NEUROPHYSICS 2015

Prof. Richard H.R. Hahnloser and Prof. Jean-Pascal Pfister

Lecture I Exercises

Teaching assistants: Anja Zai (zaia@ini.ethz.ch)

Arjun Bharadwaj (arjunb@ini.uzh.ch)

Anna Kutschireiter (annak@ini.uzh.ch)

1. Show how to obtain the derivative ΔJ_w from the error J(w) in:

$$J_n = \frac{1}{2}(\mathbf{y}^*[n] - \mathbf{y}[n])^T(\mathbf{y}^*[n] - \mathbf{y}[n])$$

$$\nabla_{\mathbf{w}} J_n = -\frac{\partial \mathbf{u}^T}{\partial \mathbf{w}} \frac{\partial \mathbf{y}^T}{\partial \mathbf{u}} (\mathbf{y}^*[n] - \mathbf{y}[n])$$

   You need to take the first order derivative of the first equation (for error J_n) to obtain the second equation. Try to write it down explicitly (writing out all terms), first for one dimension (i.e. all variables y, y*, u and w are all 1-dimensional) and then generalize it for vectors (y, y* and u) and matrices (W) of arbitrary dimensions.

2. Give a step-wise proof of convergence for the Inverse Model (using the gradient descent method utilizing $\nabla_w \boldsymbol{J}$) :

$$\boldsymbol{u} = \boldsymbol{W}\,\boldsymbol{y}$$

   with the knowledge that $\boldsymbol{y} = \boldsymbol{Q}\,\boldsymbol{u}$

$$\boldsymbol{where}\ \boldsymbol{u} \in \mathbb{R}^{P\times 1} \quad \boldsymbol{y} \in \mathbb{R}^{N\times 1} \quad \boldsymbol{W} \in \mathbb{R}^{P\times N} \quad \boldsymbol{Q} \in \mathbb{R}^{N\times P}$$

   Concretely, show that:

   For a mean squared error function J(w), and gradient ΔJ_w

$$J(w) = \frac{1}{2} E\{(u - \hat{u})^T (u - \hat{u})\}$$

$$\nabla_w J = -\frac{\partial \hat{u}^T}{\partial w}(u - \hat{u})$$

$$where \quad \hat{u} = W \; y$$

At convergence i.e. when $\nabla_w J$ is set to zero,

$$W = Q^{-1}$$

3. Give a step-wise proof of convergence for the Inverse Model trained by a **fully trained Forward** model:

   $$V = Q$$

   where the error is given by

   $$J(w) = \frac{1}{2} E\{(y^* - \hat{y})^T (y^* - \hat{y})\}$$

   Using similar logic as in question 1, show that, at convergence

   $$W = Q^{-1}$$