

Guideline for submitting your homework: In submitting your homework:

- i. Comment your codes clearly.
- ii. All your code should be written in Python language.
- iii. **Do not send your HWs via e-mail. No exception!**

Assignment 1: In this assignment you will work with 3D rotations.

Task 1- Plotting a 3D Object [5 pts]

Using numpy-stl, mpl_toolkits and matplotlib libraries read the “cow.stl” and plot the 3D cow as in the Figure 1, using the skeleton code given below.

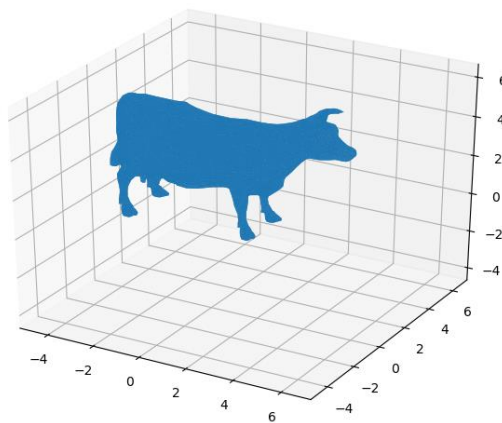


Figure 1: The cow in 3D grid

```
from stl import mesh
from mpl_toolkits import mplot3d
from matplotlib import pyplot
import numpy as np

figure = pyplot.figure()
axes = mplot3d.Axes3D(figure)
mesh_cow = mesh.Mesh.from_file('cow.stl')

print(mesh_cow.points.shape) #(5804, 9) Each triangular face of the
cow in column view
print(mesh_cow.vectors.shape) #(5804, 3, 3) Each triangular face of
the cow in 3x3 view. Each row represents a vertex.

axes.add_collection3d(mplot3d.art3d.Poly3DCollection(mesh_cow.vectors
))
#Add the 3D faces to the created matplotlib axes

min = np.min(mesh_cow.vectors.reshape(-1))
```

```
max = np.max(mesh_cow.vectors.reshape(-1)) #Find minimum and maximum
units to place the cow in a cubular grid.

axes.auto_scale_xyz([min, max], [min, max], [min, max])
pyplot.show()
```

Then, use a **homogenous transform matrix** on each vertex to squeeze the model a bit to add a “calf” nearby the cow as given in Figure 2.

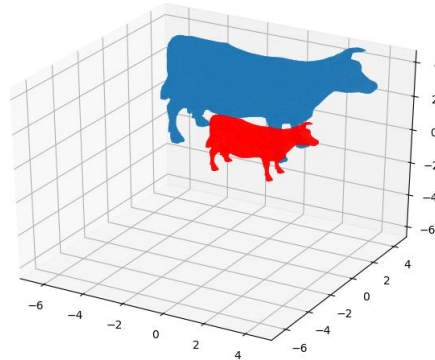


Figure 2: The cow and the calf

Task 2 - Exponential Coordinate Representation of 3D Rotations [20 pts]

(i) Start with a rotation angle of 45 degrees around z-axes. Calculate your 3x3 rotation matrix R_0 , using the corresponding w_0 vector in R^3 and using Rodrigues formula.

Rotate the cow object by the 3D rotation matrix R_0 and display your original object and the rotated object superimposed on each other.

Visualization: Use the rendering given in the previous part with transparency and different colors for each cow as given in Figure 3.

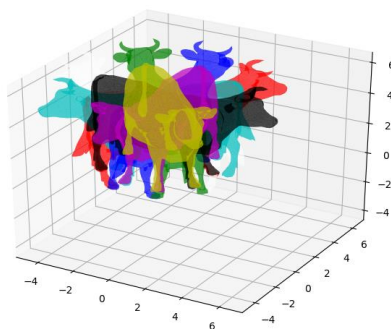


Figure 3: Rotated Cows

(ii) Repeat (i) with a rotation around the axis: $w_2 = [0.3, 0.7, -1]$. Use norm of the axis vector as the amount of rotation. **Use default camera view.**

Typically, you have to choose the center of rotation appropriately. If the object is already centered, you will not have to worry about that.

Task 3: Tańcząca Polska Krowa [15 pts]

i. Rotate your object around $w1 = [1, 0.5, 0]$, then rotate the result around $w2 = [0.3, 0.7, -1]$ vector. Use the norm of each w vector as the amount of radians for rotation. Show your result.

ii. Now rotate your object first around $w2$ then around $w1$ vector given in (i).

Do you obtain the object in the same position after (i) and (ii) ? Can you say that 3D rotations are commutative or not?

Task 4: Perturbed Rotations [20 pts]

Now, choose an axis vector w , and perturb w vector in each of its 3 components by 2 times (e.g. ± 5). Using Rodrigues formula, calculate your 3x3 rotation matrix R_i for $i=1, \dots, 6$.

Rotate your 3D object by each of the 3D rotation matrices R_i and display all the rotated objects superimposed with different colors and transparency.

Task 5: Quaternion Representation of 3D Rotations [20 pts]

Carry out the Task 3 by quaternions. Convert those rotations given by their axis vectors to quaternion representations $q1$ and $q2$. Carry out the consecutive quaternion multiplications to obtain the resulting quaternions and their corresponding rotation matrices. Comment on the result.

Task 6: Explain by a few sentences [10 pts]

- I) Is there a singularity in the Exponential Coordinates? If yes, can it be overcome?
- II) Is there a singularity in Quaternions?
- III) Why is the quaternion representation preferred over Euler angle representation for 3D rotations?

Task 7: Computing Angles [10 pts]

Read Dr. Gregory G. Slabaugh's report titled "Computing Euler angles from a rotation matrix"ⁱ. Find which Euler angles are used in the transformation matrix given below. Apply the rotation to the object

- I. Directly
- II. Using rotation matrices R_x , R_y , R_z .

and show that your findings are true.

$$\begin{bmatrix} 0.8365 & -0.5245 & -0.1585 \\ 0.2241 & 0.5915 & -0.7745 \\ 0.5000 & 0.6124 & 0.6124 \end{bmatrix}$$

ⁱ Slabaugh, G. G. (1999). Computing Euler angles from a rotation matrix. Retrieved on August, 6(2000), 39-63.