ECON42720 Causal Inference and Policy Evaluation

7 Fixed Effects and Difference-in-Differences

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Resources for Fixed Effects

Textbook chapter

- Huntington-Klein, The Effect: Ch. 16

Resources for Difference-in-Differences

Textbook chapters

- Cunningham, Causal Inference: The Mixtape, Ch. 9
- Huntington-Klein, The Effect: Ch. 18

YouTube Videos

- Videos 17-21 of my Causal Inference Playlist

Fixed Effects

Start with a regression:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

If there are unobserved confounders, we have the problem that $E[u_i|X_i] \neq 0$

If we could **observe these confounders**, we could include them in the regression

$$Y_i = \beta_0 + \beta_1 X_i + \boldsymbol{S_i'}\boldsymbol{\delta} + u_i$$

If S_i includes all confounders, $E[u_i|X_i, \boldsymbol{S_i}] = 0$ holds and we have an unbiased and consistent estimator for β_1.

Fixed Effects: Controlling for Unobservables

Problem: We usually can't observe all confounders

Fixed effects allow us to **control for (some) unobserved and observed confounders**

What we need:

- **Panel data**: multiple observations per unit
- or **Grouped data**: multiple units in each group

Fixed Effects with Panel Data

Panel data is data with multiple observations per unit i

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Now **add unit fixed effects**:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \alpha_i + u_{it}$$

The fixed effects α_i can be viewed as **separate dummies for each unit i**

What Fixed Effects Do

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \alpha_i + u_{it}$$

The fixed effects α_i isolate the **within-unit variation in Y_{it} and X_{it}**

Suppose i are countries and t are years. **Interpretation of β_1**:

- If X_{it} goes up in a given country, how does Y_{it} change within the same country?
- So β_1 measures the **average within-country effect** of X_{it} on Y_{it}

The fixed effects α_i **control for all time-invariant observables AND unobservables**

Fixed Effects: Example

We will now go through a simple example: **crime rates and police presence in cities**

- Here, a *group* is a city
- There is *within-city variation* in crime rates and police presence *over time*
- This is the classic use of *fixed effects with panel data*

Data are (to some extent) made up for illustration purposes

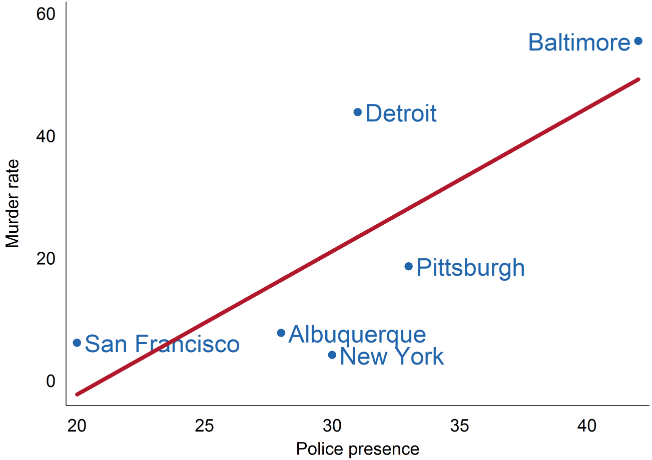
Fixed Effects: Example

Causal relationship of interest

$$\text{crime}_i = \alpha + \beta \text{ police presence}_i + u_i$$

City	Year	Murder rate	Police presence
Baltimore	2009	55.4	42
Albuquerque	2009	7.7	28
New York	2009	4.1	30
Pittsburgh	2009	18.6	33
San Francisco	2009	6.1	20
Detroit	2009	43.8	31

The Cross-sectional Relationship is Positive. . .



Fixed Effect Regressions

Logic of Fixed Effect Regressions: exploit **variation within subjects over time**

In our case: how does the **murder rate in a city change** when in the same city the police presence increases by 1 unit?

Advantage:

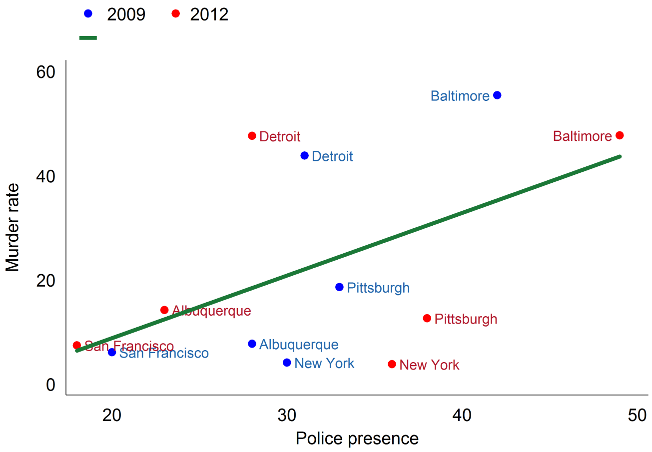
- **fixed city characteristics** are **held constant**
- And as such many determinants why Baltimore has a higher crime rate *and* police presence than San Francisco
- We circumvent an important *selection problem* $\Rightarrow$ eliminates (or reduces) **omitted variable bias**

Now Suppose You Have Panel Data

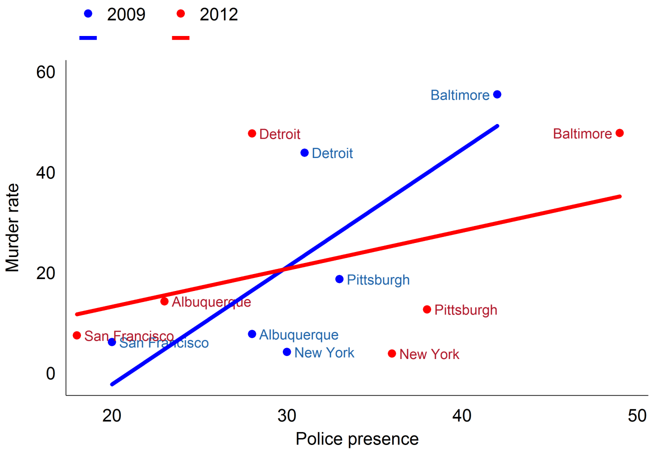
City	Year	Murder rate	Police presence
Baltimore	2009	55.4	42
Baltimore	2012	47.4	49
Albuquerque	2009	7.7	28
Albuquerque	2012	14.2	23
New York	2009	4.1	30
New York	2012	3.8	36
Pittsburgh	2009	18.6	33
Pittsburgh	2012	12.6	38
San Francisco	2009	6.1	20
San Francisco	2012	7.4	28
Detroit	2009	43.8	31
Detroit	2012	47.6	28

{Note: data are fictitious}

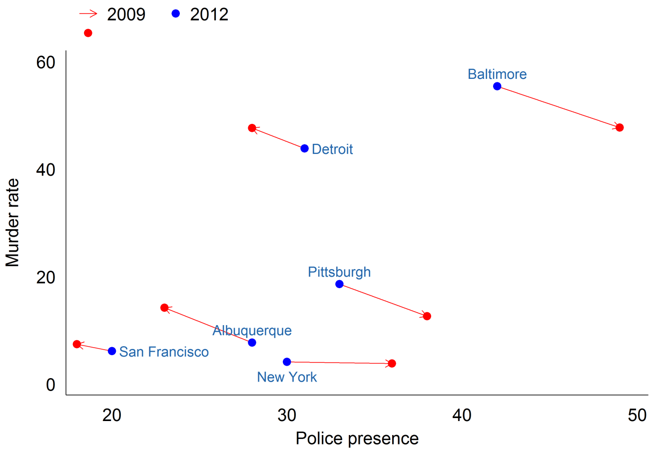
Cross-sectional Relationship in Panel Data: Still Positive



In each year we have a positive association



Now look at within-city changes



Fixed Effect Regressions

A **Fixed Effect Regressions** only relies on the **within-variation**

$$y_{it} = \beta x_{it} + \alpha_i + \varepsilon_{it}$$

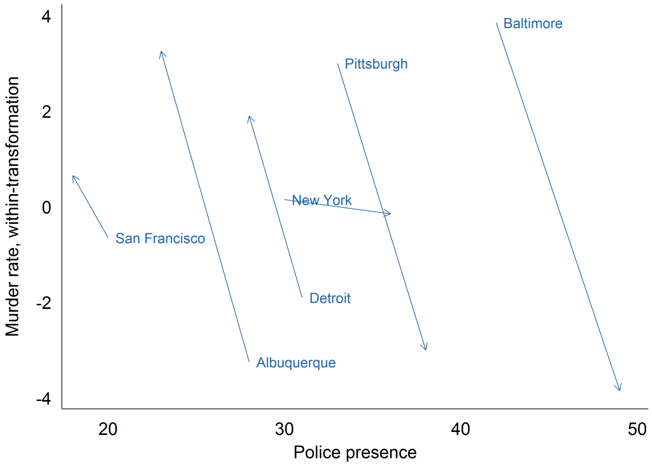
The **between-variation** will be netted out

At the core of the FE regression lies a **within-transformation**

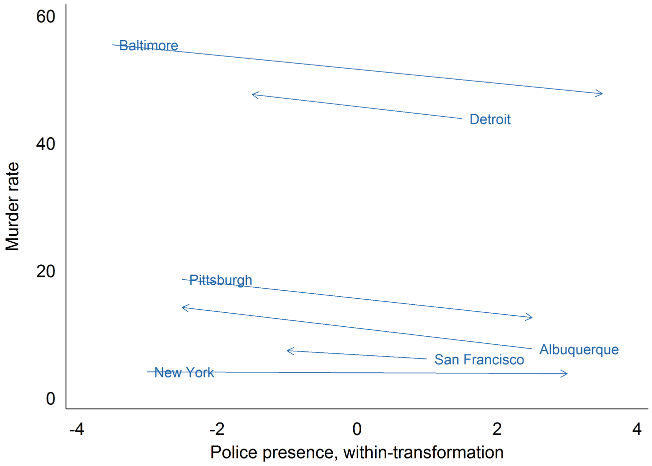
$$y_{it} - \overline{y}_i = \beta(x_{it} - \overline{x}_i) + \varepsilon_{it} - \varepsilon_i$$

Takes from each variable the **deviation from the mean**

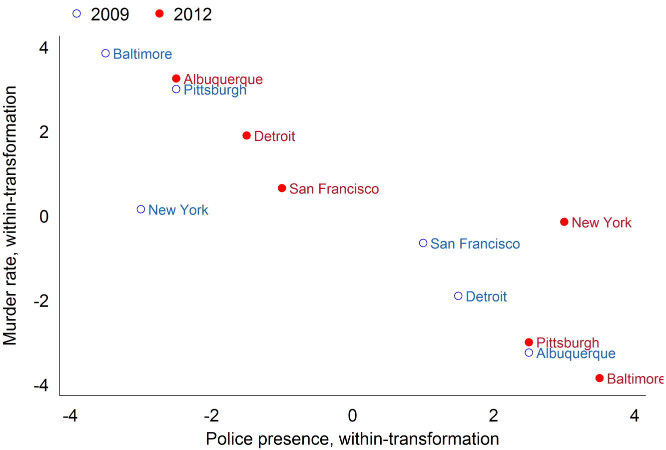
Within-transformation of Y



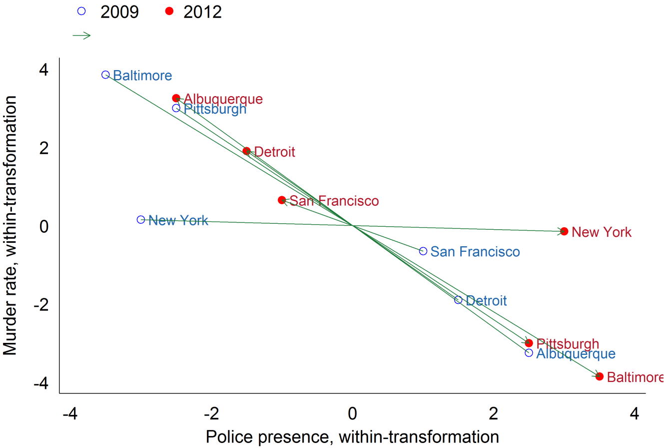
Within-transformation of X



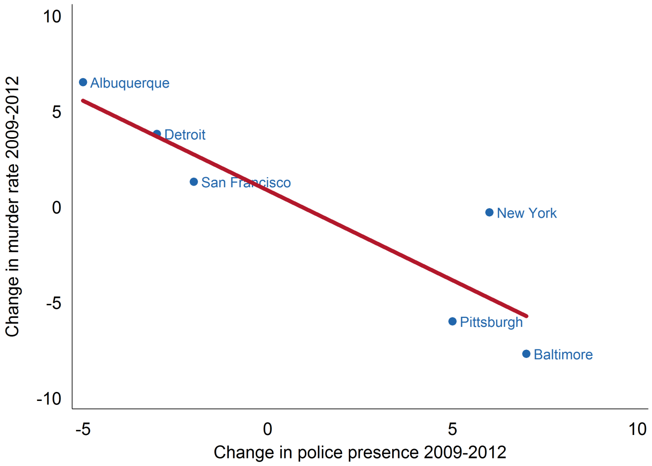
Within-transformation of X and Y



The within-effects in each city



Average within-effect: NEGATIVE!



Summary of the example

The **cross-sectional relationship** between police presence and crime rates is **positive**

- This is **between-city variation**
- It is driven by **differences in city characteristics**
- More crime-prone cities choose to hire more police officers...

We can learn a lot from **within-city variation**

- The city fixed effects **eliminate all time-invariant differences between cities**
- They isolate the **within-city variation** in **all variables**
- The **within-effect** of police presence on crime rates is **negative**

Interpretation: if within a city the police presence goes up by 1 unit, the crime rate goes down by β units

Another way to look at fixed effects

The fixed effects **split the data into many units** – here a unit is a city

A **fixed effect regression performs two tasks** at the same time:

1. it estimates the effect of X on Y within each unit
2. it averages these effects across all units

Multiple units and time periods

It is common to have **panel data with many units and many time periods**

- Example: 50 US states over 20 years

We often use **two-way fixed effects**:

- Unit fixed effects (δ_i) absorb all time-invariant differences between units
- Time fixed effects (δ_t) absorb all time trends that are common to all units

The **regression equation is then**

$$Y_{it} = \beta X_{it} + \delta_i + \delta_t + u_{it}$$

Fixed Effects with Grouped Data

Grouped data is data with multiple units $i = 1, \ldots, N$ which belong to distinct groups $g = 1, \ldots, G$.

Example: students in schools, workers in firms, patients in hospitals

Classic case: stratified experiments in within schools

- It is not random who goes to which school
- But within schools, treatment assignment is random

To estimate the treatment effect, we can use **fixed effects for groups**

$$Y_{ig} = \beta X_{ig} + \alpha_g + u_{ig}$$

Implementation of Fixed Effects Regressions in R

First of all, you need to **have panel data in "long form"**

- Each **row** is an observation for a unit at a certain time

City	Year	Murder rate	Police presence
Baltimore	2009	55.4	42
Baltimore	2012	47.4	49
Albuquerque	2009	7.7	28
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San Francisco	2012	7.4	28
Detroit	2009	43.8	31
Detroit	2012	47.6	28

Data must not be in wide format!

City	Murder Rate 2009	Police 2009	Murder Rate 2012	Police 2012
Baltimore	55.4	42	47.4	49
Albuquerque	7.7	28	14.2	23
New York	4.1	30	3.8	36
Pittsburgh	18.6	33	12.6	38
San Francisco	6.1	20	7.4	28
Detroit	43.8	31	47.6	28

Can't work with that! If you have such data, use the `pivot` commands from `dplyr` to bring your panel data into long form.

Difference-in-Differences: a Quasi-Experimental Design

Some units get treated, some don't. . . we've heard that before

What's different about difference-in-differences?

- **Treatment assignment** does **NOT** need to be as good as **random**
- The **TREND in outcomes** of the control group is a good **counterfactual** for the trend of the treated group

DiD is arguably one of the **most popular designs in empirical economics**

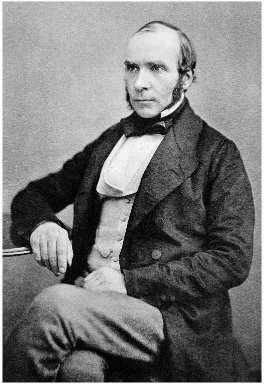
Historical DiD Example: The Cholera Hypothesis

19th century: Cholera was a major disease in Europe

Dominant hypothesis: Cholera is **transmitted through the air**

John Snow in 1854: Cholera is **transmitted through water**

Research design: **Difference-in-differences**



John Snow (1813-1858)
(Source: Wikipedia)

Broad Street Pump in London (Soho)

(Source: Wikipedia)

The Cholera Hypothesis

Snow's theory: **Cholera is transmitted through water**

- People drink contaminated water that contains the cholera bacterium
- The bacterium enters the digestive system and causes cholera
- Through vomiting and diarrhea, the bacterium is excreted and contaminates the water supply further

Some observations:

- Sailors got sick when they went on land but not when staying docked
- Cholera was more prevalent in poor areas with bad hygiene
- Some apartment blocks were affected, other neighbouring ones not

The Cholera Hypothesis

How could Snow test his theory?

- Mind you: experiments were only established in 1935 by Fisher as a means to prove causality
- And you couldn't run an experiment (drink from the Thames if heads, from another source if tails)

Snow's research design

- Some areas in London had their water supply from the Thames
- Others had their water supply from other sources
- Problem: areas were different in many ways

Snow's Research Design

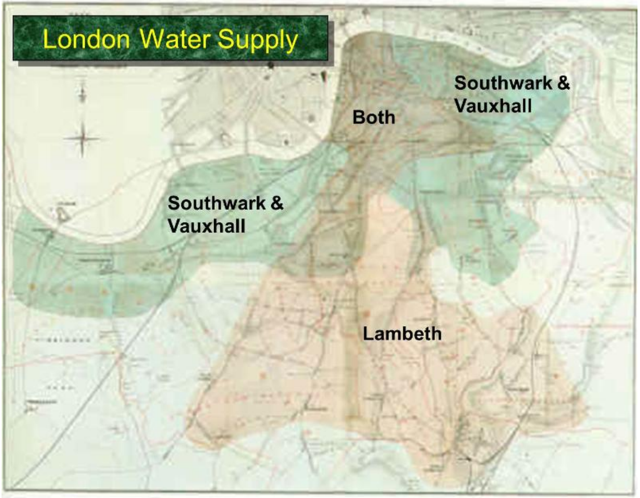
Different boroughs in London had different **water supplies, all from the Thames**

But: in 1849 the Lambeth Water Company switched to a new water source upstream

- This turned out to be cleaner and not contaminated cholera
- The Southwark and Vauxhall Water Company did not switch

Did cholera cases decline in Lambeth after the switch relative to Southwark and Vauxhall?

Lambeth vs. Southwark and Vauxhall Water Supply



(Source: inferentialthinking.com)

John Snow's Data

Much of the data on water suppliers was hand-collected (!) by Snow

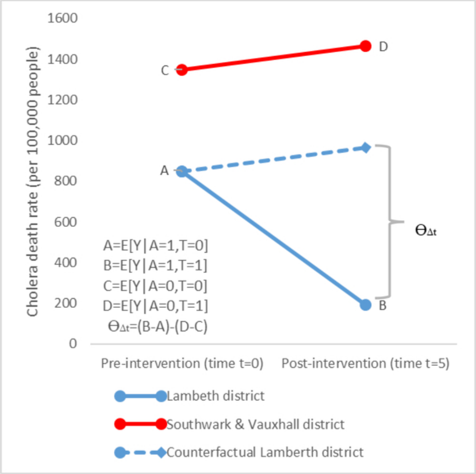
Cholera deaths per 10,000 households in the mid-1850s

Company Name	**1849** Before Switch	**1854** After Switch
Southwark and Vauxhall	135	147
Lambeth	85	19

Things to note

- There were **more deaths in both years in Southwark and Vauxhall**
- **Death rates in Lambeth dropped dramatically** after the switch
- Death rates in Southwark and Vauxhall stayed roughly the same

John Snow Discovered Difference-in-Differences



Source: Caniglia & Murray (2020)

John Snow Discovered Difference-in-Differences

Difference 1: Lambeth vs. Southwark and Vauxhall

- Solid blue vs red line: differences in cholera deaths between the two areas

Difference 2: Before vs. after the switch

- Dotted blue line: projects the trend in Lambeth if the switch had not happened
- This is just the trend of Southwark and Vauxhall

Difference-in-differences: The difference between the solid and dotted blue line

- relative to the counterfactual, the switch reduced cholera deaths by 78 per 10,000 households

John Snow Discovered Difference-in-Differences

Company Name	**1849** Before Switch	**1854** After Switch	**Difference 2**
Southwark and Vauxhall	135	147	+12
Lambeth	85	19	-66
Difference 1	-50	-128	-78

The difference-in-differences is 78 cholera deaths per 10,000 households

References

Caniglia, Ellen C., & Murray, Eleanor J. 2020. Difference-in-Difference in the Time of Cholera: a Gentle Introduction for Epidemiologists. *Current Epidemiology Reports*, **7**, 203–210.

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