The Impact of CBDC on Bank Deposits and the Interbank Market*

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Abstract

This paper investigates how the introduction of a central bank digital currency (CBDC) impacts the banking sector. The deposit market is modeled as a Salop circle, leading to imperfect substitutability between deposits issued by different institutions. The model features liquidity shocks. Absent a CBDC the interbank market is able to redistribute liquidity between banks. However, the central bank does not take part in the interbank market and the introduction of CBDC leads to greater reliance of the banking sector on central bank standing facilities. The model distinguishes between both the short-run and long-run impact of introducing a CBDC. The model highlights the need to consider the impact of CBDC on the banking sector. In particular, adjusting the remuneration rate of CBDC has little pass-through to the deposit rate set by banks but may affect concentration in the deposit market significantly.

Keywords: Central bank digital currency, banking, money, interbank Market

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1 Introduction

An increasing number of central banks are entertaining the possibility of issuing a Central Bank Digital Currency (CBDC) as a modern alternative to the physical currency that they currently issue. From the perspective of a retail depositor, a CBDC is likely to have technical features that make it a closer substitute to bank deposits than physical currency would be and thus would likely be a greater source of competition for banks in the deposit market than physical currency. This is the starting position this paper adopts. Specifically, this paper adopts a rather narrow view of CBDC in order to analyze in detail the possible implications of introducing a retail CBDC on the structure of the banking market. In particular, this paper defines a CBDC as a liability of the central bank held by retail depositors and that is an imperfect substitute for deposits issued by private banks. The model also assumes imperfect substitutability between the deposits issued by the private banks. I model two features that makes a CBDC distinct from bank deposits. First, the remuneration rate, which could be fixed at zero, is determined exogenously by the central bank and not according to the profit maximization objective of a private institution. Second, in order to maintain convertibility between private bank deposits, there operates an interbank market which I assume the central bank does not take part in. Instead, CBDC requires banks to increase their use of the central bank standing facilities. While Barrdear and Kumhof (2021) suggest that there should be no guarantee of convertibility between CBDC and private bank deposits, they do so due to financial stability concerns. This paper focuses on the everyday impact of CBDC, that is in times of low financial stress, where we would expect convertibility to hold in practice even if convertibility could be suspended in times of extreme financial stress.

The key predictions of the model are that in the short-run, the introduction of CBDC will result in an increase in interest rates on bank deposits and a reduction of the market shares of banks in the deposit market and bank profitability. In the long-run, the model predicts that the number of banks active in the deposit market will fall following the introduction of CBDC. While the interest rate on bank deposits increases in the long-run, it does so to a lesser extent than in the short-run. This paper also casts doubt on the remuneration rate of CBDC to be used as an additional independent policy rate. In the model, increasing the remuneration rate of CBDC will have no long-run impact on the interest rate on bank deposits but instead will lead to a fall in the number of banks and an increase in market concentration.

This paper can be seen as complementary to the growing literature on the policy implications of CBDC. A large literature focuses on issues of financial stability, in particular both Böser and Gersbach (2020) and Fernández-Villaverde et al. (2021) consider the increased risk of bank runs that may occur if bank depositors had access to a CBDC that they could transfer their deposits to in times of financial stress. While both Brunnermeier and Niepelt

(2019) and Niepelt (2020) discuss equivalence results where appropriate transfers from the central bank to the financial system neutralizes the impact of introducing a CBDC and mitigate the risk of CBDC induced bank runs.

Another strand of literature focuses on the implications of CBDC for monetary policy. For example Keister and Sanches (2019) suggests that while CBDC can promote efficient exchange, it may also raise funding costs. Meaning et al. (2021) provide a detailed discussion on the monetary transmission mechanism in general as well as other possible policy implications. Both Barrdear and Kumhof (2021) and Kumhof and Noone (2021) discuss the remuneration of CBDC in detail and its possible use for monetary policy. Kumhof and Noone (2021) propose a two-tier remuneration system while Barrdear and Kumhof (2021)propose both a quantity rule and a price rule for CBDC. This paper does not consider the impact of a quantity rule, but I analyze the impact of changing the CBDC remuneration rate independent of other policy rates and find it affects the structure of the banking sector while having only a short-term impact on bank deposit rates.

This paper is most closely related to the literature on the impact of CBDC on the banking sector. In a macroeconomic framework, Bacchetta and Perazzi (2021) assume a constant elasticity of substitution between a CBDC and a continuum of monopolistically competitive banks. While Andolfatto (2021) analyzes the case of a single monopoly bank where CBDC and bank deposits are perfect substitutes but there is a fixed cost for depositors to switch between the two. Chiu et al. (2019) study a model of Cournot oligopoly with a finite number of banks where banks compete in the quantity rather than the remuneration of deposits. CBDC is assumed to be a perfect substitute for bank deposits and so imposes a minimum remuneration rate on bank deposits.

This paper is also related to the literature on spatial models of imperfect competition. Competition in the deposit market before the introduction of CBDC is modeled as in the classic paper by Salop (1979). The structure of competition following the introduction of CBDC is closely related to Salop Circle models with a center such as Bouckaert (2000) and Madden and Pezzino (2011).

Finally, this paper is also related to the literature on interbank markets. In particular, the theoretical treatment of the interbank market in this paper is closest to that of Hauck and Neyer (2014) and Bucher et al. (2020) who both study the operation of an interbank market within the framework of the euro area.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium in the interbank market. In Section 4 the equilibrium in the deposit market is analyzed in the short-run, where the number of banks is fixed while Section 5 analyzes the equilibrium in the long-run where the number of banks varies to satisfy free-entry. Section 6 provides a discussion of the model's policy implications and Section 7 concludes.

2 Model

I consider a three period model of the retail deposit market. The economy consists of three types of agents, risk-neutral banks, a central bank and a continuum of bank customers.

In the model, banks provide loans which earn an exogenously given interest rate of R_L. Issuing these loans generates a cost for the bank which is assumed to be quadratic in the quantity of loans issued. The banks obtain funding for its lending activities through issuing debt and providing retail deposits.

The bank issues debt quantity $B_{B,i}$, this debt is assumed to be risk free and remunerated at the same interest rate as the central bank's refinancing operations,R_{RO}.

The retail deposit market is modeled as a Salop circle as in Salop (1979). There is a continuum of depositors located around a circle, the mass of depositors is denoted by $\bar{D}$. I denote the number of banks that wish to obtain deposit financing by $N \geq 2$, and that these banks locate equidistant from each other around the circle. Bank i chooses a deposit rate $R_{D,i}$. A depositor located at a distance x away from the bank must pay a linear transportation cost $t_B x \geq 0$ in order to deposit their funds at the bank.

The central bank may also choose to enter the deposit market through issuing a Central Bank Digital Currency (CBDC). Should the central bank issue a CBDC, it chooses a remuneration rate R_{CB} and locates itself at the center of the Salop circle. Depositors are assumed to pay a fixed transportation cost to obtain CBDC and this transport cost is drawn from the interval $t_{CB} \in [\underline{t}, \bar{t}]$ according to a uniform distribution. The entry of CBDC into the deposit market is know to all participants and banks choose whether to enter the deposit market and set their deposit rates with full knowledge of whether they will be competing against a CBDC. I assume that the CBDC is only held by households and that banks cannot deposit into CBDC. This assumption allows the central bank to set R_{CB} larger than the interest rate on the deposit facility R_{DF}. In cases where $R_{CB} \leq R_{DF}$ this assumption is rendered unnecessary as from the bank's perspective, the return on CBDC is weakly dominated by the central bank's deposit facility.

Deposits are subject to liquidity shocks which take the following form. A fraction $\xi \in (0, 1)$ of all deposits, distributed evenly around the circle relocate to a specific point on the circle, chosen at random. The structure of the liquidity shock is such that a single institution will receive a positive inflow of liquidity while the remaining institutions will see a deposit outflow. While this liquidity shock is very stylized, it has some important properties. First, it generates a role for an interbank market to redistribute liquidity among the banking sector. Second, it links the probability of receiving a positive inflow of liquidity to a bank's share of the deposit market; banks that have a larger market share are more likely to receive the positive inflow of liquidity.

Following the realization of the liquidity shock, banks that face a liquidity shortfall are able to obtain additional funding from the central bank or through the interbank market. The liquidity shock is assumed to occur at a time horizon that makes it impossible for the bank to access additional liquidity through debt issuance or through the central bank's refinancing operations. Instead, as in Hauck and Neyer (2014) the banks are able to access a lending facility to borrow at a rate $R_{LF} > R_{RO}$ and are able to access a deposit facility where they are able to deposit excess liquidity, earning a rate $R_{DF} < R_{RO}$. For simplicity, I assume that the central bank operates a symmetric corridor so that

$$R_{RO} = \frac{1}{2}\left(R_{DF} + R_{LF}\right). \tag{1}$$

In addition, the banks are able to trade liquidity between themselves in an interbank market where banks are able to borrow at rate R_{IB}. To simplify the model, I do not assume any borrowing frictions arising in the interbank market. The central bank interest rate corridor sets an upper- and lower-bound on the interbank rate.

The model timing is as follows. In the first period, $t = 1$, both commercial banks and the central bank decide on whether to enter the deposit market. Should the central bank decide to enter the deposit market by issuing a CBDC, it will also set the remuneration rate R_{CB}. In the second period, $t = 2$, the commercial banks choose the quantity of loans $L_{B,i}$, the amount of debt financing they issue $B_{B,i}$ and compete in prices in the deposit market. In the third period, the liquidity shock is realized and commercial banks are able to trade liquidity between themselves in an interbank market as well as borrow and deposit liquidity using central bank facilities in order to obtain a liquidity neutral position. I solve the game for its Subgame Perfect Nash Equilibrium in pure strategies using backward induction.

3 The Interbank Market

I begin the analysis of the model with the final period, $t = 3$. At this point in time the $N \geq 2$ banks which are indexed by i have made their decisions regarding the quantity of lending $L_{B,i}$ as well as their funding structure. The bank's funding structure consists of a quantity of deposits $D_{B,i}$ and bank debt $B_{B,i}$ as well as the interest rate that each bank sets on its deposits.

The banks are all subject to a liquidity shock. The liquidity shock is modeled in the following way; a fraction $\xi \in (0, 1)$ of all deposit holdings moves to a specific point on the Salop circle. This point is chosen randomly and each point on the circle has an equal point of being chosen. Modeling the process in this way generates a link between deposit market competition and liquidity risk. Specifically, the probability that this point lies within the deposit holdings of bank i is equal to bank $i's$ share of the deposit market $q_{B,i}$ where

$$q_{B,i} = \frac{D_{B,i}}{\bar{D}}. \tag{2}$$

With probability $1 - q_{B,i}$, bank i is hit by a net outflow of liquidity, denoted by ϵ_i^- where

$$\epsilon_i^- \equiv L_{B,i} - B_{B,i} - (1-\xi)\, q_{B,i}\bar{D}. \tag{3}$$

With probability $q_{B,i}$, bank i is hit by a net inflow of liquidity, denoted by ϵ_i^+ where

$$\epsilon_i^+ \equiv L_{B,i} - B_{B,i} - (1-\xi)\, q_{B,i}\bar{D} - \xi\bar{D}. \tag{4}$$

In the case where a bank has a liquidity deficit, they must obtain additional liquidity to ensure that they maintain a neutral liquidity position by the end of the final period. In order for a bank with a liquidity deficit to return to a neutral liquidity position, banks are able to access the interbank market at an interest rate equal to R_{IB} or obtain liquidity from the central bank's liquidity facility at an interest rate of R_{LF}. Should the bank have a liquidity surplus it may attempt to lend liquidity in the interbank market at an interest rate equal to R_{IB} or it may deposit liquidity at the central bank's deposit facility earning a rate of R_{DF}.

The aggregate liquidity deficit of the banking sector when bank i receives a net inflow of liquidity is simply $\Gamma \equiv \sum_{j\neq i} \epsilon_i^- + \epsilon_i^+$. Whenever $\Gamma > 0$, there is an aggregate liquidity deficit in the banking sector and thus at least one bank must obtain liquidity from the central bank lending facility at a rate of R_{LF}. This implies that the interest in the interbank market will be equal to the interest rate of the lending facility, $R_{IB} = R_{LF}$. Furthermore, as $R_{LF} > R_{RO}$ the cost of obtaining liquidity from either the interbank market or the central bank lending facility is more costly than it would have been to issue more debt in the prior period. It follows that in equilibrium $\epsilon_i^+ \leq 0$ and the bank that receives a net inflow of liquidity will have a surplus of liquidity.

Alternatively, when $\Gamma \leq 0$, there is an aggregate liquidity surplus in the banking sector and so long as R_{DF} is above the effective lower bound, at least one bank will deposit liquidity at the central bank using the deposit facility. Despite this, the structure of the liquidity shocks is such that should trade occur in the interbank market, there will be a single bank supplying liquidity to banks with a liquidity deficit and thus this bank will have a monopoly on liquidity provision. This implies that the interest rate in the interbank market will still be set equal to the interest rate of the liquidity facility and $R_{IB} = R_{LF}$ even in cases where there is an aggregate liquidity surplus. Furthermore, as $R_{DF} < R_{RO}$ depositing liquidity at the central bank's deposit facility is less efficient than reducing the amount of debt the bank issued in the prior period. It follows that in equilibrium $\epsilon_i^- \geq 0$ and the banks that receive a net outflow of liquidity will have a liquidity deficit.

From this point I will refer to the bank that receives a net inflow of liquidity as the surplus bank and the banks that receive a net outflow of liquidity as deficit banks.

Given the interbank rate is equal toR_{LF} independent of the aggregate liquidity position of the banking sector, the cost that a deficit bank incurs in order to reach a neutral liquidity position is simply $C_{D,i}^- = R_{LF}\epsilon_i^- > 0$. It is worth noting at this point the implications of $R_{IB} = R_{LF}$ on the liquidity cost of deficit banks. In particular, as the central bank is assumed not to take part directly in the interbank market, should there be a net inflow of liquidity into CBDC, deficit banks would only be able to obtain liquidity from the central bank's liquidity facility as no bank would have a liquidity surplus in which to lend in the interbank market.

The surplus bank will lend out as much of its excess liquidity as possible in the interbank market at a rate equal to R_{LF} while depositing any remaining excess liquidity at the central bank, earning a rate of R_{DF}. The surplus bank's cost in reaching a neutral liquidity position will depend on the aggregate liquidity position of the banking sector as this determines whether the surplus bank is able to lend all of its excess liquidity in the interbank market or not. This cost will be negative and can be written as

$$C_{D,i}^+ = \begin{cases} -R_{LF}\epsilon_i^+ & \text{if } \Gamma > 0 \\ -R_{LF}\sum_{j\neq i}\epsilon_j^- + R_{DF}\left(\epsilon_i^+ + \sum_{j\neq i}\epsilon_j^-\right) & \text{otherwise.} \end{cases} \tag{5}$$

The expected liquidity cost that bank i faces prior to the realization of the liquidity shock is then

$$E\left[C_{D,i}\right] = \begin{cases} \left(1-q_{B,i}\right)R_{LF}\epsilon_i^- + q_{B,i}R_{LF}\epsilon_i^+ & \text{if } \Gamma > 0 \\ \left(1-q_{B,i}\right)R_{LF}\epsilon_i^- - q_{B,i}R_{LF}\sum_{j\neq i}\epsilon_j^- + q_{B,i}R_{DF}\left(\epsilon_i^+ + \sum_{j\neq i}\epsilon_j^-\right) & \text{otherwise.} \end{cases} \tag{6}$$

As the probability that a bank becomes a surplus bank depends positively on the bank's market share $q_{B,i}$, it follows that the expected cost of the liquidity shock is increasing in $q_{B,i}$. Thus, to the extent that the entry of CBDC into the deposit market reduces the market share of banks, these banks will also face larger expected costs from the liquidity shock as a result.

4 Banking Sector

I now turn to the banks problem in the intermediate period, $t = 2$, taking the number of banks N as given. In this period, bank i chooses the amount it lends, $L_{B,i}$, the amount of debt it issues $B_{B,i}$ and sets the interest rate it offers to depositors $R_{D,i}$ in order to maximize its profits. The profit function of bank i is

$$\pi_i = \max_{L_{B,i},B_{B,i},R_{D,i}} \left\{ R_L L_{B,i} - \frac{1}{2}\lambda L_{B,i}^2 - R_{RO}B_{B,i} - R_{D,i}q_i\bar{D} - E\left[C_{D,i}\right] - F \right\}, \quad (7)$$

where $E\left[C_{D,i}\right]$ is defined in equation (6) and $F > 0$ is the fixed cost that banks are assume to pay in order to enter the deposit market. Banks compete for depositors in prices à la Bertrand, taking as given the deposit rates set by other banks and the funding structure of other banks as given.

4.1 Demand for Deposits

In the case where the central bank does not implement a CBDC, competition between banks in the deposit market is identical to Salop's circle model. If bank i offers a deposit rate equal to $R_{D,i}$, and the other banks offer a deposit rate equal to $R_{D,-i}$ then a depositor that is located at a distance x from bank i, where $x \in \left[0, \frac{1}{N}\right]$ will choose to deposit their funds at bank i rather than the bank neighboring bank so long as

$$R_{D,i} - t_B x \geq R_{D,-i} - t_B\left(\frac{1}{N} - x\right), \quad (8)$$

where t_B is the linear transport cost that is incurred by consumers. Bank i thus faces the following demand function

$$q_{B,i} = \frac{1}{N} + \frac{1}{t_B}\left(R_{D,i} - R_{D,-1}\right). \quad (9)$$

In the case where the central bank does implement a CBDC, bank i faces competition not just the two banks that neighbor it but also from the CBDC. I assume that the central bank sets a fixed interest rate R_{CB} and that depositors incur a transport cost t_{CB} if they deposit funds in the CBDC. The transport costs associated with CBDC are assumed to be drawn randomly from a uniform distribution from an interval $[\underline{t}, \bar{t}]$. Thus for a depositor located at distance x from bank i would prefer to deposit funds in bank i rather than in the CBDC so long as

$$R_{D,i} - t_B x \geq R_{CB} - t_{CB}. \quad (10)$$

In addition, in order for a depositor to deposit funds in bank i they must also satisfy equation (8) and preferring bank i to other banks.

To calculate the demand function bank i faces in the presence of CBDC first define the point $x_{B,i}^* \in \left[0, \frac{1}{N}\right]$ as the point where a depositor is indifferent between bank i and the neighboring bank. I make the assumption that $\bar{t}$ is sufficiently large that

$$\bar{t} > t_B x^*_{B,i} + R_{CB} - R_{D,i} \ \forall i, \tag{11}$$

which ensures that there is always competition between neighboring banks.[1] Similarly, I make the assumption $\underline{t}$ is sufficiently small that in equilibrium

$$\underline{t} \leq t_B x^*_{B,i} + R_{CB} - R_{D,i} \ \forall i, \tag{12}$$

which ensures that some depositors will hold CBDC in equilibrium. If this condition did not hold, the model would be equivalent to the case where the central bank did not implement CBDC. Given these assumptions, there exists an $\hat{x}_{B,i} \in \left[0, x^*_{B,i}\right]$ such that the following condition holds

$$\hat{x}_{B,i} \equiv \frac{1}{t_B}\left(R_{D,i} - R_{CB} - \underline{t}\right), \tag{13}$$

where all depositors located at a distance $\hat{x}_{B,i}$ or closer to bank i will prefer to hold deposits at bank i over CBDC. For the interval $x \in (\hat{x}_{B,i}, x^*_{B,i}]$ there exists a function $t^*_i(x) \in (\underline{t}, \bar{t})$ such that the following condition holds

$$t^*_i(x) = t_B x + R_{CB} - R_{D,i}, \tag{14}$$

where the function $t^*_i(x)$ defines that smallest value of t_{CB} that depositors that are located at a distance x away from bank i must have in order to prefer depositing in bank i rather than CBDC.

Given the above definitions and assumptions, the demand function that bank i faces is

$$q_{B,i} = 2\left(\int_{\hat{x}_{B,i}}^{x^*_{B,i}} \left(\frac{\bar{t} - t^*(x)}{\bar{t} - \underline{t}}\right) dx + \hat{x}_{B,i}\right), \tag{15}$$

where bank i receives deposits from all depositors located at a distance under $\hat{x}_{B,i}$ and a fraction $\left(\frac{\bar{t}-t^*(x)}{\bar{t}-\underline{t}}\right)$ of depositors located at a distance $x \in (\hat{x}_{B,i}, x^*_{B,i}]$.

4.2 Lending and Debt Issuance

As the demand for deposits is independent of the quantity of loans made by the bank $L_{B,i}$, as well as the quantity of debt issued $B_{B,i}$, both of these decisions can be solved for independently of the deposit rate that bank i sets. This also implies that the decision rules are the same regardless of whether the central bank implements a CBDC or not.

[1] If this condition is violated, banks will have local monopoly power subject to depositrs having an outside option, CBDC. In this case the solution to the model will be comparable to a Salop circle with incomplete coverage where in this case gaps in coverage would be filled by CBDC.

The bank chooses $L_{B,i}$ and $B_{B,i}$ in order to maximize its profit function, as defined by equation (7) subject to the condition that $\epsilon_i^- \geq 0$ and $\epsilon_i^+ \leq 0$. The first-order conditions of this problem yields the following proposition.

Proposition 1. *In an equilibrium with N banks, bank $i \in \{1, \ldots, N\}$ chooses its lending $L_{B,i}$ and debt level $B_{B,i}$ such that*
i) $L_{B,i} = \frac{1}{\lambda}(R_L - R_{RO})$ *and*
ii) $\epsilon_i^- = 0$.

Proof. See Appendix.

As $\epsilon_i^- = 0$ holds for all i, Proposition 1 implies the obvious but important corollary. □

Corollary 1. *In equilibrium the banking sector features an aggregate surplus with $\Gamma < 0$.*

The result that in equilibrium $\epsilon_i^- = 0$ may appear somewhat surprising at first as it implies that banks accumulate excess liquidity in order not to incur any liquidity costs should they be hit by a net outflow of liquidity. This is the case despite the fact that excess liquidity comes at a cost as $R_{RO} > R_{DF}$. This is the case because it is assumed that banks take the liquidity position of other banks $\sum_{j \neq i} \epsilon_j^-$ as given when choosing their own liquidity position. Thus the banks do not endogenize the impact of their own funding position on the aggregate liquidity position of the banking sector as a whole. This leads to an over-accumulation of liquidity relative to the social optimum.

4.3 Deposit Rates

Once the aggregate liquidity position is known, the first order condition of the bank's optimization problem yields the following equation for $R_{D,i}$

$$R_{D,i} = (1 - \xi) R_{RO} + \xi R_{DF} - \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i}. \tag{16}$$

The demand function the banks face depends on whether or not the central bank implements a CBDC.

Absent CBDC, in a symmetric equilibrium each bank obtains an equal share of deposits and as depositors have no outside option, it follows that $q_{B,i} = \frac{1}{N}$. The equilibrium is summarized by the following proposition.

Proposition 2. *If the central bank chooses not to implement a CBDC in a symmetric equilibrium each bank $i \in \{1, \ldots, N\}$ chooses $L_{B,i}$, $B_{B,i}$ and $R_{D,i}$ such that*
i) $L_{B,i} = \frac{1}{\lambda}(R_L - R_{RO})$

ii) $q_{B,i} = \frac{1}{N}$
iii) $R_{D,i} = (1-\xi) R_{RO} + \xi R_{DF} - \frac{1}{N} t_B$
iv) $B_{B,i} = L_{B,i} - (1-\xi) \frac{1}{N} \bar{D}$.

Proof. See Appendix. □

In the case where the central bank implements a CBDC, banks face a different demand function. The equilibrium is summarized by the following proposition.

Proposition 3. *If the central bank chooses to implement a CBDC and $t_{CB} \in [\underline{t}, \bar{t}]$ satisfies equations (11) and (12) in a symmetric equilibrium each bank $i \in \{1, \ldots, N\}$ chooses $L_{B,i}$, $B_{B,i}$ and $R_{D,i}$ such that chooses $L_{B,i}$, $B_{B,i}$ and $R_{D,i}$ such that*
i) $L_{B,i} = \frac{1}{\lambda} (R_L - R_{RO})$
ii) $q_{B,i} = \frac{1}{N} - \frac{1}{t_B} \left(\frac{1}{\bar{t}-\underline{t}}\right) \left(\frac{1}{2} t_B \frac{1}{N} - (R_{D,i} - R_{CB} + \underline{t})\right)^2$
iii) $R_{D,i} = (1-\xi) R_{RO} + \xi R_{DF} - \frac{1}{3} t_B q_{B,i}$
iv) $B_{B,i} = L_{B,i} - (1-\xi) q_{B,i} \bar{D}$.

Proof. See Appendix. □

From Proposition 2 and Proposition 3 several conclusion can be drawn regarding the impact of a CBDC in the model. First, as the market share of CBDC is simply $q_{CB} = 1 - \sum_i q_{B,i}$, there will be positive demand for CBDC in equilibrium so long as the transport cost t_{CB} satisfies equations (11) and (12) and thus each bank will have a market share of less than $\frac{1}{N}$ as a result. Furthermore, holding N fixed, the introduction of CBDC will increase deposit rates that banks offer depositors. As a result of the fall in market share of the deposit market, banks will substitute deposit finance with additional debt financing. Lending in this model is unaffected by the introduction of CBDC.

5 Entry

In the previous section, the number of banks competing in the deposit market was assumed to be fixed. I now consider the entry problem for banks, given banks have to pay a fixed cost $F > 0$ in order to enter the deposit market.

Given the optimal decisions of the banks in $t = 2$, the bank's profit in the case where no CBDC enters the market can be written as

$$\pi^*_{B,i} = \bar{\pi} + \frac{1}{N} t_B \bar{D}, \tag{17}$$

where

$$\bar{\pi} = \frac{1}{2\lambda}\left(R_L - R_{RO}\right)^2 - F. \tag{18}$$

It is worth noting that equation (18) is the profit level of a bank that lends the optimal quantity, $L_{B,i} = \frac{1}{\lambda}\left(R_L - R_{RO}\right)$ while having a zero market share in the deposit market and thus fully finances its lending using debt. Assuming free entry into the deposit market, the number of banks entering the market increases until profits are driven down to zero. For a finite number of banks to enter the market, a key requirement is that $\bar{\pi} \leq 0$. Should this condition hold, $N^{N,LR}$the number of banks that will enter when CBDC is not introduced is given by the following equation

$$\frac{1}{N^{N,LR}} = \sqrt{-\frac{\bar{\pi}}{t_B \bar{D}}}. \tag{19}$$

If the central bank chooses to introduce a CBDC, the equilibrium profit of bank i can be written as

$$\pi_{B,i} = \bar{\pi} + \frac{1}{3} q_{B,i}^2 t_B \bar{D}, \tag{20}$$

where $\bar{\pi}$ is defined by equation (18). Free entry of banks into the deposit market yields the following equation for $q_{B,i}^{C,LR}$ for the market share of banks

$$q_{B,i}^{C,LR} = \sqrt{-\frac{\bar{\pi}}{\frac{1}{3} t_B \bar{D}}}. \tag{21}$$

Combining equation (21) with the equilibrium market share given in proposition 3 would yield the number of banks entering the deposit market when the central bank introduces a CBDC. The following proposition highlights the key changes in market structure following the introduction of CBDC.

Proposition 4. *Following the introduction of a CBDC such that $t_{CB} \in [\underline{t}, \bar{t}]$ satisfies equations (11) and (12) and allowing banks to enter subject to the free entry condition given by equation (21)*
i) The number of banks N falls relative to the economy without CBDC
ii) The market share of each bank $q_{B,i}$ is larger than in the case without CBDC.

Proof. For part i) note that absent CBDC, the number of firms active in the economy is defined by equation (19). From equation (21) it is clear that the market share of each bank must be larger than the left hand side of equation (19). In any economy with CBDC, from Proposition 3 we know that the bank market shares are strictly less than $\frac{1}{N}$, thus it follows that the number of banks must be smaller. Part ii) Follows immediately by noting that the market shares absent CBDC is simply the inverse of the number of banks, while the market shares of banks following the introduction of CBDC are given by equation (21). □

Proposition 4 highlights the impact on the concentration in the deposit market of introducing CBDC. The added competitive pressure on banks following the introduction of CBDC forces some banks out of the market while requiring banks to have a larger market share in order to remain profitable.

6 Discussion

The model allows us to distinguish between the short-run impact of the introduction of CBDC where the number of banks N is fixed from the long-run impact where the number of banks is able to adjust. identified the short-run impact of the introduction of CBDC while Section 5 identified the long-run impact.

As discussed in Section 4, following the introduction of CBDC, the market share of banks will fall in the short-run as $q_{B,i}^{C,SR} < \frac{1}{N}$, where I denote equilibrium variables with a superscript C, SR as referring to the short-run (fixed-N) equilibrium with CBDC. Adopting a similar notation, a superscript N, SR refer to the short-run (fixed-N) equilibrium without CBDC. The change in the equilibrium deposit rate can be written as follows:

$$R_{D,i}^{C,SR} - R_{D,i}^{N,SR} = \left(\frac{1}{N} - \frac{1}{3}q_{B,i}^{C,SR}\right) t_B, \tag{22}$$

where it follows from $q_{B,i}^{C,SR} < \frac{1}{N}$ that this change in the interest rate is strictly positive.

The short-run equilibrium will not persist in the long-run as banks must pay higher deposit rates while receiving a smaller share of the deposit market. This will negatively affect bank profitability. Section 5 identified the long-run impact of the introduction of CBDC. In particular, the number of banks in the deposit market falls so that banks are able to return to profitability. However, the market share of each bank will actually be larger in the long-run following the introduction of CBDC with $q_{B,i}^{C,SR} > q_{B,i}^{N,LR}$, where denoting variables with a superscript C, SR and superscript N, LR refer to the short-run and long-run equilibria respectively.

The difference in the interest rates on deposits in the long-run can be written as follows

$$R_{D,i}^{C,LR} - R_{D,i}^{N,LR} = \left(\sqrt{\frac{-\bar{\pi}}{t_B \bar{D}}} - \sqrt{\frac{1}{3}}\sqrt{\frac{-\bar{\pi}}{t_B \bar{D}}}\right) t_B, \tag{23}$$

where it is clear that as in the short-run, there is an increase in the interest rates on deposits in the long-run following the introduction of CBDC. Noting that $\sqrt{\frac{-\bar{\pi}}{t_B \bar{D}}} = \frac{1}{N^{N,LR}}$ equation (23) can be written as

$$R_{D,i}^{C,LR} - R_{D,i}^{N,LR} = \left(\frac{1}{N^{N,LR}} - \sqrt{\frac{1}{3}}\frac{1}{N^{N,LR}}\right) t_B. \tag{24}$$

Comparing equation (22) with equation (24) highlights that the short-run increase in the interest rate on deposits is much larger than the long-run increase.

One possible benefit of CBDC is that the central bank is able to adjust the remuneration rate of CBDC, R_{CB}, and thus would have access to an additional policy instrument. I now consider the impact of a change in R_{CB} has on the deposit market equilibrium. First, consider the market share of a bank in the model with CBDC which can be expressed as

$$q_{B,i}^{C,j} = \frac{1}{N} - \frac{1}{t_B}\left(\frac{1}{\bar{t}-\underline{t}}\right)\left(\frac{1}{2}t_B\frac{1}{N} - \left(R_{D,i}^{C,j} - R_{CB} + \underline{t}\right)\right)^2 \qquad \forall j \in \{SR, LR\}. \tag{25}$$

Note that as as $x_{B,i}^{*} \geq \hat{x}_{B,i}$ by assumption, it follows that $\frac{1}{2}t_B\frac{1}{N} - (R_{D,i} - R_{CB} + \underline{t}) \geq 0$ and thus $q_{B,i}^{C,j}$ is decreasing in R_{CB}, holding $R_{D,i}^{C,j}$ fixed. Thus an increase in the remuneration rate of CBDC puts downward pressure on the market share of banks.

Substituting in the short-run equilibrium deposit rate $R_{D,i}^{C,SR}$ yields the following quadratic equation for $q_{B,i}^{C,SR}$

$$q_{B,i}^{C,SR} = \frac{1}{N} - \frac{1}{t_B}\left(\frac{1}{\bar{t}-\underline{t}}\right)\left(\frac{1}{2}t_B\frac{1}{N} - \left((1-\xi)R_{RO} + \xi R_{DF} + \frac{1}{3}q_{B,i}^{C,SR}t_B - R_{CB} + \underline{t}\right)\right)^2. \tag{26}$$

Application of the implicit function theorem to equation (26) shows that $q_{B,i}^{C,SR}$ will be strictly decreasing in R_{CB} and thus, $R_{D,i}^{C,SR}$ will be increasing in R_{CB}. Thus in the short-run, there will be pass-through from the remuneration rate of CBDC to the interest rate deposits and a fall in the market share of banks in the deposit market. As N is fixed in the short-run it follows that the market share of CBDC in the deposit market will also increase following an increase in R_{CB}. Thus following an increase in R_{CB} the market share of banks falls as more depositors instead substitute from bank deposits to CBDC.

Consider now the equation for $R_{B,i}^{C,LR}$, which can be written as

$$R_{D,i}^{C,LR} = (1-\xi)R_{RO} + \xi R_{DF} - \sqrt{\frac{1}{3}}\sqrt{\frac{-\bar{\pi}}{t_B\bar{D}}}t_B. \tag{27}$$

It is clear from equation (27)that in the long-run the interest rate on deposits is unaffected by changes in R_{CB} and thus $q_{B,i}^{C,LR}$ will be strictly decreasing in R_{CB}. Inspection of equation (25) shows that when the deposit rate is fixed, an increase in R_{CB} will necessarily lead to a fall in N in the long-run. In the long-run, there is no pass-through from the remuneration rate of CBDC to the interest rate on deposits. In addition, an increase in R_{CB} will force banks to exit the deposit market.

The model highlights the possible downsides of using the remuneration of CBDC as a policy instrument independent of other policy rates. While there may be some short-run pass through of interest rates, it also affects the profitability of the banking sector which in the long-run would affect the market concentration.

One caveat that should be expressed in relation to the discussion presented in this section is that the model does not allow for pass-through of of the higher cost of funds banks may face to the lending rate, R_L. Should banks are able to pass the increased borrowing costs to borrowers, then it is likely that the introduction of CBDC or a rise in its remuneration rate may also increase borrowing costs. This effect would be exacerbated in the long-run if a more concentrated market in retail deposits also lead to a more concentrated loan market.

7 Conclusion

As the policy debate surrounding the potential introduction of a retail CBDC grows, so does the need for further analysis of its potential implications. This paper focuses on the impact of CBDC on the structure of the market for retail bank deposits and on the interbank market. In this paper, CBDC is modeled as a source of direct competition for bank deposits. Competition in the deposit market is modeled using a Salop circle model and thus there is imperfect substitutability between deposits of different banks and the CBDC. This framework allows us to distinguish between the short-run impact of CBDC, where the number of banks is fixed from the long-run impact where the number of banks may adjust.

The model suggests that in the short-run, the introduction of CBDC will result in an increase in interest rates on bank deposits and a reduction of the market shares of banks in the deposit market. Banks substitute these deposits by issuing more debt and as their cost of funds increases, bank profitability falls. In the long-run, the model suggests that the introduction of CBDC will reduce the number of banks active in the deposit market and lead to greater concentration in the banking sector. While the interest rate on bank deposits increases in the long-run, it does so to a lesser extent than in the short-run.

This paper also casts doubt on the remuneration rate of CBDC to be used as an additional independent policy rate. In the model, increasing the remuneration rate of CBDC will have no long-run impact on the interest rate on bank deposits but instead will lead to a fall in the number of banks and an increase in market concentration.

While this paper makes no claims regarding the welfare implications of the introduction of CBDC, it would be prudent for policymakers to take into account the welfare implications of a more concentrated banking sector that may follow the introduction of a CBDC.

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Appendix

Proof of Proposition 1

The Lagrangian for the bank's problem can be written as follows

$$\begin{aligned}\mathcal{L}_i = & R_L L_{B,i} - \frac{1}{2}\lambda L_{B,i}^2 - R_{RO} B_{B,i} - R_{D,i} q_{B,i} \bar{D} - E\left[C_{D,i}\right] \\ & + \mu_i^- \left(L_{B,i} - B_{B,i} - q_{B,i}\left(1-\xi\right)\bar{D}\right) \\ & + \mu_i^+ \left(\xi \bar{D} + q_{B,i}\left(1-\xi\right)\bar{D} + B_{B,i} - L_{B,i}\right),\end{aligned} \tag{A.28}$$

where μ_i^- is the Kuhn-Tucker multiplier on the inequality constraint $\epsilon_i^- \geq 0$ and μ_i^+ is the Kuhn-Tucker multiplier on the inequality constraint $\epsilon_i^+ \leq 0$ described by equations (3) and (4) respectively.

There are two cases to consider, depending on whether there is an aggregate liquidity deficit $\Gamma \geq 0$ or an aggregate liquidity deficit $\Gamma < 0$. First consider the case where $\Gamma \geq 0$ and there is an aggregate liquidity deficit.

Case 1: $\Gamma \geq 0$ In the case where $\Gamma \geq 0$, the banking sector has an aggregate liquidity deficit and the expected liquidity cost is

$$E\left[C_{D,i}\right] = \left(1 - q_{B,i}\right) R_{LF} \epsilon_i^- + q_{B,i} R_{LF} \epsilon_i^+. \tag{A.29}$$

As the first-order conditions of the Lagrangian with respect to $L_{B,i}$ and $B_{B,i}$ do not depend on the derivative of the demand functions it follows that the first order conditions can be stated independently of whether or not a CBDC has been introduced. Specifically, the following first order conditions must hold

$$\frac{\partial \mathcal{L}_i}{\partial L_{B,i}} : R_L - \lambda L_{B,i} + R_{RO} - R_{LF} + \mu_i^- - \mu_i^+ = 0, \tag{A.30}$$

$$\frac{\partial \mathcal{L}_i}{\partial B_{B,i}} : - R_{RO} + R_{LF} - \mu_i^- + \mu_i^+ = 0. \tag{A.31}$$

Adding equation (A.30) and (A.31) and rearranging yields the following equation for $L_{B,i}$

$$L_{B,i} = \frac{1}{\lambda}\left(R_L - R_{RO}\right). \tag{A.32}$$

Now consider equation (A.31) on its own. Rearranging this equation so that the Kuhn-Tucker multipliers are on the right-hand side yields

$$R_{LF} - R_{RO} = \mu_i^- - \mu_i^+. \tag{A.33}$$

Given at most one of equations (3) and (4) must hold with equality, it follows from the fact that $R_{LF} > R_{RO}$ that equation (3) must hold with strict equality and thus $\epsilon_i^- = 0$.

Case 2: $\Gamma < 0$ In the case where $\Gamma < 0$, the banking sector has an aggregate liquidity surplus and the expected liquidity cost is

$$E\left[C_{D,i}\right] = \left(1 - q_{B,i}\right) R_{LF} \epsilon_i^- - q_{B,i} R_{LF} \sum_{j \neq i} \epsilon_j^- + q_{B,i} R_{DF} \left(\epsilon_i^+ + \sum_{j \neq i} \epsilon_j^-\right). \tag{A.34}$$

Taking the first-order conditions of the Lagrangian with respect to $L_{B,i}$ and $B_{B,i}$ yields the following two equations that must hold in equilibrium

$$\frac{\partial \mathcal{L}_i}{\partial L_{B,i}} : R_L - \lambda L_{b,i} + R_{RO} - \left(1 - q_{B,i}\right) R_{LF} - q_{B,i} R_{DF} + \mu_i^- - \mu_i^+ = 0, \tag{A.35}$$

$$\frac{\partial \mathcal{L}_i}{\partial B_{B,i}} : - R_{RO} + \left(1 - q_{B,i}\right) R_{LF} + q_{B,i} R_{DF} - \mu_i^- + \mu_i^+ = 0. \tag{A.36}$$

Adding equation (A.35) and (A.36) and rearranging yields an identical equation for $L_{B,i}$ as defined by equation (A.32). Thus this equation holds regardless of the value that Γ takes.

Next, consider equation (A.36). Rearranging this equation as in the previous case so that the Kuhn-Tucker multipliers are on the right-hand side yields

$$\left(1 - q_{B,i}\right) R_{LF} + q_{B,i} R_{DF} - R_{RO} = \mu_i^- - \mu_i^+. \tag{A.37}$$

As $R_{RO} = \frac{1}{2}\left(R_{LF} + R_{DF}\right)$ it follows that

$$\left(1 - q_{B,i}\right) R_{LF} + q_{B,i} R_{DF} \geq R_{RO} \Longleftrightarrow q_{B,i} \leq \frac{1}{2} \tag{A.38}$$

and as $q_{B,i} \leq \frac{1}{N}$ and $N \geq 2$, it follows that the first-order condition holds only when equation (3) holds with strict equality and thus $\epsilon_i^- = 0$. In cases where $\Gamma \geq 0$ and $\Gamma < 0$, it is always optimal for firms to set $\epsilon_i^- = 0$. Thus Proposition 1 follows from the solution to bank i's optimization problem independently of the aggregate liquidity of the banking sector.

Corollary 1

Recall the definition of Γ,

$$\Gamma \equiv \sum_{j \neq i} \epsilon_i^- + \epsilon_i^+ \tag{A.39}$$

From proposition 1, in equilibrium banks accumulate excess liquidity so that $\epsilon_i^- = 0$ and this must hold for all banks and thus it follows that $\sum_i \epsilon_i^- = 0$. Substituting this into equation (A.39) yields $\Gamma = \epsilon_i^+$. Furthermore, with $\epsilon_i^- = 0$, it follows from equation (4) that

$$\Gamma = -\xi \bar{D} < 0. \tag{A.40}$$

Proof of Proposition 2

I now present the proof of each part of Proposition 2, which describes the solution to the equilibrium of the model without CBDC.

i) $L_{B,i} = \frac{1}{\lambda} \left(R_L - R_{RO}\right)$

This is proven in Proposition 1.

ii) $q_{B,i} = \frac{1}{N}$

In a symmetric equilibrium, $R_{D,i} = R_{D,-i}$ and thus the result follows immediately from equation (9).

iii) $R_{D,i} = (1 - \xi) R_{RO} + \xi R_{DF} - \frac{1}{N} t_B$

From the Lagrangian set out in equation (A.28) the first order condition with respect to $R_{D,i}$ can be written as

$$\frac{\partial \mathcal{L}_i}{\partial R_{D,i}} : -q_i \bar{D} - \frac{\partial q_{B,i}}{\partial R_{D,i}} \left[R_{D,i} \bar{D} + \frac{\partial E\left[C_{D,i}\right]}{\partial q_{B,i}} + (1 - \xi) \bar{D} \left(\mu_i^- - \mu_i^+\right) \right] = 0. \tag{A.41}$$

From equation (A.37) and given $\mu_i^+ = 0$, μ_i^- can simply be expressed as

$$\mu_i^- = \left(1 - q_{B,i}\right) R_{LF} + q_{B,i} R_{DF} - R_{RO}. \tag{A.42}$$

Next, note that as stated in Corollary 1, $\Gamma < 0$ and thus the expected liquidity cost faced by banks is described by equation (A.34) and thus $\frac{\partial E[C_{D,i}]}{\partial R_{D,i}}$ can be written as

$$\frac{\partial E\left[C_{D,i}\right]}{\partial q_{B,i}} = -R_{LF}\epsilon_i^- - R_{LF}\sum_{j\neq i}\epsilon_j^- + R_{DF}\left(\epsilon_i^+ + \sum_{j\neq i}\epsilon_j^-\right) - \left(q_{B,i}R_{DF} + \left(1-q_{B,i}\right)R_{LF}\right)\left(1-\xi\right)q_i\bar{D}. \tag{A.43}$$

Substituting equation (A.43) into the first-order condition described by equation (A.41) and rearranging yields the following equation

$$R_{D,i}\bar{D} = R_{LF}\sum_i \epsilon_i^- - R_{DF}\left(\sum_i \epsilon_i^- - \xi\bar{D}\right) + \left(1-\xi\right)\bar{D}R_{RO} - \left[\frac{\partial q_i}{\partial R_{D,i}}\right]^{-1} q_i\bar{D}, \tag{A.44}$$

which can be simplified noting that in equilibrium $\sum_i \epsilon_i^- = 0$ and so after some further simplification we have the following equation for $R_{D,i}$

$$R_{D,i} = \left(1-\xi\right)R_{RO} + \xi R_{DF} - \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i}. \tag{A.45}$$

It should be noticed at this point that equation (A.45) will hold regardless of whether or not the central bank implements a CBDC. However, now note that in the equilibrium without CBDC, the derivative of the demand function is described by equation (9) and thus the derivative is simply

$$\frac{\partial q_{B,i}}{\partial R_{D,i}} = \frac{1}{t_B}. \tag{A.46}$$

Combining this with equilibrium market shares equaling $\frac{1}{N}$ yields the proposition equation

$$R_{D,i} = \left(1-\xi\right)R_{RO} + \xi R_{DF} - \frac{1}{N}t_B. \tag{A.47}$$

iv) $B_{B,i} = L_{B,i} - \left(1-\xi\right)\frac{1}{N}\bar{D}$.

Finally, we note that as $\epsilon_i^- = 0$ and rearranging equation (3) the equilibrium debt holdings will be equal to

$$B_{B,i} = L_{B,i} - \left(1-\xi\right)\frac{1}{N}\bar{D}. \tag{A.48}$$

Proof of Proposition 3

I now present the proof of each part of Proposition 3, which describes the solution to the equilibrium of the model following the implementation of a CBDC.

i) $L_{B,i} = \frac{1}{\lambda}\left(R_L - R_{RO}\right)$

This is proven in Proposition 1.

ii) $q_{B,i} = \frac{1}{N} - \frac{1}{t_B}\left(\frac{1}{\bar{t}-\underline{t}}\right)\left(\frac{1}{2}t_B\frac{1}{N} - \left(R_{D,i} - R_{CB} + \underline{t}\right)\right)^2$

The the total demand function for the bank is

$$q_{B,i} = 2\left(\int_{\hat{x}_{B,i}}^{x^*_{B,i}}\left(\frac{\bar{t} - t^*_i\left(x\right)}{\bar{t}-\underline{t}}\right)dx + \hat{x}_{B,i}\right) \tag{A.49}$$

where

$$t^*_i\left(x\right) = t_B x + R_{CB} - R_{D,i} \tag{A.50}$$

$$\hat{x}_{B,i} \equiv \frac{1}{t_B}\left(R_{D,i} - R_{CB} + \underline{t}\right) \tag{A.51}$$

$$x^*_{B,i} = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_{D,i} - R_{D,-i}\right)\right) \tag{A.52}$$

Substituting out $t^*\left(x\right)$ we can write this as

$$q_{B,i} = 2\left(\int_{\hat{x}_{B,i}}^{x^*_{B,i}}\left(\frac{\bar{t} - t_B x + R_{D,i} - R_{CB}}{\bar{t}-\underline{t}}\right)dx + \frac{1}{t_B}\left(R_{D,i} - R_{CB} - \underline{t}\right)\right). \tag{A.53}$$

We can also rewrite the quantity equation by evaluating the definite integral, yielding the following equation

$$\begin{aligned} q_{B,i} = &\left(\frac{1}{\bar{t}-\underline{t}}\right)\left[\bar{t} - \frac{1}{4}t_B\frac{1}{N} + \frac{1}{4}R_{D,-i} + \frac{3}{4}R_{D,i} - R_{CB}\right]\frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_{D,i} - R_{D,-i}\right)\right) \\ &- \frac{1}{t_B}\left(\frac{1}{\bar{t}-\underline{t}}\right)\left[\bar{t} - \frac{1}{2}\underline{t} + \frac{1}{2}R_{D,i} - \frac{1}{2}R_{CB}\right]\left(R_{D,i} - R_{CB} + \underline{t}\right). \end{aligned} \tag{A.54}$$

Next, imposing a symmetric equilibrium and thus $R_{D,i} = R_{D,-i}$, by collecting terms the above can be simplified to

$$q_{B,i} = \left(\frac{1}{\bar{t}-\underline{t}}\right)\left(\left(\bar{t} + R_{D,i} - R_{CB} - \frac{1}{4}t_B\frac{1}{N}\right)\frac{1}{N} - \frac{1}{t_B}\left(R_{D,i} - R_{CB} + \underline{t}\right)^2\right). \tag{A.55}$$

Adding and subtracting $\frac{1}{N}$to the right hand side of this equation allows us, after some rearranging, the demand function as

$$q_{B,i} = \frac{1}{N} - \frac{1}{t_B}\left(\frac{1}{\bar{t}-\underline{t}}\right)\left(\frac{1}{2}\frac{1}{N}t_B - \left(R_{D,i} - R_{CB} + \underline{t}\right)\right)^2, \tag{A.56}$$

which is the main expression given in the paper.

iii) $R_{D,i} = \left(1-\xi\right)R_{RO} + \xi R_{DF} - \frac{1}{3}t_b q_{B,i}$

Note that we have, by Leibniz's integral rule to equation (A.53) yields

$$\frac{\partial}{\partial R_{D,i}}\left\{\int_{\hat{x}_{B,i}}^{x^{*}_{B,i}}\left(\frac{\bar{t}-t_{B}x+R_{D,i}-R_{CB}}{\bar{t}-\underline{t}}\right)dx\right\}=\frac{1}{2}\frac{1}{t_{B}}, \tag{A.57}$$

thus from equation (A.53) it follows immediately that

$$\frac{\partial q_{B,i}}{\partial R_{D,i}}=3\frac{1}{t_{B}}. \tag{A.58}$$

As noted earlier equation (A.45) holds regardless of whether or not the central bank implements a CBDC. Taking this equation and substituting out the derivative of the demand function specified by equation (A.58) yields

$$R_{D,i}=(1-\xi)\,R_{RO}+\xi R_{DF}-\frac{1}{3}t_{B}q_{B,i}. \tag{A.59}$$

iv) $B_{B,i}=L_{B,i}-(1-\xi)\,q_{B,i}\bar{D}$.

Finally, we note that as $\epsilon_{i}^{-}=0$ and rearranging equation (3) the equilibrium debt holdings will be equal to

$$B_{B,i}=L_{B,i}-(1-\xi)\,q_{B,i}\bar{D}, \tag{A.60}$$

where $q_{B,i}$ is specified by equation (A.56).