

$$\tan \theta = \frac{h_2 - h_1}{w_2 - w_1}$$

$$= \frac{r_2 \cos \theta_2 - r_1 \cos \theta_1}{r_2 \sin \theta_2 - r_1 \sin \theta_1}$$

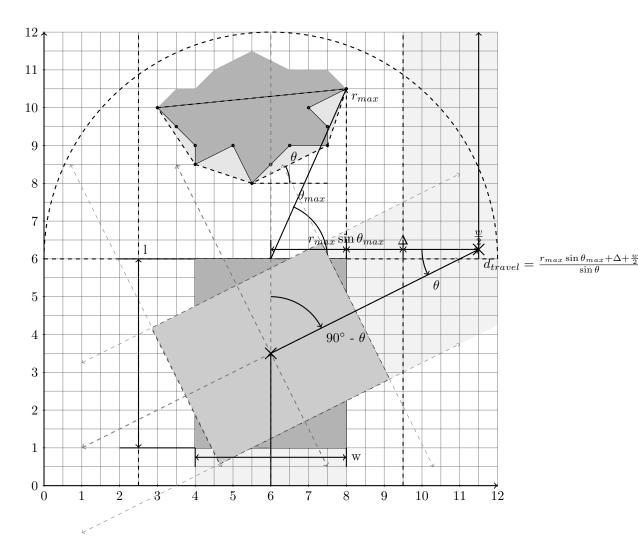
$$\theta = \arctan \frac{r_2 \cos \theta_2 - r_1 \cos \theta_1}{r_2 \sin \theta_2 - r_1 \sin \theta_1}$$
(2)
$$\alpha = 90^\circ - \theta$$
(3)

$$=\frac{r_2\cos\theta_2 - r_1\cos\theta_1}{r_2\sin\theta_2 - r_1\sin\theta_1}\tag{2}$$

$$\theta = \arctan \frac{r_2 \cos \theta_2 - r_1 \cos \theta_1}{r_2 \sin \theta_2 - r_1 \sin \theta_1} \tag{3}$$

$$\alpha = 90^{\circ} - \theta \tag{4}$$

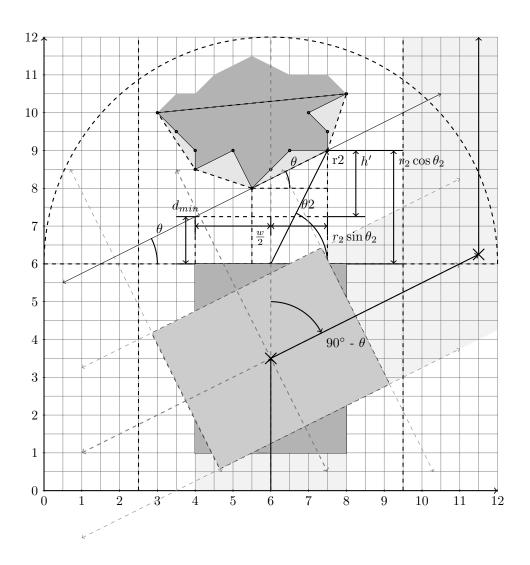
$$= \arctan \frac{r_2 \sin \theta_2 - r_1 \sin \theta_1}{r_2 \cos \theta_2 - r_1 \cos \theta_1}$$
 (5)



$$d_{travel}\sin\theta = r_{max}\sin\theta_{max} + \Delta + \frac{w}{2} \tag{6}$$

$$d_{travel} \sin \theta = r_{max} \sin \theta_{max} + \Delta + \frac{w}{2}$$

$$d_{travel} = \frac{r_{max} \sin \theta_{max} + \Delta + \frac{w}{2}}{\sin \theta}$$
(6)



$$h' = \left(\frac{w}{2} + r_2 \sin \theta_2\right) \tan \theta \tag{8}$$

$$d_{min} = r_2 \cos \theta_2 - h' \tag{9}$$

$$= r_2 \cos \theta_2 - \left(\frac{w}{2} + r_2 \sin \theta_2\right) \tan \theta \tag{10}$$

$$= r_2 \cos \theta_2 - \left(\frac{w}{2}\right) \tan \theta - r_2 \sin \theta_2 \tan \theta \tag{11}$$

$$= r_2 \cos \theta_2 - r_2 \sin \theta_2 \left(\frac{\sin \theta}{\cos \theta}\right) - \left(\frac{w}{2}\right) \tan \theta \tag{12}$$

$$= \frac{r_2}{\cos \theta} \left(\cos \theta_2 \cos \theta - \sin \theta_2 \sin \theta\right) - \left(\frac{w}{2}\right) \tan \theta \tag{13}$$

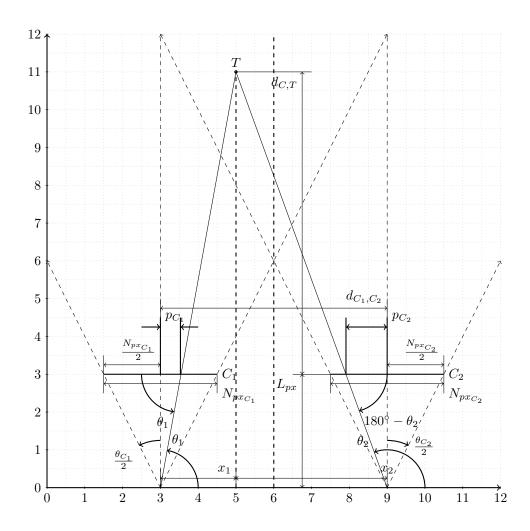
$$= \left(\frac{r_2}{\cos \theta}\right) \cos \left(\theta_2 + \theta\right) - \left(\frac{w}{2}\right) \tan \theta \tag{14}$$

$$d_{min} = \left(r_2 \cos\left(\theta_2 + \theta\right) - \left(\frac{w}{2}\right) \sin\theta\right) \frac{1}{\cos\theta} \tag{15}$$

$$d_{min} \ge \Delta_{min} \tag{16}$$

$$t_{hit} = \frac{d_{min}}{v_{robo}} \tag{17}$$

$$\implies t_{decision} \le t_{hit}$$
 (18)



$$\tan\left(\frac{\theta_{C_1}}{2}\right) = \frac{\left(\frac{N_{px_{C_1}}}{2}\right)}{L_{px}} \tag{19}$$

$$\implies L_{px} = \left(\frac{N_{px_{C_1}}}{2}\right) \frac{1}{\tan\left(\frac{\theta_{C_1}}{2}\right)} \tag{20}$$

$$=\frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)}\tag{21}$$

$$\tan \theta_1 = \frac{L_{px}}{p_{C_1}} \tag{22}$$

$$= \frac{\left(\frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)}\right)}{p_{C_1}}$$

$$= \frac{N_{px_{C_1}}}{2p_{C_1}\tan\left(\frac{\theta_{C_1}}{2}\right)}$$
(23)

$$=\frac{N_{px_{C_1}}}{2p_{C_1}\tan\left(\frac{\theta_{C_1}}{2}\right)}\tag{24}$$

$$\implies \frac{1}{\tan \theta_1} = \frac{2p_{C_1} \tan\left(\frac{\theta_{C_1}}{2}\right)}{N_{px_{C_1}}} \tag{25}$$

$$\tan(180^{\circ} - \theta_2) = \frac{L_{px}}{p_{C_2}} \tag{26}$$

$$= \frac{\left(\frac{N_{px_{C_2}}}{2\tan\left(\frac{\theta_{C_2}}{2}\right)}\right)}{p_{C_2}}$$

$$= \frac{N_{px_{C_2}}}{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}$$
(27)

$$=\frac{N_{px_{C_2}}}{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}\tag{28}$$

$$\implies \tan \theta_2 = -\left(\frac{N_{px_{C_2}}}{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}\right) \tag{29}$$

$$\implies \frac{1}{\tan \theta_2} = -\left(\frac{2p_{C_2} \tan\left(\frac{\theta_{C_2}}{2}\right)}{N_{px_{C_2}}}\right) \tag{30}$$

 $Let: \sigma_{px/unit} be density of pixels per unit of measurement$ (31)

$$Let: L_{units} = \frac{L_{px}}{\sigma_{px/unit}} \tag{32}$$

$$\tan \theta_1 = \frac{d_{C,T} + L_{units}}{x_1} \tag{33}$$

$$\tan \theta_1 = \frac{d_{C,T} + L_{units}}{x_1} \tag{34}$$

$$\implies x_1 = \frac{d_{C,T} + L_{units}}{\tan \theta_1} \tag{35}$$

$$\tan(180^{\circ} - \theta_2) = \frac{d_{C,T} + L_{units}}{x_2}$$
 (36)

$$\implies x_2 = \frac{d_{C,T} + L_{px}}{\tan\left(180^\circ - \theta_2\right)} \tag{37}$$

$$\implies x_1 + x_2 = \frac{d_{C,T} + L_{units}}{\tan \theta_1} + \frac{d_{C,T} + L_{units}}{\tan (180^\circ - \theta_2)}$$

$$(38)$$

$$= (d_{C,T} + L_{units}) \left(\frac{1}{\tan \theta_1} + \frac{1}{\tan (180^\circ - \theta_2)} \right)$$
 (39)

$$= (d_{C,T} + L_{units}) \left(\frac{1}{\tan \theta_1} - \frac{1}{\tan \theta_2} \right)$$
 (40)

$$= (d_{C,T} + L_{units}) \left(\frac{\tan \theta_2 - \tan \theta_1}{\tan \theta_1 \tan \theta_2} \right) \tag{41}$$

(42)

$$x_1 + x_2 = d_{C_1, C_2} (43)$$

$$\implies d_{C_1,C_2} = (d_{C,T} + L_{units}) \left(\frac{\tan \theta_2 - \tan \theta_1}{\tan \theta_1 \tan \theta_2} \right)$$
(44)

$$\implies d_{C,T} + L_{units} = d_{C_1,C_2} \left(\frac{\tan \theta_1 \tan \theta_2}{\tan \theta_2 - \tan \theta_1} \right) \tag{45}$$

$$=\frac{d_{C_1,C_2}\tan\theta_1\tan\theta_2}{\tan\theta_2-\tan\theta_1}\tag{46}$$

$$\implies d_{C,T} = \frac{d_{C_1,C_2} \tan \theta_1 \tan \theta_2}{\tan \theta_2 - \tan \theta_1} - L_{units}$$

$$\tag{47}$$

$$\tan \theta_2 - \tan \theta_1$$

$$\Rightarrow d_{C,T} = \frac{d_{C_1,C_2} \tan \theta_1 \tan \theta_2}{\tan \theta_2 - \tan \theta_1} - L_{units}$$

$$\Rightarrow d_{C,T} = \frac{d_{C_1,C_2} \tan \theta_1 \tan \theta_2}{\tan \theta_2 - \tan \theta_1} - \frac{N_{px_{C_1}}}{2 \tan \left(\frac{\theta_{C_1}}{2}\right) \sigma_{px/unit}}$$
(48)

$$\implies d_{C,T} = \frac{d_{C_1,C_2}}{\left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}\right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}} \tag{49}$$

$$= \frac{d_{C_1,C_2}}{\left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}\right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}}$$
(50)

$$= \frac{d_{C_1,C_2}}{\left(\frac{2p_{C_1}\tan\left(\frac{\theta_{C_1}}{2}\right)}{N_{px_{C_1}}} + \frac{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}{N_{px_{C_2}}}\right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}}$$
(51)

$$Let: \tan\left(\frac{\theta_{C_1}}{2}\right) = \tan\left(\frac{\theta_{C_2}}{2}\right) = \tan\left(\frac{\theta_C}{2}\right)$$
 (52)

$$Let: N_{px_{C_1}} = N_{px_{C_2}} = N_{px_C} \tag{53}$$

$$Let: N_{px_{C_1}} = N_{px_{C_2}} = N_{px_C}$$

$$\implies d_{C,T} = \frac{d_{C_1,C_2}}{\left(\frac{2p_{C_1}\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}} + \frac{2p_{C_2}\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}}\right)} - \frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)\sigma_{px/unit}}$$

$$(53)$$

$$= \frac{d_{C_1,C_2}}{\left(\frac{2\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}}\right)(p_{C_1} + p_{C_2})} - \frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)\sigma_{px/unit}}$$
(55)

$$= \frac{d_{C_1,C_2} N_{px_C}}{2 \tan \left(\frac{\theta_C}{2}\right) \left(p_{C_1} + p_{C_2}\right)} - \frac{N_{px_C}}{2 \tan \left(\frac{\theta_C}{2}\right) \sigma_{px/unit}}$$

$$(56)$$

$$= \left(\frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)}\right) \left(\frac{d_{C_1,C_2}}{p_{C_1} + p_{C_2}} - \frac{1}{\sigma_{px/unit}}\right)$$

$$(57)$$

$$If: \frac{1}{\sigma_{px/unit}} \ll \frac{d_{C_1, C_2}}{p_{C_1} + p_{C_2}} \tag{58}$$

$$d_{C,T} \approx \frac{N_{px_C} d_{C_1,C_2}}{2 \tan\left(\frac{\theta_C}{2}\right) (p_{C_1} + p_{C_2})}$$
 (59)