

This page shows how you can derive the simple form of the LSLR slope using only algebra:

$$b = \frac{s_y}{s_x} r$$

from its original definition.

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{(x - \bar{x})^2}$$

Denominator looks a bit like variance:

$$s_x = \sqrt{\frac{\sum (x - \bar{x})^2}{(n - 1)}}$$

$$s_x^2 = \frac{\sum (x - \bar{x})^2}{(n - 1)}$$

$$s_x^2 (n - 1) = \sum (x - \bar{x})^2$$

So we can rewrite b as:

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{s_x^2(n - 1)}$$

And the factors of the numerator look a bit like z-scores...

$$z_x = \frac{(x - \bar{x})}{s_x}$$

$$z_x s_x = (x - \bar{x})$$

$$z_y = \frac{(y - \bar{y})}{s_y}$$

$$z_y s_y = (y - \bar{y})$$

So we can rewrite the numerator:

$$\sum (x - \bar{x})(y - \bar{y}) = \sum z_x s_x z_y s_y$$

And now we can rewrite b, using our simplified b from above:

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{s_x^2(n - 1)}$$

$$b = \frac{\sum z_x s_x z_y s_y}{s_x^2(n - 1)}$$

But since standard deviation is a scalar (just a number, not a list of numbers), we can take it outside the summation:

$$b = \frac{s_x s_y \sum z_x z_y}{s_x^2(n - 1)}$$

$$b = \frac{s_y \sum z_x z_y}{s_x(n - 1)}$$

But this summation above should remind you of the simple form of Pearson's correlation formula!

$$r = \frac{\sum z_x z_y}{(n - 1)}$$

So we can finally rewrite the slope of the LSLR line as:

$$b = \frac{s_y}{s_x} r$$

Note that if  $x$  has zero standard deviation, then  $b$  is undefined! But that's ok, because that means  $x$  is a constant, and therefore has no predictive power for  $y$  anyway.

In R code,  $b$  is given by:

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lslr_slope = (sd(y)/sd(x))*cor(x, y)
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