1 November 2021

an MST / the MST

recognize it might not be unique

there is only one

Why is it ok to assume $\exists!$ MST?

→ If we fix an (arbitrary) order on the vertices, then we can break ties using the product order. ~~[illegible]~~
So, this breaks all ties ⇒ a unique poset of edges, all with "different" weights.

→ See the beg. of Ch 7 for a full explanation

MST problem:

- given: weighted graph $G=(V,E,\omega)$
- want: $T \subseteq G$ such that T is a tree w/ vertex set V and among all such trees, T has minimum weight, where the weight of a tree is: $\omega(T) := \sum_{e \in E_T} \omega(e)$

many sol'ns build T iteratively & a given pt have a spanning forest $F \subseteq T$. Here, note $\mathcal{F}$ has 3 types of edges:

1. useless: edge that has 2 endpts in same component. (Lem 7.3)
2. safe: for each cc (a tree), look at all outgoing edges. The min weight one is safe. (Lem. 7.2)
3. undecided

3 spins on the algorithm

$F = (V, \emptyset)$

① Kruskal's Algorithm

Sort edges by increasing weight. Scan through + add safe edges as we encounter them.

② Jarnik / Prim Algorithm

repeatedly add safe edges. (one at a time)

③ Borůvka's Algorithm

add <u>all</u> safe edges + repeat

In groups, → walk through ② and ③.
→ how much better is ③? How do we even describe this?

Initially, $F = (V, \emptyset)$

what are our safe edges? $S = \{1, 2, 3, 4, 5\}$

useless edges? $\emptyset$

unsure edges: $E \setminus S$

after Prim: add edge 3

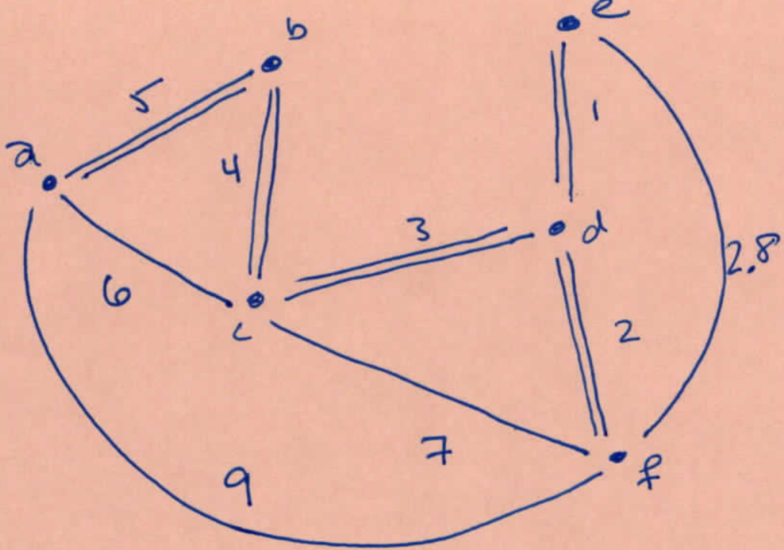
safe edges: $S = \{1, 2, 4, 5\}$

useless: $\emptyset$

unsure: $E \setminus (S \cup E_F)$

What can happen in general? New safe edges would be out of the cc containing the new edge!

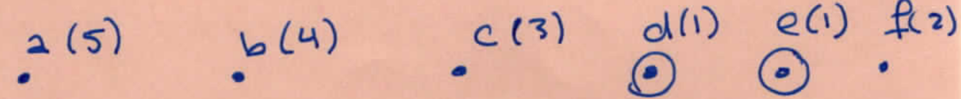
Example:



1 Kruskal

edges, sorted: [1, 2, 2.8, 3, 4, 5, 6, 7, ~~8~~, 9]

init $F = (V, \emptyset)$

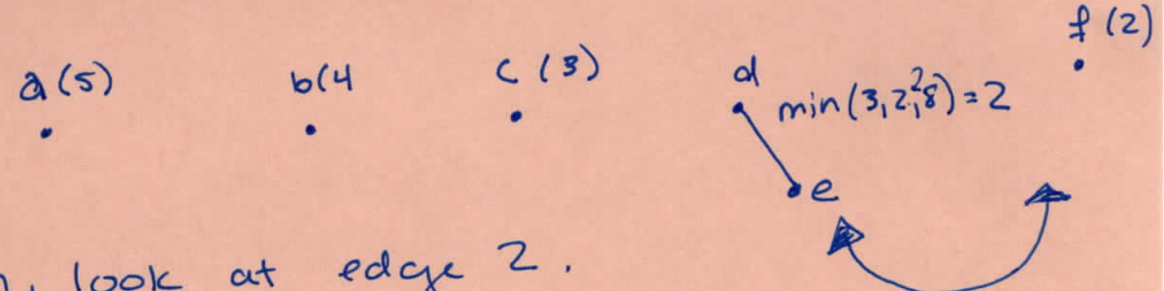


1st: edge 1 connects [e] w/ [d]

1, 8 — ✓ 1 is safe

1, 2, 3, — ✓ 1 is safe

F:

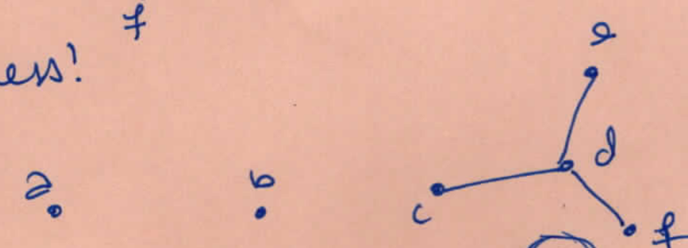


Nex in scan, look at edge 2.

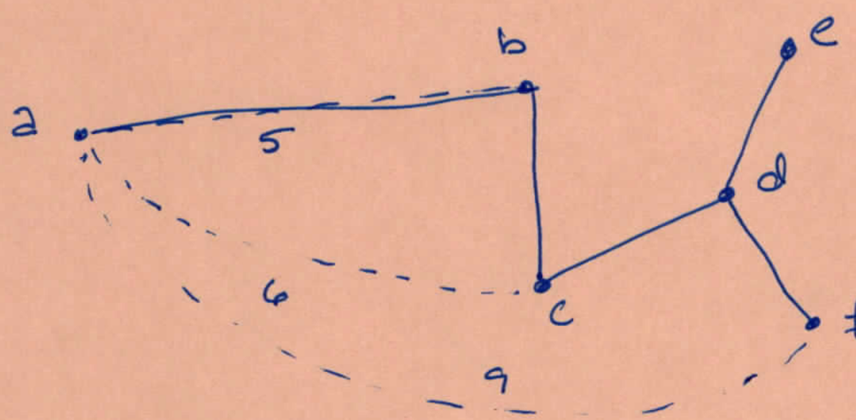
safe, so add to ~~MST~~ F

Next up: edge 2.8 is useless!

edge 3: safe, so add it



Next: edge 4 is safe, so F becomes:



Next: edge 5 is safe, so add it

Next: edge 6 useless
7 useless
9 useless

once I've established ~~any~~ spanning tree, everything else is useless.