

$$S = 0 + q + 2q^2 + 3q^3 + 4q^4 + \dots \quad (1)$$

$$S = q(1 + 2q + 3q^2 + 4q^3 + \dots) \quad (2)$$

$$S = q(1 + q + 2q^2 + 3q^3 + \dots + q + q^2 + q^3 + \dots) \quad (3)$$

$$S = q\left(S + \frac{1}{1-q}\right) \quad (4)$$

$$S = qS + \frac{q}{1-q} \quad (5)$$

$$S - qS = \frac{q}{1-q} \quad (6)$$

$$S(1-q) = \frac{q}{1-q} \quad (7)$$

$$S = \frac{q}{(1-q)^2} \quad (8)$$