

Suffix Array Longest Common Prefix

Time	n
Space	n

Data Structures

Name	Type	Initial Value
suffix	int[]	suffixArr(str)
str	string	input string (length n)

Algorithm

```
// phi contains the index of the next shortest suffix in the array
int phi[n];
phi[suffix[0]] = -1;
for(int i = 1; i < n; i++) {
    phi[suffix[i]] = suffix[i-1];
}

int plcp[n]; // Permuted least common prefix
int len = 0;
for(int i = 0; i < n; i++) {
    if(phi[i] == -1) {
        plcp[i] = 0;
        continue;
    }

    // Compute length of common prefix between adjacent suffixes
    while(str.charAt(i + len) == str.charAt(phi[i] + len)) len++;
    plcp[i] = len;
    len = max(0, len - 1);

    int lcp[n];
    for(int i = 0; i < n; i++) lcp[i] = plcp[suffix[i]];

    int longest = 0;
    int index = 0;
    for(int i = 0; i < n; i++) {
        if(lcp[i] > longest) {
            longest = lcp[i];
            index = i;
        }
    }

    return str.substring(suffix[index], suffix[index] + longest);
}
```

Results

- Returns the longest prefix between any two suffixes in `str`. This is also the longest repeated substring.

Suffix Array (Fast)

Time	$n \cdot \log^2 n$
Space	n

Data Structures

Name	Type	Initial Value
suffix	List<Integer>	[0, ..., n]
rank	List<Integer>	[str.charAt[0], ..., str.charAt[n]]

Algorithm

```
// Sort array log_2(n) times according to the first 1, 2, 4, ... characters of each suffix
int tempRank[str.length()];
for(int k = 1; k < n; k <= 1) {
    // For each part, sort the array 2 times: First by the next k elements that have not been sorted,
    // then stable sort the first k elements again
    sort(suffix, (a, b) -> Integer.compare(a + k < n ? rank[a + k] : 0, b + k < n ? rank[b + k] : 0));
    sort(suffix, (a, b) -> Integer.compare(rank[a], rank[b]));

    // Reorder the ranks of the suffixes based on the order they are now sorted in
    int curRank = 0;
    tempRank[suffix[0]] = curRank;
    for(int i = 1; i < n; i++) {
        // Only if two contiguous suffixes don't have the same rank pair, increase the rank
        if(!(rank[suffix[i]] == rank[suffix[i-1]] && rank[suffix[i]+k] == rank[suffix[i-1]+k])) {
            curRank++;
        }
        tempRank[suffix[i]] = curRank;
    }

    for(int i = 0; i < n; i++) rank[i] = tempRank[i];

    // All ranks unique; sort done early
    if(rank[n-1] == n-1) break;
}
```

Results

- `suffix[i]` is the starting index of the `i`th lexicographical suffix of `str`. Suffixes that share common starting runs will be stored in contiguous blocks of the array.

Notes

- Only works for single strings. If a suffix data structure is needed for multiple strings, look into an actual tree structure.
- This is fast enough for $n \leq 100,000$. If more speed is needed, implement a radix sort.
- A special character (often `$`) should be appended to the string to help this construction.

Suffix Array Contains

Time	$m \cdot \log n$
Space	1

Data Structures

Name	Type	Initial Value
suffix	int[]	suffixArr(str)
str	string	input string (length n)
target	string	target string (length m)

Algorithm

```
int low = 0;
int high = n - 1;
while(low < high) {
    int mid = (low + high) / 2;
    if(str.substring(suffix[mid]).startsWith(target)) {
        return true;
    }
    if(str.substring(suffix[mid]).compareTo(part) > 0) {
        hi = mid;
    } else {
        low = mid + 1;
    }
}
return false;
```

Results

- Returns true iff target is contained in str.

Suffix Array (Naive)

Time	$n^2 \cdot \log n$
Space	n

Data Structures

Name	Type	Initial Value
arr	int[]	[0]*n
str	string	input

Algorithm

```
int arr[str.length()];
for(i = 0; i < n; i++) {
    arr[i] = i;
}
Arrays.sort(arr, (a, b) -> str.substring(a).compareTo(str.substring(b)));
return arr;
```

Results

- arr[i] is the starting index of ith lexicographical suffix of str. Because they are sorted in lexicographical order, suffixes that share common starting runs will be stored in contiguous blocks of the array.

Notes

- This will only work for single strings. If a suffix data structure is needed for multiple strings, look into an actual tree structure
- Because of the large construction time, this naive method should only be used for strings up to 1000 chars. If this is satisfied, this is easier to write.
- A special character, usually \$ should be appended to the string to help this construction.

Convex Hull

TODO

Point

```
#define _sca(op) \
Point &operator op##=(double t) { x op##= t; y op##= t; return *this; } \
Point operator op(double t) { return Point(*this) op##= t; }

#define _vec(op) \
Point &operator op##=(Point p) { x op##= p.x; y op##= p.y; return *this; } \
Point operator op(Point p) { return Point(*this) op##= p; }

struct Point {
    double x, y;

    Point(double x, double y): x(x), y(y) {}

    Point operator-() { return Point(-x, -y); }

    _vec(+)
    _vec(-)
    _sca(*)
    _sca(/)
};

Point operator*(double t, Point p) { return p * t; }
string _s(Point p,...) { return _s(make_pair(p.x, p.y)); }

/* Dot product */
double dot(Point p, Point q) { return p.x * q.x + p.y * q.y; }
double norm(Point p) { return dot(p, p); }
double abs(Point p) { return sqrt(norm(p)); }
double dist(Point p, Point q) { return abs(p - q); }
Point unit(Point p) { return p / abs(p); }
bool eq(Point p, Point q) { return dist(p, q) < eps; }
double proj(Point p, Point q) { return dot(p, q) / abs(q); }
Point project(Point p, Point q) { return dot(p, q) / norm(q) * q; }
double angle(Point p, Point q) { return acos(proj(p, q) / abs(p)); }

/* Cross product */
double cross(Point p, Point q) { return p.x * q.y - p.y * q.x; }
double pgram(Point p, Point q) { return abs(cross(p, q)); }
double triangle(Point p, Point q) { return pgram(p, q) / 2; }
```

Line

```
#define _orient(u, v) \
double dist(u x, v y) { return abs(sdist(x, y)); } \
int side(u x, v y) { double s = sdist(x, y); return (s > eps) - (s < -eps); } \
bool incident(u x, v y) { return side(x, y) == 0; }

struct Line {
    double a, b, c;

    Line(double a, double b, double c): a(a), b(b), c(c) { normalize(); }
    Line(Point p, Point q): a(p.y - q.y), b(q.x - p.x), c(-a * p.x - b * p.y) { normalize(); }

    void normalize() {
        double z = sqrt(a * a + b * b);
        if (a < 0 && b < 0) z = -z;
        a /= z; b /= z; c /= z;
    }
};

/* Line */
string _s(Line m,...) { return '(' + _s(m.a) + "x + " + _s(m.b) + "y + " + _s(m.c) + " = 0)"; }
Point normal(Line m) { return Point(m.a, m.b); }
Point tangent(Line m) { return Point(m.b, -m.a); }

/* Point, Line */
double sdist(Point p, Line m) { return m.a * p.x + m.b * p.y + m.c; }
_orient(Point, Line)
Point project(Point p, Line m) { return p - normal(m) * sdist(p, m); }

/* Line, Line */
bool eq(Line m, Line n) { return eq(normal(m), normal(n)) && abs(m.c - n.c) < eps; }
double cross(Line m, Line n) { return cross(normal(m), normal(n)); }
double angle(Line m, Line n) { double a = angle(normal(m), normal(n)); return min(a, pi - a); }
bool parallel(Line m, Line n) { return abs(cross(m, n)) < eps; }
Point intersect(Line m, Line n) {
    return Point(m.b * n.c - m.c * n.b, m.c * n.a - m.a * n.c) / cross(m, n);
}
```

Circle

```
struct Circle {
    Point o; double r;

    Circle(Point o, double r): o(o), r(r) {}
}

/* Circle */
string _s(Circle c) { return _s(make_pair(c.o, c.r)); }
double area(Circle c) { return pi * c.r * c.r; }
double circum(Circle c) { return 2 * pi * c.r; }

/* Point, Circle */
double sdist(Point p, Circle c) { return dist(c.o, p) - r; }
_orient(Point, Circle)
Point project(Point p, Circle c) { return c.o + unit(p - c.o) * r; }

/* Line, Circle */
double sdist(Line l, Circle c) { return dist(c.o, l) - r; }
_orient(Line, Circle)
void intersect(Line l, Circle c, Point &p, Point &q) {
    Point m = project(c.o, l);
    Point v = tangent(l) * sqrt(c.r * c.r - norm(m - c.o));
    p = m + v;
    q = m - v;
}
```

Notes

- A point is inside a circle when `side(p, c) == -1`.
- A line intersects a circle when `side(l, c) <= 0` (in two points when `side(l, c) == -1`).
- A line is tangent to a circle when `incident(l, c)` (at `project(c.o, l)`).

Polygon

// **TODO**

Traveling Salesman

Find a minimum-cost hamiltonian path or cycle in a complete graph.

Held-Karp algorithm

Time	$2^n n^2$
Space	$2^n \sqrt{n}$ (store $\text{dp}[y][S]$ for $ S = n - 1$)

Let $\text{dp}[y][S]$ be the minimum cost of a path from 0 to y through the intermediate vertices in S .

- $\text{dp}[y][\emptyset] = W[0 \rightarrow y]$
- $\text{dp}[y][S] = \min_{x \in S} \{ \text{dp}[x][S - \{x\}] + W[x \rightarrow y] \}$

Notes

- If $V \subseteq \{0, \dots, 63\}$, use an integer bitset for S .
- To reconstruct the hamiltonian path or cycle, track $\text{prev}[y][S]$.
- For the hamiltonian cycle, compute $\min_{x \in V} \{ \text{dp}[x][V - \{x\}] + W[x \rightarrow 0] \}$.

Knapsack

Given $0 \leq W$, $[1 \leq v_1, \dots, v_N]$, and $[1 \leq w_1, \dots, w_N]$, maximize $\sum v_i x_i$ with $\sum w_i x_i \leq W$.

0/1 Knapsack

When $0 \leq x_i \leq 1$.

Time	NW
Space	W (store $\text{dp}[n - 1]$)

Let $\text{dp}[n][w]$ be the solution when $W = w$ and $N = n$.

- $\text{dp}[0][w] = 0$
- $\text{dp}[n][w] = \max \left(\text{dp}[n - 1][w], \max_{1 \leq i \leq N} \{ \text{dp}[n - 1][w - w_i] + v_i : w \geq w_i \} \right)$

Bounded Knapsack

When $0 \leq x_i \leq k$.

Time	kNW
Space	W (store $\text{dp}[n - 1]$)

Solve the 0/1 knapsack problem where each v_i, w_i implicitly appears k times ($N' = kN$).

Unbounded Knapsack

When $0 \leq x_i < \infty$.

Time	NW
Space	W

Let $\text{dp}[w]$ be the solution when $W = w$.

- $\text{dp}[0] = 0$
- $\text{dp}[w] = \max_{1 \leq i \leq N} \{ \text{dp}[w - w_i] + v_i : w \geq w_i \}$

Notes

- Divide w_1, \dots, w_n, W by their GCD to improve complexity in some cases.

Edmonds-Karp

Time	$ V \cdot E ^2$
Space	$ V $

Data Structures

Name	Type	Initial Value
residual	Map<Vertex, Map<Vertex, Capacity>>	$G + E\{v: \{u: 0\}\}$

Algorithm

```
Flow flow = 0;
for (;;) {
    // Find an augmenting path
    // PathBFS skips (u → v) where residual[u][v] == 0 (and stops at sink)
    Map<Vertex, Vertex> prev;
    if (!PathBFS(residual, source, sink, &prev) break;

    // Find the bottleneck capacity of the augmenting path
    Capacity cap = INF;
    for (Vertex v = sink, u = v; prev.has(u); v = u) {
        u = prev[u];
        cap = min(cap, residual[u][v]);
    }

    // Send flow down the augmenting path
    for (Vertex v = sink, u = v; prev.has(u); v = u) {
        u = prev[u];
        residual[u][v] -= cap;
        residual[v][u] += cap;
    }

    flow += cap;
}
```

Results

- `flow` is the maximum flow from `source` to `sink`.
- `residual[v][u]` is the flow through `u → v` in **some** maximum flow.

Notes

- Fails on graphs with self-loops, parallel edges, and bidirectional edges.
- Can be fixed for those graphs by defining `struct Edge { Vertex v, Capacity, Flow, Edge *rev }.`

Bellman-Ford

Time	$ V \cdot E $
Space	$ V $

Data Structures

Name	Type	Initial Value
prev?	Map<Vertex, Vertex>	{}
dist	Map<Vertex, Distance>	{start: 0}

Algorithm

```
for (|V| - 1) {
  for ((Vertex u, Vertex v, Weight w) : E) {
    if (!dist.has(u)) continue;

    if (!dist.has(v) || dist[v] > dist[u] + w) {
      dist[v] = dist[u] + w;
      prev[v] = u;

      // Relax u → v
    }
  }
}

// Extra iteration
for ((Vertex u, Vertex v, Weight w) : E) {
  if (dist.has(u) && dist[v] > dist[u] + w) {
    return false; // Negative cycle detected
  }
}

return true;
```

Results

- `dist[v]` is the distance from `start` to `v` (if they are connected).
- `prev[v]` is the penultimate vertex on **some** shortest path from `start` to `v` (if they are connected).

Notes

- The extra iteration will relax some vertex iff a negative cycle is reachable from `start`.
- $|V| - 1$ extra iterations will relax `v` iff there is a negative cycle between `start` and `v`.
- Fails on negative cycles (detected).

DFS

Time	$ V + E $
Space	$ V $

Data Structures

Name	Type	Initial Value
back	Stack<Vertex>	[start]
visited	Set<Vertex>	{}

Algorithm

```
while (!back.empty()) {
  Vertex u = back.top();

  if (!visited.has(u)) {
    visited.add(u);

    // Start visiting u
  } else {
    // Backtrack to u
  }

  bool follow = false;
  for (Vertex v : E[u]) {
    if (visited.has(v)) continue;

    // Follow u → v

    back.push(v);
    follow = true;
    break;
  }
  if (follow) continue;

  // Finish visiting u

  back.pop();
}
```

Results

- `visited` is the set of vertices connected to `start`.

Dinitz's Algorithm

Time	$ V ^2 \cdot E $
Space	$ V $

Data Structures

Name	Type	Initial Value
residual	Map<Vertex, Map<Vertex, Capacity>>	G + E{v: {u: 0}}

Algorithm

```
Flow flow = 0;
for (;;) {
    // LevelBFS skips (u → v) where residual[u][v] == 0 (and stops at sink)
    // Returns a DAG of edges seen from each vertex traversed
    Map<Vertex, Stack<Vertex>> layered;
    if (!LevelBFS(residual, source, sink, &layered)) break;

    for (;;) {
        // PathDFS pops edges when it backtracks (and stops at sink)
        Map<Vertex, Vertex> prev;
        if (!PathDFS(layered, source, sink, &prev)) break;

        // Get the bottleneck capacity of the augmenting path
        Capacity cap = INF;
        for (Vertex v = sink, u = v; prev.has(u); v = u) {
            u = prev[u];
            cap = min(cap, residual[u][v]);
        }

        // Send flow down the augmenting path
        for (Vertex v = sink, u = v; prev.has(u); v = u) {
            u = prev[u];
            residual[u][v] -= cap;
            residual[v][u] += cap;

            if (residual[u][v] == 0) layered[u].pop();
        }

        flow += cap;
    }
}
```

Results

- flow is the maximum flow from source to sink.
- residual[v][u] is the flow through u → v in some maximum flow.

Notes

- Fails on graphs with self-loops, parallel edges, and bidirectional edges.
- Can be fixed for those graphs by defining struct Edge { Vertex v, Capacity, Flow, Edge *rev }.

BFS

Time	$ V + E $
Space	$ V $

Data Structures

Name	Type	Initial Value
front	Queue<Vertex>	[start]
seen	Set<Vertex>	{start}
prev?	Map<Vertex, Vertex>	{}

Algorithm

```
while (!front.empty()) {
  Vertex u = front.top();
  front.pop();

  // Visit u

  for (Vertex v : E[u]) {
    if (seen.has(v)) continue;
    seen.add(v);
    prev[v] = u;

    // See u → v

    front.push(v);
  }
}
```

Results

- seen is the set of vertices connected to start.
- prev[v] is the penultimate vertex on **some** shortest path from start to v (if they are connected).

Notes

- seen may be redundant if prev is used.
- Finds shortest paths by number of edges (not weight).

Johnson's Algorithm

Time	$(E + V) V \log V $
Space	$ V ^2$

Data Structures

Name	Type	Initial Value
adjusted	Map<Vertex, List<(Weight, Vertex)>>	G + {q: [V(0, v)]}
height	Map<Vertex, Distance>	{}
prev?	Map<Vertex, Map<Vertex, Vertex>>	{}
dist	Map<Vertex, Map<Vertex, Distance>>	{}

Algorithm

```
if (!BellmanFord(adjusted, &height, q)) return false;
adjusted.remove(q);

// Reweighting
for (Vertex u : V) {
    for ((Weight w, Vertex v) : adjusted[u]) {
        w += height[u] - height[v];
    }
}

// Repeated Dijkstra
for (Vertex v : V) {
    (dist[v], prev[v]) = Dijkstra(adjusted, v);
}

return true;
```

Results

- `dist[u][v] - height[u] + height[v]` is the distance from `u` to `v` (if they are connected).
- `prev[u][v]` is the penultimate vertex on **some** shortest path from `u` to `v` (if they are connected).

Notes

- Bellman-Ford & reweighting can be skipped for graphs with non-negative edges.
- Fails on negative cycles (detected during Bellman-Ford).

Floyd-Warshall

Time	$ V ^3$
Space	$ V ^2$

Data Structures

Name	Type	Initial Value
next?	Map<(Vertex, Vertex), Vertex>	{E(u, v): v}
dist	Map<(Vertex, Vertex), Distance>	{E(u, v): w, V(v, v): 0}

Algorithm

```
for (Vertex m : V) {
  for (Vertex u : V) {
    for (Vertex v : V) {
      if (!dist.has((u, m)) || !dist.has((m, v))) continue;

      if (dist[u, v] > dist[u, m] + dist[m, v]) {
        dist[u, v] = dist[u, m] + dist[m, v];
        next[u, v] = next[u, m];

        // Relax u → v through m
      }
    }
  }
}

for (Vertex v : V) {
  if (dist[v, v] < 0) {
    return false; // Negative cycle detected
  }
}

return true;
```

Results

- `dist[u, v]` is the distance from `u` to `v` (if they are connected).
- `next[u, v]` is the second vertex on **some** shortest path from `u` to `v` (if they are connected and distinct).

Notes

- Johnson’s Algorithm is faster for sparse graphs.
- Fails on negative cycles (detected).

Dijkstra's Algorithm

Time	$(E + V) \log V $
Space	$ V ^2$

Data Structures

Name	Type	Initial Value
front	PriorityQueue<(Distance, Vertex)>	[(0, start)]
prev?	Map<Vertex, Vertex>	{}
dist	Map<Vertex, Distance>	{start: 0}

Algorithm

```
while (!front.empty()) {
  (Distance d, Vertex u) = front.top();
  front.pop();

  if (dist.has(u) && d > dist[u]) continue;

  // Visit u

  for ((Vertex v, Weight w) : E[u]) {
    Distance r = d + w;
    if (!dist.has(v) || dist[v] > r) {
      dist[v] = r;
      prev[v] = u;

      // Relax u → v

      front.push((r, v));
    }
  }
}
```

Results

- `dist[v]` is the distance from `start` to `v` (if they are connected).
- `prev[v]` is the penultimate vertex on **some** shortest path from `start` to `v` (if they are connected).

Notes

- Fails on graphs with negative edges (use Bellman-Ford).

Topological Sort

Time	$ V + E $
Space	$ V $

Data Structures

Name	Type	Initial Value
sorted	Set<Vertex>	{}
topo	List<Vertex>	[]

Algorithm

```
for (Vertex start : V) {
    if (sorted.has(start)) continue;

    Set<Vertex> visited = {};
    Stack<Vertex> backtrack = [start];

    while (!backtrack.empty()) {
        Vertex u = backtrack.top();
        visited.add(u);

        bool follow = false;
        for (Vertex v : E[u]) {
            if (sorted.has(v)) continue;
            if (visited.has(v)) return false; // Cycle detected

            backtrack.push(v);
            follow = true;
            break;
        }
        if (follow) continue;

        sorted.add(u);
        topo.push(u);
        backtrack.pop();
    }
}

return true;
```

Results

- $i < j$ implies there is no path from `topo[i]` to `topo[j]` (reverse topological order).

Notes

- Impossible with cycles (detected).

Prim's Algorithm

Time	$(E + V) \log V $
Space	$ V ^2$

Data Structures

Name	Type	Initial Value
front	PriorityQueue<(Weight, Vertex)>	[(0, start)]
parent	Map<Vertex, Vertex>	{}
cost	Map<Vertex, Weight>	{}
tree	Map<Vertex, List<(Weight, Vertex)>>	{}

Algorithm

```
while (!front.empty()) {
    (Weight w, Vertex u) = front.top();
    front.pop();

    if (cost.has(u) && w > cost[u]) continue;

    // Visit u

    if (parent.has(u)) {
        tree[u].push((w, parent[u]));
        tree[parent[u]].push((w, u));

        // Connect parent[u] to u
    }

    for ((Vertex v, Weight x) : E[u]) {
        if (!cost.has(v) || cost[v] > x) {
            cost[v] = x;
            parent[v] = u;

            // Relax u → v

            front.push((x, v));
        }
    }
}
```

Results

- `tree` is **some** MST of `start`'s connected component.

Notes

- Fails on directed graphs.

Sample Graphs

```
10
0 0 (no vertices)
9 0 (no edges)
1 1 (self-loop)
0 0 5
2 2 (parallel edges)
0 1 0
0 1 2
3 3 (cycle)
0 1 0
1 2 2
2 0 4
4 2 (disconnected)
0 1 0
2 3 2
3 3 (non-consecutive)
9 3 0
7 3 2
7 9 4
3 3 (negative edge)
0 1 -2
1 2 4
2 0 0
3 3 (negative cycle)
0 1 -4
1 2 2
2 0 0
5 10 (complete graph)
0 1
0 2
0 3
0 4
1 2
1 3
1 4
1 5
2 3
2 4
2 5
3 4
3 5
4 5
```

Other possibilities

- Dense or sparse
- Multiple solutions (e.g., shortest paths, MSTs, topological sorts)

C++ Prelude

```
#include <bits/stdc++.h>
using namespace std;

#define DEBUG 1
#define dbg(x) (DEBUG ? _d((#x), (x)) : (x))
#define mod(x, m) (((x) % (m)) + (m)) % (m)
#define _f(k ? '\n' + string(f, ' ') : "")

template <class T> auto _s(T x,...) -> decltype(to_string(x)) { return to_string(x); }
string _s(char x,...) { return string("") + x + ""; }
string _s(string x,...) { return "" + x + ""; }

template <class P, class Q> string _s(pair<P, Q> x, int f=0, int k=0) {
    return _f + '(' + _s(x.first) + ", " + _s(x.second) + ')';
}

template <class T> auto _s(T x, int f=0, int k=0) -> decltype(end(x), string()) {
    string s; int i = 0; auto b = begin(x), e = end(x);
    while (b != e) s += _s(*b++, f+1, i++), s += (b == e ? "" : ", ");
    return _f + '[' + s + ']';
}

template <class T> T& _d(string s, T& x) {
    cout << s + " = " + _s(x, s.size() + 3) + '\n'; return x;
}

typedef vector<int> vi;
typedef pair<int, int> ii;
typedef int64_t i64;
typedef uint64_t u64;

const double eps = 1e-9;
const double pi = 2 * acos(0);
const double dinf = 1 / 0.0;
const int inf = numeric_limits<int>::max() >> 2;
const long linf = numeric_limits<i64>::max() >> 2;
```

C++ Tricks

- `set` is better than `priority_queue` for Prim's and Dijkstra's:

	<code>set<T></code>	<code>priority_queue<T, vector<T>, greater<T>></code>
Insert	<code>q.insert(x)</code>	<code>q.push(x)</code>
Top	<code>*q.begin()</code>	<code>q.top()</code>
Pop	<code>q.erase(q.begin())</code>	<code>q.pop()</code>
Delete	<code>q.erase(q.find(x))</code>	N/A