10 Sept 2021

H-1 due on Monday!!

- → group submission (1 sub per group per platform)
  - Gradescope: select group members
  - D2L: just <u>one</u> person submits
  - no upper bound on group size
- → Q5 (recursions)
  - all <u>base cases</u> are 1.
  - use floor for getting the closed form.
    so, think of $T(n/4)$ as $T(\lfloor n/4 \rfloor)$

Formally writing up that extra condition for case 3 looks like:

Let $n_0 = 1$ and $c = 2/3$. Then, $\forall n \geq n_0$,

$$
\begin{aligned}
af(n/b) &= 2f(n/4) && \text{by substitution} \\
&= 2(n/4) && \text{by sub.} \\
&= n/2 \\
&\leq 2/3\, n
\end{aligned}
$$

, as was to be shown. $\boxplus$

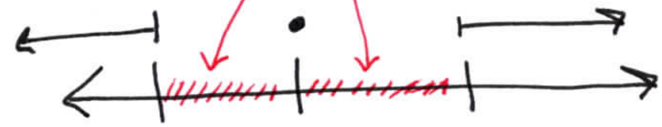
In-Class Exercise 03

CSCI 432

September 10, 2021

Name:
Who did you work with today?

if $f(n)$ is here, we can't solve it



Master's Theorem

Master's theorem allows us to quickly solve recurrence relations of the form:

$$\boxed{T(n) = aT(n/b) + f(n),}$$

where $a, b \in \mathbb{N}$ such that $a \geq 1$ and $b > 0$ and $f(n)$ is asymptotically positive. Then, we can determine the closed-form of $T(n)$ as follows:

case 1. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $\boxed{f(n)} \in \boxed{O(n^{\log_b a - \varepsilon})}$, THEN $T(n) \in \Theta(n^{\log_b a})$. $\leq$

case 2. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $f(n) \in \boxed{\Theta(n^{\log_b a})}$, THEN $T(n) \in \Theta(n^{\log_b a} \log n)$. =

case 3. IF (1) there exists $\varepsilon \in \mathbb{R}_+$ such that $f(n) \in \Omega\boxed{(n^{\log_b a + \varepsilon})}$ and (2) there exists $c \in (0, 1)$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $af(n/b) \leq cf(n)$, THEN $T(n) \in \Theta(f(n))$. $\geq$

	a	b	$\log_b a$	$n^{\log_b a}$	$f(n)$	Potential Case?	ε, if Case 1 or 3	Asymptotic ~~Closed~~ Form
$T(n) = T(n/2) + 1$ binary search!	1	2	0	$1 =$	1	2	n/a	$\Theta(\log n)$ ← cool! It's what we expect
$T(n) = 2T(n/4) + \sqrt{n}$	2	4	1/2	$n^{1/2} =$	$n^{1/2}$	2	n/a	$\Theta(n^{1/2} \log n)$
$T(n) = 2T(n/4) + n$	2	4	1/2	$n^{1/2} <$	n^1	3	1/2 (*)	$\Theta(n)$
$T(n) = 2T(n/4) + 6n^2$	2	4	1/2	$n^{1/2}$	$6n^2 \in \Theta(n^2)$	3	$\varepsilon = 1/2$	$\Theta(n^2)$
$T(n) = 3T(n/3) + \Theta(1)$	3	3	1	$n >$	$\Theta(1)$	1	want: $n^{1-\varepsilon} > n^0$ $\varepsilon = 1/3$	$\Theta(n)$

← compare $n^{\log_b a}$ to $f(n)$ to guess here.

Remember, Case 3 has an additional condition to check! Do that in the space provided below, or on the back of this page.

we need n_0, c s.t. $\forall n \geq n_0$,
$a \cdot f(n/b) \leq c f(n)$. Since $f(n) = n$, ~~this~~ $a = 2$, $b = 4$, this means:
$2 \cdot (n/4) \leq c \cdot n \iff n/2 \leq c \cdot n$.
So, $n_0 = 1$, $c = 2/3$ works.

scratch work

(*) Here, anything in the half-open interval $(0, 0.5]$ works. But, pick a value + give it to me in your proof! I need ε s.t. $\Theta(n^{1/2 + \varepsilon}) \leq \Theta(n^1)$