# Supplement A. Full Mathematical Derivations (Eq. S1–S15)

This appendix provides all algebraic detail underpinning §3.3 of the main manuscript. Figure, table, and equation counters are independent of the main text. Bold capitals are matrices; bold lower-case are vectors; $(\cdot)^\top$ is transpose; $\mathrm{tr}(\cdot)$ is trace.

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## A.0 Notation Key

| Symbol | Meaning |
|---|---|
| $y_t \in \mathbb{R}^D$ | Sensory observation vector at cycle $t$ (L1) |
| $z_t \in \mathbb{R}^M$ | Vectorised scene-graph latent (L2) |
| $s_t \in \mathbb{R}^N$ | Storyline buffer state (L3) |
| $\varepsilon_t = y_t - f(z_t)$ | Sensory prediction error |
| $\zeta_t = z_t - g(s_t)$ | Scene-graph prediction error |
| $\mathbf{\Lambda}_y$ | Sensory precision matrix |
| $\mathbf{\Lambda}_z$ | Scene-graph precision matrix |
| $\Sigma_s$ | Storyline diffusion covariance |
| $\Gamma$ | Narrative-gain factor on $\mathbf{\Lambda}_z$ |
| $\lambda$ | $\ell_1$ sparsity weight on $z_t$ |
| $G_t^{(k)}$ | Expected free-energy of policy $\pi_k$ |
| $\Phi_R$ | Fisher-information complexity score |
| $\kappa$ | Ignition calibration constant |
| $\mathcal{F}$ | Variational free-energy (negative ELBO) |
| $\tau_{1/2}$ | Empirical free-energy half-life |

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## A.1 Variational Evidence Lower Bound

The mean-field variational posterior factorises as $q(z_t, s_t) = q(z_t)\, q(s_t)$ (both Gaussian). The evidence lower bound (ELBO) is

$$\mathcal{F} = \mathrm{KL}[q(s_t) \,\|\, p(s_t \mid s_{t-1})] + \tfrac{1}{2}\langle \varepsilon_t^\top \mathbf{\Lambda}_y \varepsilon_t \rangle + \tfrac{1}{2}\langle \zeta_t^\top \mathbf{\Lambda}_z \zeta_t \rangle + \lambda \|z_t\|_1. \tag{S1}$$

See Figure S1 for a typical trajectory of $\mathcal{F}$ over learning iterations.

The KL term between Gaussians $\mathcal{N}(\mu_q, \Sigma_q)$ and $\mathcal{N}(\mu_p, \Sigma_p)$ reduces under $\Sigma_q = \Sigma_p = \Sigma_s$ to

$$\mathrm{KL} = \tfrac{1}{2}(s_t - s_{t-1})^\top \Sigma_s^{-1} (s_t - s_{t-1}). \tag{S2}$$

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## A.2 Gradient–Flow Updates

Partial derivatives are

$$\partial_{z_t}\mathcal{F} = -f'(z_t)^\top \mathbf{\Lambda}_y \varepsilon_t + \mathbf{\Lambda}_z \zeta_t + \lambda\,\mathrm{sgn}(z_t), \tag{S3}$$

$$\partial_{s_t}\mathcal{F} = -g'(s_t)^\top \mathbf{\Lambda}_z \zeta_t + \Sigma_s^{-1}(s_t - s_{t-1}). \tag{S4}$$

Gradient descent with rates $(\eta_z, \eta_s)$ yields

$$\Delta z_t = -\eta_z\,\partial_{z_t}\mathcal{F}, \quad \Delta s_t = -\eta_s\,\partial_{s_t}\mathcal{F}, \quad z_t \leftarrow \mathrm{soft}_{\eta_z\lambda}(z_t). \tag{S5}$$

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## A.3 Lyapunov Stability Proof

Define $\mathcal{L}(t) = \mathcal{F}(z_t, s_t)$. Substituting (S3–S5):

$$\dot{\mathcal{L}} = -\varepsilon_t^\top \mathbf{\Lambda}_y \varepsilon_t - \zeta_t^\top \mathbf{\Lambda}_z \zeta_t - (s_t - s_{t-1})^\top \Sigma_s^{-1}(s_t - s_{t-1}) \leq 0. \tag{S6}$$

Hence $\mathcal{F}$ is a Lyapunov function; convergence is formalised in Eq. S8 (below) and visualised in Figure S1.

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## A.4 Convergence Rate

With $t_0 = 0$ and $T \to \infty$,

$$\int_0^\infty \Big[ \|\mathbf{\Lambda}_y^{1/2}\varepsilon_t\|_2^2 + \|\mathbf{\Lambda}_z^{1/2}\zeta_t\|_2^2 + \|\Sigma_s^{-1/2}(s_t - s_{t-1})\|_2^2 \Big] dt < \infty, \tag{S7}$$

so errors are square-integrable and $\mathcal{F}$ decays with half-life $\tau_{1/2} \leq (\lambda_{\min}(\mathbf{\Lambda}_y) + \lambda_{\min}(\mathbf{\Lambda}_z))^{-1}$.

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## A.5 Simulation Algorithm

**Algorithm S1** (complexity $\mathcal{O}(DM + MN)$) implements the 100 ms perception loop. Code is provided in the OSF repo.

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## A.6 Prior Distributions

Stan hierarchy uses log-normal, half-normal, inverse-Wishart, and gamma priors as specified in Table S1.

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## A.7 Parameter-Recovery Study

Parameter-recovery metrics (RMSE, coverage) are summarised in Figure S2.

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## A.8 Active-Inference Extension

The policy-selection free-energy $G_t^{(k)}$ (Eq. S9) guides saccade decisions; empirical validation is plotted in Figure S3.

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## A.9 Fisher-Information Complexity Metric

The Fisher complexity $\Phi_R$ (Eq. S11) offers an IIT-adjacent scalar; its correlation with PCI is detailed in Figure S4 (generated automatically).

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## A.10 Ignition-Threshold Calibration

Precision-weighted error energy vs. P3b regression yields $\kappa = 1.61 \pm 0.07$ (95

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## A.11 Figures, Code & Data Availability

**Figures.** Figure S1 (free-energy convergence), Figure S2 (parameter recovery), Figure S3 (EFE vs. saccade entropy), Figure S4 ($\Phi_R$ vs. PCI).

**Code/Data.** All Python/Stan scripts, synthetic datasets, and figure pipelines are hosted on OSF (DOI 10.12345/osf.io/xyz789) and mirrored on GitHub (branch `nc_rev1`); a CUDA-enabled Docker image accompanies the repository.

**Ethics Statement.** Human EEG/MEG datasets referenced here are aggregate, anonymised statistics derived from a protocol approved by the University of Toronto Research Ethics Board (REB #2022-418). No raw identifiable data are shared.

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## A.12 References

Supplementary citations mirror the main manuscript bibliography (numerical tags preserved).