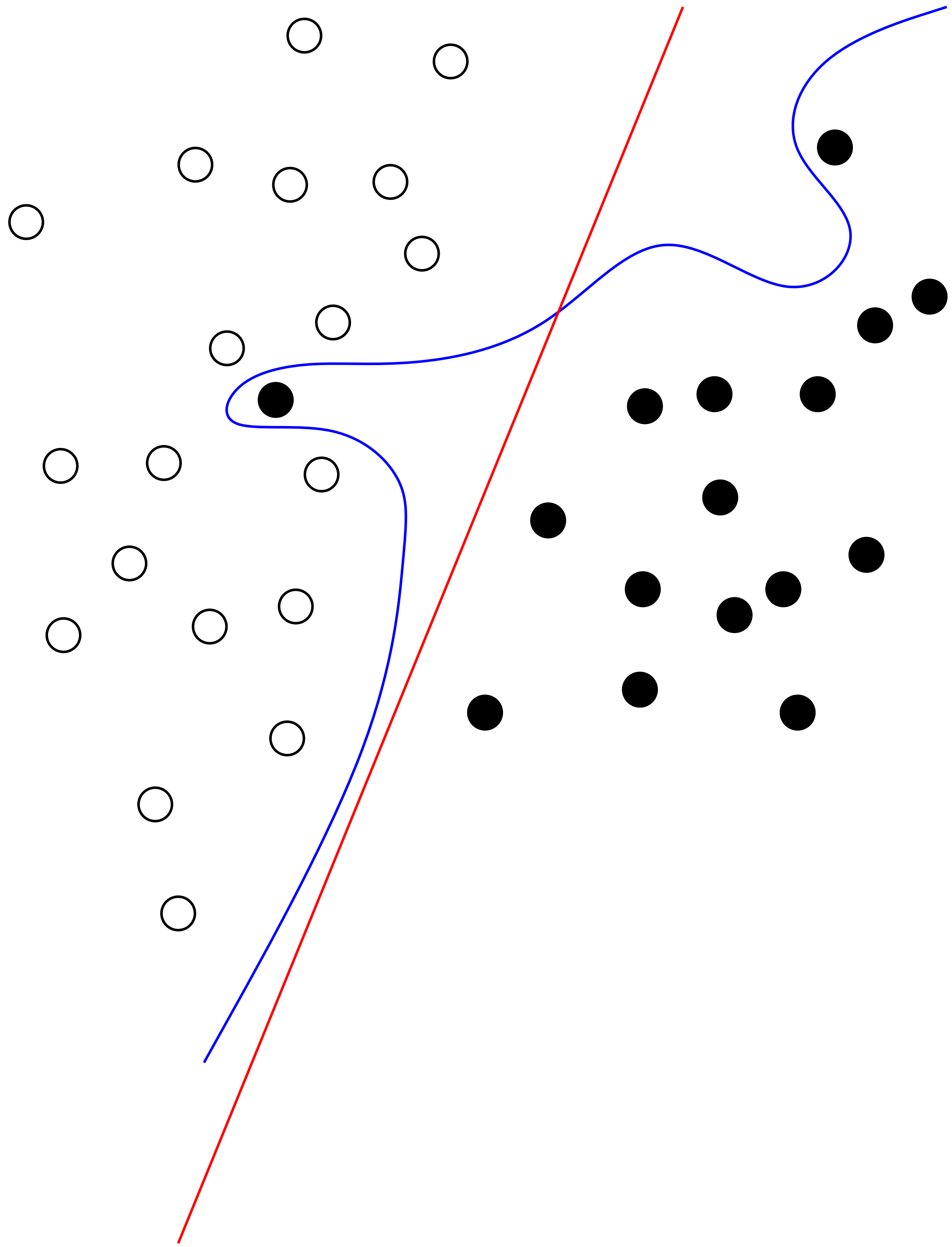
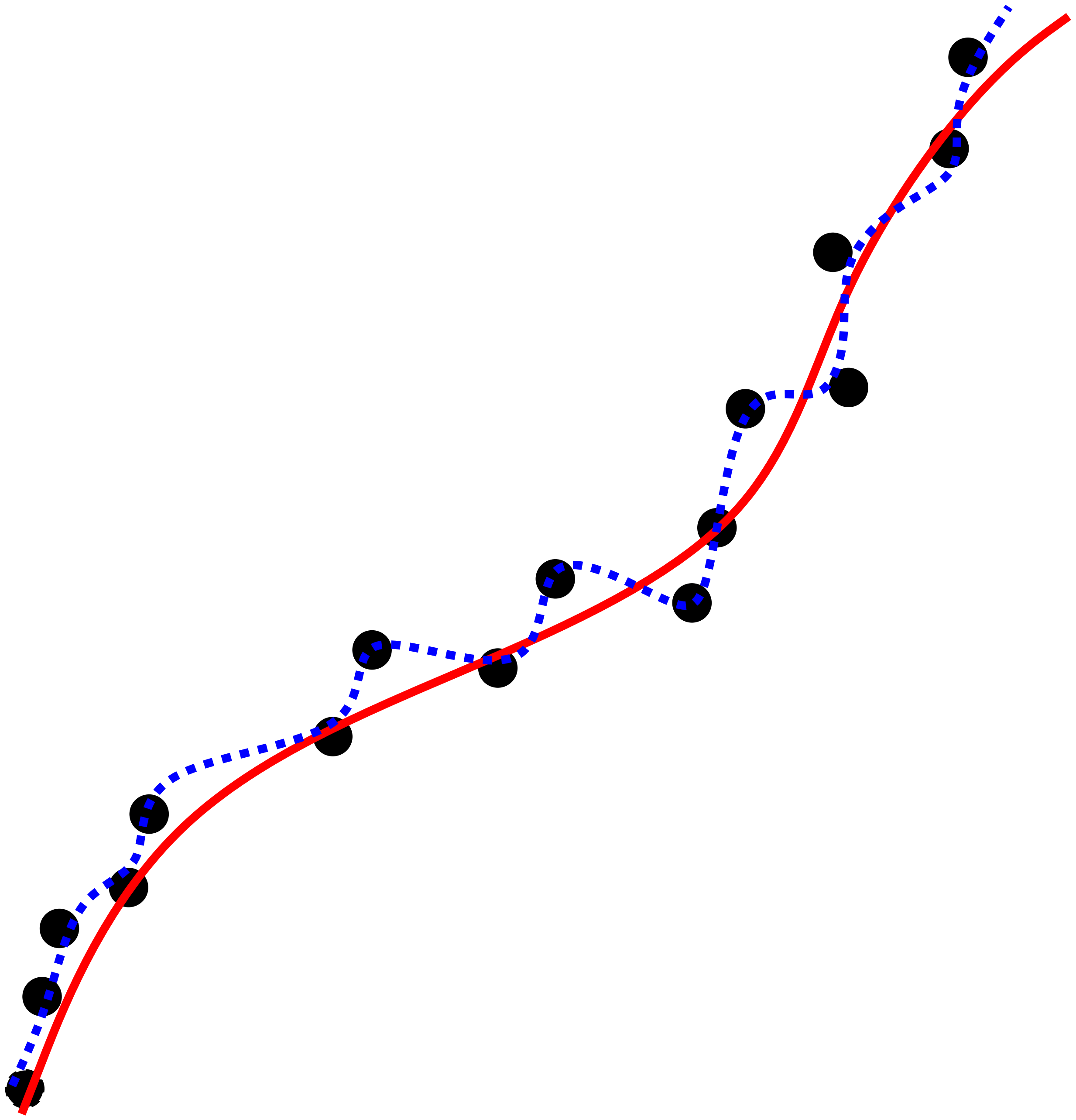
Recap of Classification Problem



- we wanted to pick a function f amongst a class $\mathcal{F}$ of functions which hopefully not only classifies some given *sample* S (picked according to some probability distribution) but also future points that come along according to the same prob. dist.
- it seemed reasonable not to pick a very complicated function that fits the sample S exactly as this might decrease performance on future points that come along

The Regression Problem



- we want to pick a function f amongst a class $\mathcal{F}$ of functions which 'well approximates' the given *sample* S (picked according to some probability distribution) and also lies close to future points

- again it seems reasonable not to pick a very complicated function that fits the sample S exactly as this might decrease performance on future points that come along

General plan

- same idea as for classification problem:
  - consider linear case first
  - apply kernel trick to deal with non-linear data

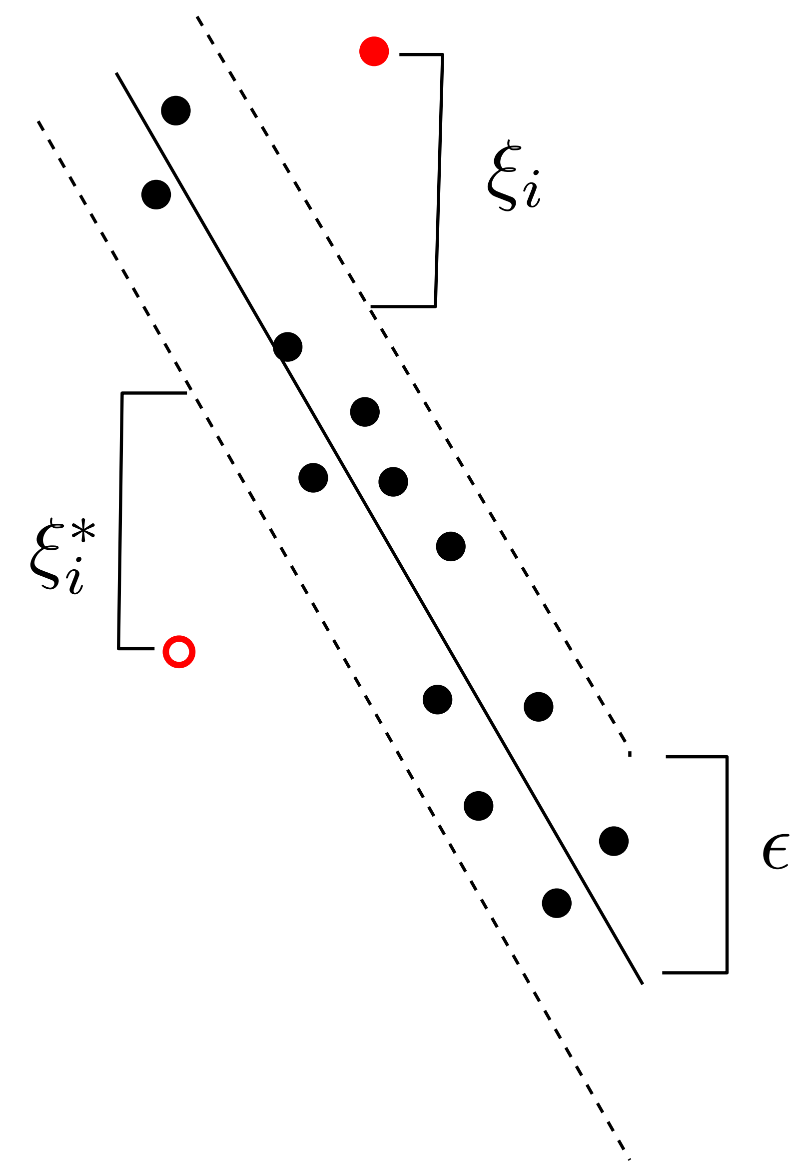
Linear Regression SVM

- we want to determine a hyperplane of the form $\mathbf{w} \cdot x + \mathbf{b} = y$ which closely fits all data points, i.e.

  $y_i - (x_i \cdot \mathbf{w} + \mathbf{b}) \leq \epsilon$
  $x_i \cdot \mathbf{w} + \mathbf{b} - y_i \leq \epsilon$

  assuming such a function exists which approximates the data up to ϵ-precision

- analogously to the 'soft margin' classifier, we can also allow for 'outliers' with penalties
  $\min \frac{1}{2}||\mathbf{w}||^2 + C \sum(\xi_i + \xi_i^*)$
  $y_i - (x_i \cdot \mathbf{w} + \mathbf{b}) \leq \epsilon + \xi_i$
  $x_i \cdot \mathbf{w} + \mathbf{b} - y_i \leq \epsilon + \xi_i^*$
  $\xi_i, \xi_i^* \geq 0$ C determines the weight of the penalties for the outliers



Lagrangian Formulation

- We replace the original problem by its Lagrangian formulation (penalizing violations of the constraints)

$$L_P \equiv \frac{1}{2}||\mathbf{w}||^2 + C\sum(\xi_i + \xi_i^*) - \sum_{i=1}^{l} \alpha_i(\epsilon + \xi_i - y_i + x_i \cdot \mathbf{w} + b)$$

$$- \sum_{i=1}^{l} \alpha_i^*(\epsilon + \xi_i^* + y_i - x_i \cdot \mathbf{w} - b) - \sum \eta_i \xi_i + \eta_i^* \xi_i^*$$

  with dual variables $\alpha_i, \alpha_i^*, \eta_i, \eta_i^* \geq 0$.

  which needs to be minimized (wrt $\mathbf{w}, \mathbf{b}, \xi_\mathbf{i}, \xi_\mathbf{i}^*$; requiring that the partial derivatives wrt $\alpha_i, \alpha_i^*, \eta_i, \eta_i^*$ vanish)

Lagrangian Dual

- in the respective dual we *maximize* L_P s.t. the gradients wrt to w, b, ξ_i, ξ_i^* vanish, i.e. we obtain the conditions

$$\sum(\alpha_i^* - \alpha_i) = 0$$

$$w - \sum(\alpha_i - \alpha_i^*)x_i = 0$$

$$C - \alpha_i^{(*)} - \eta_i^{(*)} = 0$$

- Substitution yields

$$\max L_D = -\frac{1}{2}\sum_{i,j}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)x_i \cdot x_j$$

$$-\epsilon\sum(\alpha_i + \alpha_i^*) + \sum y_i(\alpha_i - \alpha_i^*)$$

subject to

$$\sum(\alpha_i - \alpha_i^*) = 0$$

$$\alpha_i, \alpha_i^* \in [0, C]$$

Lagrangian Dual

- the y-coordinate of new points can be predicted as

$$f(x) = w^T x + b = \sum(\alpha_i - \alpha_i^*) x_i \cdot x + b$$

- where b can be computed via the Karush-Kuhn-Tucker complementary slackness conditions

- again, we have a formulation where the input only appears as dot products

- the kernel trick can be applied as in the case of the classifier SVMs

Surface Reconstruction via Regression SVMs

- Paper by Steinke,Schölkopf, Blanz (EUROGRAPHICS'05)

- Goal: given a set of points (samples) from the surface of a volume, compute a function which implicitly represents the surface (surface $\equiv$ zero set)

- Solution via Regression SVM by mapping the data to some space $\mathcal{H}$, where the original surface becomes a hyperplane

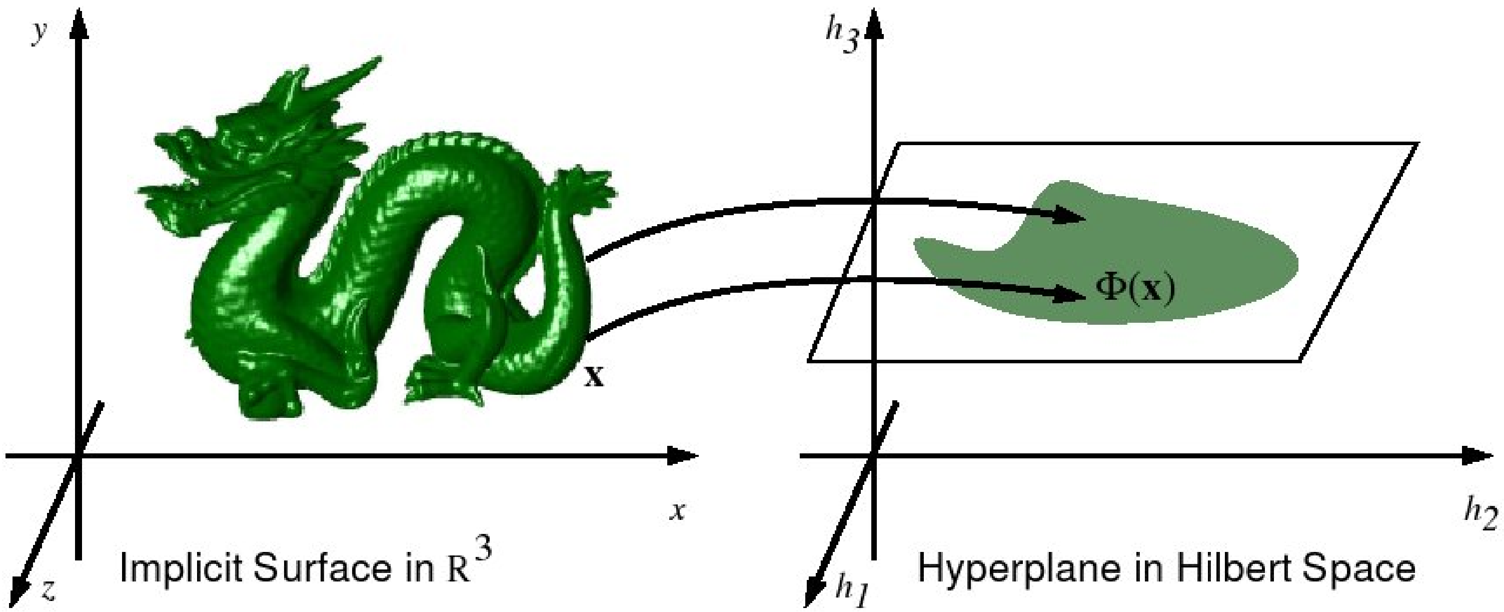


Figure 1: *The mapping Φ defined by the kernel function k (Equation 1) transforms the 3D surface to a hyperplane in a high-dimensional Hilbert space.*

Setting Up the SVM

- represent the surface as the zero set of

$$f(x) = w \cdot \Phi(x) + b$$

  in the space $\mathcal{H}$

- this hyperplane will approximately pass through the given surface points mapped to $\mathcal{H}$ via Φ

- Idea: training data consists of points $x_1, \ldots, x_n$ in the original space with "offsets" $y_1, \ldots y_n$ from the surface; ($y_i = 0 \Rightarrow x_i$ is on the surface)

- Why not just use orignal point sample with $y_i = 0$?

Generating Training data

- Problem: this would support the 'trivial' function $f(x) = 0$
- Rescue: Generate additional, artificial data points with non-zero y_i values by displacing surface points along their surface normals by distance y_i
- $\Rightarrow$ prevents learning of the zero function
- Disadvantage: requires estimation/knowledge of surface normals and orientation
- apply standard SVM regression with a polynomial kernel

Examples

Figure 3: *An implicit surface reconstruction of the dragon of the Stanford 3D Scanning Repository. The original mesh has more than 400k data points. Our reconstruction uses 280k kernel centers.*

Figure 4: *Holes due to occlusions in the scanning process (left) are filled by the implicit surface (right).*

- surprisingly fast: Dragon model (405k points) takes around 5min on a 2.2 GHz machine
- implicit representation to some degree repairs insufficient surface scans
- also behaves nicely wrt outliers/noise; smoothing can easily be incorporated

Face detection via SVMs

- Paper by E. Osuna, R. Freund, and F. Girosi. at CVPR'97
- Goal: decide for small 19×19 grayscale bitmaps 'face or not?'
- training using database of face/non-face pixel patterns after histogram normalization
- interpretation of 19×19 pixel patterns as elements in 361-dimensional space
- polynomial kernel of degree 2
- retraining using misclassified images

Results

- routine built into face recongition system, which scales and decomposes image into 19×19 tiles searching for faces
- system is reported to achieve a detection rate between 75% and 97% with very few false positives
- compares apparently favourably to state of the art FR systems at the time
- running time not clearly stated