Vacation Sheet 5

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December 20, 2011

Submit neat answers to all the questions you attempt (include all the working you wish to be marked, e.g. calculations, diagrams). Partial answers should also be handed in: these may be awarded partial credit.

1. We test knowledge mathematical language. Decide which of the following are true statements.
   (a) Graph the following transformations for a constant a.
      i. a is positive implies that a^2 is positive
      ii. a^2 is positive implies that a is positive
      iii. Whenever a^2 is odd, $a+1$ is even
      iv. The discriminant being positive is a sufficient condition for a quadratic to have two distinct roots.
2. Briefly sketch the sine and cosine functions on the interval $[0°, 720°]$.
   (a) Let $f(x)$ be the sine function. Sketch on a separate graph the transformation $f(x+90°)$
   (b) Comparing your transformation to the cosine function you sketched, write down a relationship between sine and cosine of the form $sin(x+a) = cos(x)$ for some constant a.
   (c) If $f(x) = f(x+b)$ for all x and some constant b, we say that f has **period** b. Intuitively, this means that the graph of $f(x)$ "repeats" itself every b. Only very special functions are **periodic** in this way. Is sine periodic? If so, find its period.
3. We test differentiation and integration technique. Differentiate and integrate the following.
   (a) $f(x) = x^{\frac{1}{1}} + x^{\frac{1}{2}} + \cdots + x^{\frac{1}{100}}$
   (b) $g(x) = x^{\frac{1}{\sqrt{2}}}$
   (c) $h(x) = \dfrac{(x+2)^2}{x^{-\frac{1}{2}}}$

4. Consider the quadratic $z(x) = x^2 + kx + k$

   (a) Write down the interval for k for which:

      i. $z(x)$ has two distinct roots

      ii. $z(x)$ has a repeated root

      iii. $z(x)$ has no real roots

   (b) From now on, suppose that $z(x)$ has two distinct roots. What is the minimum point of $z(x)$?

   (c) Does $z(x)$ have a maximum point?

   (d) We define the **second derivative** of $z(x)$ to be the derivative of the derivative and write it as $\frac{d^2z}{dx^2}$. Therefore, $\frac{d^2z}{dx^2} = \frac{d}{dx}\frac{dz}{dx}$. Find the second derivative of $z(x)$.