Correctness of Local Probability Propagation in Graphical Models with Loops

by

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Outline

1. Thoughts
2. Introduction
   - Definitions
   - The Problem

1. bayesian networks with no loops converge to correct posterior probability
2. empirical studies show that bayes nets with loops also converge
3. we don't know why it works in theory
4. certain single-looped bayes nets can provably be shown to converge
5. graphical model - lauritzen 1996
6. bayesian network, markov network
7. singly-connected networks - belief propagation, pearl 1988
8. BU - Belief Update, BR - Belief Revision, MM - Maximum Marginal, MAP - Maximum a Posteriori
9. explanation + example of belief update and belief revision on singly-connected markov net

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Probabilistic Graphical Models

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Definition

A Probabilistic Graphical Model (PGM) is a graph, either directed or undirected, in which the nodes correspond to random variables, and the edges correspond to direct probabilistic interactions between them.

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A Bayes network is a directed acyclic PGM whose edges could be seen as "cause" and "effect", e.g. an edge $X \to Y$ means that X directly influences Y, or X is the cause of Y.

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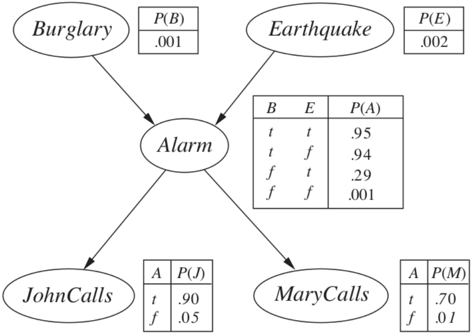

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A Markov Random Field (MRF), or a Markov network, is an undirected PGM which is used when the relations between random variables are symmetric, rather than hierarchical, e.g. pixels in an image.

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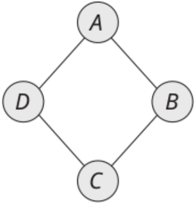

$\phi_1(A,B)$			$\phi_2(B,C)$			$\phi_3(C,D)$			$\phi_4(D,A)$		
a^0	b^0	30	b^0	c^0	100	c^0	d^0	1	d^0	a^0	100
a^0	b^1	5	b^0	c^1	1	c^0	d^1	100	d^0	a^1	1
a^1	b^0	1	b^1	c^0	1	c^1	d^0	100	d^1	a^0	1
a^1	b^1	10	b^1	c^1	100	c^1	d^1	1	d^1	a^1	100

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Belief Propagation

content...

"Loopy" Belief Propagation

content...

- How far is the steady-state belief from the correct posterior when the update rules (equations 2.7-2.8) are applied in a loopy network?
- What are the conditions under which the BU assignment equals the MM assignment when the update rules are applied in a loopy network?
- What are the conditions under which the BR assignment equals the MAP assignment when the update rules are applied in a loopy network?