29 Nov 2021

announcements

- project progress report due today
- project (next week)
  - → video (file, URL) in D2L only * MON
  - → project write-up FRI
  - → optional: code (due w/ write-up) FRI → zip file or git URL
  - → INDIVIDUAL assessment/contribution stmt)
- group homework H7 → last "normal" HW
  - → as per H6, need to have 20% to be eligible to drop. (due WED the 8th)
- the (n+1)st assignment → don't forget an appendix!!!

(due finals week)

- MISC assignment: Wed after class job talk discuss data science -vs- computer science
- Don't forget to come for the 3rd exam Dec. 15 (Wed) 2pm – 4pm

RQS(A, s, t)

A: an array of real-#s
s: start index, incl.
t: end index, incl.

```
if t-s < 2
|   return A
endif
i ← randInt[s,t]
p ← Pivot(A, s, t, i)
A ← RQS(A, s, p-1)
A ← RQS(A, p+1, t)
return A
```

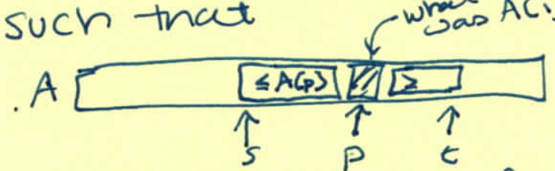
base case. No sorting needed!

randInt[s,t]: returns a rand. int btwn s & t, inclusive.

Pivot (A, s, t, i)

A: array
s: start index, inc.
t: end index, inc.
i: pivot ($s \le i \le t$)

- return, reorder elts of A such that



and returns pivot index

- Question: do an inplace pivot algorithm.

1st call: for array A with $|A| = n$

RQS(A, 1, n)

practice:

A =

index:	1	2	3	4	5	6	7	8
A	8	3	4	1	5	2	6	7

$i = 5$

Pivot (A, 1, 8, 5)

↳ end result of A: $\underbrace{\boxed{3\ 4\ 1\ 2}}_{\le 5}$ $\boxed{A[5]=5}$ $\underbrace{\boxed{8\ 7\ 6}}_{\ge 5}$

p=5

Pivot (A, 1, 8, 4)

$\boxed{1}$ $\boxed{2\ 3\ 4\ 5\ 6\ 7\ 8}$

p=1

What is the worst-case runtime?

Picking the smallest elt. as the pivot each time?

The recurrence relation is then:

$$T(n) = \underbrace{\Theta(1)}_{\text{lines 1-4}} + \underbrace{\Theta(n)}_{\text{pivot}} + \underbrace{T(0)}_{\Theta(1)} + T(n-1)$$

$$= \Theta(n) + T(n-1)$$ can you prove this?

$$= \boxed{\Theta(n^2)}$$

↑ n^2 both upper & lower bounds the worst-case

So, the worst-case runtime of RQS is quadratic. But, why do we like RQS?

① Linearity of Expectation

$$\mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$

↑ expected value of

② What is the expected value?

- in the long run, what is the average value?

$$\mathbb{E}(X) = \sum_{e \in \Omega} \mathbb{P}(e)\, V(e)\ , \quad \Omega = \text{set of all possible outcomes}$$

e.g.) coin flip: you get \$1 if H \$0 T
what are your expected winnings?

$$\mathbb{E}(\text{Coin Flip}) = \mathbb{P}(H)\,V(H) + \mathbb{P}(T)\,V(T)$$

$$= \frac{1}{2}\cdot\$1 + \frac{1}{2}\cdot\$0$$

$$= \$0.50$$

(3) $H_n = \frac{1}{1} + \frac{1}{2} + \frac{3}{3}\!\!\!\!\!\!\phantom{\frac{1}{3}}$

is the nth Harmonic number

$$\mathbb{H}\ \sum_{i=1}^{n} H_i \in [n\log n,\ 1+n\log n]$$

$$\Rightarrow \mathbb{H} \in \Theta(n\log n)$$