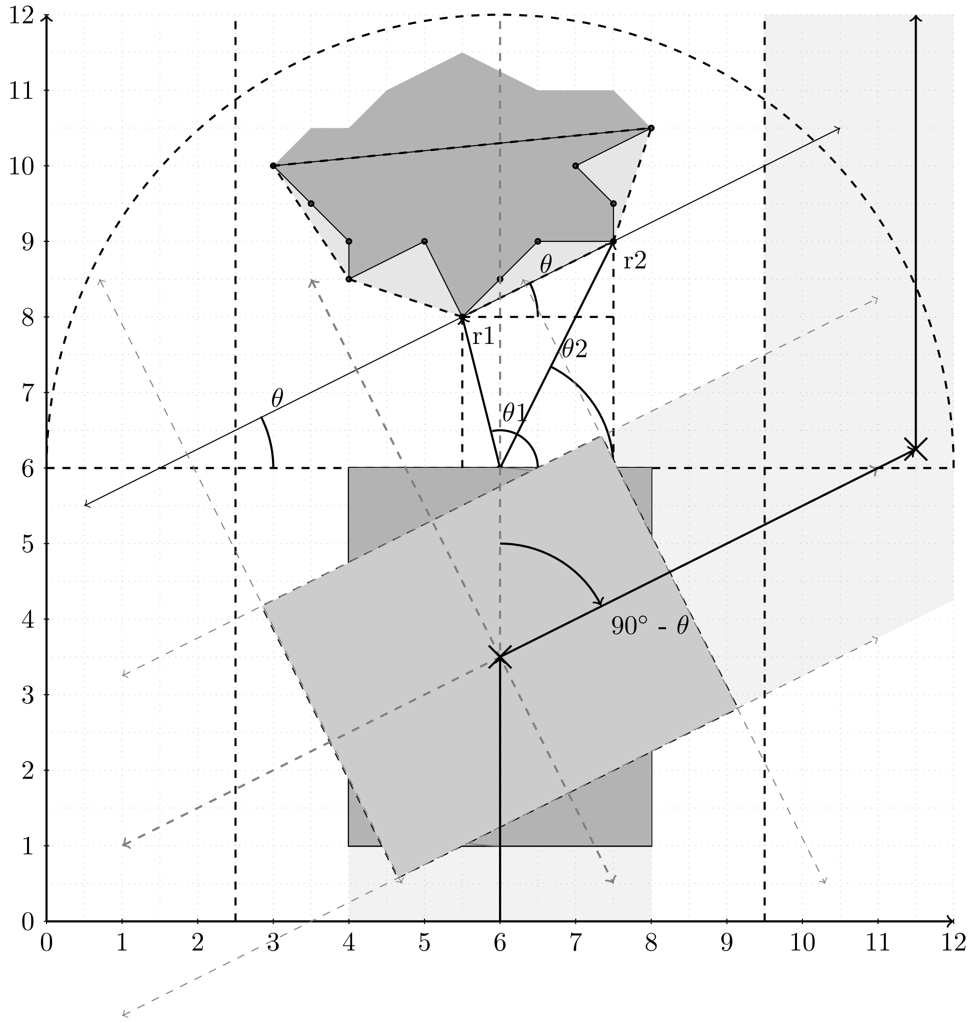


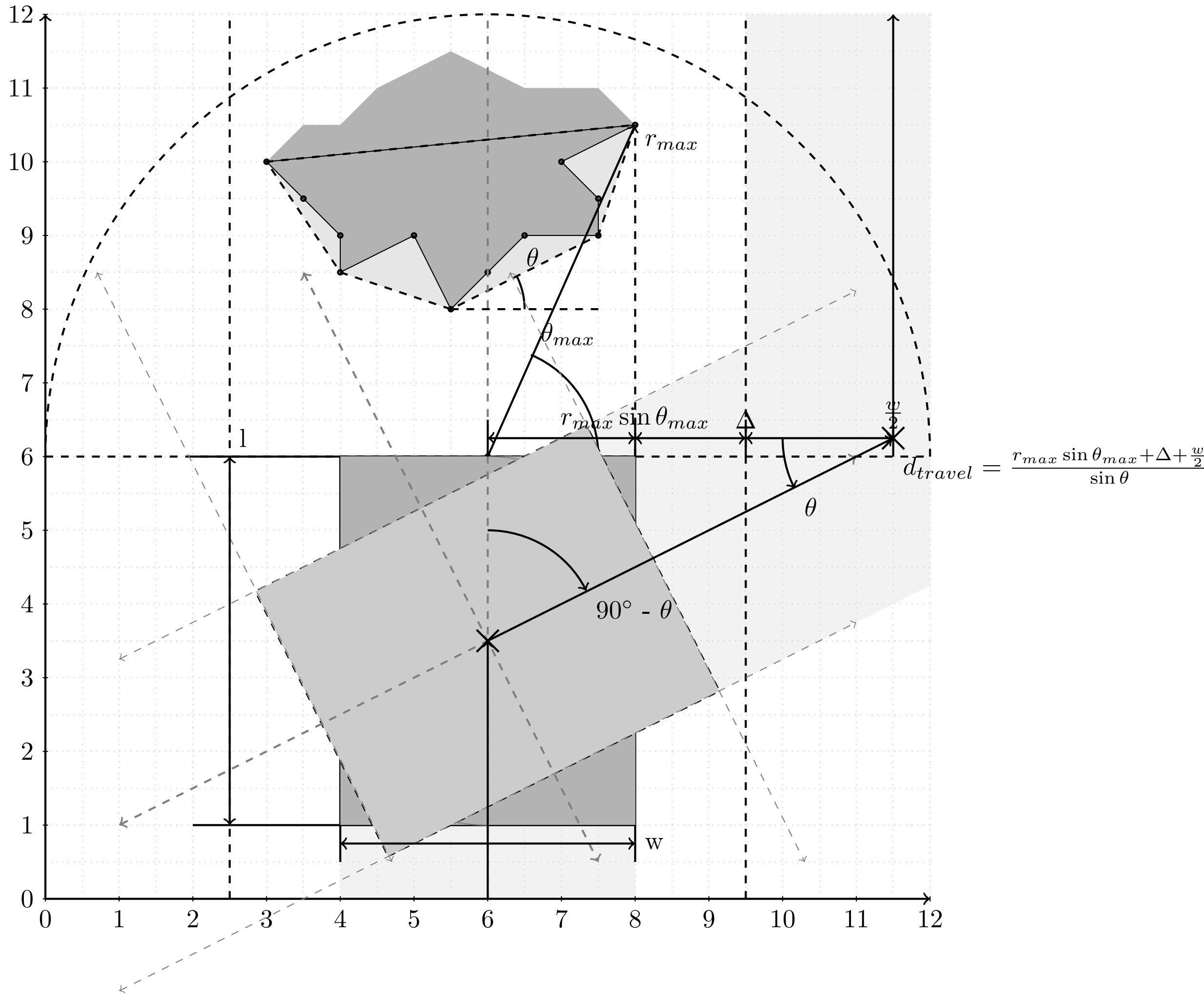
$$\tan\theta = \frac{h_2 - h_1}{w_2 - w_1} \tag{1}$$

$$= \frac{r_2\cos\theta_2 - r_1\cos\theta_1}{r_2\sin\theta_2 - r_1\sin\theta_1} \tag{2}$$

$$\theta = \arctan\frac{r_2\cos\theta_2 - r_1\cos\theta_1}{r_2\sin\theta_2 - r_1\sin\theta_1} \tag{3}$$

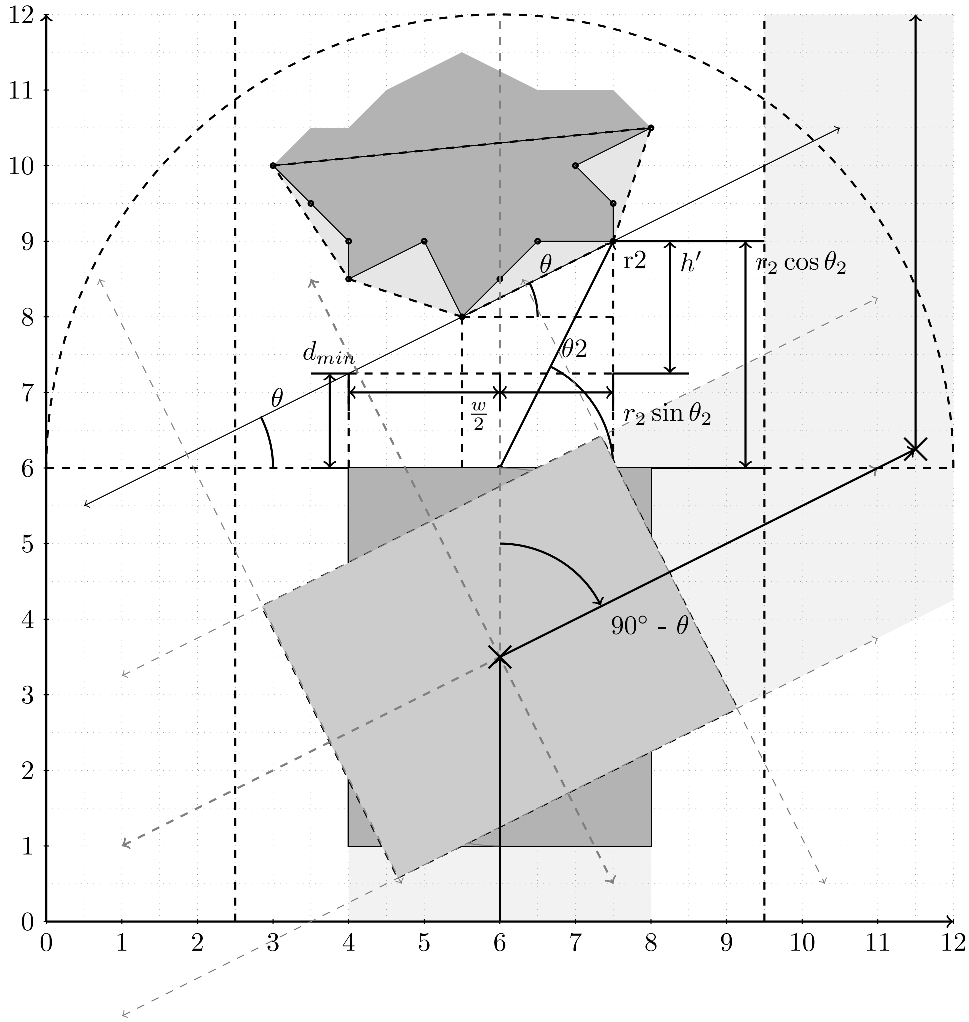
$$\alpha = 90^\circ - \theta \tag{4}$$

$$= \arctan\frac{r_2\sin\theta_2 - r_1\sin\theta_1}{r_2\cos\theta_2 - r_1\cos\theta_1} \tag{5}$$



$$d_{travel} \sin \theta = r_{max} \sin \theta_{max} + \Delta + \frac{w}{2} \tag{6}$$

$$d_{travel} = \frac{r_{max} \sin \theta_{max} + \Delta + \frac{w}{2}}{\sin \theta} \tag{7}$$

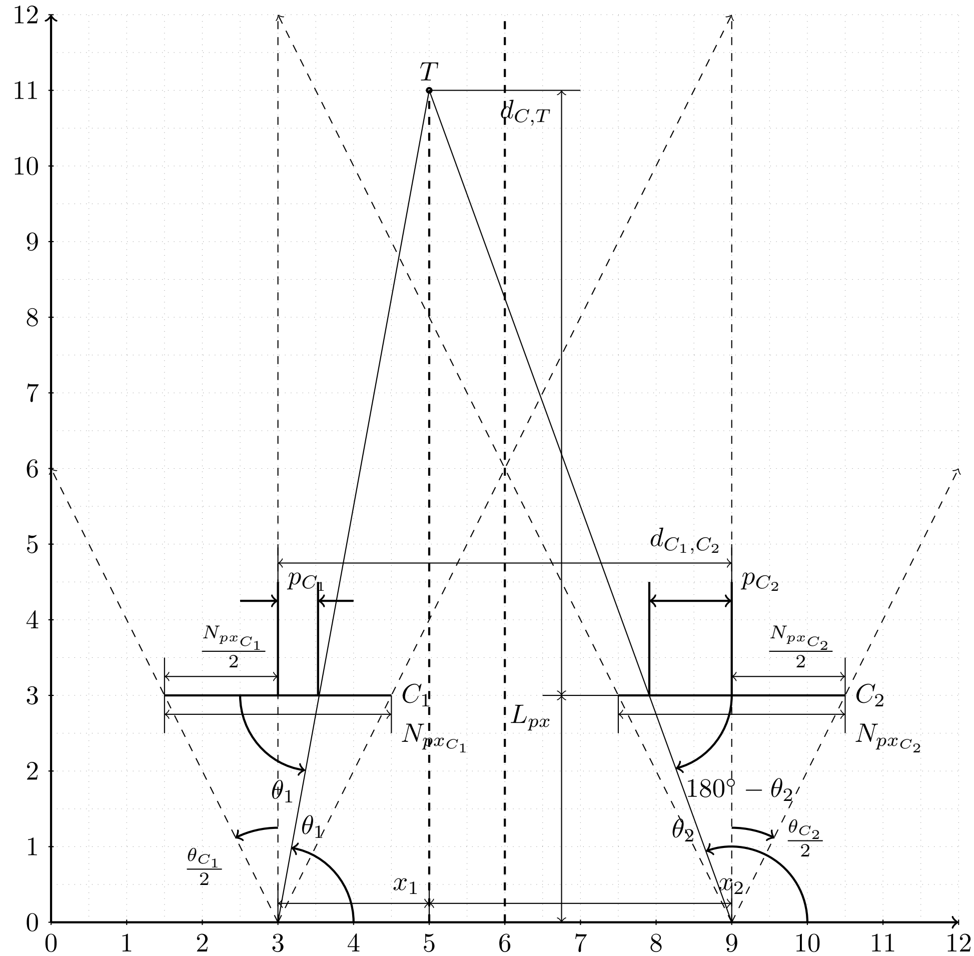


d_{min}
θ
θ
$\theta 2$
r2
h'
$r_2 \cos \theta_2$
$r_2 \sin \theta_2$
$\frac{w}{2}$
90° - θ
0
1
2
3
4
5
6
7
8
9
10
11
12

$$
\begin{aligned}
h' &= \left(\frac{w}{2} + r_2 \sin\theta_2\right)\tan\theta && (8)\\
d_{min} &= r_2\cos\theta_2 - h' && (9)\\
&= r_2\cos\theta_2 - \left(\frac{w}{2} + r_2\sin\theta_2\right)\tan\theta && (10)\\
&= r_2\cos\theta_2 - \left(\frac{w}{2}\right)\tan\theta - r_2\sin\theta_2\tan\theta && (11)\\
&= r_2\cos\theta_2 - r_2\sin\theta_2\left(\frac{\sin\theta}{\cos\theta}\right) - \left(\frac{w}{2}\right)\tan\theta && (12)\\
&= \frac{r_2}{\cos\theta}\left(\cos\theta_2\cos\theta - \sin\theta_2\sin\theta\right) - \left(\frac{w}{2}\right)\tan\theta && (13)\\
&= \left(\frac{r_2}{\cos\theta}\right)\cos\left(\theta_2+\theta\right) - \left(\frac{w}{2}\right)\tan\theta && (14)\\
d_{min} &= \left(r_2\cos\left(\theta_2+\theta\right) - \left(\frac{w}{2}\right)\sin\theta\right)\frac{1}{\cos\theta} && (15)
\end{aligned}
$$

$$
d_{min} \geq \Delta_{min} \tag{16}
$$

$$
\begin{aligned}
t_{hit} &= \frac{d_{min}}{v_{robo}} && (17)\\
\implies t_{decision} &\leq t_{hit} && (18)
\end{aligned}
$$



T
$d_{C,T}$
d_{C_1,C_2}
p_{C_1}
p_{C_2}
$\frac{N_{pxC_1}}{2}$
$\frac{N_{pxC_2}}{2}$
C_1
C_2
N_{pxC_1}
N_{pxC_2}
L_{px}
θ_1
θ_1
θ_2
$180° - \theta_2$
$\frac{\theta_{C_1}}{2}$
$\frac{\theta_{C_2}}{2}$
x_1
x_2
0 1 2 3 4 5 6 7 8 9 10 11 12

$$\tan\left(\frac{\theta_{C_1}}{2}\right) = \frac{\left(\frac{N_{px_{C_1}}}{2}\right)}{L_{px}} \tag{19}$$

$$\implies L_{px} = \left(\frac{N_{px_{C_1}}}{2}\right)\frac{1}{\tan\left(\frac{\theta_{C_1}}{2}\right)} \tag{20}$$

$$= \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)} \tag{21}$$

$$\tan\theta_1 = \frac{L_{px}}{p_{C_1}} \tag{22}$$

$$= \frac{\left(\frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)}\right)}{p_{C_1}} \tag{23}$$

$$= \frac{N_{px_{C_1}}}{2p_{C_1}\tan\left(\frac{\theta_{C_1}}{2}\right)} \tag{24}$$

$$\implies \frac{1}{\tan\theta_1} = \frac{2p_{C_1}\tan\left(\frac{\theta_{C_1}}{2}\right)}{N_{px_{C_1}}} \tag{25}$$

$$\tan(180^\circ - \theta_2) = \frac{L_{px}}{p_{C_2}} \tag{26}$$

$$= \frac{\left(\frac{N_{px_{C_2}}}{2\tan\left(\frac{\theta_{C_2}}{2}\right)}\right)}{p_{C_2}} \tag{27}$$

$$= \frac{N_{px_{C_2}}}{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)} \tag{28}$$

$$\implies \tan\theta_2 = -\left(\frac{N_{px_{C_2}}}{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}\right) \tag{29}$$

$$\implies \frac{1}{\tan\theta_2} = -\left(\frac{2p_{C_2}\tan\left(\frac{\theta_{C_2}}{2}\right)}{N_{px_{C_2}}}\right) \tag{30}$$

$$Let : \sigma_{px/unit} be density of pixels per unit of measurement \tag{31}$$

$$Let : L_{units} = \frac{L_{px}}{\sigma_{px/unit}} \tag{32}$$

$$\tan\theta_1 = \frac{d_{C,T} + L_{units}}{x_1} \tag{33}$$

$$\tan\theta_1 = \frac{d_{C,T} + L_{units}}{x_1} \tag{34}$$

$$\implies x_1 = \frac{d_{C,T} + L_{units}}{\tan\theta_1} \tag{35}$$

$$\tan(180^\circ - \theta_2) = \frac{d_{C,T} + L_{units}}{x_2} \tag{36}$$

$$\implies x_2 = \frac{d_{C,T} + L_{px}}{\tan(180^\circ - \theta_2)} \tag{37}$$

$$\implies x_1 + x_2 = \frac{d_{C,T} + L_{units}}{\tan\theta_1} + \frac{d_{C,T} + L_{units}}{\tan(180^\circ - \theta_2)} \tag{38}$$

$$= (d_{C,T} + L_{units}) \left(\frac{1}{\tan\theta_1} + \frac{1}{\tan(180^\circ - \theta_2)} \right) \tag{39}$$

$$= (d_{C,T} + L_{units}) \left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2} \right) \tag{40}$$

$$= (d_{C,T} + L_{units}) \left(\frac{\tan\theta_2 - \tan\theta_1}{\tan\theta_1 \tan\theta_2} \right) \tag{41}$$

$$\tag{42}$$

$$
\begin{aligned}
x_1 + x_2 &= d_{C_1,C_2} && (43)\\
\implies d_{C_1,C_2} &= (d_{C,T} + L_{units}) \left(\frac{\tan\theta_2 - \tan\theta_1}{\tan\theta_1 \tan\theta_2} \right) && (44)\\
\implies d_{C,T} + L_{units} &= d_{C_1,C_2} \left(\frac{\tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1} \right) && (45)\\
&= \frac{d_{C_1,C_2} \tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1} && (46)\\
\implies d_{C,T} &= \frac{d_{C_1,C_2} \tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1} - L_{units} && (47)\\
\implies d_{C,T} &= \frac{d_{C_1,C_2} \tan\theta_1 \tan\theta_2}{\tan\theta_2 - \tan\theta_1} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}} && (48)\\
\implies d_{C,T} &= \frac{d_{C_1,C_2}}{\left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}\right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}} && (49)\\
&= \frac{d_{C_1,C_2}}{\left(\frac{1}{\tan\theta_1} - \frac{1}{\tan\theta_2}\right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}} && (50)\\
&= \frac{d_{C_1,C_2}}{\left(\frac{2p_{C_1} \tan\left(\frac{\theta_{C_1}}{2}\right)}{N_{px_{C_1}}} + \frac{2p_{C_2} \tan\left(\frac{\theta_{C_2}}{2}\right)}{N_{px_{C_2}}} \right)} - \frac{N_{px_{C_1}}}{2\tan\left(\frac{\theta_{C_1}}{2}\right)\sigma_{px/unit}} && (51)
\end{aligned}
$$

$$Let : \tan\left(\frac{\theta_{C_1}}{2}\right) = \tan\left(\frac{\theta_{C_2}}{2}\right) = \tan\left(\frac{\theta_C}{2}\right) \tag{52}$$

$$Let : N_{px_{C_1}} = N_{px_{C_2}} = N_{px_C} \tag{53}$$

$$\implies d_{C,T} = \frac{d_{C_1,C_2}}{\left(\frac{2p_{C_1}\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}} + \frac{2p_{C_2}\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}}\right)} - \frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)\sigma_{px/unit}} \tag{54}$$

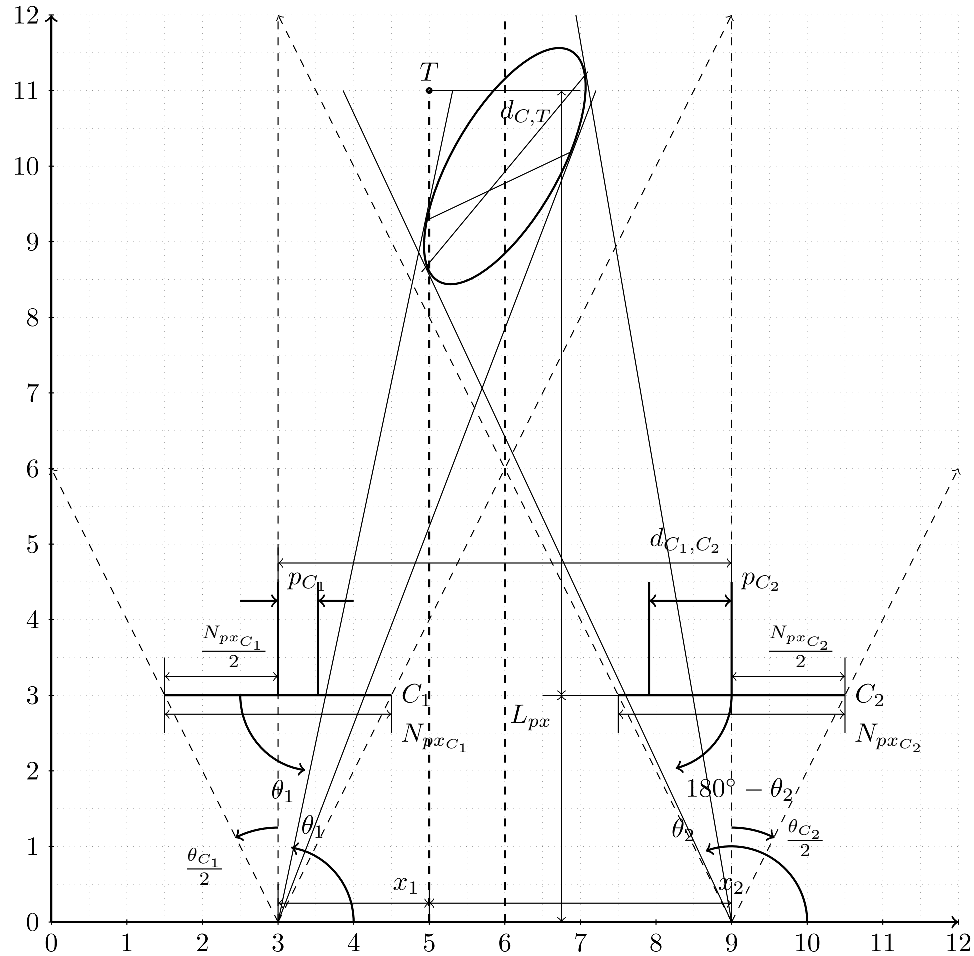
$$= \frac{d_{C_1,C_2}}{\left(\frac{2\tan\left(\frac{\theta_C}{2}\right)}{N_{px_C}}\right)(p_{C_1} + p_{C_2})} - \frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)\sigma_{px/unit}} \tag{55}$$

$$= \frac{d_{C_1,C_2}N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)(p_{C_1} + p_{C_2})} - \frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)\sigma_{px/unit}} \tag{56}$$

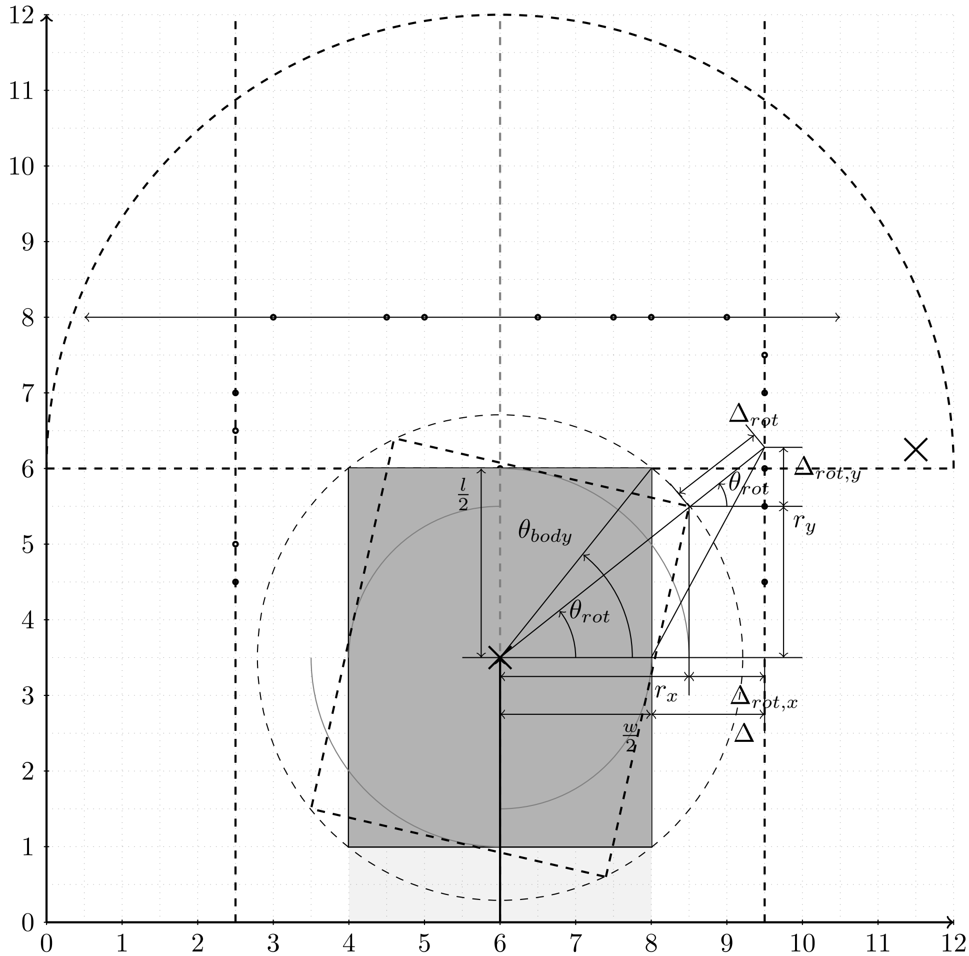
$$= \left(\frac{N_{px_C}}{2\tan\left(\frac{\theta_C}{2}\right)}\right)\left(\frac{d_{C_1,C_2}}{p_{C_1} + p_{C_2}} - \frac{1}{\sigma_{px/unit}}\right) \tag{57}$$

$$If : \frac{1}{\sigma_{px/unit}} \ll \frac{d_{C_1,C_2}}{p_{C_1} + p_{C_2}} \tag{58}$$

$$d_{C,T} \approx \frac{N_{px_C}d_{C_1,C_2}}{2\tan\left(\frac{\theta_C}{2}\right)(p_{C_1} + p_{C_2})} \tag{59}$$



T
$d_{C,T}$
d_{C_1,C_2}
p_{C_1}
p_{C_2}
$\frac{N_{px_{C_1}}}{2}$
$\frac{N_{px_{C_2}}}{2}$
C_1
C_2
$N_{px_{C_1}}$
$N_{px_{C_2}}$
L_{px}
θ_1
$180^\circ - \theta_2$
θ_2
$\frac{\theta_{C_1}}{2}$
$\frac{\theta_{C_2}}{2}$
x_1
x_2



Δ_{rot}
$\Delta_{rot,y}$
θ_{rot}
r_y
$\frac{l}{2}$
θ_{body}
θ_{rot}
r_x
$\Delta_{rot,x}$
Δ
$\frac{w}{2}$

$$r\cos(\theta_{rot}) = \frac{w}{2} \tag{60}$$

$$r\sin(\theta_{rot}) = \frac{l}{2} \tag{61}$$

$$r = \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{w}{2}\right)^2} \tag{62}$$

$$\implies r = \frac{1}{2}\sqrt{l^2 + w^2} \tag{63}$$

$$\frac{w}{2} + \Delta \geq r + \Delta_{min} \tag{64}$$

$$\implies \Delta \geq \frac{1}{2}\sqrt{l^2 + w^2} + \Delta_{min} - \frac{w}{2} \tag{65}$$

$$(r + \Delta_{rot})\cos(\theta_{rot}) = \frac{w}{2} + \Delta \tag{66}$$

$$\cos(\theta_{rot}) = \left(\frac{\frac{w}{2} + \Delta}{r + \Delta_{rot}}\right) \tag{67}$$

$$\sin(\theta_{rot}) = \sqrt{1 - \left(\frac{\frac{w}{2} + \Delta}{r + \Delta_{rot}}\right)^2} \tag{68}$$

$$= \frac{\sqrt{(r + \Delta_{rot})^2 - \left(\frac{w}{2} + \Delta\right)^2}}{r + \Delta_{rot}} \tag{69}$$

$$= \frac{\sqrt{r^2 + 2r\Delta_{rot} + \Delta_{rot}^2 - \left(\frac{w}{2}\right)^2 - 2\left(\frac{w}{2}\right) - \Delta^2}}{r + \Delta_{rot}} \tag{70}$$

$$= \frac{\sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{w}{2}\right)^2 + 2r\Delta_{rot} + \Delta_{rot}^2 - \left(\frac{w}{2}\right)^2 - w - \Delta^2}}{r + \Delta_{rot}} \tag{71}$$

$$= \frac{\sqrt{\left(\frac{l}{2}\right)^2 + 2r\Delta_{rot} + \Delta_{rot}^2 - w - \Delta^2}}{r + \Delta_{rot}} \tag{72}$$

$$\implies (r + \Delta_{rot})\sin(\theta_{rot}) = \sqrt{\left(\frac{l}{2}\right)^2 + 2r\Delta_{rot} + \Delta_{rot}^2 - w - \Delta^2} \tag{73}$$

$$= \sqrt{\frac{l^2}{4} + 2r\Delta_{rot} + \Delta_{rot}^2 - w - \Delta^2} \tag{74}$$

$$y_{gap} = (r + \Delta_{rot})\sin(\theta_{rot}) \tag{75}$$

$$= \sqrt{\frac{l^2}{4} + 2r\Delta_{rot} + \Delta_{rot}^2 - w - \Delta^2} \tag{76}$$

$$y_{gap} \geq \sqrt{\frac{l^2}{4} + 2r\Delta_{min} + \Delta_{min}^2 - w - \Delta^2} \tag{77}$$

$$y_{gap} \geq \frac{w}{2} + \Delta_{min} \tag{78}$$

$$\Delta = r_{gap} \cos(\theta_{gap}) \tag{79}$$

$$\implies y_{gap} \geq \sqrt{\frac{l^2}{4} + 2r\Delta_{min} + \Delta_{min}^2 - w - r_{gap}^2 \cos^2(\theta_{gap})} \tag{80}$$

$$\implies y_{gap} \geq \sqrt{\frac{l^2}{4} + \Delta_{min}\sqrt{l^2 + w^2} + \Delta_{min}^2 - w - r_{gap}^2 \cos^2(\theta_{gap})} \tag{81}$$

Algorithm 1: Find rotation direction

Result: r if right rotation is possible, l if left rotation is possible, *none* if rotation is not possible

compute $C_{right} = \sqrt{\frac{l^2}{4} + \Delta_{min}\sqrt{l^2 + w^2} + \Delta_{min}^2 - w - r_{right,gap}^2 \cos^2(\theta_{right,gap})}$;

compute $C_{left} = \sqrt{\frac{l^2}{4} + \Delta_{min}\sqrt{l^2 + w^2} + \Delta_{min}^2 - w - r_{left,gap}^2 \cos^2(\theta_{left,gap})}$;

compute $y_{right,gap} = r_{right,gap} \sin(\theta_{right,gap})$;

compute $y_{left,gap} = r_{left,gap} \sin(\theta_{left,gap})$;

compute $y_{gap,min} = \left(\frac{w}{2} + \Delta_{min}\right)$;

if $y_{right,gap} \geq \max(C_{right}, y_{gap,min})$ **then**

  return r;

else

  if $y_{left,gap} \geq \max(C_{left}, y_{gap,min})$ **then**

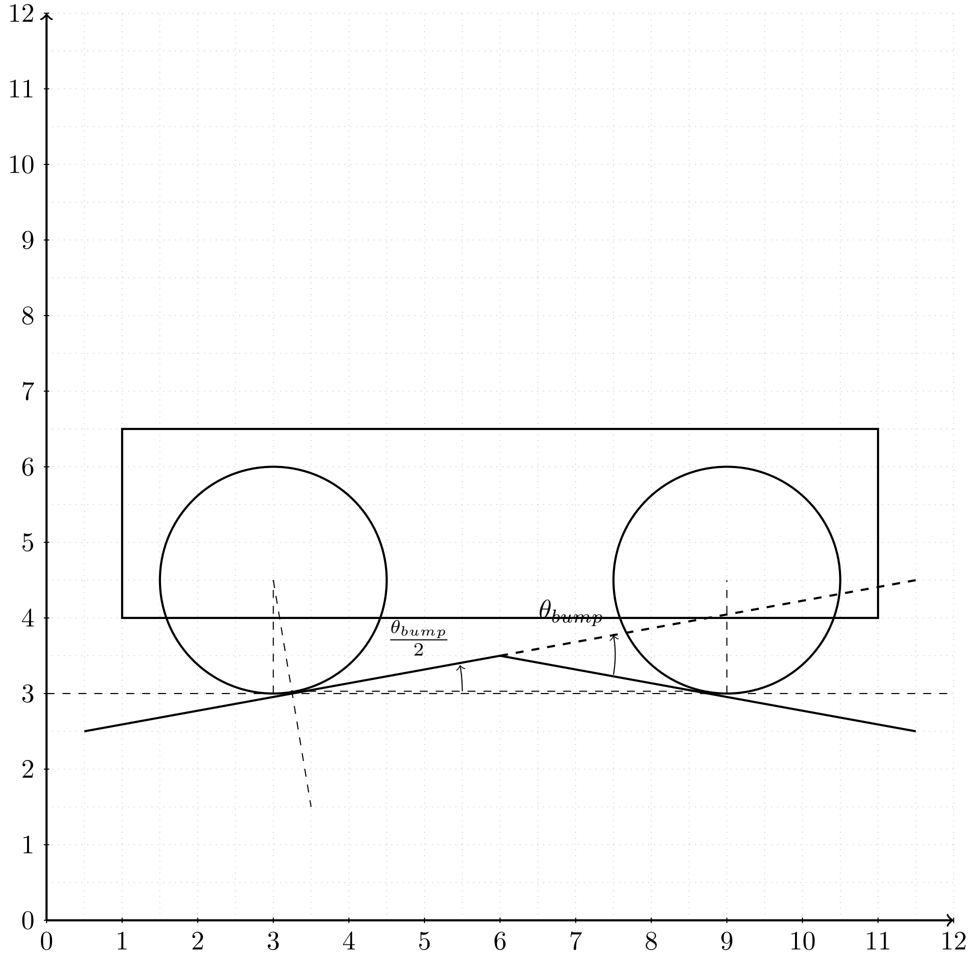
    return l;

  else

    return *none*;

  end

end



0
1
2
3
4
5
6
7
8
9
10
11
12
θ_{bump}
$\frac{\theta_{bump}}{2}$

$$r\cos\left(\frac{\theta_{bump}}{2}\right) = (r-h) + \Delta_{h,min} + p \tag{82}$$

$$\implies p = r\cos\left(\frac{\theta_{bump}}{2}\right) - r + h - \Delta_{h,min} \tag{83}$$

$$r\sin\left(\frac{\theta_{bump}}{2}\right) + q = \frac{d}{2} \tag{84}$$

$$\implies q = \frac{d}{2} - r\sin\left(\frac{\theta_{bump}}{2}\right) \tag{85}$$

$$\tan\left(\frac{\theta_{bump}}{2}\right) = \frac{p}{q} \tag{86}$$

$$\implies \tan\left(\frac{\theta_{bump}}{2}\right) = \frac{r\cos\left(\frac{\theta_{bump}}{2}\right) - r + h - \Delta_{h,min}}{\frac{d}{2} - r\sin\left(\frac{\theta_{bump}}{2}\right)} \tag{87}$$

$$\left(\frac{d}{2} - r\sin\left(\frac{\theta_{bump}}{2}\right)\right)\tan\left(\frac{\theta_{bump}}{2}\right) = r\cos\left(\frac{\theta_{bump}}{2}\right) - r + h - \Delta_{h,min} \tag{88}$$

$$\left(\frac{d}{2} - r\sin\left(\frac{\theta_{bump}}{2}\right)\right)\frac{\sin\left(\frac{\theta_{bump}}{2}\right)}{\cos\left(\frac{\theta_{bump}}{2}\right)} = r\cos\left(\frac{\theta_{bump}}{2}\right) - r + h - \Delta_{h,min} \tag{89}$$

$$\left(\frac{d}{2}\right)\sin\left(\frac{\theta_{bump}}{2}\right) - r\sin^2\left(\frac{\theta_{bump}}{2}\right) = r\cos^2\left(\frac{\theta_{bump}}{2}\right) + (-r + h - \Delta_{h,min})\cos\left(\frac{\theta_{bump}}{2}\right) \tag{90}$$

$$\implies \left(\frac{d}{2}\right)\sin\left(\frac{\theta_{bump}}{2}\right) = r\cos^2\left(\frac{\theta_{bump}}{2}\right) + r\sin^2\left(\frac{\theta_{bump}}{2}\right) + (-r + h - \Delta_{h,min})\cos\left(\frac{\theta_{bump}}{2}\right) \tag{91}$$

$$\left(\frac{d}{2}\right)\sin\left(\frac{\theta_{bump}}{2}\right) = r\left(\cos^2\left(\frac{\theta_{bump}}{2}\right) + \sin^2\left(\frac{\theta_{bump}}{2}\right)\right) + (-r + h - \Delta_{h,min})\cos\left(\frac{\theta_{bump}}{2}\right) \tag{92}$$

$$\left(\frac{d}{2}\right)\sin\left(\frac{\theta_{bump}}{2}\right) = r + (-r + h - \Delta_{h,min})\cos\left(\frac{\theta_{bump}}{2}\right) \tag{93}$$

$$\left(\left(\frac{d}{2}\right)\sin\left(\frac{\theta_{bump}}{2}\right)\right)^2 = \left(r + (-r + h - \Delta_{h,min})\cos\left(\frac{\theta_{bump}}{2}\right)\right)^2 \tag{94}$$

$$\left(\frac{d}{2}\right)^2\sin^2\left(\frac{\theta_{bump}}{2}\right) = r^2 + 2r\left(-r + h - \Delta_{h,min}\right)\cos\left(\frac{\theta_{bump}}{2}\right) + \left(-r + h - \Delta_{h,min}\right)^2\cos^2\left(\right. \tag{95}$$

$$r^2 + 2r\left(-r+h-\Delta_{h,min}\right)\cos\left(\frac{\theta_{bump}}{2}\right) + \left(-r+h-\Delta_{h,min}\right)^2\cos^2\left(\frac{\theta_{bump}}{2}\right) - \left(\frac{d}{2}\right)^2\sin^2\left(\frac{\theta_{bump}}{2}\right) = \tag{96}$$

$$r^2 + 2r\left(-r+h-\Delta_{h,min}\right)\cos\left(\frac{\theta_{bump}}{2}\right) + \left(-r+h-\Delta_{h,min}\right)^2\cos^2\left(\frac{\theta_{bump}}{2}\right) - \left(\frac{d}{2}\right)^2\left(1-\cos^2\left(\frac{\theta_{bump}}{2}\right)\right) = \tag{97}$$

$$r^2 + 2r\left(-r+h-\Delta_{h,min}\right)\cos\left(\frac{\theta_{bump}}{2}\right) + \left(-r+h-\Delta_{h,min}\right)^2\cos^2\left(\frac{\theta_{bump}}{2}\right) - \left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2\cos^2\left(\frac{\theta_{bump}}{2}\right) = \tag{98}$$

$$\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)\cos^2\left(\frac{\theta_{bump}}{2}\right) + 2r\left(-r+h-\Delta_{h,min}\right)\cos\left(\frac{\theta_{bump}}{2}\right) + r^2 - \left(\frac{d}{2}\right)^2 = \tag{99}$$

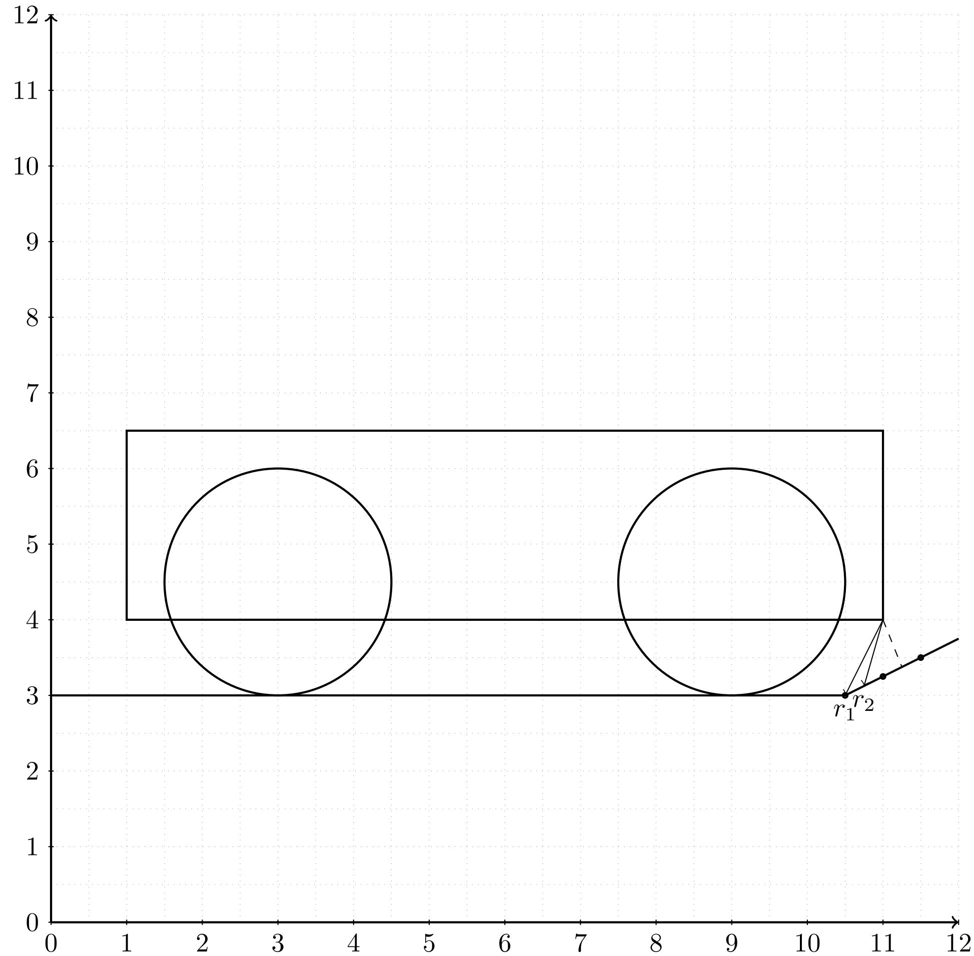
$$\cos\left(\frac{\theta_{bump}}{2}\right) = \frac{-2r\left(-r+h-\Delta_{h,min}\right) \pm \sqrt{\left(2r\left(-r+h-\Delta_{h,min}\right)\right)^2 - 4\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)\left(r^2 - \left(\right.\right.}}{2\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)} \tag{100}$$

$$\implies \frac{\theta_{bump}}{2} = \cos^{-1}\left(\frac{-2r\left(-r+h-\Delta_{h,min}\right) \pm \sqrt{\left(2r\left(-r+h-\Delta_{h,min}\right)\right)^2 - 4\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}}{2\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}\right. \tag{101}$$

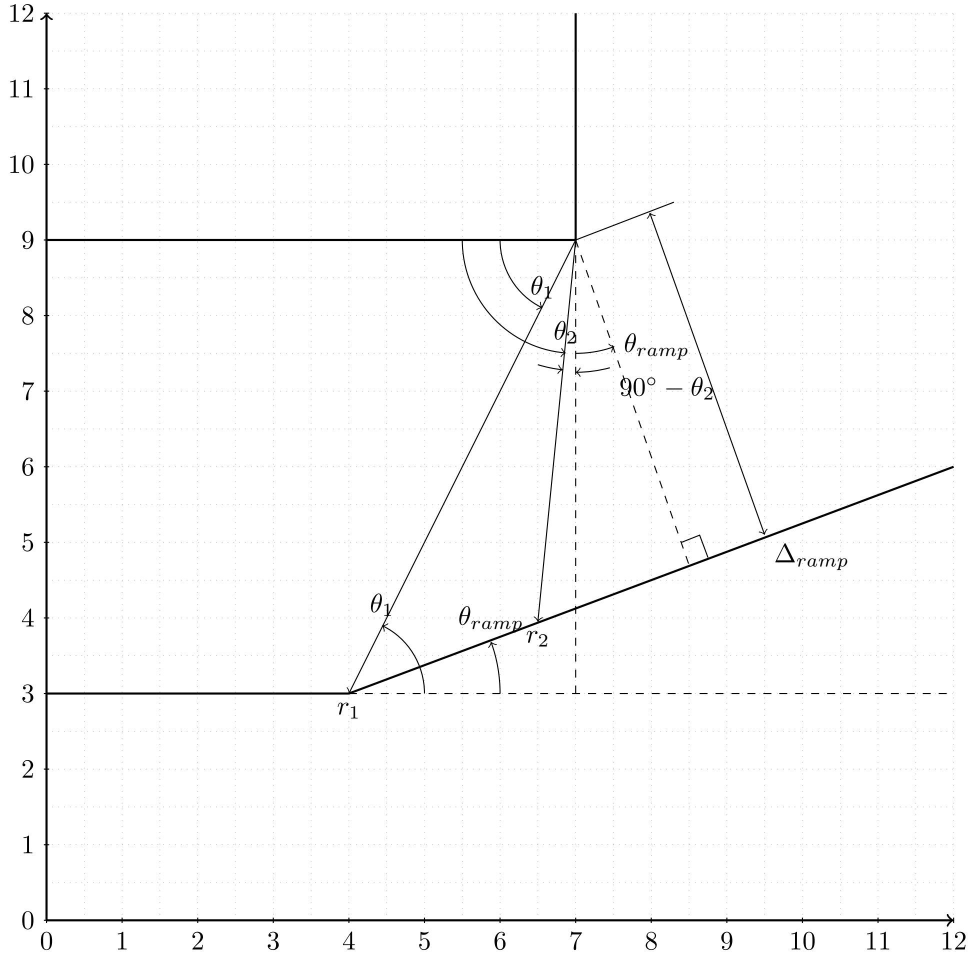
$$\therefore \theta_{bump} = 2\cos^{-1}\left(\frac{-2r\left(-r+h-\Delta_{h,min}\right) \pm \sqrt{\left(2r\left(-r+h-\Delta_{h,min}\right)\right)^2 - 4\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right.}}{2\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}\right. \tag{102}$$

$$= 2\cos^{-1}\left(\frac{-2r\left(-r+h-\Delta_{h,min}\right) \pm 2\sqrt{\left(r\left(-r+h-\Delta_{h,min}\right)\right)^2 - \left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}}{2\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}\right. \tag{103}$$

$$= 2\cos^{-1}\left(\frac{-r\left(-r+h-\Delta_{h,min}\right) \pm \sqrt{\left(r\left(-r+h-\Delta_{h,min}\right)\right)^2 - \left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)\left(\right.}}{\left(\left(\frac{d}{2}\right)^2 + \left(-r+h-\Delta_{h,min}\right)^2\right)}\right. \tag{104}$$



r_1
r_2



θ_1
θ_2
θ_{ramp}
$90° - \theta_2$
Δ_{ramp}
θ_1
θ_{ramp}
r_2
r_1
0
1
2
3
4
5
6
7
8
9
10
11
12

$$r_2 \cos\left(90^\circ - \theta_2 + \theta_{ramp}\right) = \Delta_{ramp} \tag{105}$$

$$\implies r_2 \cos\left(90^\circ - \left(\theta_2 - \theta_{ramp}\right)\right) = \Delta_{ramp} \tag{106}$$

$$r_2 \sin\left(\theta_2 - \theta_{ramp}\right) = \Delta_{ramp} \tag{107}$$

$$r_2 \left(\sin\left(\theta_2\right) \cos\left(\theta_{ramp}\right) - \cos\left(\theta_2\right) \sin\left(\theta_{ramp}\right)\right) = \Delta_{ramp} \tag{108}$$

$$\tan\left(\theta_{ramp}\right) = \frac{r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)}{r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)} \tag{109}$$

$$\implies \sin\left(\theta_{ramp}\right) = \frac{r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} \tag{110}$$

$$\implies \cos\left(\theta_{ramp}\right) = \frac{r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} \tag{111}$$

$$\therefore r_2 \frac{\left(\sin\left(\theta_2\right)\left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right) - \cos\left(\theta_2\right)\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{112}$$

$$r_2 \frac{\left(\sin\left(\theta_2\right)\left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right) - \cos\left(\theta_2\right)\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{113}$$

$$r_2 \frac{\left(\left(r_1 \sin\left(\theta_2\right)\cos\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\cos\left(\theta_2\right)\right) - \left(r_1 \sin\left(\theta_1\right)\cos\left(\theta_2\right) - r_2 \sin\left(\theta_2\right)\cos\left(\theta_2\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{114}$$

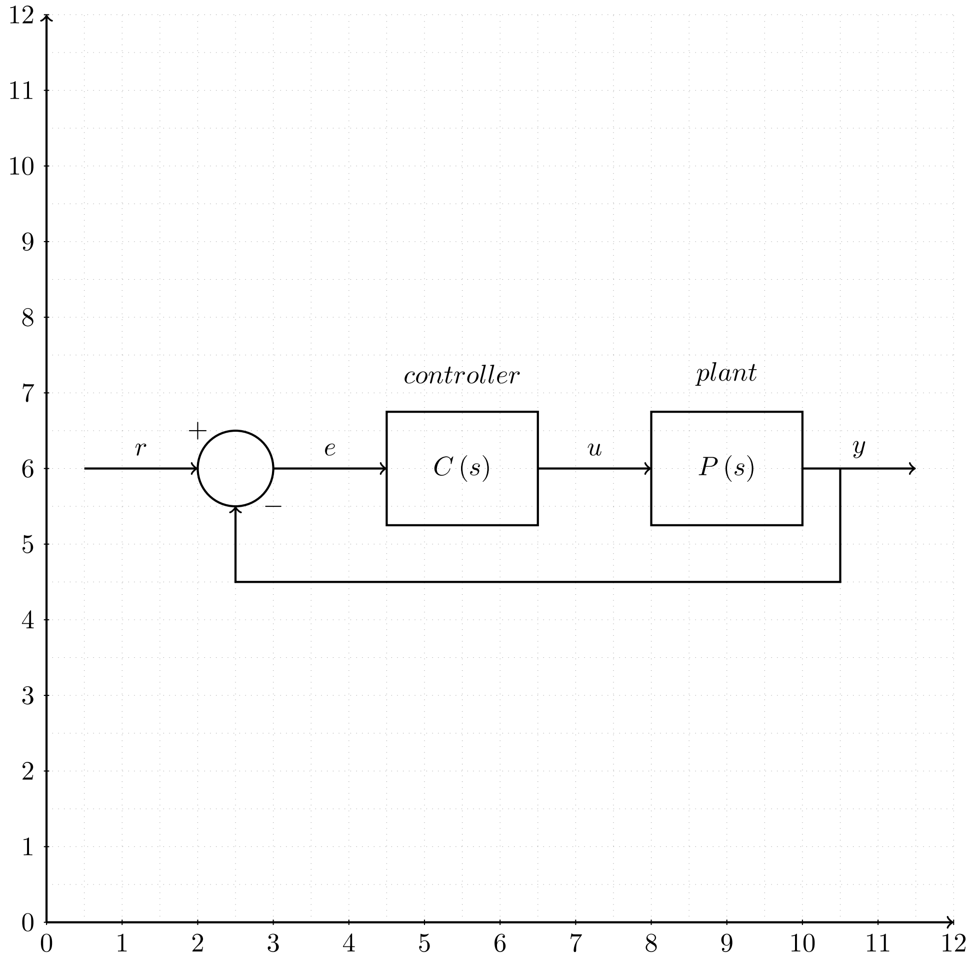
$$r_2 \frac{\left(\left(r_1 \sin\left(\theta_2\right)\cos\left(\theta_1 - r_1 \sin\left(\theta_1\right)\cos\left(\theta_2\right)\right)\right) - \left(r_2 \sin\left(\theta_2\right)\cos\left(\theta_2\right) - r_2 \sin\left(\theta_2\right)\cos\left(\theta_2\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{115}$$

$$r_1 r_2 \frac{\left(\left(r_1 \sin\left(\theta_2\right)\cos\left(\theta_1 - r_1 \sin\left(\theta_1\right)\cos\left(\theta_2\right)\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{116}$$

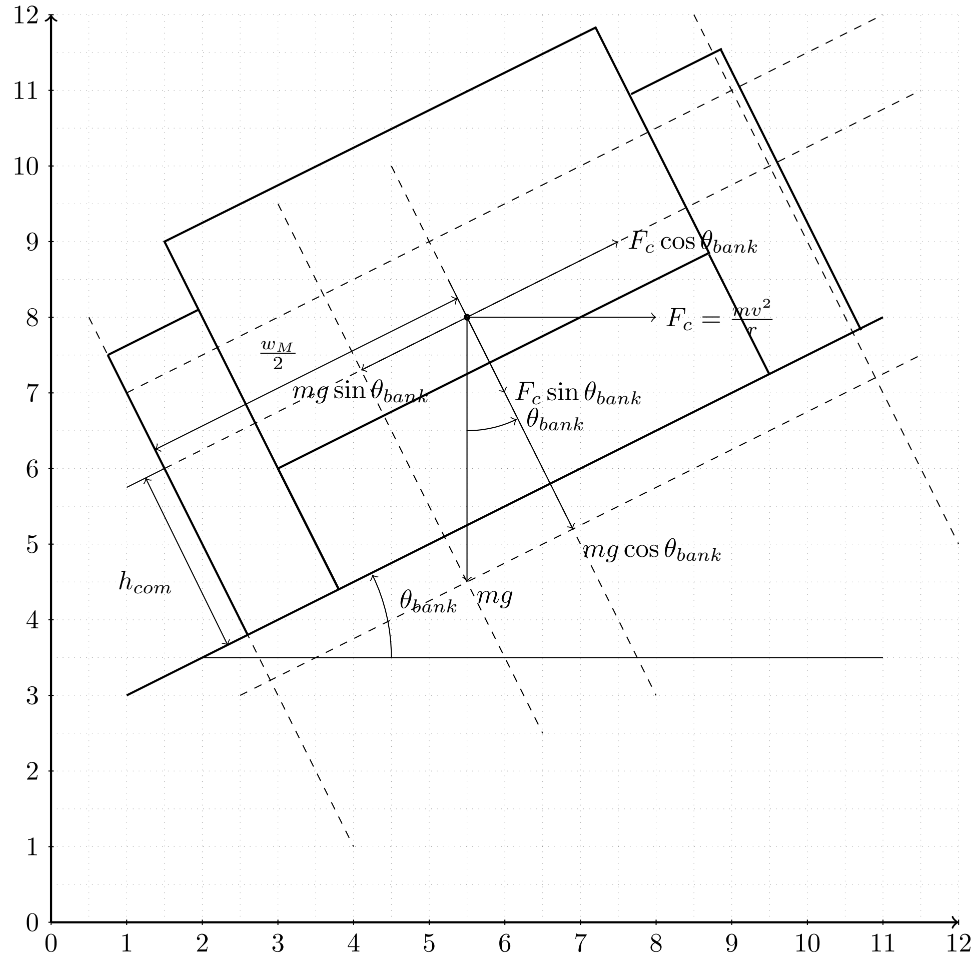
$$r_1 r_2 \frac{\left(\left(\sin\left(\theta_2\right)\cos\left(\theta_1\right) - \sin\left(\theta_1\right)\cos\left(\theta_2\right)\right)\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{117}$$

$$\frac{r_1 r_2 \sin\left(\theta_2 - \theta_1\right)}{\sqrt{\left(r_1 \sin\left(\theta_1\right) - r_2 \sin\left(\theta_2\right)\right)^2 + \left(r_1 \cos\left(\theta_1\right) - r_2 \cos\left(\theta_2\right)\right)^2}} = \Delta_{ramp} \tag{118}$$

$$\frac{r_1 r_2 \sin\left(\theta_2 - \theta_1\right)}{\sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\left(\theta_2 - \theta_1\right)}} = \Delta_{ramp} \tag{119}$$



$$u\left(t\right) = K_p e\left(t\right) + K_i \int e\left(t\right) dt + K_d \frac{de\left(t\right)}{dt} \tag{120}$$


$F_c \cos\theta_{bank}$
$F_c = \frac{mv^2}{r}$
$\frac{w_M}{2}$
$mg \sin\theta_{bank}$
$F_c \sin\theta_{bank}$
θ_{bank}
$mg \cos\theta_{bank}$
h_{com}
θ_{bank}
mg
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12

$$\left(mg\cos\theta_{bank} + F_c\sin\theta_{bank}\right)\frac{w_M}{2} = \left(mg\sin\theta_{bank} - F_c\cos\theta_{bank}\right)h_{com} \tag{121}$$

$$\left(mg + F_c\tan\theta_{bank}\right)\frac{w_M}{2} = \left(mg\tan\theta_{bank} - F_c\right)h_{com} \tag{122}$$

$$mgh_{com}\tan\theta_{bank} - F_c\frac{w_M}{2}\tan\theta_{bank} = mg\frac{w_M}{2} + F_c h_{com} \tag{123}$$

$$\left(mgh_{com} - F_c\frac{w_M}{2}\right)\tan\theta_{bank} = mg\frac{w_M}{2} + F_c h_{com} \tag{124}$$

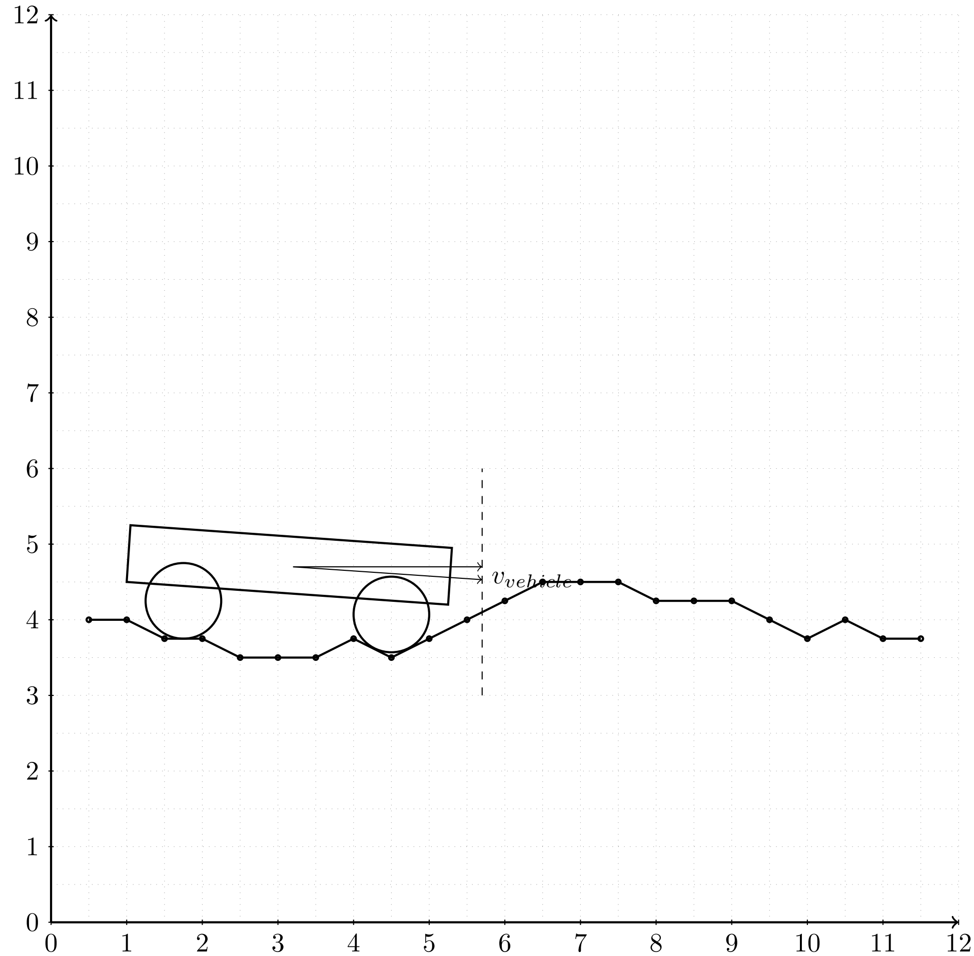
$$\tan\theta_{bank} = \frac{mg\frac{w_M}{2} + F_c h_{com}}{mgh_{com} - F_c\frac{w_M}{2}} \tag{125}$$

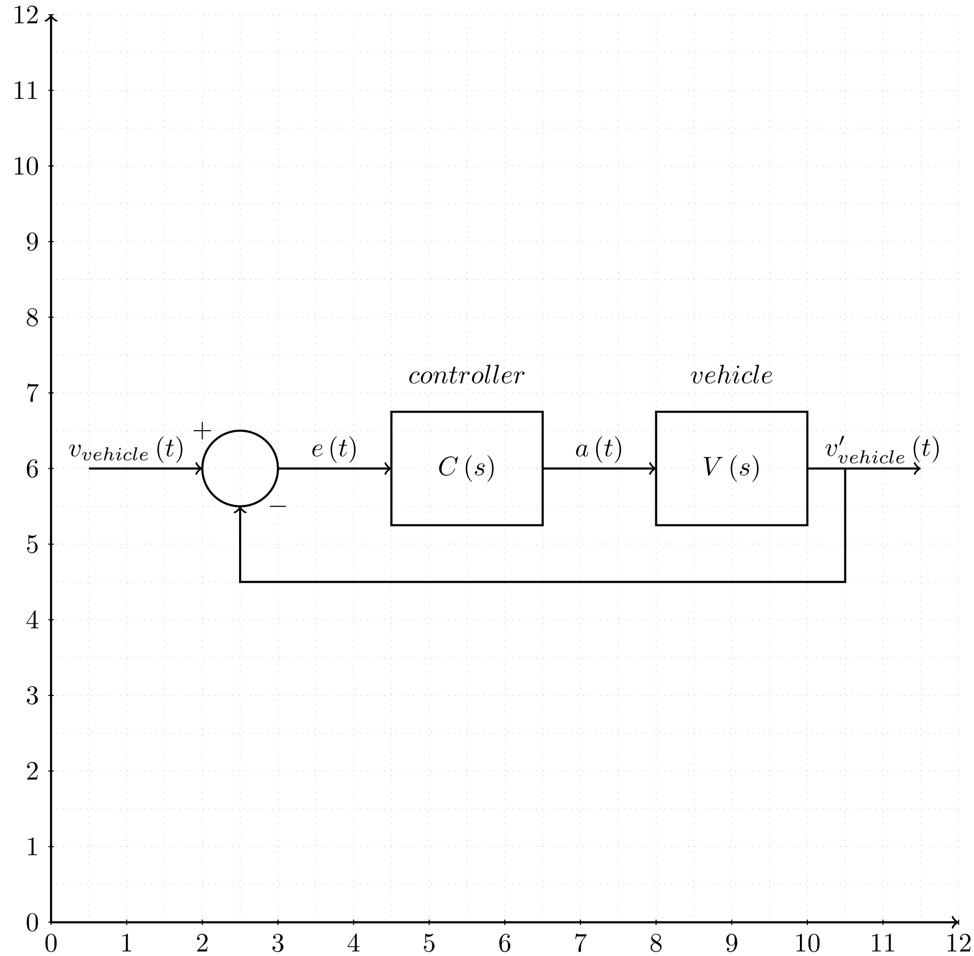
$$\theta_{bank} = \tan^{-1}\left(\frac{mg\frac{w_M}{2} + F_c h_{com}}{mgh_{com} - F_c}\right) \tag{126}$$

$$\theta_{bank} = \tan^{-1}\left(\frac{mg\frac{w_M}{2} + \frac{mv^2}{r}h_{com}}{mgh_{com} - \frac{mv^2}{r}}\right) \tag{127}$$

$$\theta_{bank} = \lim_{r\to\infty}\left(\tan^{-1}\left(\frac{mg\frac{w_M}{2} + \frac{mv^2}{r}h_{com}}{mgh_{com} - \frac{mv^2}{r}}\right)\right) \quad = \tan^{-1}\left(\frac{mg\frac{w_M}{2}}{mgh_{com}}\right) \tag{128}$$

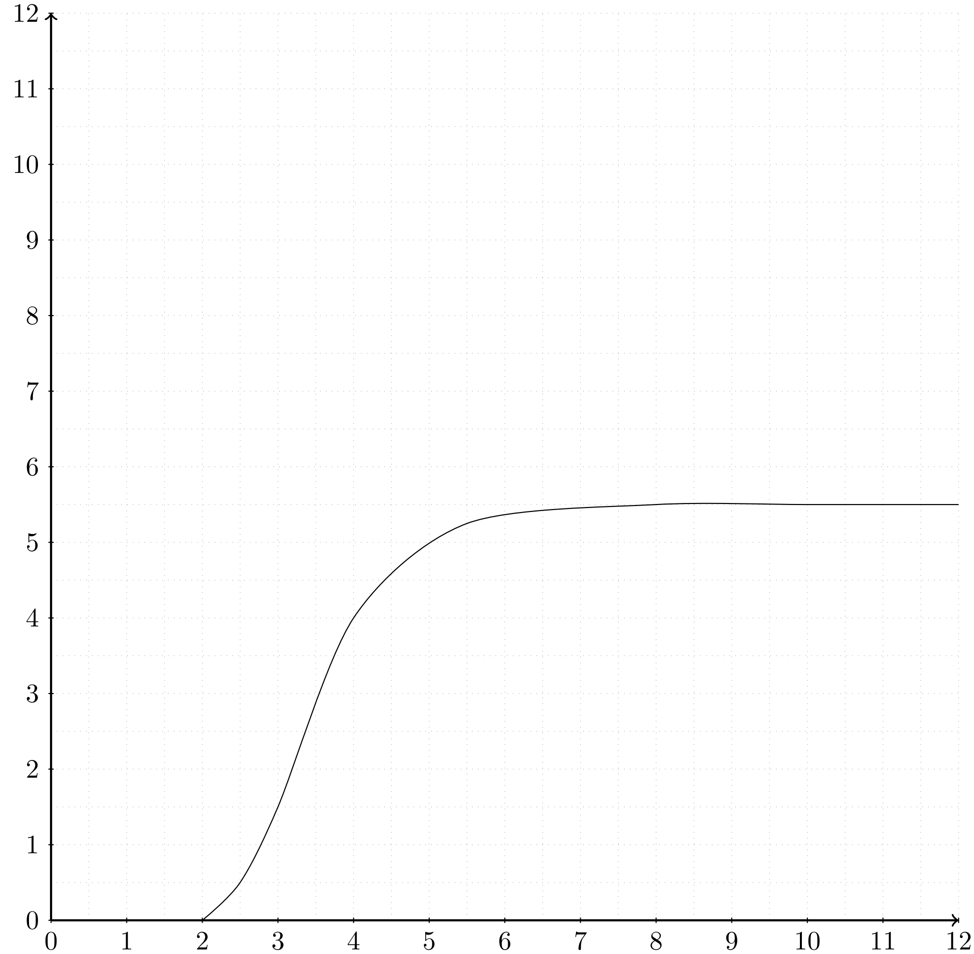
$$= \tan^{-1}\left(\frac{w_M}{2h_{com}}\right) \tag{129}$$


$v_{vehicle}$



$$Let : e\left(t\right) = \left(v'_{vehicle}\left(t\right) - v_{vehicle}\left(t\right)\right) \tag{130}$$

$$a\left(t\right) = K_p e\left(t\right) + K_i \int e\left(t\right) dt + K_d \frac{de\left(t\right)}{dt} \tag{131}$$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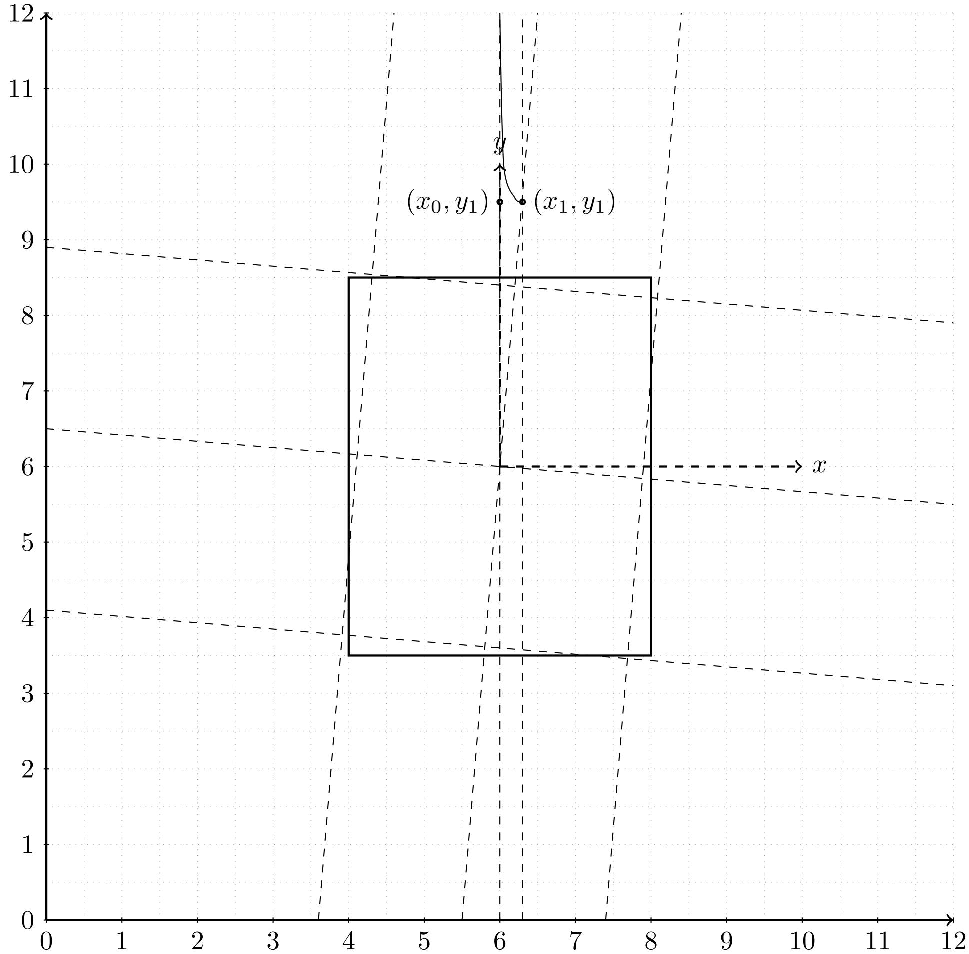
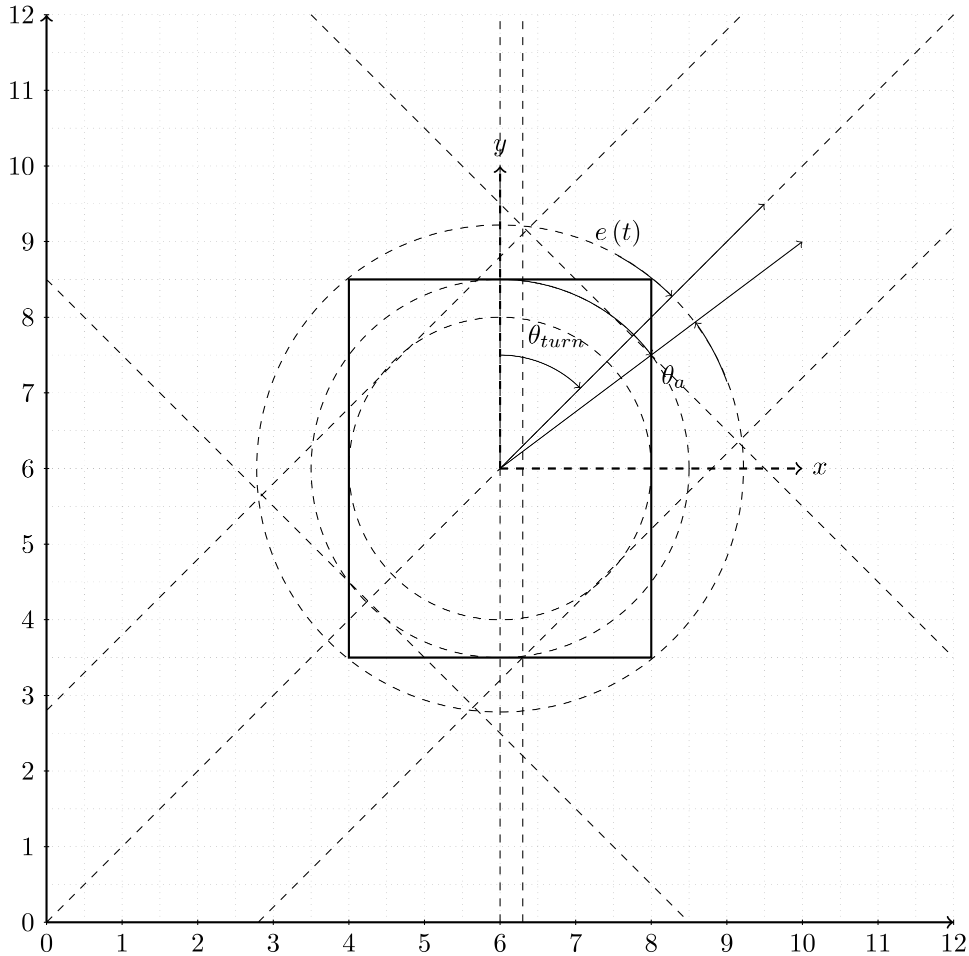


$$Let : e(t) = (x_1(t) - x_0(t)), x_0(t) = 0 \tag{132}$$

$$\implies e(t) = x_1(t) \tag{133}$$

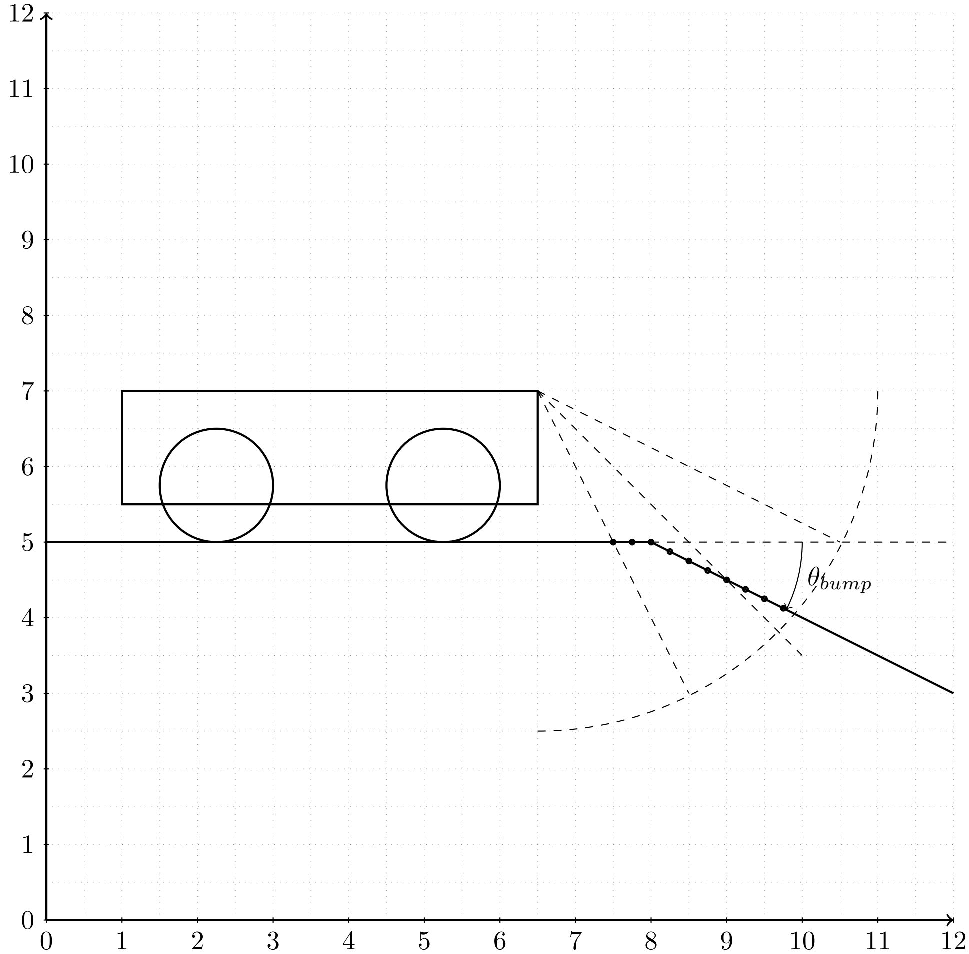
$$\theta(t) = K_p e(t) + K_i \int e(t)\, dt + K_d \frac{de(t)}{dt} \tag{134}$$

$$\implies \theta(t) = K_p x_1(t) + K_i \int_0^{x_1} x_1(t)\, dt + K_d \frac{dx_1(t)}{dt} \tag{135}$$



$$Let : e\left(t\right) = \left(\theta_a\left(t\right) - \theta_{turn}\left(t\right)\right) \tag{136}$$

$$\alpha\left(t\right) = K_p e\left(t\right) + K_i \int e\left(t\right) dt + K_d \frac{de\left(t\right)}{dt} \tag{137}$$



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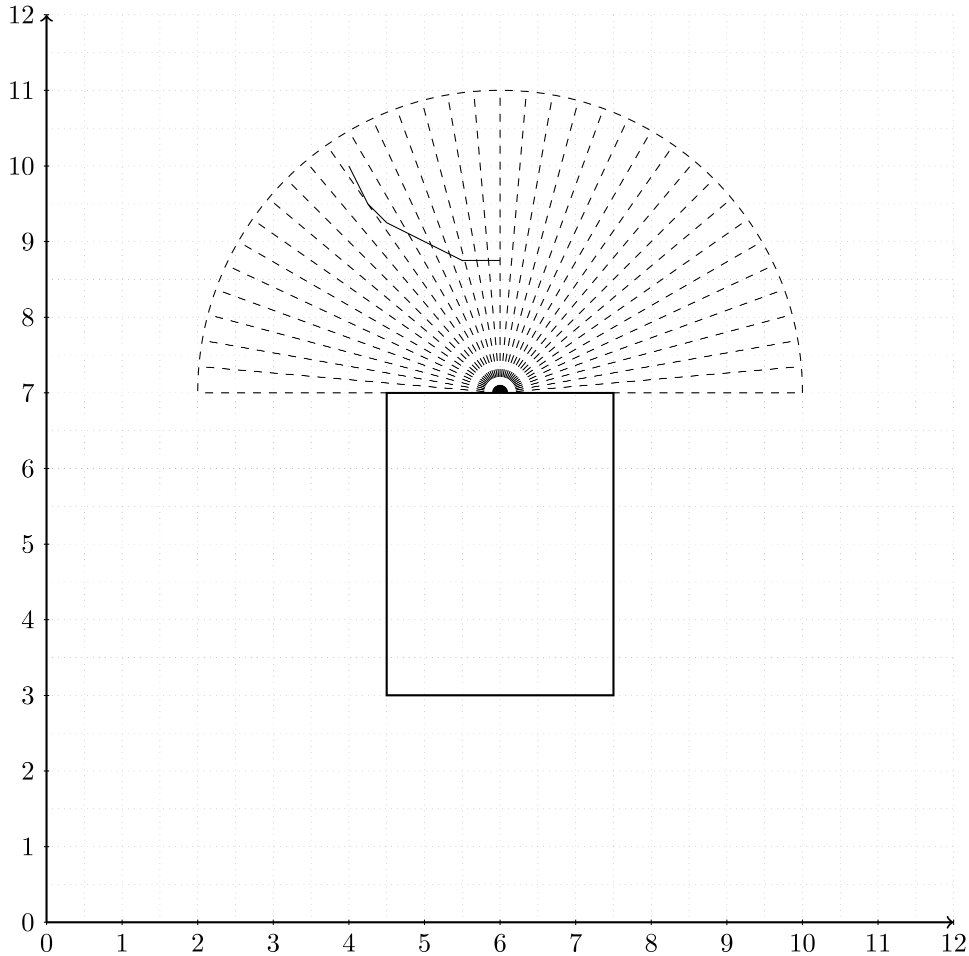
θ_{bump}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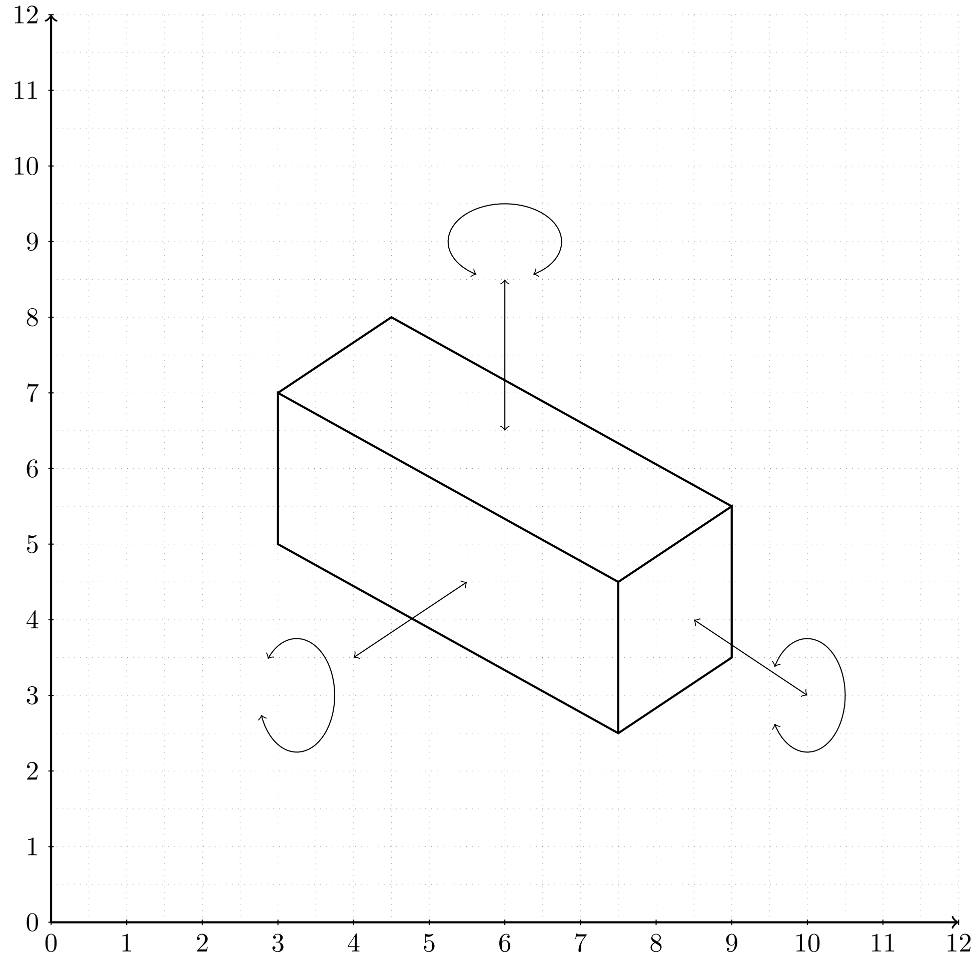

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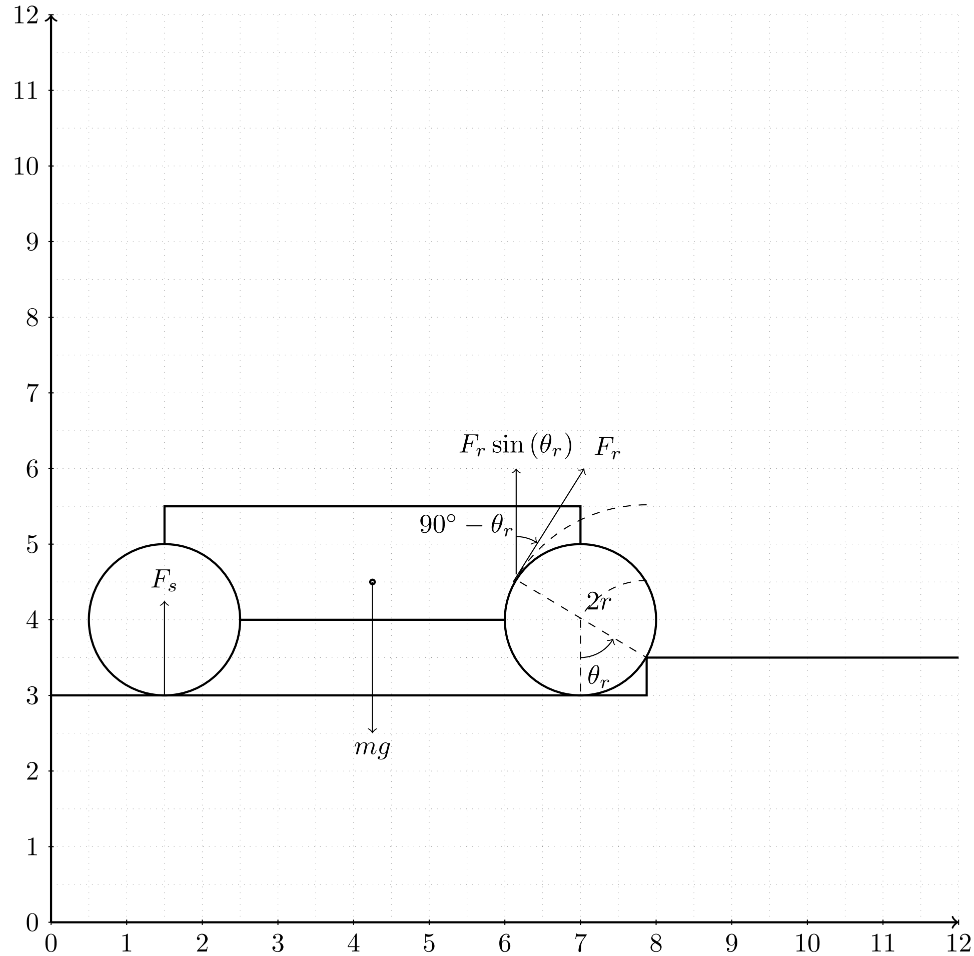


$F_r \sin(\theta_r)$
F_r
$90° - \theta_r$
F_s
$2r$
θ_r
mg

$$mg = F_s + F_r \sin(\theta_r) \tag{138}$$

$$\implies F_s = mg - F_r \sin(\theta_r) \tag{139}$$

$$F_s \times (l + r\sin(\theta_r)) + F_r \times 2r = mg \times \left(\frac{l}{2} + r\sin(\theta_r)\right) \tag{140}$$

$$\implies (mg - F_r \sin(\theta_r)) \times (l + r\sin(\theta_r)) + F_r \times 2r = mg \times \left(\frac{l}{2} + r\sin(\theta_r)\right) \tag{141}$$

$$\implies mg \times (l + r\sin(\theta_r)) - F_r \sin(\theta_r) \times (l + r\sin(\theta_r)) + F_r \times 2r = mg \times \left(\frac{l}{2} + r\sin(\theta_r)\right) \tag{142}$$

$$\implies mg \times (l + r\sin(\theta_r)) - mg \times \left(\frac{l}{2} + r\sin(\theta_r)\right) = F_r \sin(\theta_r) \times (l + r\sin(\theta_r)) - F_r \times 2r \tag{143}$$

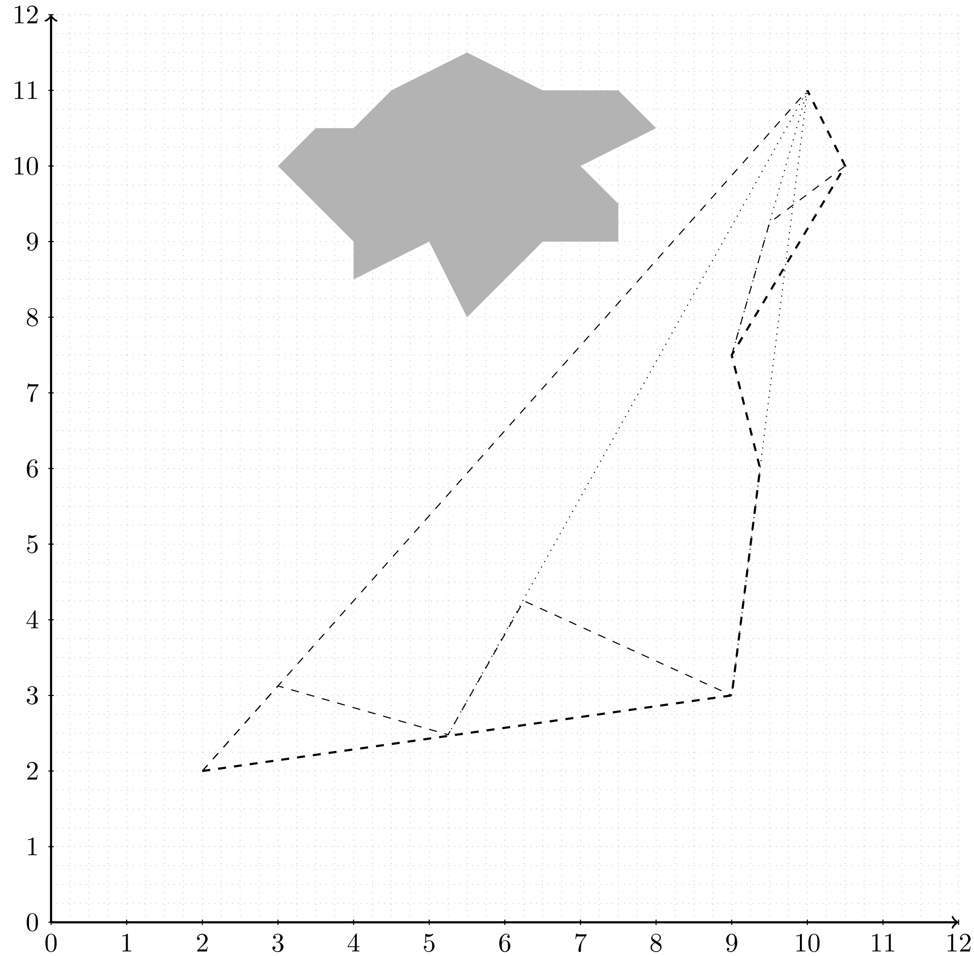
$$\implies mg \times \frac{l}{2} = F_r (\sin(\theta_r) \times (l + r\sin(\theta_r)) - 2 \times r) \tag{144}$$

$$\implies mg \times \frac{l}{2} = F_r \left(l\sin(\theta_r) + r\sin^2(\theta_r) - 2r\right) \tag{145}$$

$$\implies F_r = \frac{mgl}{2\left(l\sin(\theta_r) + r\sin^2(\theta_r) - 2r\right)} \tag{146}$$

$$\tau = F_r \times 2r \tag{147}$$

$$\implies \tau = 2F_r r \tag{148}$$



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