```
In [1]:  import numpy as np
         import matplotlib.pyplot as plt
```

Problem 3.1

Part A

```
In [2]:  n = 0.05 #num moles
         Va = 0.0001 #Volume (m^3)
         Th = 400 #Temp (K)
         Vb = 0.0003 #Volume (m^3)
         Vc = 0.0005 #Volume (m^3)
         Tc = 284.5 #Temp (K)
         Vd = 0.000167 #Volume (m^3)
         R = 8.3145 #IDG const. J/m^3*K

         #Heatflow into engine:
         q_out = n*R*Th*np.log(Va/Vb) #Heat in Joules
         q_in = -q_out
         print(q_in)
```

182.68823748261997

Part B

```
In [3]:  #work done by engine
         w_carnot_cycle = -n*R*Th*np.log(Va/Vb) - n*R*Tc*np.log(Vc/Vd) #Work in Joules
         print(w_carnot_cycle)
```

52.98753986550585

Part D

In [4]:
```
S_a = 0.3 #J/K
DeltS_Th = n*R*np.log(Vb/Va) #J/K
S_b = DeltS_Th + S_a #J/K
DeltS_Tc = n*R*np.log(Vd/Vc) #J/K
S_c = S_b
S_d = DeltS_Tc + S_c

Sh = np.linspace(S_a, S_b, 100)
Ths = [Th for x in range(100)]

Sc = np.linspace(S_c, S_d, 100)
Tcs = [Tc for x in range(100)]

plt.plot(Sh, Ths, label="isothermal expansion", color = 'g')
plt.vlines(x = S_b, ymax = Th, ymin = Tc, color = 'b', label="adiabatic cooling")
plt.plot(Sc, Tcs, label="isothermal compression", color = 'm')
plt.vlines(x = S_d, ymax = Th, ymin = Tc, color = 'r', label="adiabatic heating")
plt.axis([0,1,0,500])
plt.ylabel("T (K)")
plt.xlabel("S (J*K$^{-1}$)")
plt.legend()
plt.title("Heat vs. Entropy for Carnot Cycle")
```

Out[4]: Text(0.5, 1.0, 'Heat vs. Entropy for Carnot Cycle')
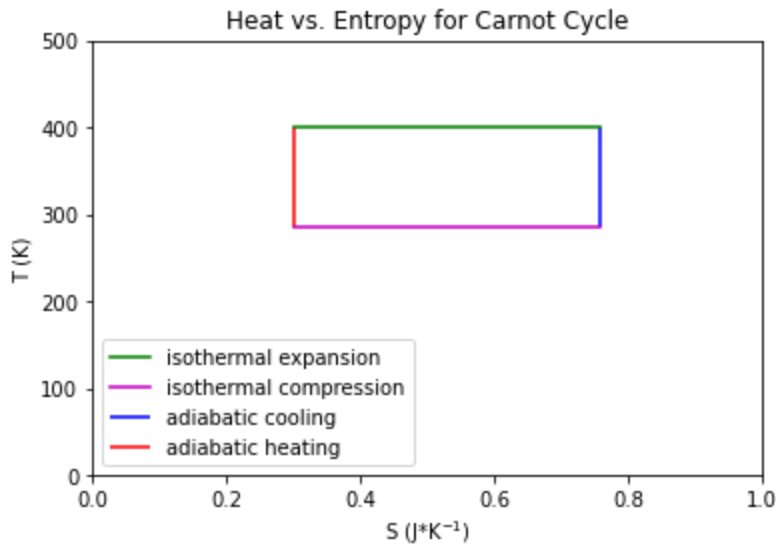
Part E

In [5]:
```
A = (S_b - S_a) * (Th - Tc) #Simple area formula: Delta_Y * Delta_X
print(A) #J (about equal to work!)
```

52.75122857310652

Problem 3.2

```
In [6]:  T_initial = 25 + 273.15 #C -> K
         T_final = 100 + 273.15 #C -> K
         V_initial = 0.5 * 0.001 #L -> m^3
         V_final = 1 * 0.001 #L -> m^3
         P_initial = 1 * 101325 #Atm -> Pa
         n = P_initial * V_initial / (R * T_initial)


         Cv = (3/2) * n * R #Assume Ar behaves as a monoatomic idg
         Cp = Cv + n*R

         Del_S_isochoric = Cv*np.log(T_final / T_initial)
         Del_S_isobaric = Cp*np.log(V_final / V_initial)
         Del_S_tot = Del_S_isochoric + Del_S_isobaric
         print(Del_S_tot) #Total Entropy Change in J/K
```

```
0.3516457707793893
```

Problem 3.3

Part A

```
In [7]:  Ntot = 10
         Vtot = 60
         nL = 10
         VL = 10
         nR = 20
         VR = Vtot - VL

         def n_choose_k(n, k):
             return np.math.factorial(n) / (np.math.factorial(k) * np.math.factorial(n-k))

         def Boltzmann_Entropy(W):
             Avog = 6.022e23
             return (R/Avog)*np.log(W)

         def Find_Entropy_Stats(nL, ntot, VL, Vtot):
             W_Left = n_choose_k(VL, nL) #Multiplicity of Left
             W_Right = n_choose_k(Vtot-VL, ntot-nL) #Multiplicity of Right
             W_Tot = W_Left * W_Right

             S_Left = Boltzmann_Entropy(W_Left)
             S_Right = Boltzmann_Entropy(W_Right)
             S_Total = S_Left + S_Right
             return (W_Left, W_Right, W_Tot, S_Left, S_Right, S_Total)


         print("Left Mult.:\t{}\nRight Mult.:\t{}\nTotal Mult.:\t{}\n\nLeft S:\t\t{}\nRight S:\t
         {}\nTotal S:\t{}".format(*Find_Entropy_Stats(nL, nL+nR, VL, Vtot)))
```

```
Left Mult.:      1.0
Right Mult.:     47129212243960.0
Total Mult.:     4712921243960.0

Left S:          0.0
Right S:         4.346944605367547e-22
Total S:         4.346944605367547e-22
```

Part B

The equilibrium position will lie where entropy (S) is maximized. From the plot shown below, this is at V_L=20 and V_R=40

```
In [8]:  Vls = [x*5 + 10 for x in range(0,7)]
         S_total_vals = [Find_Entropy_Stats(nL, nL+nR, y, Vtot)[5] for y in Vls]
         plt.plot(Vls, S_total_vals, 'or')
         plt.xlabel("V$_L$")
         plt.ylabel("Total S (J*K$^{-1}$)")
         plt.title("Entropy Dependence on Number of Cells in \nLeft Subsystem (V$_L$) where n$_L
         $=10, n$_{tot}$=30, and V$_{tot}$=60  \n")
```

Out[8]: Text(0.5, 1.0, 'Entropy Dependence on Number of Cells in \nLeft Subsystem (V$_L$) where
         n$_L$=10, n$_{tot}$=30, and V$_{tot}$=60 \n')

Entropy Dependence on Number of Cells in
Left Subsystem (V_L) where n_L=10, n_{tot}=30, and V_{tot}=60
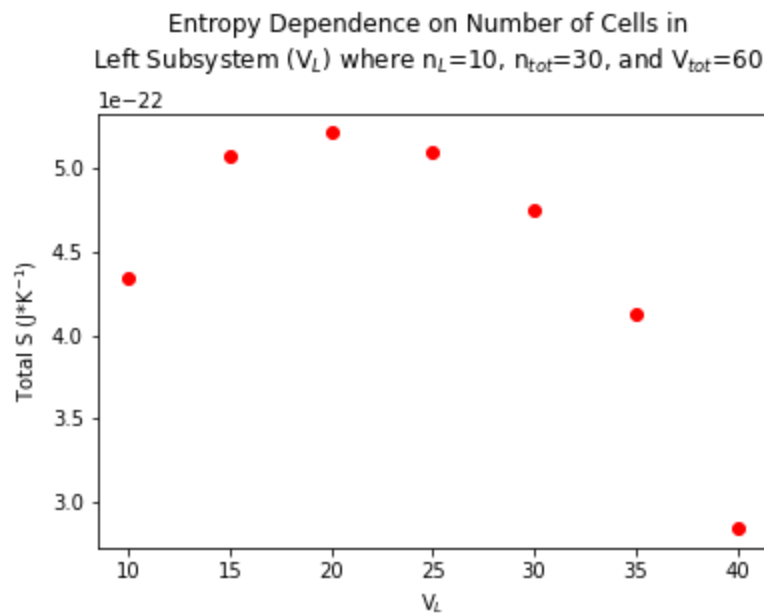
Problem 3.5

Part C

Interpretation:

The entropy of the system increases up until around E_{tot} =6 and then begins to decrease. The corresponding Temperatures for a change in E (always 1) and the change in entropy calculated below is maximized when Entropy is maximized which is reflected in the dS being smallest around E_{tot} =6. Since internal energy changes are constantly 1 on the Y axis, for this system, large changes in S correspond to low temperatures and smaller changes in S correspond to high temperatures. This is consistent with what we should expect from the given relationship T = dU/dS. Small entropy changes reflect high temperature when internal energy is low, but when it is high, larger entropy changes are required to achieve the same temperature.

```
In [9]: E_tots = [3, 4, 5, 6, 7, 8, 9]
        W_tots = [1, 3, 6, 7, 6, 3, 1]
        T_tots = []

        Boltzmann_S = [R/6.022e23 * np.log(x) for x in W_tots]
        prev = 0
        dS = []
        for i in Boltzmann_S:
            sub = i - prev
            dS.append(sub)
            prev = i

        T = [abs(1/j) for j in dS if j != 0 ]

        plt.plot(Boltzmann_S, E_tots, 'r-o')
        plt.ylabel("E$_{tot}$")
        plt.xlabel("S (J*K$^{-1}$)")
        plt.title("Plot of E$_{tot}$ [U] vs. Entropy for System of 3 Molecules and Energy Level
        s")

        print("Temperature From Changes:")
        print(T)
```
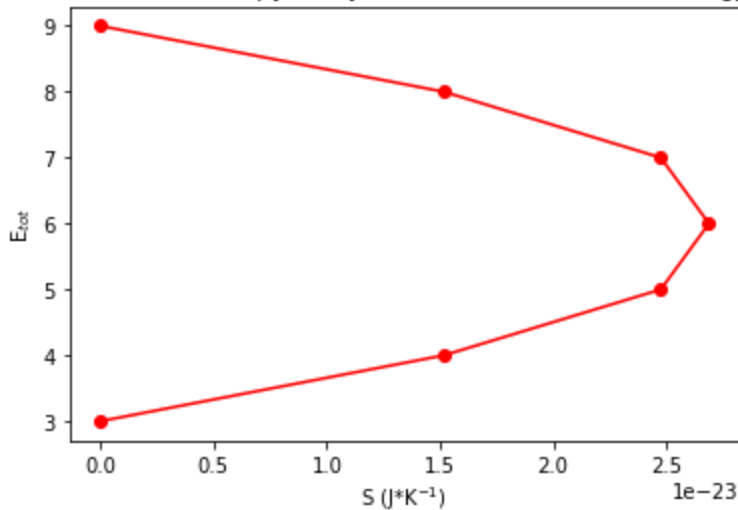
```
Temperature From Changes:
[6.592652141135143e+22, 1.0449106423998241e+23, 4.698499328891361e+23, 4.69849932889136
1e+23, 1.0449106423998241e+23, 6.592652141135143e+22]
```


Plot of E$_{tot}$ [U] vs. Entropy for System of 3 Molecules and Energy Levels

Secret Code **20:**

3.1 A)

	q_{carnot}	w
Isothermal Expansion	$nRT_H \ln(V_A/V_B) = q_{in}$	$nRT_H \ln(V_A/V_B)$
Adiabatic expansion	0	$C_v(T_H - T_c)$
Isothermal Compression	$nRT_c \ln(V_c/V_D)$	$nRT_c \ln(V_c/V_D)$
Adiabatic Compression	0	$C_v(T_c - T_H)$

$$q_{out} @ T_H = nRT_H \ln(V_A/V_B) = -q_{in}$$

$$\boxed{q_{in} = 182.7 \text{ J}}$$

3.1 B)

$$W_{tot} = nRT_H \ln(V_A/V_B) + nRT_c \ln(V_c/V_D)$$

$$= -52.99 \text{ J}$$

$$\boxed{= 52.99 \text{ J done by System}}$$

3.1 C) Total Area: (assuming axes the same as above)

$$(0.0005 \text{ m}^3)(2.0 \times 10^6 \text{ pa}) = 1000 \text{ J}$$

∴ Paper energy density: $\dfrac{1000 \text{ J}}{1467.8 \text{ mg}} = 0.6813 \text{ J/mg}$

Carnot weight = 78.7 mg

Carnot energy = $0.6813 \dfrac{\text{J}}{\text{mg}} \times 78.7 \text{ mg} = \boxed{53.62 \text{ J}}$

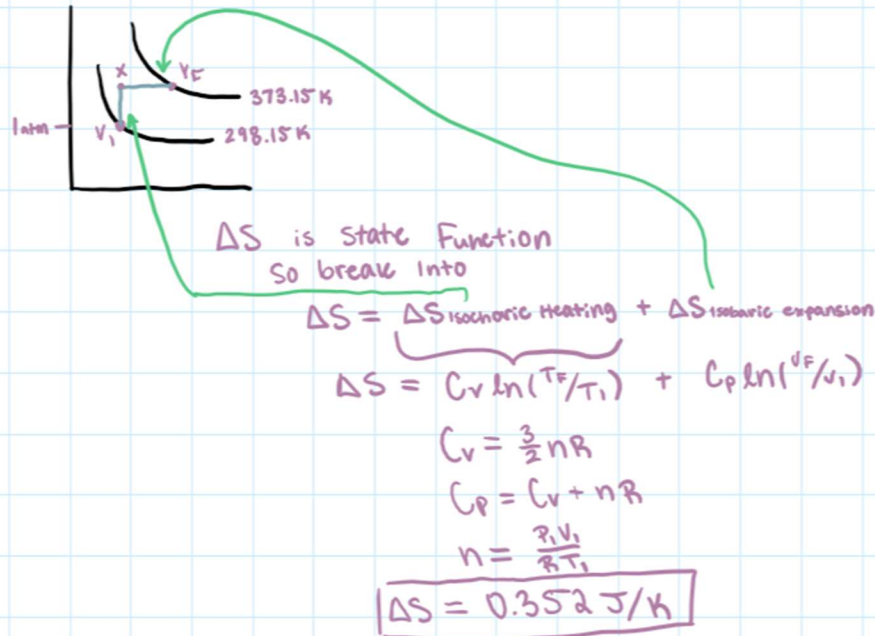**3.1 D)** Isothermal ΔS: $nR \ln(V_F/V_1) = S_F - S_1$

given $S_1 = 0.3 \text{ J/k}$

$S_F = nR\ln(V_F/V_1) + S_I$

Adiabatic ΔS: 0 $\quad \Delta T = T_H - T_c$

3.1E)

$$A = \Delta S \times \Delta T$$
$$= (S_F - S_i) \cdot (T_H - T_c) \quad = [J/\cancel{K} \cdot \cancel{K}]$$
$$= \boxed{52.8 \, J = W \text{ done by cycle}}$$

3.2)

373.15 K

298.15 K

1 atm — V_1

ΔS is state Function
So break into

$$\Delta S = \Delta S_{\text{Isochoric Heating}} + \Delta S_{\text{Isobaric expansion}}$$

$$\Delta S = C_V \ln(T_F/T_i) + C_p \ln(V_F/V_i)$$

$$C_V = \tfrac{3}{2} nR$$

$$C_p = C_V + nR$$

$$n = \frac{P_1 V_1}{R T_1}$$

$$\boxed{\Delta S = 0.352 \, J/K}$$

3.4A) $dB = edK - JdL$

$$\boxed{dB = \left(\frac{\partial B}{\partial K}\right) dK - \left(\frac{\partial B}{\partial L}\right) dL}$$

3.4B) $e = \left(\frac{\partial B}{\partial K}\right)$

$$J = \left(\frac{\partial B}{\partial L}\right)$$

$$dB = e \, dK + -J dL$$
$$\parallel \qquad \frac{\partial e}{\partial K} \qquad \frac{-\partial J}{\partial K}$$
$$edK$$
$$+ \qquad\qquad\qquad //$$
$$-JdL \qquad \frac{\partial e}{\partial K} \qquad \frac{-\partial J}{\partial L}$$

3.4 C)

$$\left(\frac{\partial e}{\partial L}\right)_K = \left(\frac{-\partial J}{\partial K}\right)_L$$

3.5A)

$\varepsilon_3 = 3$

$\varepsilon_2 = 2$

$\varepsilon_1 = 1$

| $W_{3,1} = 1$ | $W_{4,1} = \dfrac{3!}{2!}$ | $W_{5,1} = \dfrac{3!}{2!}$ | $W_{6,1} = 3!$ | $W_{7,1} = \dfrac{3!}{2!}$ | $W_{8,1} = \dfrac{3!}{2!}$ | $W_{9,1} = 1$ |

| $W_{3,2} = 0$ | $W_{4,2} = 0$ | $W_{5,2} = \dfrac{3!}{2!}$ | $W_{6,2} = 1$ | $W_{7,2} = \dfrac{3!}{2!}$ | $W_{8,2} = 0$ | $W_{9,2} = 0$ |

| $W_{3,3} = 0$ | $W_{4,3} = 0$ | $W_{5,3} = 0$ | $W_{6,3} = 0$ | $W_{7,3} = 0$ | $W_{8,3} = 0$ | $W_{9,3} = 0$ |

| $E_{tot} = 3$ | $E_{tot} = 4$ | $E_{tot} = 5$ | $E_{tot} = 6$ | $E_{tot} = 7$ | $E_{tot} = 8$ | $E_{tot} = 9$ |

3.5B)

| $W_{3,tot} = 1$ | $W_{4,tot} = 3$ | $W_{5,tot} = 6$ | $W_{6,tot} = 7$ | $W_{7,tot} = 6$ | $W_{8,tot} = 3$ | $W_{9,tot} = 1$ |