The impact of central bank digital currency on bank deposits and the interbank market*

Benjamin Hemingway[†]

May 2023

Abstract

This paper proposes a theoretical model in which a central bank digital currency (CBDC) and bank deposits are imperfect substitutes. Deposits are subject to liquidity shocks. In the absence of a CBDC, the interbank market can redistribute liquidity between banks. The introduction of CBDC leads to a greater reliance of the banking sector on central bank standing facilities. Calibrating the model to the euro area, the model shows that adjusting the remuneration rate of CBDC has little pass-through to the deposit rate set by banks and also has implications for the transmission of monetary policy.

Keywords: central bank digital currency, banking, money, interbank Market

JEL codes: E42, E52, E58, G21

*I would like to thank Marius Jurgilius, Jack Meaning and Sigitas Siaudinis as well as the participants of the CEBRA Annual Conference 2022 and the members of the Eurosystem Monetary Policy Committee Expert Group on CBDC. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of Lithuania or the Eurosystem.

†CEFER, Bank of Lithuania and Vilnius University. b.hemingway@st-annes.oxon.org

1 Introduction

The introduction of a retail central bank digital currency (CBDC) is currently under active consideration by central banks around the world. A key motivation for the introduction of a central bank digital currency (CBDC) is the decrease in cash use. According to the ECB Study on the payment attitudes of consumers in the euro area (SPACE) from 79% of all point-of-sale transactions in the euro zone in 2016 to 72% in 2019 and 59% in 2022, (ECB, 2022). The 2022 wave of the SPACE survey suggests that cash may no longer be the preferred means of payment within the Eurozone; 55% of consumers within the euro zone stated a preference for using cards and other cashless payments in stores, while only 22% preferred to use cash.

In this context, the introduction of a CBDC can be seen as a way to modernize fiat currency for the digital age. In addition to adapting fiat currency to the declining usage of physical cash, a CBDC is likely to have technical features that make it a closer substitute for bank deposits than physical cash. Thus, CBDC is likely to be a greater source of competition for banks in the deposit market. This gives rise to potential risks associated with the introduction of a CBDC, for example, the financial stability impact of an increase in the cost of bank funding highlighted by Broadbent (2016).

This paper focuses on one aspect of financial stability that has not been widely discussed. Namely, the impact on the operation of the interbank market of allowing households to hold a bank account directly with the central bank. In order to maintain a well-functioning payment system, the introduction of a CBDC is likely to require additional transactions between the banking sector and the central bank.

I propose a theoretical model where CBDC and bank deposits are imperfect substitutes and where deposits are subject to liquidity shocks. Banks are able to transfer liquidity between themselves through an interbank market. I assume that the central bank would choose not to participate directly in the interbank market, and thus the introduction of a CBDC increases the banking sector's use of the central bank standing facilities. In this setting, CBDC raises the cost of bank funding in two ways; directly by competing for depositors and indirectly by increasing the number of transactions with the central bank.

The deposit market model is based on the spatial competition model of Salop (1979) with the addition of a central bank. A continuum of atomistic depositors choose to deposit their funds at one of a finite number of banks or, through a CBDC, at the central bank. This deviates from the existing literature on CBDC, where households would have a portfolio consisting of both bank deposits and CBDC. There is some evidence that the majority of households currently do not hold multiple bank accounts, at least with respect to deposits issued by banks. As part of the UK Competition and Markets Authority's investigation into the retail banking market, they commissioned a survey by GfK NOK which found that only 22% of UK households actively used a personal current account at more than one bank, (Moon et al., 2015).

In order to study the impact of a CBDC on the structure of the deposit market, I consider two equilibria; a short-run equilibrium where the number of banks is fixed and a long-run equilibrium where the number of banks adjusts according to a free entry condition. This distinction has an implication for the results. With the number of banks fixed, banks respond to the introduction of CBDC by increasing the deposit rate as banks attempt to maintain market share. On the other hand, if the number of banks adjusts, the introduction of CBDC, the deposit market becomes more concentrated which dampens the effect on deposit rates.

Using this model, I study the effect introducing a CBDC has on the structure of the deposit market and the implications for monetary policy. In particular, I focus on two parts of the policy debate around CBDC. First, the effectiveness of the CBDC remuneration rate as an additional tool in the monetary policy toolkit and second, the implications CBDC has on the transmission of the policy rate through the deposit market. To make these results empirically relevant, I calibrate the model to the Euro zone.

The model makes several predictions with important policy implications. First, if the banks do not face liquidity risk from deposit financing, then in the short-run the introduction of a CBDC results in an increase in interest rates on bank deposits and a fall in the market shares of banks in the deposit market. This leads to a decrease in bank profitability, so the model predicts that in the long run the number of banks active in the deposit market will fall following the introduction of CBDC.

As the banking sector becomes more concentrated in the long-run, the interest rate on bank deposits may not be increasing in the remuneration rate of CBDC, as a more concentrated banking sector places downward pressure on the bank deposit rate. This paper also highlights the importance of the liquidity risk channel for monetary policy transmission in general. In the absence of liquidity risk, the bank deposit rate increases one for one after an increase in the policy rate, even after the introduction of a CBDC.

However, if banks face liquidity risk in the deposit market, introducing a CBDC will impact the transmission of monetary policy, as there will be imperfect pass-through of the policy rate to the deposit rate. Furthermore, the impact of monetary policy will now impact the structure of the deposit market, and thus monetary policy will impact the deposit rate to differing degrees in the short-run and long-run.

This paper is complementary to the growing literature on the policy implications of CBDC. A large literature focuses on financial stability issues; in particular, both Böser and Gersbach (2020) and Fernández-Villaverde et al. (2021) consider the increased risk of bank runs that can occur if bank depositors had access to a CBDC so that they could transfer their deposits in times of financial stress. Both Brunnermeier and Niepelt (2019) and Niepelt (2020) discuss an equivalence result where appropriate transfers from the central bank to the financial system are capable of neutralizing the impact of introducing a CBDC and mitigate the risk of a CBDC-induced bank run. This paper also introduces liquidity risk of deposits; the focus is not on bank runs, but on the costs imposed on banks when they obtain liquidity from a central bank lending facility.

This paper is also related to the literature on how CBDC should be remunerated. Agur et al. (2022) consider the welfare trade-off for the central bank when choosing a non-interest-bearing versus an interest-bearing CBDC. Barrdear and Kumhof (2022) find that a countercyclical remuneration rate rule for CBDC can contribute to stabilizing the business cycle. Similarly, Bordo (2021) finds that an interest-bearing CBDC may improve the transmission mechanism of monetary policy. On the other hand, Chiu and Davoodalhosseini (2021) find that a non-interest-bearing CBDC increases bank intermediation and thus welfare, while an interest-bearing CBDC results in bank disintermediation and lower welfare. Williamson (2022) studies various implementations of CBDC and shows how an interest-bearing CBDC can increase welfare by competing with private means of payment. This paper casts doubt on the use of the CBDC remuneration rate in the monetary toolkit. Instead, the paper sets out a model that predicts imperfect pass-through of the CBDC remuneration rate to the bank deposit rat and proposes a channel through which it also affects the structure of the banking sector.

Another related strand of literature focuses on the implications of CBDC for monetary policy. A summary of the possible monetary policy implications of CBDC can be found in Bindseil (2019). For example, Keister and Sanches (2019) suggests that, while CBDC can promote efficient exchange, it can also increase funding costs. Meaning et al. (2021) provide a detailed discussion on the monetary transmission mechanism in general, as well as other possible policy implications. Burlon et al. (2022) study the welfare implications of a CBDC and attempt to characterize the welfare-maximizing CBDC policy rules. Kumhof and Noone (2021) discuss the remuneration of CBDC in detail and its possible use for monetary policy. Kumhof and Noone (2021) propose a two-tier remuneration system, while Barrdear and Kumhof (2022) propose a quantity rule and a price rule for CBDC. This paper does not consider the impact of a quantity rule, but does consider how in the presence of deposit liquidity risk a CBDC may impact the transmission of monetary policy through the bank deposit rate.

This paper is most closely related to the literature on the impact of CBDC on the banking sector. In a macroeconomic framework, Bacchetta and Perazzi (2021) assume a constant elasticity of substitution between a CBDC and a continuum of monopolistically competitive banks. While Andolfatto (2021) analyzes the case of a single monopoly bank where CBDC and bank deposits are perfect substitutes, but there is a fixed cost for depositors to switch between the two. Chiu et al. (2019) study a model of Cournot oligopoly with a finite number of banks where banks compete in quantity rather than the remuneration of deposits. CBDC is assumed to be a perfect substitute for bank deposits, and so imposes a minimum remuneration rate on bank deposits.

This paper is also related to the literature on spatial models of imperfect competition, as the deposit market is based on the classic paper by Salop (1979). Spatial competition models have been widely used to study deposit markets. For example, Chiappori et al. (1995) study the regulation of deposit rates using a Salop circle model of both loans and deposit markets while Matutes and Vives (1996) study the impact of deposit insurance in a model of spatial competition in the deposit market. Along similar lines Repullo (2004) investigates the effect of capital requirements on bank behavior when imperfect competition in the deposit market is modeled using a Salop circle. Empirical support for spatial models of the deposit market is provided by Park and Pennacchi (2008) and Ho and Ishii (2011), among others. The structure of competition after the introduction of a CBDC is closely related to Salop Circle models with a center such as Bouckaert (2000) and Madden and Pezzino (2011).

Finally, this paper is also related to the literature on interbank markets. In particular, the theoretical treatment of the interbank market in this paper is closest to that of Hauck and Neyer (2014) and Bucher et al. (2020), who both study the operation of an interbank market within the framework of the euro area.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 goes into further detail on how the banking sector is modeled. In Section 4 the equilibrium is presented. Section 5 provides comparative statics on both the impact of the CBDC remuneration rate on the bank deposit rate and the implications for the transmission of monetary policy. Section 6 calibrates the model and provides a quantitative assessment. Section 7 concludes.

2 Model

I consider a three period model of the retail deposit market. The economy consists of three types of agents; risk-neutral banks, a central bank, and a continuum of depositors.

There are three discrete periods, $t = 1, 2, 3$.

In the first period $t = 1$, banks enter the deposit market, paying a fixed cost $F > 0$. I consider two different equilibria: in a short-run equilibrium a fixed number $N \geq 2$ of banks enter in $t = 1$. In a long-run equilibrium the number of banks adjusts endogenously subject to a free-entry condition. Distinguishing between short- and long-run equilibria allows the study of two channels separately. The impact on the demand for bank deposits is isolated in the short-run equilibrium. While in the long-run equilibrium, the supply side is able to adjust through a change in the concentration of banks in the deposit market.

Banks have access to a technology which yields an exogenously given return R_L on liquidity. Banks must obtain an exogenously given quantity of liquidity $L > 1$ in order to operate this technology. Banks obtain liquidity in period $t = 2$, either from the central bank or from depositors. Bank i obtains liquidity B_i from the central bank at time $t = 2$ through open market operations at an interest rate R_f. The retail deposit market is subject to liquidity risk that is realized in $t = 3$.

At the end of $t = 3$ banks must return to a liquidity neutral position. In order to do so, banks may borrow or lend liquidity either through the interbank market or the central bank standing facilities. The central bank offers a deposit facility with an interest rate R_{DF} and a lending facility with an interest rate R_{LF}. The central bank charges penalty rates on these standing facilities such that $R_{DF} < R_f < R_{LF}$. Banks are able to trade liquidity between themselves in an interbank market. Trade in the interbank market takes place at a state-dependent interbank rate R^s_{IB}, where the superscript s denotes the state. The interest rates on the central bank standing facilities define a corridor that sets an upper and lower bound on the interbank rate.

The retail deposit market is modeled as a Salop circle as in Salop (1979). There is a continuum of depositors located around a circle with unit mass. Banks locate equidistant from each other around the circle. A depositor located at a distance $x \geq 0$ away from the bank must pay a linear transportation cost $t_B x \geq 0$ in order to deposit their funds. Banks compete in prices *à la* Bertrand. The interest rate paid on deposits by bank i, denoted as R_i.

The central bank may also enter the deposit market by issuing a CBDC and setting a remuneration rate R_{CB} in $t = 1$. Entry of CBDC into the deposit market and its remuneration rate are known to all participants in advance. All bank decisions are made in full knowledge of whether they will compete against a CBDC and are conditional on R_{CB}.

It is assumed that each depositor pays a fixed transport cost to obtain CBDC with this transport cost drawn uniformly from the interval $t_{CB} \in [0, t_B]$. The structure of

CBDC transport costs serves two purposes. First, it captures the idea that CBDC may have specific features that differentiate it from retail deposits. Examples given in the literature include privacy concerns or preferences over additional security of deposited funds. Second, it allows for competition between neighbor banks and CBDC to occur simultaneously. This would not be the case if CBDC transport costs were identical among depositors.

I assume that the CBDC is only held by households and that banks cannot deposit into CBDC. This assumption allows the central bank to set R_{CB} higher than the interest rate on the deposit facility R_{DF}. In the cases where $R_{CB} \leq R_{DF}$ this assumption is rendered unnecessary because from the bank's perspective, the return on CBDC is weakly dominated by the central bank's deposit facility.

Deposits are subject to liquidity shocks that occur at $t = 3$. With probability λ, a fraction $\xi \in [0, 1]$ of bank deposits relocate to other locations, evenly distributed around the circle, while CBDC depositors do not relocate. With probability $1 - \lambda$, CBDC depositors are subject to the same liquidity shocks as banks and a fraction ξ of all depositors relocate around the circle.

The liquidity shocks are structured such that absent CBDC, the law of large numbers ensures that each bank receives a liquidity inflow equal to its liquidity outflow. The presence of CBDC introduces additional liquidity risk in the banking sector. While these liquidity shocks are similar in spirit to those in papers such as Fernández-Villaverde et al. (2021) that focus on the possibility of CBDC generated bank runs, here there is no risk of bank runs. Instead, liquidity risk generates additional costs of deposits for banks. If CBDC depositors are not subject to the liquidity shock, the aggregate liquidity of the banking sector falls, and banks will need to increase their use of the central bank standing facilities.

To summarize the timing of the model, in the first period $t = 1$, the central bank decides whether to enter the deposit market and will also set the remuneration rate R_{CB}. In a short-run equilibrium, a fixed number N banks enter in $t = 1$ while in a long-run equilibrium, N banks enter subject to a free-entry condition. In the second period, $t = 2$, commercial banks compete in the deposit market by setting a deposit rate R_i and obtaining liquidity B_i from the central bank. In the third period, $t = 3$, the liquidity shock is realized and commercial banks use the central bank standing facilities, as well as the interbank market, to obtain a liquidity neutral position. In what follows, I focus on the symmetric equilibrium and solve for the Subgame Perfect Nash Equilibrium in pure strategies using backward induction.

3 Banking Sector

3.1 Bank Liquidity

I begin the analysis of the banking sector with the final period, $t = 3$. The $N \geq 2$ banks, indexed by i, have made their decisions about their funding structure. The bank's funding structure consists of a quantity of deposits q_i and central bank liquidity B_i.

The bank's choice of liquidity B_i and deposits q_i implies that before the realization of the liquidity shocks the banks have the following ex ante liquidity deficit

$$\epsilon_i \equiv L - B_i - q_i. \tag{1}$$

With probability $1-\lambda$, a fraction ξ of all depositors relocate to locations evenly distributed around the Salop circle. Here banks face the same liquidity inflows as liquidity outflows and their ex post liquidity deficit is simply equal to their ex ante liquidity deficit

$$\epsilon_i^0 = \epsilon_i. \tag{2}$$

With probability λ, a fraction ξ of bank depositors relocate while CBDC depositors do not. Bank i receives a liquidity outflow equal to $q_i\xi$, while each bank receives an inflow of liquidity equal to $(1 - q_{CB})\, q_i\xi$. Thus each bank will receive a net outflow of liquidity and has an ex post liquidity deficit equal to

$$\epsilon_i^+ = \epsilon_i + q_{CB} q_i \xi. \tag{3}$$

Banks must return to a liquidity neutral position by the end of $t = 3$. How much liquidity they must trade in order to achieve this depends on their ex post liquidity deficit ϵ_i^s, where $s \in \{0, +\}$ denotes the realization of the liquidity shock. If $\epsilon_i^s > 0$, banks will need to obtain additional liquidity through the interbank market or through the central bank liquidity facility, while if $\epsilon_i^s < 0$ banks will reduce their liquidity by lending in the interbank market or depositing liquidity at the central bank deposit facility.

The interest rate in the interbank market depends on the aggregate liquidity deficit of the banking system, $\sum_i \epsilon_i^s$. This in turn depends on the realization of the liquidity shock. The interest rates on the central bank standing facilities act as upper- and lower-bounds on the interbank rate and thus the interbank rate is

$$R_{IB}^{s}\begin{cases} = R_{LF} & \text{if } \sum_i \epsilon_i^s > 0 \\ = R_{DF} & \text{if } \sum_i \epsilon_i^s < 0 \\ \in [R_{DF}, R_{LF}] & \text{otherwise.} \end{cases} \tag{4}$$

The expected liquidity cost of deposits is simply the expected cost of returning to a liquidity neutral position

$$E\left[C_i\right] = (1-\lambda)\, R_{IB}^{0}\epsilon_i^{0} + \lambda R_{IB}^{+}\epsilon_i^{+}. \tag{5}$$

As the central bank standing facilities are more costly for banks to access compared to the interbank market, this generates a higher liquidity cost of deposits that increases in the size of the liquidity shock.

3.2 Bank's Problem

Consider the bank's problem in the intermediate period, $t = 2$, taking the number of banks N as given. In this period, the bank decides on its funding structure by obtaining liquidity from the central bank and sets the interest rate it offers to depositors R_i. Banks are risk neutral and maximize expected profits. The profit function of bank i is

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - E\left[C_i\right] - F \right\}, \tag{6}$$

where $E\left[C_i\right]$ is defined in equation (5) and $F > 0$ is the fixed cost that banks are assume to pay in order to enter the deposit market. Banks compete for depositors in prices à la Bertrand, taking as given both the deposit rates set by other banks and the funding structure of other banks.

In the case where the central bank does not implement a CBDC, competition between banks in the deposit market is identical to Salop's circle model. If bank i offers a deposit rate equal to R_i and other banks offer a deposit rate equal to R_{-i}, then a depositor located at a distance x from bank i, where $x \in \left[0, \frac{1}{N}\right]$, will choose to deposit their funds at bank i rather than the neighboring bank so long as

$$R_i - t_B x \geq R_{-i} - t_B \left(\frac{1}{N} - x \right), \tag{7}$$

where t_B is the linear transport cost that is incurred by depositors. Bank i thus faces the following demand function

$$q_i = \frac{1}{N} + \frac{1}{t_B}\left(R_i - R_{-1}\right). \tag{8}$$

In the case where the central bank implements a CBDC, bank i faces competition not only from the two banks that neighbor it, but also from the CBDC. I assume that the central bank sets a fixed interest rate R_{CB} and that depositors incur a transport cost t_{CB} if they deposit funds in the CBDC. The transport costs associated with CBDC are assumed to be drawn randomly from a uniform distribution over the interval $[0, t_B]$. Thus a depositor located at distance x from bank i would prefer to deposit funds in bank i rather than in the CBDC so long as

$$x \leq \frac{1}{t_B}\left(R_i - R_{CB} + t_{CB}\right). \tag{9}$$

Following the introduction of CBDC, for a depositor to deposit funds in bank i they must satisfy both equation (7) and equation (9) and must prefer bank i to the CBDC as well as all other banks.

The market share of deposits obtained by bank i depends on the deposit rate that it offers, R_i, relative to the deposit rate offered by the neighboring banks, R_{-i} and the CBDC remuneration rate R_{CB}. In order to characterize bank i's demand function, it is helpful to define some additional variables.

First, define the distance x_i^* as the point where a depositor is indifferent between bank i and the neighboring bank $-i$. The equation for x_i^* follows from equation (7) as in the standard Salop model and is given by

$$x_i^* = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_i - R_{-i}\right)\right). \tag{10}$$

Now consider a depositor with the lowest possible CBDC related cost, $t_{CB} = 0$. From equation (9) this depositor's preference for bank i's deposits depends on their location relative to bank i as well as the ratio of the spread between bank deposits and CBDC and the deposit transport cost. I now introduce a variable, z_i that will be important in describing the equilibrium with CBDC that is defined as

$$z_i \equiv x_i^* - \frac{1}{t_B}\left(R_i - R_{CB}\right). \tag{11}$$

If $z_i < 0$, then the CBDC remuneration rate is low enough that any depositor that prefers bank i deposits to those of another bank would also prefer bank i deposits to CBDC.

Competition occurs only between neighboring banks and in a symmetric equilibrium where $R_i = R_{-i}$ each bank obtains a market share of $1/N$ as in the standard Salop setup and the demand for deposits follows from (10) as

$$q_i = 2x_i^* \text{ if } z_i < 0. \tag{12}$$

If $0 \leq z_i \leq x_i^*$, bank i faces competition from neighboring banks and partial competition from CBDC in the sense that all depositors located at a distance $x_i^* - z_i > 0$ or closer to bank i prefer bank i deposits to CBDC. However, for depositors located at some distance $x \in [x_i^* - z_i, x_i^*]$ some depositors with a low realization of the cost of CBDC t_{CB} will prefer CBDC to bank i deposits. For these depositors, there exists a function $t_i^*(x)$ that defines the smallest value of t_{CB} that depositors located a distance x from bank i must have in order to prefer depositing in bank i rather than depositing in the CBDC. Thus, the demand function facing bank i is

$$q_i = 2\left(\int_{x_i^*-z_i}^{x_i^*}\left(\frac{t_B - t^*(x)}{t_B}\right)dx + x_i^* - z_i\right) \text{ if } 0 \leq z_i \leq x_i^* \tag{13}$$

If $x_i^* < z_i \leq 1$, bank i faces competition from neighboring banks and full competition from CBDC, as some depositors with a low realization of the CBDC cost t_{CB} will prefer CBDC to bank i deposits regardless of the distance they are located from bank i. In this case, some fraction of all depositors located at a distance of x_i^* or closer to bank i will choose CBDC over depositing at bank i. For there to still be meaningful competition between banks, some depositors located equidistant between bank i and its neighboring bank must prefer to deposit at bank i over CBDC. From equation (9) this occurs whenever

$$x_i^* \leq \frac{1}{t_B}\left(R_i - R_{CB} + t_B\right).. \tag{14}$$

Combining this with the definition of z_i, equation (11), this yields the condition that $z_i \leq 1$. The demand function facing bank i is then

$$q_i = 2\left(\int_0^{x_i^*}\left(\frac{t_B - t^*(x)}{t_B}\right)dx\right) \text{ if } x_i^* < z_i \leq 1 \tag{15}$$

If $1 < z_i \leq 1 + x_i^*$, bank i does not compete directly with its neighboring banks and instead operates a local monopoly where it attracts a fraction of depositors located a distance z_i from it. In this case, bank i competes only with CBDC for deposits, and only depositors who have a high realization of the CBDC cost t_{CB} will prefer deposits to

CBDC. Banks are able to attract some depositors so long as $z_i \leq 1 + x_i^*$. The demand function facing bank i is then

$$q_i = 2\left(\int_0^{z_i}\left(\frac{t_B - t^*(x)}{t_B}\right)dx\right) \text{ if } 1 < z_i \leq 1 + x_i^* \tag{16}$$

Finally, in the case where $z_i > 1 + x_i^*$ this implies that the CBDC rate is sufficiently high that $R_{CB} \geq R_i + t_B$ and CBDC dominates the deposit market. Here, the CBDC remuneration rate is sufficiently high that no depositor would prefer to deposit in bank i over CBDC and all depositors hold CBDC. Banks do not obtain a market share in this situation and thus $q_i = 0$.

Explicitly evaluating the integrals, the demand function bank i faces can be specified in a piece-wise fashion as

$$q_i = \begin{cases} 2x_i^* & \text{if } z_i < 0 \\ 2x_i^* - z_i^2 & \text{if } 0 \leq z_i \leq x_i^* \\ 2x_i^* - x_i^*\left(2z_i - x_i^*\right) & \text{if } x_i^* < z_i \leq 1 \\ \left(1 + x_i^* - z_i\right)^2 & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{17}$$

As depositors are assumed not to have an outside option, there will be full coverage in the deposit market and all deposits will be deposited either at a retail bank or at the central bank. As a result, the market share of CBDC can be written as

$$q_{CB} = 1 - \sum_i q_i. \tag{18}$$

3.3 Bank Entry

As discussed in section 2, I consider two types of equilibria. In a short-run equilibrium, the number of banks competing in the deposit market is fixed exogenously at some $N \geq 2$. In a long-run equilibrium, the number of banks is set according to a free-entry condition. In $t = 1$, since banks have to pay a fixed cost $F > 0$ to enter the deposit market, banks choose to enter as long as the expected profits defined by equation (6) are weakly positive. Then, in a long-run equilibrium the number of banks that enter in $t = 1$, N adjusts until expected profits are driven to zero.

4 Equilibrium

I focus on solving for a symmetric equilibrium in which all banks make the same decision about their funding structure B_i and set the same deposit rate R_i. As banks set the same deposit rate, they obtain the same market share of deposits, q_i. I distinguish between a short-run equilibrium, where the number of banks N is fixed, and a long-run equilibrium where $N \geq 2$ adjusts according to a free-entry condition such that all banks make zero expected profits.

4.1 Interbank Market and Bank Funding Structure

I begin by characterizing the equilibrium funding structure of the bank chosen in $t = 2$ and the equilibrium interest rate of the interbank market in $t = 3$. In choosing their funding structure banks take the interest rates in the interbank market, the policy rate and the interest rates on standing facilities as given. Conditional on the deposit rate they set, banks perfectly anticipate the market share of deposits they obtain. Obtaining one additional unit of liquidity from the central bank in $t = 2$ has a marginal cost of R_f, but also reduces by one unit the bank's *ex ante* liquidity deficit ϵ_i. Therefore, in equilibrium, the bank will adjust B_i so that the marginal cost of increasing B_i is equal to the expected marginal cost of increasing its *ex ante* liquidity deficit ϵ_i and thus

$$\frac{\partial E\left[C_i\right]}{\partial \epsilon_i} = R_f. \tag{19}$$

In equilibrium, the interbank rates are such that the bank funding decision in $t = 2$ is consistent. Given a bank's choice of B_i and its market share q_i, a bank's ex post liquidity deficit ϵ_i^s is conditional on the realization of the liquidity shock $s \in \{0, +\}$. In equilibrium, the interbank rate conditional on liquidity shock s can be found in equation (4). The equilibrium interbank rate and the bank's equilibrium funding structure are summarized in Proposition 1.

Proposition 1. *In both a long-term and short-term equilibrium, banks obtain liquidity B_i from the central bank in $t = 2$ such that*

I. *When $\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - q_i$. The equilibrium interbank market rates are then $R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda}\right)\left(R_{LF} - R_f\right)$ and $R_{IB}^+ = R_{LF}$.*

II. *When $\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - \left(1 - q_{CB}\xi\right) q_i$. The equilibrium interbank market rates are then $R_{IB}^0 = R_{DF}$ and $R_{IB}^+ = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right)$.*

Proof. See the Appendix. □

One implication of proposition 1 is that the interest rates in the interbank market depends on λ, the probability the bank is hit by a net outflow of liquidity. If λ is low then $\epsilon_i^0 = 0$ and banks would have a neutral liquidity position if they are not hit by an outflow of liquidity. However, if λ is sufficiently high, then $\epsilon_i^+ = 0$ and the banks would have a neutral liquidity position if they are hit by an outflow of liquidity and therefore would hold surplus liquidity should they not be hit by a liquidity outflow. As the probability of being hit by a liquidity shock increases, the incentive banks have a greater incentive to accumulate liquidity in $t = 2$ and thus the supply of liquidity in the banking sector in $t = 3$ increases. As a result, the conditional interbank rates are weakly decreasing in λ.

By combining the bank's equilibrium funding decision with equation (5), the bank's expected cost of deposits can be written as

$$E[C_i] = \epsilon_i R_f + \lambda \xi R_{IB}^+ q_{CB} q_i. \tag{20}$$

An important property of equation (20) is that if $\lambda > 0$ the expected cost of deposits is strictly increasing in the market share of the central bank. This is an important feedback mechanism of the model. An increase in the market share of CBDC increases the liquidity risk of deposits, and thus increases the expected cost of deposits for banks.

4.2 Deposit Market Equilibrium without CBDC

I now turn to the equilibrium in the deposit market. In $t = 2$ bank i sets a deposit rate R_i that in combination with its funding decision set out in Proposition 1 maximizes the bank's expected profit.

The equilibrium deposit rate can be found by differentiating the bank's profit function given by equation (6) with respect to the deposit rate chosen by the bank, R_i yielding

$$-q_i - \frac{\partial q_i}{\partial R_i}\left(R_i + \frac{\partial E[C_i]}{\partial q_i}\right) - \frac{\partial q_{CB}}{\partial R_i}\frac{\partial E[C_i]}{\partial q_{CB}} = 0. \tag{21}$$

First, I focus on the case where CBDC does not have a share of the deposit market, $q_{CB} = 0$. By combining equations (10) and (11), it follows that there exists a sufficiently low CBDC remuneration rate, $\underline{R}_{CB}$ such that no depositor prefers CBDC to bank deposits. In a symmetric equilibrium with $R_{CB} \leq \underline{R}_{CB}$, all banks receive an equal market share. As depositors are assumed to have no outside option, absent CBDC, they will always deposit their funds in a bank and thus bank i's market share is simply $q_i = \frac{1}{N}$.

In the case where $q_{CB} = 0$, the first-order condition for the deposit rate described in equation (21) can be simplified, leading to a closed-form solution for the deposit rate as in a textbook Salop circle model.

The profit bank i makes in the case where $R_{CB} \leq \underline{R}_{CB}$ can be found by substituting the equilibrium deposit rate and market share into equation (6), yielding

$$\pi_i = \bar{\pi} - \left(R_i - R_f + \lambda \xi R_{IB}^{+} q_{CB}\right) q_i, \tag{22}$$

where

$$\bar{\pi} \equiv \left(R_L - R_f\right) L - F. \tag{23}$$

A necessary requirement for $N \geq 2$ banks to enter the market in an equilibrium with $q_{CB} = 0$ is that the fixed cost of entry is such that $-\frac{1}{4}t_B \leq \bar{\pi} < 0$. Then, in a long-run equilibrium the number of banks adjusts subject to a free entry condition such that banks make zero profit in expectation. I assume that this parameter restriction is satisfied and focus on the case where long-run competition in the deposit occurs in the absence of CBDC.

The equilibrium without CBDC is fully characterized by Proposition 2.

Proposition 2. *If $R_{CB} \leq \underline{R}_{CB} \equiv R_f - \frac{3}{2}t_B\frac{1}{N}$ then:*

- **I.** *When the number of banks is fixed at $N \geq 2$ there exists a unique symmetric short-run equilibrium where banks compete such that for all i: $R_i = R_f - \frac{1}{N}t_B$, $q_i = \frac{1}{N}$ and $q_{CB} = 0$.*
- **II.** *When banks enter subject to $\pi_i \geq 0$ and if $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ then there exists a unique symmetric long-run equilibrium where banks compete such that for all i: $R_i = R_f - \frac{1}{N}t_B$, $q_i = \frac{1}{N}$, $N = t_B^{\frac{1}{2}}\left(F - \left(R_L - R_f\right) L\right)^{-\frac{1}{2}}$, and $q_{CB} = 0$.*

Proof. See the Appendix. □

With $q_{CB} = 0$, the bank's equilibrium funding decision combined with equation (20) implies that $E\left[C_i\right] = 0$ and thus the expected liquidity cost of deposits is zero. This is a direct consequence of the structure of the liquidity shock. In an economy without CBDC banks face net inflows of liquidity that exactly offset the net outflows of liquidity regardless of the realization of s. Thus, banks are able to accumulate sufficient liquidity so that they do not need to make use of the central bank's standing facilities.

The cutoff CBDC remuneration rate $\underline{R}_{CB}$ is increasing in N, thus the more concentrated the banking sector, the lower the threshold for CBDC to obtain a positive market share. With fewer banks active in the deposit market, banks offer lower deposit rates and, therefore, CBDC poses greater competitive pressure to banks at a given R_{CB}.

4.3 Deposit Market Equilibrium with CBDC

I now consider the bank's choice of deposit rate when $R_{CB} > \underline{R}_{CB}$ and thus the CBDC remuneration rate is sufficiently high that it poses meaningful competition to banks. With $q_{CB} > 0$, the market share of each bank in a symmetric equilibrium is no longer equal to $\frac{1}{N}$ and instead depends on the deposit rate offered by the banks. As a consequence, the short-run deposit rate is now determined by a system of two equations; the first-order condition for the deposit rate, equation (21), and the definition of $\hat{x}_i$ set out by equation (11). The long-run equilibrium will also require that the free-entry condition of banks holds.

To simplify the analysis, I focus on the case where the liquidity cost banks face is not too large that banks are forced out of the deposit market almost immediately. Specifically, I assume that the following parameter restriction holds

$$\frac{1}{t_B}\lambda\xi R^{+}_{IB} \leq 1. \tag{24}$$

This assumption is discussed further in the appendix.

In a symmetric short-run equilibrium, from equation (10) that $x_i^* = \frac{1}{N}$. When $R_{CB} > \underline{R}_{CB}$, through combining equations (11) and (21) the short-run equilibrium can be found as the z_i that solves the following equation

$$\Gamma \equiv -q_i - \frac{\partial q_i}{\partial R_i}\left(R_{CB} + t_B\left(x_i^* - z_i\right) - R_f + \lambda\xi R^{+}_{IB} q_{CB}\right) - \frac{\partial q_{CB}}{\partial R_i}\lambda\xi R^{+}_{IB} q_i = 0, \tag{25}$$

where q_i and q_{CB} are functions of x_i^* and z_i given by equations (17) and (18), respectively.

The bank takes the deposit rates set by the other banks as well as the CBDC remuneration rate as given. It chooses its deposit rate R_i taking into account the effect that a change in the deposit rate has on both its own market share, q_i and on the market share of CBDC, q_{CB}.

Equation (25) also depends on the impact of an increase of R_i on the market share of CBDC, holding the deposit rates of other banks fixed. To obtain this, consider the definition of q_{CB} set out by equation (18). As all depositors are assumed to deposit their funds somewhere, the market share of CBDC is simply the mass of depositors that choose not to deposit funds at any bank. An increase in R_i affects the market share not only of bank i, but also of neighboring banks; the impact of R_i on the market share of CBDC can be calculated from

$$\frac{\partial q_{CB}}{\partial R_i} = -\frac{\partial q_i}{\partial R_i} - \frac{\partial q_{i+1}}{\partial R_i} - \frac{\partial q_{i-1}}{\partial R_i}, \tag{26}$$

where q_{i+1} and q_{i-1} denote the market shares of neighboring banks.

The short-run equilibrium can be summarized as z_i that solves equation (25). In cases where the deposit rate is sufficiently high $R_{CB} > \underline{R}_{CB}$, equation (25) ensures that $z_i > 0$ and CBDC obtains a positive share of the deposit market, $q_{CB} > 0$. From equations (10) and (11) there exists a threshold CBDC remuneration rate R^*_{CB} above which banks will not directly compete with each other and instead would operate a local monopoly competing only against the CBDC. It also follows from equation (11) that should the CBDC remuneration rate increase above some upper limit $\bar{R}_{CB}$, then banks will not operate in the deposit market and all depositors hold CBDC.

This result is summarized in the following proposition.

Proposition 3. *Given that $\frac{1}{t_B}\lambda\xi R^+_{IB} \leq 1$ and the number of banks is fixed at $N \geq 2$ then if $R_{CB} > R_f - \frac{3}{2}t_B\frac{1}{N}$*

- **I.** *When $R_{CB} \leq R^*_{CB} \equiv R_f + t_B\left(1 - \frac{3}{4N}\right) - \lambda\xi R^+_{IB}\left(1 - \frac{1+N}{4N^2}\right)$ there is short-run competition between banks and some depositors hold CBDC ($q_{CB} > 0$). The market share of banks, q_i is strictly decreasing in R_{CB} while the market share of CBDC is strictly increasing in R_{CB}.*
- **II.** *When $R_{CB} > \bar{R}_{CB} \equiv R_f + t_B - \lambda\xi R^+_{IB}$ banks do not operate in the deposit market in the short-run and CBDC dominates ($q_{CB} = 1$)*
- **III.** *When $R^*_{CB} < R_{CB} \leq \bar{R}_{CB}$ banks operate local monopolies in the short-run and some depositors hold CBDC ($q_{CB} > 0$). The market share of banks, q_i is strictly decreasing in R_{CB} while the market share of CBDC is strictly increasing in R_{CB}.*

Proof. See the Appendix. □

In a short-run equilibrium, the market share of banks is strictly decreasing in R_{CB} over the interval $\left(\underline{R}_{CB}, \bar{R}_{CB}\right]$. As the number of banks in the short-run equilibrium is fixed, this also results in an increase in the market share of CBDC. A higher remuneration rate of CBDC leads to more depositors choosing CBDC over bank deposits. As banks face stiffer competition from CBDC and declining market shares, their profit also falls.

In the case where $q_{CB} > 0$, the profit a bank makes by setting the deposit rate at the profit maximizing level can be written as a function of z_i and x^*_i by substituting equation (11) into (22) and noting that both q_{CB} and q_i will be functions of z_i and x^*_i in equilibrium. This yields

$$\pi_i = \bar{\pi} - \left(R_{CB} - R_f + t_B\left(x^*_i - z_i\right) + \lambda\xi R^+_{IB}q_{CB}\right)q_i. \tag{27}$$

The long-run equilibrium in the deposit market can be summarized as the $\{x^*_i, z_i\}$ pair that satisfies equations (25) and (27).

I focus on the case where $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ and thus at least two banks compete in the long-run equilibrium without CBDC. From Proposition 3 if the number of banks is held fixed, bank profits fall. In the long-run equilibrium, an increase in R_{CB} results in an increase in market concentration, the number of banks in the deposit market will fall, so that banks return to profitability. As the CBDC remuneration rate increases, the number of banks falls further. At some point, it becomes unprofitable for a single banks to enter in the deposit market in the long-run equilibrium. I denote this threshold value by R^{**}_{CB}, the formal definition of which is set out in the following Proposition which describes the long-run equilibrium.

Proposition 4. *Given that $\frac{1}{t_B}\lambda\xi R^{+}_{IB} \leq 1$, $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ and banks enter subject to $\pi_i \geq 0$, there exists some $R^{**}_{CB} > R_f - t_B\frac{3}{2N}$ such that if $R_{CB} = R^{**}_{CB}$, a single bank ($N = 1$) is indifferent between entering in $t = 1$ or not.*

- **I.** *When $R_{CB} \leq R^{**}_{CB}$ there is long-run competition between banks and some depositors hold CBDC ($q_{CB} \geq 0$).*
- **II.** *When $R_{CB} > R^{**}_{CB}$ in the long-run equilibrium banks do not operate in the deposit market and CBDC dominates ($q_{CB} = 1$).*

Proof. See the Appendix. □

In contrast to the short-run equilibrium, a long-run equilibrium where multiple banks operating local monopolies does not occur. From equation (20), as long as $\frac{1}{t_B}\lambda\xi R^{+}_{IB} > 0$, the liquidity cost increases in the market share of CBDC, which would lower bank profits.

While the central bank's balance sheet is not explicitly modeled, Proposition 1 highlights that in equilibrium, each bank increases their holdings of central bank liquidity (B_i) as their market share falls. Summing over all N banks and using the definition of q_{CB} given in equation 18 yields the following equation for the aggregate liquidity borrowed from the central bank by the banking sector

$$\sum_i B_i = NL - (1 - q_{CB})\,. \tag{28}$$

This shows that as the market share of CBDC increases, the aggregate banking sector holds more central bank liquidity, and therefore the introduction of a CBDC will see the central bank balance sheet increase both its liabilities (q_{CB}) and its assets ($\sum_i B_i$).

5 Comparative Statics

5.1 Impact of CBDC on deposit rates

I now present the impact of a change in the CBDC remuneration rate, R_{CB}, on the equilibrium deposit rate R_i offered by the banks. The impact of the CBDC remuneration rate will depend on whether the bank faces liquidity costs in its use of deposits. In particular, I consider the impact of a change in R_{CB} when $\lambda\xi R_{IB}^{+} = 0$ versus when $\lambda\xi R_{IB}^{+} > 0$. The modeling framework I use also allows me to distinguish between the change in R_{CB} in a short-run equilibrium, where the number of banks is held fixed, versus a long-run equilibrium, where the number of banks adjusts according to the free-entry condition. The results presented in this section are especially important in terms of whether the remuneration rate of a CBDC can be used as an additional tool in the central bank's toolbox as has been discussed among others in Meaning et al. (2021).

In the case where $R_{CB} \leq \underline{R}_{CB}$ and $q_{CB} = 0$, CBDC has no market share and the deposit rate is given by Proposition 2. An increase in the CBDC remuneration rate will have no impact on the bank deposit rate. This holds regardless of the value $\lambda\xi R_{IB}^{+}$ takes.

If $R_{CB} > \underline{R}_{CB}$ so that in an equilibrium CBDC has a positive market share, $q_{CB} > 0$, the impact of an increase in R_{CB} on the deposit rate in the short-run equilibrium can be found by using the implicit function Theorem and rearranging equation (11) and differentiating with respect to R_{CB} which produces

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B \left. \frac{dz_i}{dR_{CB}} \right|_{\Gamma=0}. \tag{29}$$

The impact of an increase in R_{CB} in a long-run equilibrium with $q_{CB} > 0$ can be calculated in a similar way by applying the Implicit Function Theorem to equations (25) and (27).

In general, the pass-through of an increase in R_{CB} to the deposit rate will be imperfect as $\left(\frac{\partial R_i}{\partial R_{CB}} < 1\right)$. In a short-run equilibrium with $\lambda\xi R_{IB}^{+} = 0$, the equilibrium deposit rate will be strictly increasing in R_{CB} for all $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$. An increase in the CBDC remuneration rate will result in banks losing market share to CBDC and banks will raise their deposit rates in response to this additional competition.

In a long-run equilibrium, the pass-through of the CBDC remuneration rate to the deposit rate is always lower than in the short-run equilibrium. Furthermore, in a long-run equilibrium, even when $\lambda\xi R_{IB}^{+} = 0$, the deposit rate is not guaranteed to be strictly increasing in the CBDC remuneration rate. In a long-run equilibrium, as R_{CB} increases, the number of banks in the deposit market falls. Thus while banks face additional competition from higher CBDC remuneration rates, this is counteracted in the long-run equilibrium by a more concentrated deposit market resulting in lower competition from other banks.

If $\lambda\xi R_{IB}^{+} > 0$ then an increase in the market share of CBDC also results in banks facing a higher expected liquidity cost from holding deposits. Thus, an increase in R_{CB} not only increases the competition that banks face in the deposit market but also makes deposits a less desirable form of liquidity for banks to hold. As a consequence of this, the pass-through of the CBDC remuneration rate to the deposit rate is lower than it would be in the case where $\lambda\xi R_{IB}^{+} = 0$. Furthermore, the deposit rate is not guaranteed to increase in the CBDC remuneration rate, even in the short-run equilibrium.

These results are summarized in the following proposition.

Proposition 5. *Given that* $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ *then*

I. *For any equilibrium that features* $q_i > 0$*, the pass-through of the CBDC rate to the deposit rate is imperfect* $\left(\frac{\partial R_i}{\partial R_{CB}} < 1\right)$.

II. *For any equilibrium, the pass-through is positive for any* R_{CB} *larger but sufficiently close to* $\underline{R}_{CB}$. $\left(\lim_{R_{CB}\downarrow\underline{R}_{CB}} \left\{\frac{\partial R_i}{\partial R_{CB}}\right\} > 0\right)$

III. *For a short-run equilibrium if* $\lambda\xi R_{IB}^{+} = 0$ *the pass-through is positive* $\left(\frac{\partial R_i}{\partial R_{CB}} > 0\right)$ *for all* $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$.

IV. *The pass-through* $\left(\frac{\partial R_i}{\partial R_{CB}}\right)$ *will be strictly lower in a long-run equilibrium than in a short-run equilibrium.*

Proof. See the Appendix. □

The results set out in Proposition 5 have important policy implications. In particular, regarding the use of the CBDC remuneration rate as an additional tool in the central bank's toolkit. Even in the most benign scenario where there is no liquidity risk and without considering the long-run impact on the banking sector, the pass-through of the CBDC remuneration rate to the bank deposit rate is imperfect. In this scenario, banks do raise their deposit rates in response to increased competition from CBDC but as competition is imperfect, they do so less than one-for-one. This section also highlights that if the central bank chose to use the CBDC remuneration rate as a policy tool, there may be long-run consequences on the banking sector which would serve to dampen the pass-through to the bank deposit rate. Finally, in the case where there is risk of liquidity flowing from bank deposits to a CBDC, the additional cost this imposes on banks further weakens the pass-through of the CBDC remuneration rate to the bank deposit rate.

5.2 Implications for Monetary Policy Transmission

In this section, I consider the implication of CBDC for the transmission of monetary policy within the context of the model. To this end, I add some additional structure to the model in the following way. First, I assume that the spreads on the central bank standing facilities are held fixed and that the interest rate on the liquidity facility and on the deposit facility are of the form

$$R_{LF} = R_f + \Delta_{LF}, \tag{30}$$

and

$$R_{DF} = R_f - \Delta_{DF}, \tag{31}$$

with $\Delta_{LF} > 0$ and $\Delta_{DF} > 0$. Second, I assume that the interest rate on bank loans is equal to the policy rate plus a fixed mark-up such that

$$R_L = R_f + \Delta_L, \tag{32}$$

with $\Delta_L > 0$. Finally, I assume that the central bank sets the remuneration rate of CBDC such that it is a fixed distance from the policy rate such that

$$R_{CB} = R_f + \Delta_{CB}. \tag{33}$$

Here, Δ_{CB} could be positive or negative, depending on the remuneration rate of CBDC. It should also be noted that this is just one possible remuneration policy that central banks could choose for CBDC. However, the remuneration policy considered here can be interpreted as the most neutral implementation of CBDC remuneration in the model. Other remuneration policies can be obtained through combining a change in the policy rate with a change in R_{CB}.

In the case where $q_{CB} = 0$, it follows from Proposition 2 that the deposit rate increases one for one with the policy rate. The more interesting case occurs where $q_{CB} > 0$. Given the above assumptions on interest rates, the two key equations that determine the short- and long-run equilibrium, equations (25) and (27), can be rewritten as follows

$$\tilde{\Gamma} \equiv -q_i - \frac{\partial q_i}{\partial R_i}\left(\Delta_{CB} + t_B\left(x_i^* - z_i\right) + \lambda\xi\left(R_f + \Delta_{IB}^+\right)q_{CB}\right) - \frac{\partial q_{CB}}{\partial R_i}\lambda\xi\left(R_f + \Delta_{IB}^+\right)q_i = 0, \tag{34}$$

and

$$\tilde{\pi} \equiv \Delta_L L - F - \left(\Delta_{CB} + t_B\left(x_i^* - z_i\right) + \lambda\xi\left(R_f + \Delta_{IB}^+\right)q_{CB}\right)q_i = 0, \tag{35}$$

where

$$\Delta_{IB}^{+} = \begin{cases} \Delta_{LF} & \text{if } R_{IB}^{+} = R_{LF} \\ \left(\frac{1-\lambda}{\lambda}\right)\Delta_{DF} & \text{if } R_{IB}^{+} = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right). \end{cases}$$

In the case where $R_{CB} > \underline{R}_{CB}$, the deposit rate can be written in terms of R_f as

$$R_i = t_B\left(x_i^* - z_i\right) + R_f + \Delta_{CB}. \tag{36}$$

Equation (36) emphasizes that if the spread between the deposit rate and the CBDC rate, z_i is held fixed, then the deposit rate will move one-for-one with the policy rate.

If $\lambda\xi R_{IB}^{+} = 0$, the two equations that describe the equilibria, equations (34) and (35) do not respond to a change in the policy rate R_f. Thus it follows that in both a short-run and long-run equilibrium, the value of z_i is also unchanged and so the deposit rate will increase one-for-one with the policy rate.

If, on the other hand $\lambda\xi R_{IB}^{+} > 0$, equations (34) and (35) are now affected by changes in policy rate R_f. This in turn will impact the equilibrium value of z_i and thus the market share of CBDC will also be impacted by the policy rate. The deposit rate will no longer increase one-for-one with the policy rate. Furthermore, since both equations are affected by R_f, the pass-through of the policy rate to the deposit rate will be lower in the long-run equilibrium than in the short-run equilibrium.

These results are summarized in the following proposition.

Proposition 6. *Given that $0 < \frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ then for any equilibrium that features $q_i > 0$, the pass-through of the policy rate to the deposit rate is imperfect $\left(\frac{\partial R_i}{\partial R_f} \neq 1\right)$ and the pass-through will be lower in a long-run equilibrium than in a short-run equilibrium.*

Proof. See the Appendix. □

The key mechanism that drives the imperfect pass-through of the policy rate is the liquidity cost of deposits described in equation (20). If $\lambda\xi R_{IB}^{+} > 0$, an increase in the policy rate also increases the cost of obtaining additional liquidity should the bank require it. This results in lower profits and thus a more concentrated banking sector in the long-run equilibrium. If deposits become less desirable for the bank to hold, there is now a downward pressure on bank deposit rates as banks require larger spreads to compensate for the additional liquidity risk.

Proposition 6 highlights a possible risk the introduction of CBDC poses for the transmission of monetary policy to the economy. In the model, monetary transmission happens solely through pass-through of the policy rate to the deposit rate offered by banks. In the case without deposit liquidity risk, a CBDC can be introduced without impacting the normal workings of monetary policy. However, if banks face a liquidity risk in obtaining liquidity from retail deposits, this cost will increase in the deposit rate, and this, in turn, will affect the transmission of monetary policy through the deposit rate. This occurs because the cost of this liquidity risk that banks face depends on the cost of obtaining additional liquidity through the central bank lending facility. The cost of obtaining this liquidity increases with the policy rate.

6 Quantitative Analysis

The previous section provides some theoretical results regarding the impact of a CBDC on both the deposit rate and on the pass-through of monetary policy. To address whether these theoretical results are quantitatively important, I present a simple calibration of the model to the Eurozone economy without CBDC.

6.1 Calibration

Data are obtained from the ECB Statistical Data Warehouse. The data obtained is the average for the year 2021, which is the last year that data is available for all of the series. The policy rates in the model are calibrated to the corresponding ECB rates. The main policy rate R_f is calibrated to the ECB's Main Refinancing Rate which was 0 throughout 2021. The interest rates on the standing facilities, R_{LF} and R_{DF} are calibrated to the ECB's Lending Facility Rate and Deposit Facility Rate which were 25 basis points and -50 basis points, respectively.

The parameters affecting the banking sector are calibrated such that there is no CBDC ($q_{CB} = 0$) and the free-entry condition holds. The number of banks in this equilibrium is set to $N = 7$. This is chosen to match the Herfindahl Hirschman Index (HHI) of Eurozone credit institutions which averaged 0.145 in 2021. Given the model assumes banks of equal size and thus the HHI corresponds to $1/N$.

Given $q_{CB} = 0$, the equilibrium deposit rate is given by Proposition 2 as

$$R_i = R_f - \frac{1}{N} t_B. \tag{37}$$

Therefore, the transport cost t_B can be set such that the equilibrium deposit rate R_i matches the average deposit rate in the Eurozone, which was -1.44 basis points. This deposit rate is calculated as the weighted average deposit rate on overnight household deposits and overnight corporate deposits.

In an equilibrium without CBDC, banks hold sufficient liquidity that they do not require additional liquidity from either standing facilities or the interbank market and $L = B_i + q_i$. As the size of the banks are normalized by $q_i = 1/N$, L and B_i are calibrated such that the ratio of deposits to total liabilities (q_i/L) in the model matches the Ratio of Deposit to Liabilities of Eurozone Credit institutions of 0.42.

The bank lending rate R_L is chosen to match the interest rate on short-term loans to non-financial corporations which stood at 150 basis points. Given this, F follows from the assumption that the free-entry condition binds in this calibration, which implies

$$F = (R_L - R_f) L + t_B \frac{1}{N^2}. \tag{38}$$

Finally, the size of the liquidity shock ξ is set to match the percentage of total deposit liabilities that are traded daily in the Target 2 market which in 2021 stood at 3.13%. The calibration is summarized in Table 1.

Table 1: Calibrated Parameters

Parameter	Notation	Value	Calibration Target
Main Policy Rate	R_f	1.0	ECB Main Refinancing Rate
Lending Facility Rate	R_{LF}	1.0025	ECB Lending Facility Rate
Deposit Facility Rate	R_{DF}	0.995	ECB Deposit Facility Rate
Number of Banks	N	7	Herfindahl Hirschman Index (HHI) of Eurozone credit institutions
Bank Lending Rate	R_L	1.015	Interest rate on short-term loans to non-financial corporations
Bank Deposit Rate	R_i	0.99986	Overnight deposit rate of household and corporate deposits
Deposit to Liability Ratio	q_i/L	0.42	Ratio of Deposit to Liabilities of Eurozone Credit institutions
Size of Liquidity Shock	ξ	0.00313	Ratio of Euro short-term rate volume to Eurozone Credit institutions Deposits

6.2 Impact of the CBDC remuneration rate

Using the benchmark calibration, I now plot how the deposit rate R_i changes as R_{CB} varies in the range $\underline{R}_{CB}$ to $\bar{R}_{CB}$. First I consider the case where $\lambda = 0$ and thus the liquidity shock channel is shut down. This case is plotted in Figure 1.

In Figure 1, for $R_{CB} \leq \underline{R}_{CB}$, R_i does not respond to R_{CB}. This is because the market share of CBDC is zero and at these low remuneration rates, CBDC does not pose meaningful competition to bank deposits. As R_{CB} increases above $\underline{R}_{CB}$, R_i is strictly increasing in the short-run equilibrium. In the long-run equilibrium, R_i is increasing in R_{CB} only for R_{CB} above but sufficiently close to R_{CB}. As R_{CB} increases further, R_i decreases in R_{CB} in the long-run. This follows from the stated result in Proposition 5 that the long-run response of R_i to an increase in R_{CB} will always be lower than in the short-run.

In the short-run, with the number of banks fixed, an increase in R_{CB} will lead to a decrease in the market share of banks, and banks will increase deposit rates to mitigate this loss of market share. The fall in market share and increase in cost of deposits leads to a fall in bank profits in the short-run which results in a consolidation of the banking sector in the long-run. As the number of banks falls in the long-run in response to an increase in R_{CB}, banks face less competition from their neighboring banks and thus may be able to reduce deposit rates even in the face of greater competition from CBDC.

Figure 1 also highlights two kinks that exist in the response of R_i to R_{CB}. The first of these occurs at the point where $z_i = 0$, that is the point at which CBDC starts putting competitive pressure on bank deposits. The second kink occurs at the point where $z_i = x_i^*$ which is the point at which some proportion of depositors will choose CBDC over bank deposit no matter how far away they are located from a bank. These kinks correspond to the intervals over which the piece-wise continuous function for q_i is defined by equation (17).

Setting $\lambda = 0.05$, the response of R_i to a change in R_{CB} is plotted in figure 2. Apart from λ, all other parameters remain the same as in figure 1. Figure 2 illustrates the case where the expected liquidity cost of the deposits, $\lambda R_{IB}^{+}\xi$, is large enough that even in the short-run, the deposit rate is decreasing in R_{CB} at sufficiently high levels of R_{CB}. In the previous case, with $\lambda = 0$, an increase in R_{CB} placed additional competitive pressure on the banking sector in the short-run and thus forced banks to raise the deposit rates they offer in order to compete for market share. In the case of $\lambda > 0$, increasing R_{CB} now has an additional effect, which is to increase the cost of deposits for banks. This can be seen in equation (5) where the expected liquidity cost of the deposits increases in the market

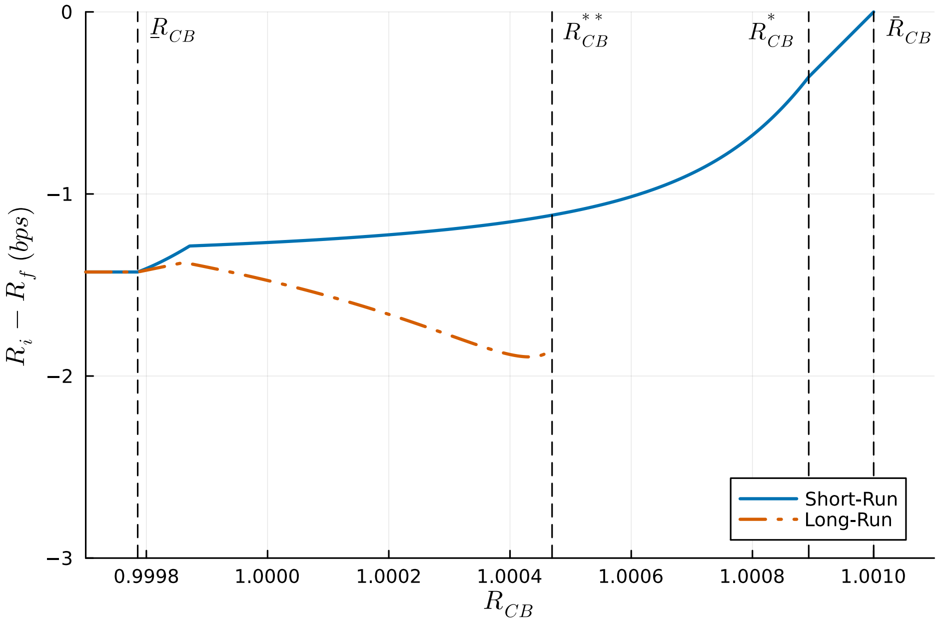

Figure 1: Impact of R_{CB} on the deposit rate ($\lambda = 0$)

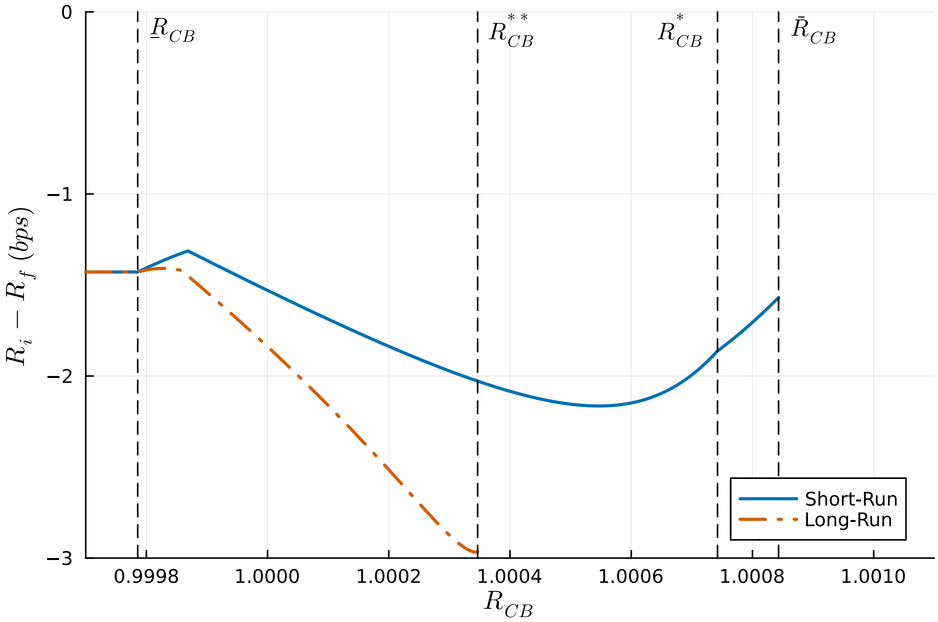

Figure 2: Impact of R_{CB} on the deposit rate ($\lambda = 0.05$)

share of CBDC and decreases in the bank's own market share q_i. Therefore, if R_{CB} is sufficiently large and the market share of CBDC is sufficiently large, banks may choose to lower their deposit rate to reduce their own market share and thus lower the liquidity cost of deposits.

6.3 Implications for Monetary Policy Pass-through

I now plot how the pass-through of the deposit rate interacts with the market share of CBDC. As discussed in the previous section, in the case where $\lambda = 0$, the policy rate has perfect pass-through to the deposit rate with $\frac{dR_i}{dR_f} = 1$. Thus, I focus only on the cases where $\lambda > 0$. In the quantitative exercise, I make the same simplifying assumption regarding the policy rate as made in the previous section.

Figure 3 provides the pass-through to the deposit rate, $\frac{dR_i}{dR_f}$ in the case where $\lambda = 0.05$. The figure shows a nonlinear response of $\frac{dR_i}{dR_f}$ to the increase in q_{CB}. The figure uses the same calibration as that of figure 2, with an increase in q_{CB} generated by a corresponding increase in R_{CB}. It should be noted that while the figure confirms the theoretical results of Proposition 6, it also suggests that the magnitude of this effect may not be very large.

It is instructive to consider what generates the non-linear response of $\frac{dR_i}{dR_f}$ to the increase in q_{CB}, especially in the short-run. From equation 34 it is clear that an increase in R_f would lead to a change in z_i and hence each bank's share of the deposit market. Therefore, in the presence of liquidity shocks, the increase in the policy rate will have an impact on the deposit market similar to the increase in R_{CB}. The main mechanism occurs as an increase in R_f will, if q_{CB} is sufficiently large, lead to an increase in z_i and therefore through equation 36, a lower increase in the bank deposit rate with $\frac{dR_i}{dR_f} < 1$. Analogously to the result stated in proposition 5, the increase in z_i for a given increase in R_f will vary with q_{CB}, hence the nonlinear response of $\frac{dR_i}{dR_f}$.

Figure 3 also shows a discontinuity in the pass-through response in the short-run equilibrium. This occurs at the point where $R_{CB} = R^*_{CB}$ and the banking sector transitions to an equilibrium without direct competition between banks. As stated in Proposition 4, this equilibrium does not occur in the long-run, hence why this discontinuity is only visible in the short-run response.

Next, I consider the reason for the low magnitude of the pass-through distortion highlighted in figure 3. From equation (34) and equation (35) it is clear that an important determinant of the pass-through distortion is $\lambda\xi$. The values of λ and ξ in the benchmark calibration are quite modest and are the key to driving the low magnitude exhibited in Figure 3. To illustrate this, in figure 4 I show the response of the policy rate pass-through

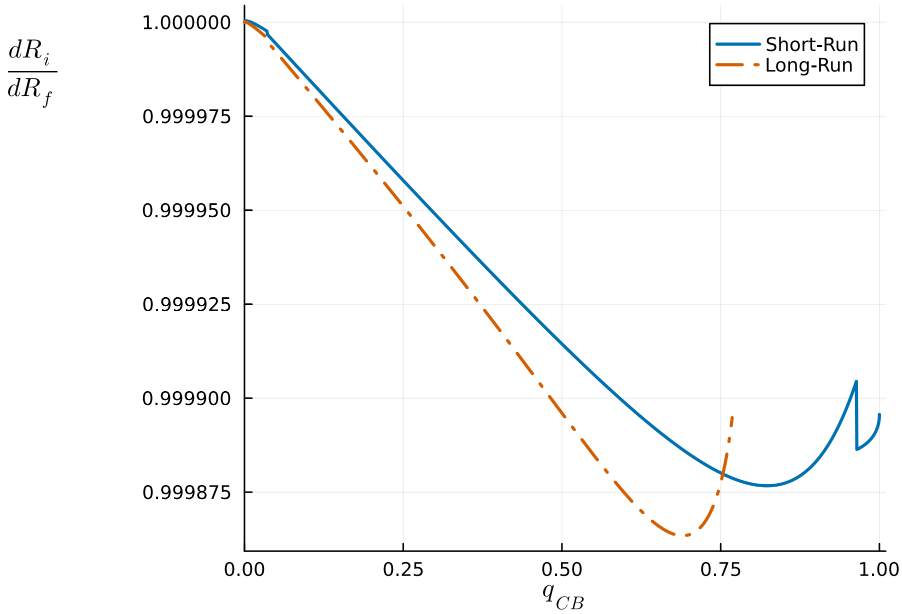

Figure 3: Impact of CBDC on Monetary Policy Transmission ($\lambda = 0.05$)

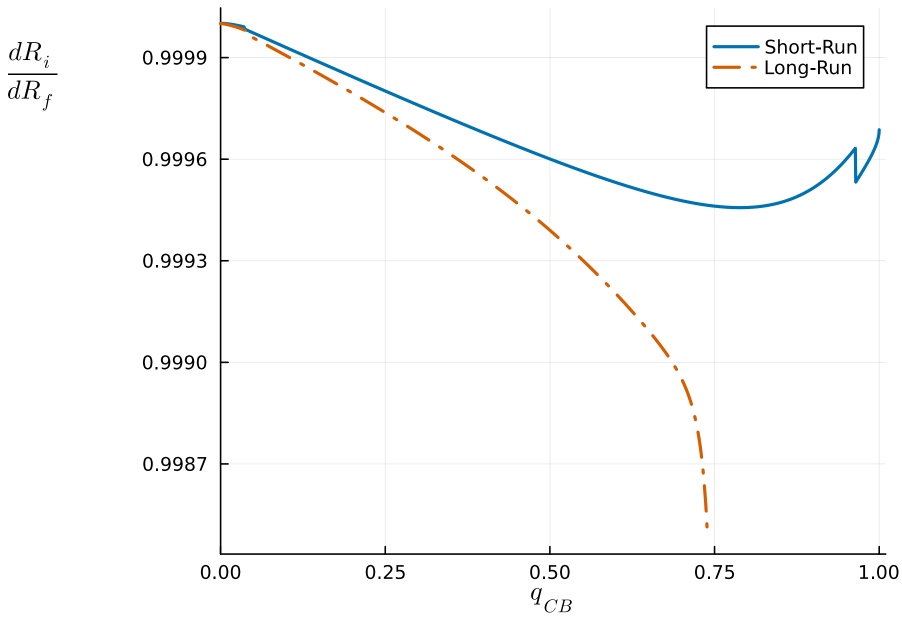

Figure 4: Impact of CBDC on Monetary Policy Transmission ($\lambda = 0.15$)

if the probability of liquidity shocks occurring took a higher value of $\lambda = 0.15$. Here, the magnitude of the effect is much stronger and the difference between the long-run and short-run responses are more stark. A similar increase in magnitude could occur if the parameter ξ took a larger value.

The quantitative analysis highlighted the importance of both allowing for long-run structural changes in the banking sector and in the size of the liquidity risk channel. This is especially true when attempting to quantify the impact of CBDC on the transmission of monetary policy.

7 Conclusion

As the policy debate surrounding the potential introduction of a retail CBDC grows, so does the need for further analysis of its potential implications. This paper focuses on the impact of CBDC on the structure of the market for retail bank deposits and on bank liquidity. In this paper, CBDC is modeled as a source of direct competition for bank deposits. Competition in the deposit market is modeled using a Salop circle model, and thus there is imperfect substitutability between deposits of different banks and the CBDC. This framework allows us to distinguish between the short-run impact of CBDC, where the number of banks is fixed from the long-run impact where the number of banks may adjust. Additionally, the model suggests a liquidity risk channel through which CBDC can further increase the costs of banks operating in the deposit market.

In the absence of liquidity risk, the model suggests that in a short-run equilibrium, the introduction of CBDC will result in an increase in interest rates on bank deposits. This leads to a reduction of the market shares of banks in the deposit market. Banks substitute these deposits by obtaining additional liquidity from the central bank through open market operations, and bank profitability falls. In the long-run, the model suggests that the introduction of CBDC will reduce the number of banks active in the deposit market and lead to greater concentration in the banking sector. In the long run, the pass-through of the CBDC remuneration rate to the bank deposit rate is lower than in the short-run, and the deposit rate may even be decreasing in the CBDC remuneration rate. This effect is amplified if banks face liquidity risk in holding deposits, and in this case the bank deposit rate may be decreasing in the CBDC remuneration even in the short-run. Thus the paper casts doubt on the use of the remuneration rate of CBDC as an additional tool in the monetary policy toolkit of central banks.

The paper also highlights the importance of the liquidity risk channel for monetary policy transmission more generally. In absence of liquidity risk, the bank deposit rate increases one-for-one following an increase in the policy rate, even after the introduction of a CBDC. However, if banks face liquidity risk in the deposit market, the introduction of a CBDC will now affect the transmission of monetary policy through the bank deposit rate. Furthermore, the impact of monetary policy will now impact the structure of the deposit market, and thus monetary policy will have a different impact in the short and long run.

Although this paper makes no claims regarding the welfare implications of the introduction of CBDC, it would be prudent for policymakers to take into account the welfare implications of a more concentrated banking sector that may follow the introduction of a CBDC as well as possible implications for the transmission of monetary policy.

References

Itai Agur, Anil Ari, and Giovanni Dell'Ariccia. Designing central bank digital currencies. *Journal of Monetary Economics*, 125:62–79, 2022.

David Andolfatto. Assessing the Impact of Central Bank Digital Currency on Private Banks. *The Economic Journal*, 131(634):525–540, March 2021.

Philippe Bacchetta and Elena Perazzi. CBDC as Imperfect Substitute for Bank Deposits: A Macroeconomic Perspective. Swiss Finance Institute Research Paper Series 21-81, Swiss Finance Institute, December 2021.

John Barrdear and Michael Kumhof. The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, 142(104148), 2022.

Ulrich Bindseil. Central bank digital currency: Financial system implications and control. *International Journal of Political Economy*, 48(4):303–335, 2019.

Michael D Bordo. Central bank digital currency in historical perspective: Another crossroad in monetary history. Working Paper 29171, National Bureau of Economic Research, August 2021.

Florian Böser and Hans Gersbach. Monetary policy with a central bank digital currency: The short and the long term. CEPR Discussion Papers 15322, C.E.P.R. Discussion Papers, September 2020.

Jan Bouckaert. Monopolistic competition with a mail order business. *Economics Letters*, 66(3):303–310, March 2000.

Ben Broadbent. Central banks and digital currencies. *Speech delivered at the London School of Economics, London, UK, 2 March 2016.*, 2016.

Markus K. Brunnermeier and Dirk Niepelt. On the equivalence of private and public money. *Journal of Monetary Economics*, 106:27–41, October 2019.

Monika Bucher, Achim Hauck, and Ulrike Neyer. Interbank market friction-induced holdings of precautionary liquidity: Implications for bank loan supply and monetary policy implementation. *Economic Theory*, 70(1):165–222, July 2020.

Lorenzo Burlon, Carlos Montes-Galdón, Manuel A. Muñoz, and Frank Smets. The optimal quantity of CBDC in a bank-based economy. Working Paper Series 2689, European Central Bank, July 2022.

Pierre-André Chiappori, David Perez-Castrillo, and Thierry Verdier. Spatial competition in the banking system: Localization, cross subsidies and the regulation of deposit rates. *European Economic Review*, 39(5):889–918, 1995.

Jonathan Chiu and Mohammad Davoodalhosseini. Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design. Staff Working Papers 21-63, Bank of Canada, December 2021.

Jonathan Chiu, Mohammad Davoodalhosseini, Janet Hua Jiang, and Yu Zhu. Bank market power and central bank digital currency: Theory and quantitative assessment. Staff Working Papers 19-20, Bank of Canada, May 2019.

European Central Bank. *Study on the payment attitudes of consumers in the euro area (SPACE).* Publications Office, December 2022.

Jesús Fernández-Villaverde, Daniel Sanches, Linda Schilling, and Harald Uhlig. Central bank digital currency: Central banking for all? *Review of Economic Dynamics*, December 2021.

Achim Hauck and Ulrike Neyer. A model of the Eurosystem's operational framework and the euro overnight interbank market. *European Journal of Political Economy*, 34: S65–S82, June 2014.

Katherine Ho and Joy Ishii. Location and competition in retail banking. *International Journal of Industrial Organization*, 29(5):537–546, 2011.

Todd Keister and Daniel R. Sanches. Should central banks issue digital currency? Working Papers 19-26, Federal Reserve Bank of Philadelphia, June 2019.

Michael Kumhof and Clare Noone. Central bank digital currencies – Design principles for financial stability. *Economic Analysis and Policy*, 71(C):553–572, 2021.

Paul Madden and Mario Pezzino. Oligopoly on a Salop Circle with Centre. *The B.E. Journal of Economic Analysis & Policy*, 11(1), January 2011.

Carmen Matutes and Xavier Vives. Competition for Deposits, Fragility, and Insurance. *Journal of Financial Intermediation*, 5(2):184–216, April 1996.

Jack Meaning, Ben Dyson, James Barker, and Emily Clayton. Broadening Narrow Money: Monetary Policy with a Central Bank Digital Currency. *International Journal of Central Banking*, 17(2):1–42, 2021.

Nick Moon, Sarah McHugh, and Natalie Cass. Personal Current AccountInvestigation: A report for the Competition and Markets Authority by GfK NOP. *GfK NOP*, 2015.

Dirk Niepelt. Reserves for all? Central bank digital currency, deposits, and their (non)-equivalence. *International Journal of Central Banking*, 16(3):211–238, 2020.

Kwangwoo Park and George Pennacchi. Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation. *The Review of Financial Studies*, 22(1): 1–40, 05 2008.

Rafael Repullo. Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156–182, 2004. ISSN 1042-9573. Bank Capital Adequacy Regulation under the New Basel Accord.

Steven C. Salop. Monopolistic Competition with Outside Goods. *The Bell Journal of Economics*, 10(1):141, 1979.

Stephen Williamson. Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861, 2022.

Appendix

Proof of Proposition 1

By combining (5) and (6) the single bank's problem can be written as follows

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - (1-\lambda) R_{IB}^0 \epsilon_i^0 - \lambda R_{IB}^+ \epsilon_i^+ - F \right\} \tag{A.39}$$

where ϵ_i^0 and ϵ_i^+ are defined by equations (2) and (3) respectively.

Differentiating with respect to B_i yields the following first-order condition

$$-R_f + (1-\lambda) R_{IB}^0 + \lambda R_{IB}^+ = 0. \tag{A.40}$$

Given $\epsilon_i^0 < \epsilon_i^+$ it follows from equation (4) and that $R_f \in (R_{DF}, R_{LF})$ that $R_{IB}^0 < R_{IB}^+$. There are two possible equilibria cases that are possible. Furthermore, since obtaining too much or too little liquidity from the central bank in $t = 2$ is costly, the following inequality constraints must hold, $\epsilon_i^0 \leq 0$ and $\epsilon_i^+ \geq 0$, with one of these inequality constraints holding with equality.

Thus, there are two cases to consider. First, if $\epsilon_i^0 = 0$, then banks will have exactly enough liquidity to ensure that if they are in a neutral liquidity position if they do not receive a net liquidity outflow. In this case, $R_{IB}^+ = R_{LF}$ and $R_{IB}^0 \in [R_{DF}, R_{LF})$. From equation (A.40) it follows that the value of R_{IB}^0 that ensures the first condition holds is

$$R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda} \right) (R_{LF} - R_f), \tag{A.41}$$

and that for $R_{IB}^0 \geq R_{DF}$ it must be the case that

$$\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}} \right). \tag{A.42}$$

Finally, for $\epsilon_i^0 = 0$ it follows from equation (2) that

$$B_i = L - q_i. \tag{A.43}$$

Thus the first part of the proposition has been obtained.

The second case to consider occurs if $\epsilon_i^+ = 0$ where banks hold exactly enough liquidity that they do not require any additional liquidity should they suffer a net outflow of liquidity. In this case, $R^0_{IB} = R_{DF}$ and $R^+_{IB} \in (R_{DF}, R_{LF})$. From equation (A.40) it follows that the value of R^+_{IB} that ensures the first condition holds is

$$R^+_{IB} = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right), \tag{A.44}$$

and for $R^+_{IB} < R_{LF}$ it follows that

$$\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right). \tag{A.45}$$

Finally, for $\epsilon^+_i = 0$ it follows from equation (3) that

$$B_i = L - \left(1 - q_{CB}\xi\right) q_i. \tag{A.46}$$

Thus the whole proposition has now been obtained.

Proof of Proposition 2

First, differentiating the bank's profit function given in equation (A.39) with respect to R_i and combining with equation (A.40) gives the following first-order condition for the bank given by equation (21).

In the case where $q_{CB} = 0$, the demand function that the bank faces is given by equation (8) and thus

$$\frac{\partial q_i}{\partial R_i} = \frac{1}{t_B}, \tag{A.47}$$

while from equation (20) if $q_{CB} = 0$ then

$$\frac{\partial E\left[C_i\right]}{\partial q_i} = -R_f, \tag{A.48}$$

and

$$\frac{\partial E\left[C_i\right]}{\partial q_{CB}} = 0. \tag{A.49}$$

In the short-run equilibrium with $q_{CB} = 0$, all banks have an equal market share and since there is full coverage, $q_i = {}^1\!/_N$. Combining the above with the first-order condition yields the following equation for the deposit rate in the short-run

$$R_i = R_f - t_B \frac{1}{N}. \tag{A.50}$$

Finally, from equation (10), the distance from bank i where a depositor is indifferent between holding a bank i deposit and the CBDC is $x^*_i = \frac{1}{2N}$. The highest possible remuneration rate in CBDC such that all depositors prefer bank deposits to CBDC $\underline{R}_{CB}$, can be found by combining equations (11) and (A.50) and substituting $x_i^* = \frac{1}{2N}$ to produce

$$\underline{R}_{CB} = R_f - \frac{3}{2} t_B \frac{1}{N}. \tag{A.51}$$

In the long-run equilibrium, N adjusts such that banks enter and make zero expected profits. The bank's profit function is given by equation (6). Substituting the expected liquidity of the deposits given by equation (20), the deposit rate given by equation (A.50) and the optimal funding decision as defined by proposition 1 produce the following equation for profit in the case where $q_{CB} = 0$

$$\pi_i = (R_L - R_f) L - F + t_B \frac{1}{N^2}. \tag{A.52}$$

Denote the number of firms that drive bank profit to zero as N^*. Assuming that $F - (R_L - R_f) L > 0$, a positive N^* exists and can be written as

$$N^* = t_B^{\frac{1}{2}} (F - (R_L - R_f) L)^{-\frac{1}{2}}. \tag{A.53}$$

If $F - (R_L - R_f) L \leq \frac{1}{4} t_B$ then $N^* \geq 2$ and it follows that when $R_{CB} \leq R_f - \frac{3}{2} t_B \frac{1}{N^*}$ there exists a long-run equilibrium with $q_{CB} = 0$ where banks obtain equal market shares with $q_i = \frac{1}{N^*}$ and set deposit rates as in equation (A.50).

Proof of Proposition 3

A short-run equilibrium with $R_{CB} > R_f - \frac{3}{2} t_B \frac{1}{N}$ can be summarized as z_i that solves equation (25).

From equation (17), the derivative of a bank's share of the deposit market with respect to the deposit rate is

$$\frac{\partial q_i}{\partial R_i} = \begin{cases} \frac{1}{t_B} & \text{if } z_i < 0 \\ \frac{1}{t_B} (1 + z_i) & \text{if } 0 \leq z_i \leq x_i^* \\ \frac{1}{t_B} (1 + 2x_i^* - z_i) & \text{if } x_i^* < z_i \leq 1 \\ 2\frac{1}{t_B} (1 + x_i^* - z_i) & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{A.54}$$

From equation (18), the derivative of the CBDC's share of the deposit market with respect to the deposit market is

$$\frac{\partial q_{CB}}{\partial R_i} = \begin{cases} 0 & \text{if } z_i < 0 \\ -2\frac{1}{t_B}\left(x_i^* - \hat{x}_i\right) & \text{if } 0 \leq z_i \leq x_i^* \\ -2\frac{1}{t_B}x_i^* & \text{if } x_i^* < z_i \leq 1 \\ -2\frac{1}{t_B}\left(1 + \hat{x}_i\right) & \text{if } 1 < z_i \leq 1 + x_i^* \\ 0 & \text{if } z_i > 1 + x_i^*. \end{cases} \tag{A.55}$$

Equation (25) can be written as

$$\Gamma = \begin{cases} R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+ \frac{z_i^2}{2x_i^*} & \\ -t_B\left(1 - 2z_i\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(\frac{2x_i^* - z_i^2}{1+z_i}\right), & \text{if } 0 \leq z_i \leq x_i^* \\ R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+\left(z_i - \frac{1}{2}x_i^*\right) & \\ -t_B\left(1 - 2x_i^*\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(\frac{2x_i^* - x_i^*\left(2z_i - x_i^*\right)}{1 - z_i + 2x_i^*}\right), & \text{if } x_i^* < z_i \leq 1 \\ R_f - R_{CB} - t_B\left(x_i^* - z_i\right) - \lambda\xi R_{IB}^+\left(1 - \frac{1}{2x_i^*}\left(1 + x_i^* - z_i\right)^2\right) & \\ -t_B\left(1 - 2\left(1 + x_i^* - z_i\right)\frac{1}{t_B}\lambda\xi R_{IB}^+\right)\left(1 + x_i^* - z_i\right), & \text{if } 1 < z_i \leq 1 + x_i^*. \end{cases} \tag{A.56}$$

A solution to $\Gamma = 0$ exists at the limit as $z_i \to 0$ which is equal to the equilibrium without CBDC set out by Proposition 2 where the solution to $\Gamma = 0$ occurs if $R_{CB} = \underline{R}_{CB}$ where $\underline{R}_{CB} \equiv R_f - t_B\frac{3}{4N}$.In the case where $R_CB < \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently low that CBDC does not compete with bank deposits and the equilibrium without CBDC set out by Proposition 2 holds.

Next, consider how the function Γ changes with z_i

$$\frac{\partial \Gamma}{\partial z_i} = \begin{cases} t_B - \lambda\xi R_{IB}^+\frac{z_i}{x_i^*} + \left(\frac{2\lambda\xi R_{IB}^+\left(2x_i^* - z_i^2\left(2z_i + 3\right)\right) + t_B\left(2x_i^* + 2z_i + z_i^2\right)}{\left(1 + z_i\right)^2}\right) & \text{if } 0 \leq z_i \leq x_i^* \\ t_B - \lambda\xi R_{IB}^+ + 3x_i^{*2}\left(\frac{t_B - 2x_i^*\lambda\xi R_{IB}^+}{\left(1 - z_i + 2x_i^*\right)^2}\right) & \text{if } x_i^* < z_i \leq 1 \\ t_B - \lambda\xi R_{IB}^+\frac{1}{x_i^*}\left(1 + x_i^* - z_i\right) + t_B - 4\left(1 + x_i^* - z_i\right)\lambda\xi R_{IB}^+ & \text{if } 1 < z_i \leq 1 + x_i^*. \end{cases} \tag{A.57}$$

At the limit as $z_i \to 0$, $\frac{\partial \Gamma}{\partial z_i} > 0$ and from the implicit function theorem there exists a solution to $\Gamma = 0$ in the neighborhood of $z_i = 0^+$. In general, as long as $\lambda\xi R_{IB}^+ \leq t_B$, $\frac{\partial \Gamma}{\partial z_i} > 0$ for all $z_i \in [0, 1 + x_i^*]$.

Thus, if $\lambda\xi R_{IB}^+ \leq t_B$, a solution to $\Gamma = 0$ exits at the upper limit, as $z_i \to 1 + x_i^*$ and occurs if $R_{CB} = \bar{R}_{CB}$ were $\bar{R}_{CB} = R_f + t_B - \lambda\xi R_{IB}^+$. It follows that a solution to $\Gamma = 0$

exists for any $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$ and that the equilibrium z_i increases in R_{CB}. From equations (17) and (18) it follows that q_i is increasing in R_{CB} and q_{CB} is decreasing in R_{CB} over this range.

In the case where $R_{CB} > \bar{R}_{CB}$, the CBDC remuneration rate is sufficiently high that CBDC dominates bank deposits and banks obtain a zero market share.

Finally, for any $z_i > 1$, banks do not directly compete in the sense that all depositors that are indifferent between holding deposits at neighboring banks strictly prefer to hold CBDC to bank deposits. In this case, banks face direct competition only from CBDC. This occurs whenever $R_{CB} > R^*_{CB}$ where R^*_{CB} can be found as the limit of equation (A.57) as $z_i \to 1$

$$R^*_{CB} = R_f + t_B\left(1 - \frac{3}{4N}\right) - \lambda\xi R^+_{IB}\left(1 - \frac{1+N}{4N^2}\right). \tag{A.58}$$

Proof of Proposition 4

A long-run equilibrium with $R_{CB} > R_f - \frac{3}{2}t_B\frac{1}{N}$ can be summarized as the tuple (x^*_i, z_i) that solves equations (25) and (27).

Equation (27) can be written explicitly in terms of x^*_i and z_i as

$$\pi = \begin{cases} \bar{\pi} - (2x^*_i - z_i^2)\left(R_{CB} + t_B(x^*_i - z_i) - R_f + \lambda\xi R^+_{IB}\frac{1}{2x^*_i}z_i^2\right) & \text{if } 0 \leq z_i \leq x^*_i \\ \bar{\pi} - (2x^*_i - x^*_i(2z_i - x^*_i))\left(R_{CB} + t_B(x^*_i - z_i) - R_f + \lambda\xi R^+_{IB}\left(z_i - \frac{1}{2}x^*_i\right)\right) & \text{if } x^*_i < z_i \leq 1 \\ \bar{\pi} - (1 + x^*_i - z_i)^2\left(R_{CB} + t_B(x^*_i - z_i) - R_f + \lambda\xi R^+_{IB}\left(1 - \frac{1}{2x^*_i}(1 + x^*_i - z_i)^2\right)\right) & \text{if } 1 < z_i \leq 1 + \end{cases} \tag{A.59}$$

As long as $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ a solution to this system of equations exists at the limit of $z_i \to 0$ where $q_{CB} = 0$. This is simply the long-run no-CBDC equilibrium set out by Proposition 2 and occurs if $R_{CB} = \underline{R}_{CB}$ where $\underline{R}_{CB}$ is defined in Proposition 2.

In the case where $R_CB < \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently low that CBDC does not compete with bank deposits and the no-CBDC equilibrium set out by Proposition 2 holds.

At the limit as $z_i \to 0$, det $\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x^*_i} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x^*_i} \end{pmatrix} > 0$ for any $N \geq 2$. From the implicit function theorem, there exists a solution to $[\Gamma, \pi] = [0, 0]$ in the neighborhood of $z_i = 0^+$. In general, for there to exist a solution to this system of equations, we require

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} = \frac{\partial\Gamma}{\partial z_i}\frac{\partial\pi}{\partial x_i^*} - \frac{\partial\Gamma}{\partial x_i^*}\frac{\partial\pi}{\partial z_i} > 0. \tag{A.60}$$

Consider first the following derivative.

$$\frac{\partial\Gamma}{\partial z_i}+\frac{\partial\Gamma}{\partial x_i^*} = \begin{cases} -t_B\left(\frac{2-2x_i^*-z_i^2}{(1+z_i)^2}\right) - \lambda\xi R_{IB}^+\left(2\left(\frac{2-2x_i^*-z_i^2}{(1+z_i)^2}\right)+\left(1-\frac{1}{2}\frac{z_i}{x_i^*}\right)\frac{z_i}{x_i^*}\right) & \text{if } 0 \le z_i \le x_i^* \\ t_B\left(1-\frac{3x_i^{*2}+6x_i^*(1-z_i)+3(1-z_i)^2}{\left(1-z_i+2x_i^*\right)^2}\right)-\lambda\xi R_{IB}^+\left(\frac{1}{2}-\frac{2x_i^*\left(x_i^{*2}+7x_i^*(1-z_i)-4(1-z_i)^2\right)}{\left(1-z_i+2x_i^*\right)^2}\right) & \text{if } x_i^* < z_i \le 1 \\ -\lambda\xi R_{IB}^+\left(\left(2+\frac{1}{x_i^*}\right)(1+x_i^*-z_i)+\frac{1}{2}\left(\frac{1+x_i^*-z_i}{x_i^*}\right)^2\right) & \text{if } 1 < z_i \le 1 + \end{cases} \tag{A.61}$$

From equation (A.61), if $\lambda R_{LF}\xi < t_B$ then for any $x_i^* < \frac{1}{2N}$ and $N \ge 2$, it follows that

$$\frac{\partial\Gamma}{\partial z_i}+\frac{\partial\Gamma}{\partial x_i^*} < 0. \tag{A.62}$$

Given this,the following two inequalities must hold

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} < \left(-\frac{\partial\Gamma}{\partial x_i^*}\right)\left(\frac{\partial\pi}{\partial x_i^*}+\frac{\partial\pi}{\partial z_i}\right) \tag{A.63}$$

and

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} > \frac{\partial\Gamma}{\partial z_i}\left(\frac{\partial\pi}{\partial x_i^*}+\frac{\partial\pi}{\partial zi}\right) \tag{A.64}$$

It follows from this that a necessary and sufficient condition for the existence of a long-run equilibrium is that

$$\frac{\partial\pi}{\partial x_i^*}+\frac{\partial\pi}{\partial z_i} > 0 \tag{A.65}$$

where

$$\frac{\partial\pi}{\partial z_i}+\frac{\partial\pi}{\partial x_i^*} = \begin{cases} \left(\frac{2x_i^*-z_i^2}{1+z_i}\right)\left(2(1-z_i)t_B-\lambda\xi R_{IB}^+\left((1+z_i)\left(1-\frac{1}{2}\left(\frac{z_i}{x_i^*}\right)\right)\left(\frac{z_i}{x_i^*}\right)+4z_i(1-z_i)\right)\right) & \text{if } 0 \le z_i \\ \left(\frac{2x_i^*-x_i^*\left(2z_i-x_i^*\right)}{1-z_i+2x_i^*}\right)\left(2t_B(1-z_i)-\lambda\xi R_{IB}^+\left(\left(\frac{1}{2}+4x_i^*\right)(1-z_i)+x_i^*\right)\right) & \text{if } x_i^* < z_i \\ -\frac{1}{x_i^*}\lambda\xi R_{IB}^+(1+x_i^*-z_i)^3\left(1+\frac{1}{x_i^*}(x_i^*+z_i-1)\right) & \text{if } 1 < z_i \end{cases} \tag{A.66}$$

From equation (A.66) The condition set out by equation (A.65) clearly holds for $0 \leq z_i \leq x_i^*$ while clearly fails to hold for $1 < z_i \leq 1 + x_i^*$. In fact, there exists some point $z_i^{**} \in (x_i^*, 1)$ such that at $z_i = z_i^{**}$ the determinant is zero and negative for $z_i > z_i^{**}$. From equation (A.66) z_i^{**} can be written as

$$z_i^{**} \leq \left(\frac{2 - \frac{1}{t_B}\lambda R_{LF}\xi\left(\frac{1}{2} + 5x_i^*\right)}{2 - \frac{1}{t_B}\lambda R_{LF}\xi\left(\frac{1}{2} + 4x_i^*\right)} \right). \tag{A.67}$$

By combining equation (A.67) with equation (A.56), an upper-bound in terms of the CBDC remuneration rate, R_{CB}^{**} can be found, where

$$R_{CB}^{**} = R_f - t_B\left(x_i^* - z_i^{**}\right) - \lambda\xi R_{IB}^{+}\left(z_i^{**} - \frac{1}{2}x_i^*\right) - t_B\left(1 - 2x_i^*\frac{1}{t_B}\lambda\xi R_{IB}^{+}\right)\left(\frac{2x_i^* - x_i^*\left(2z_i^{**} - x_i^*\right)}{1 - z_i^{**} + 2x_i^*}\right). \tag{A.68}$$

It follows from this that given $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$ and $-\frac{1}{4}t_B \leq \bar{\pi} < 0$ there exists a long-run equilibrium with $q_i > 0$ and $q_{CB} > 0$ for any $R_{CB} \in [\underline{R}_{CB}, R_{CB}^{**}]$. In the case where $R_{CB} > R_{CB}^{**}$, banks do not enter the deposit market and CBDC dominates the market with $q_{CB} = 1$.

Proof of Proposition 5

From equation (11), in a symmetric equilibrium,

$$R_i = R_{CB} + t_B\left(x_i^* - z_i\right). \tag{A.69}$$

Given that $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$, any equilibrium with $q_i > 0$ features $R_{CB} > \underline{R}_{CB}$. Applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the short-run where x_i^* is fixed

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B\frac{\partial\Gamma/\partial R_{CB}}{\partial\Gamma/\partial z_i}. \tag{A.70}$$

From proposition 4, $\frac{\partial\Gamma}{\partial z_i} > 0$ and differentiating equation (25) with respect to R_{CB} yields $\frac{\partial\Gamma}{\partial R_{CB}} = -1$. Therefore, it follows that given $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$, whenever $q_i > 0$, $\frac{\partial R_i}{\partial R_{CB}} < 1$ and the pass-through of the CBDC rate to the deposit rate is imperfect.

Next consider the limit as R_{CB} approaches $\underline{R}_{CB}$ from above of the short-run pass through.

$$\lim_{R_{CB}\downarrow \underline{R}_{CB}} \left\{ \frac{\partial R_i}{\partial R_{CB}} \right\} = \frac{\left(2\frac{1}{t_B}\lambda\xi R_{IB}^{+}+1\right)2x_i^*}{1+\left(2\frac{1}{t_B}\lambda\xi R_{IB}^{+}+1\right)2x_i^*} > 0 \tag{A.71}$$

Thus, the short-run pass-through is positive (but less than 1) for any x_i^* for R_{CB} above but sufficiently close to $\underline{R}_{CB}$.

In the special case where $\lambda\xi R_{IB}^{+}=0$, from equation (A.57) note that

$$\frac{\partial\Gamma}{\partial z_i} = \begin{cases} t_B\left(1+\frac{2x_i^*+2z_i+z_i^2}{(1+z_i)^2}\right) & \text{if } 0\le z_i\le x_i^* \\ t_B\left(1+\frac{3x_i^{*2}}{\left(1-z_i+2x_i^*\right)^2}\right) & \text{if } x_i^*<z_i\le 1 \\ 2t_B & \text{if } 1<z_i\le 1+x_i^*. \end{cases} \tag{A.72}$$

and thus $\frac{\partial\Gamma}{\partial z_i} > t_B$. From equation (A.70) it follows that in the short-run $\frac{\partial R_i}{\partial R_{CB}} > 0$.

In the long-run, x_i^* is not fixed, and applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the long-run

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B\left(\frac{\partial x_i^*}{\partial R_{CB}} - \frac{\partial z_i}{\partial R_{CB}}\right) \tag{A.73}$$

where

$$\frac{\partial x_i^*}{\partial R_{CB}} - \frac{\partial z_i}{\partial R_{CB}} = \frac{\det\begin{pmatrix} \frac{\partial\Gamma}{\partial R_{CB}} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial R_{CB}} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} - \det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial R_{CB}} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial R_{CB}} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix}}. \tag{A.74}$$

From proposition 4 we showed that for all $R_{CB} < R_{CB}^{**}$ we have

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} > 0. \tag{A.75}$$

Thus the properties of the long-run pass through depend on

$$\det\begin{pmatrix} \frac{\partial\Gamma}{\partial R_{CB}} & \frac{\partial\Gamma}{\partial x_i^*} \\ \frac{\partial\pi}{\partial R_{CB}} & \frac{\partial\pi}{\partial x_i^*} \end{pmatrix} - \det\begin{pmatrix} \frac{\partial\Gamma}{\partial z_i} & \frac{\partial\Gamma}{\partial R_{CB}} \\ \frac{\partial\pi}{\partial z_i} & \frac{\partial\pi}{\partial R_{CB}} \end{pmatrix} = \left(\frac{\partial\pi}{\partial z_i} + \frac{\partial\pi}{\partial x_i^*}\right)\frac{\partial\Gamma}{\partial R_{CB}} - \left(\frac{\partial\Gamma}{\partial z_i} + \frac{\partial\Gamma}{\partial x_i^*}\right)\frac{\partial\pi}{\partial R_{CB}} \tag{A.76}$$

and thus

$$\frac{\partial R_i}{\partial R_{CB}} = 1 - \left(\frac{1 + q_i \left(-\frac{\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}}{\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}} \right)}{\frac{1}{t_B}\frac{\partial \Gamma}{\partial z_i} + \frac{1}{t_B}\frac{\partial \pi}{\partial z_i}\left(-\frac{\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}}{\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}} \right)} \right) < 1 \tag{A.77}$$

where we note that from proposition 4 we found that have shown that for all $R_{CB} < R_{CB}^{**}$ we have

$$\left(\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*} \right) > 0 \left(\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*} \right) < 0 and thus \left(-\frac{\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}}{\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}} \right) > 0 \tag{A.78}$$

Comparing equation (A.70) to equation (A.77), the long-run pass-through will be strictly less than the short-run pass-through if

$$‘q_i \frac{\partial \Gamma}{\partial z_i} > \frac{\partial \pi}{\partial z_i} \tag{A.79}$$

This holds for all $z_i \in [0, z_i^{**}]$.

Proof of Proposition 6

Applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the policy rate to the deposit rate in the short-run where x_i^* is fixed is

$$\frac{\partial R_i}{\partial R_f} = 1 + t_B \frac{\partial \Gamma / \partial R_f}{\partial \Gamma / \partial z_i}. \tag{A.80}$$

From proposition 3 as long as $\frac{1}{t_B}\lambda \xi R_{IB}^+ \leq 1$

$$\frac{\partial \Gamma}{\partial z_i} > 0 \tag{A.81}$$

and that from differentiating equation (34)

$$\frac{\partial \Gamma}{\partial R_f} = -\lambda \xi \left(\left[\frac{\partial q_i}{\partial R_i} \right]^{-1} \frac{\partial q_{CB}}{\partial R_i} q_i + q_{CB} \right) \tag{A.82}$$

which in the case where $0 < \frac{1}{t_B}\lambda \xi R_{IB}^+$ is non-zero and in general as $\frac{\partial R_i}{\partial R_f} \neq 1$.

In the long-run, x_i^* is not fixed, and applying the Implicit Function Theorem to equation (29) yields the following equation for the pass-through of the CBDC rate to the deposit rate in the long-run

$$\frac{\partial R_i}{\partial R_f} = 1 + t_B \left(\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} \right) \tag{A.83}$$

where

$$\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} = \frac{\det\begin{pmatrix} \frac{\partial \Gamma}{\partial R_f} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial R_f} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} - \det\begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial R_f} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial R_f} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix}} \tag{A.84}$$

From Proposition 4, given $\frac{1}{t_B}\lambda\xi R_{IB}^{+} \leq 1$, we showed that for all $R_{CB} < R_{CB}^{**}$ we have

$$\det\begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} > 0. \tag{A.85}$$

Thus the properties of the long-run pass through depend on

$$\det\begin{pmatrix} \frac{\partial \Gamma}{\partial R_f} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial R_f} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix} - \det\begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial R_f} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial R_f} \end{pmatrix} = \left(\frac{\partial \pi}{\partial z_i} + \frac{\partial \pi}{\partial x_i^*}\right)\frac{\partial \Gamma}{\partial R_f} - \left(\frac{\partial \Gamma}{\partial z_i} + \frac{\partial \Gamma}{\partial x_i^*}\right)\frac{\partial \pi}{\partial R_f} \tag{A.86}$$

with

$$\frac{\partial \Gamma}{\partial R_f} = \lambda\xi\left(\left[\frac{\partial q_i}{\partial R_i}\right]^{-1}\frac{\partial q_{CB}}{\partial R_i}q_B + q_{CB}\right) \tag{A.87}$$

and

$$\frac{\partial \pi}{\partial R_f} = -\lambda\xi q_{CB} q_i \tag{A.88}$$

where the derivative will again be nonzero if $0 < \frac{1}{t_B}\lambda\xi R_{IB}^{+}$.

Now note that equation (A.84) can be rewritten as

$$\frac{\partial x_i^*}{\partial R_f} - \frac{\partial z_i}{\partial R_f} = \frac{\frac{\partial \Gamma}{\partial R_f} - \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix}\frac{\partial \pi}{\partial R_f}}{\frac{\partial \Gamma}{\partial z_i} - \begin{pmatrix} \frac{\partial \Gamma}{\partial z_i} & \frac{\partial \Gamma}{\partial x_i^*} \\ \frac{\partial \pi}{\partial z_i} & \frac{\partial \pi}{\partial x_i^*} \end{pmatrix}\frac{\partial \pi}{\partial z_i}} \tag{A.89}$$

Thus a sufficient condition for the long-run pass-through to be less than the long-run pass-through is that

$$\frac{\partial \pi}{\partial R_f} < q_i\frac{\partial \Gamma}{\partial R_f} \tag{A.90}$$

which holds for all values of $z_i \in [0, z_i^{**}]$.