x) \longrightarrow af(x)$

      iii. $f(x) \longrightarrow f(ax)$

      iv. $f(x) \longrightarrow f(x) + a$

   (b) Pick some interesting cubic and let this be f(x). Graph f(x) and each of the above transformations when $a = 3$.

2. Consider the two points $P = (a, b)$ and $Q = (c, d)$.

   (a) Suppose that $a \neq c$. Give the equation of the unique line passing through P and Q in the form $y = mx + c$. Why is it important that $a \neq c$?

   (b) Suppose now that $a = c$. Give the equation of the line *(you will not be able to write it in the form $y = mx + c$).*

   (c) Suppose again that $a \neq c$, and that P and Q lie in the upper-right quadrant (i.e. $a, b, c, d \geq 0$). Give the area of the triangle bounded by the line passing through P and Q and the coordinate axes.

3. The Greek mathematician Euclid set forth certain postulates prefacing his work *The Elements*, the most famous of which was the fifth: *"Given a line L and a point P (not on the line), there is exactly one line passing through P that is parallel to L."*

   (a) Let L be the line $y = 3x - 7$, and P the point $(1, 1)$. Give the special line passing through P, parallel to L.

(b) Consider the (Euclidean) lines L and M, given by equations $y = ax+b$ and $y = cx + d$, respectively. Give conditions on a and b for:

   i. L and M to be parallel

   ii. L and M to be perpendicular

(c) Euclid's fifth postulate is only true of "Euclidean Geometry", that is, geometry in planes. In spherical geometry (like on the surface of the globe), "lines" are actually "greater circles" going around the whole sphere at its widest point. Thus the equator and Greenwich Meridian are "lines". Considering the equator and Paris, how many "lines" can be drawn through Paris that are parallel to the Equator?

4. We test the differential and integral calculus.

   (a) Suppose $f(x)$ passes through the origin and has derivative $f'(x) = 3x^2-1$. Determine fully $f(x)$. Using the "difference of squares" trick, factorize it as far as possible.

   (b) Suppose that the minimum value of $g(x)$ is -1, and that $g(x)$ has derivative $g'(x) = 2x - 12$. Determine fully $g(x)$, factorizing your answer.

   (c) Differentiate and integrate (the Greek letters are constants):

      i. αx^{β}

      ii. $\delta + x^{\frac{\sigma}{\phi}}$

   (d) Invent two symbols for your own language, and state the simple differentiation "power rule" in terms of them. **Make them sufficiently complex so that they will not exist in any real-world alphabet!**