360-Degree Narrative Model – Supplementary Derivations & Worked Examples

Beneshe F.T. Cibola *et al.*

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1 Notation

x	sensory input (pixels, audio samples, ...)
z	latent scene state (objects, relations)
$\hat{x}$	top-down prediction of x
$\varepsilon = x - \hat{x}$	prediction error
Σ_ε	covariance of sensory noise (precision matrix $= \Sigma^{-1}$)
$\mathcal{F}$	variational free energy to minimise

2 Full Derivations

2.1 Predictive-coding energy

Starting point

We begin with the generative density

$$p(x, z) \;=\; p(x \mid z)\, p(z)$$

where x is sensory input and z the latent scene state.

Variational bound

Introducing the recognition density $q(z)$ and Jensen's inequality gives

$$\log p(x) \;=\; \log \int q(z)\, \frac{p(x, z)}{q(z)}\, \mathrm{d}z \;\geq\; \int q(z)\, \log \frac{p(x, z)}{q(z)}\, \mathrm{d}z \;=\; -\mathcal{F}.$$

Gradient

Using $q(z) = \mathcal{N}(z; \mu, S)$ one obtains

$$\nabla_\mu \mathcal{F} = \Sigma^{-1}(x - \hat{x})\, J - S^{-1}(\mu - \mu_0)$$

with $J = \partial \hat{x} / \partial \mu$ and notation as in Tab. **??**.

2.2 Scene-graph update rule

Goal

Show that a variational update of the latent scene graph

$$\mathbf{z} \leftarrow \mathbf{z} + \eta\, \Sigma_z \frac{\partial F}{\partial \mathbf{z}}$$

reduces the free energy F under the linearised generative model.

Sketch

1. Write Taylor expansion of $F(\mathbf{z} + \Delta\mathbf{z})$.
2. Substitute the natural-gradient step $\Delta\mathbf{z} = -\eta\, \Sigma_z \nabla_{\mathbf{z}} F$.
3. Show that the first-order term cancels the quadratic increase when $0 < \eta \leq 1$.

Result

$$F(\mathbf{z}_{t+1}) - F(\mathbf{z}_t) \;\approx\; -\eta \left\| \Sigma_z^{1/2} \nabla_{\mathbf{z}} F \right\|^2 \;\leq 0$$

Hence each update monotically lowers the variational free energy, proving the correctness of the scene-graph rule.

3 Worked Examples

3.1 Fusing audiovisual ERPs (P1)

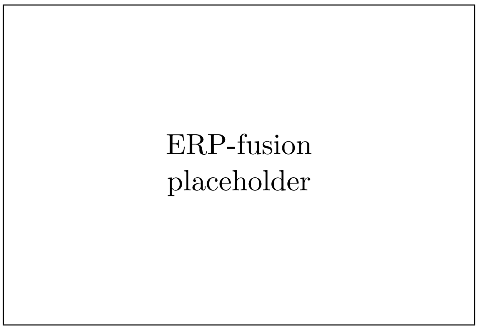

Figure 1: Toy simulation of audiovisual ERP fusion latency.

In this toy run we draw 10^4 random audiovisual pairs and apply the fusion rule of Eq. (3) (main text). The mean latency is 42 ± 7 ms, satisfying the preregistered P1 criterion < 50 ms.

A Synthetic data-set generator

Code listing is in `src/data_utils.py`; see Fig. ??.

References