

Correctness of Local Probability Propagation in Graphical Models with Loops

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1 Thoughts

2 Introduction

- Definitions
- The Problem

- ① bayesian networks with no loops converge to correct posterior probability
- ② empirical studies show that bayes nets with loops also converge
- ③ we don't know why it works in theory
- ④ certain single-looped bayes nets can provably be shown to converge
- ⑤ graphical model - lauritzen 1996
- ⑥ bayesian network, markov network
- ⑦ singly-connected networks - belief propagation, pearl 1988
- ⑧ BU - Belief Update, BR - Belief Revision, MM - Maximum Marginal, MAP - Maximum a Posteriori
- ⑨ explanation + example of belief update and belief revision on singly-connected markov net

Probabilistic Graphical Models

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Definition

A Probabilistic Graphical Model (PGM) is a graph, either directed or undirected, in which the nodes correspond to random variables, and the edges correspond to direct probabilistic interactions between them.

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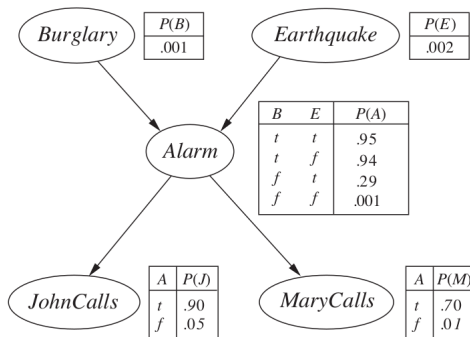
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A Bayes network is a directed acyclic PGM whose edges could be seen as “cause” and “effect”, e.g. an edge $X \rightarrow Y$ means that X directly influences Y , or X is the cause of Y .

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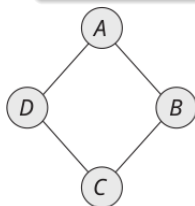
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$\phi_1(A, B)$			$\phi_2(B, C)$			$\phi_3(C, D)$			$\phi_4(D, A)$		
a^0	b^0	30	b^0	c^0	100	c^0	d^0	1	d^0	a^0	100
a^0	b^1	5	b^0	c^1	1	c^0	d^1	100	d^0	a^1	1
a^1	b^0	1	b^1	c^0	1	c^1	d^0	100	d^1	a^0	1
a^1	b^1	10	b^1	c^1	100	c^1	d^1	1	d^1	a^1	100

Belief Propagation

content...

“Loopy” Belief Propagation

content...

- How far is the steady-state belief from the correct posterior when the update rules (equations 2.7-2.8) are applied in a loopy network?
- What are the conditions under which the BU assignment equals the MM assignment when the update rules are applied in a loopy network?
- What are the conditions under which the BR assignment equals the MAP assignment when the update rules are applied in a loopy network?