# Banking regulation and collateral screening in a model of information asymmetry*

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**Abstract**

This paper explores the impact of banking regulation on a competitive credit market with ex-ante asymmetric information and aggregate uncertainty. I construct a model where the government to impose a regulatory constraint that limits the losses banks make in the event of their default. I show that the addition of banking regulation results in three deviations from the standard theory. First, collateral is demanded of both high and low risk firms, even in the absence of asymmetric information. Second, if banking regulation is sufficiently strict, there may not exist an adverse selection problem. Third, a pooling Nash equilibrium can exist.

**Keywords:** Banking; Adverse Selection; Collateral; Banking regulation.

**JEL codes:** D86; G21; G28

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# 1 Introduction

Following the Financial Crisis of 2007-2008, regulators and policy makers have increased their focus on ensuring stability in the banking sector. One key tool at the regulator's disposal is stress testing, which has become more widely used by regulators since the financial crisis. The empirical evidence suggests that the use of stress tests by the Federal Reserve, and other banking regulators can have a negative impact on the lending conditions facing firms. For example, Acharya et al. (2018), focusing on lending to large firms in the US, find that stress tested banks tend to reduce the quantity of loans supplied to firms and tend to increase borrowing rates. Similarly, Cortés et al. (2018) complement this by documenting similar negative effects of stress testing on small business loans. Specifically, they provide evidence that stress tests conducted under the Comprehensive Capital Analysis and Review (CCAR) led to a decrease in affected banks' credit supply to small business. An overview of the recent history of stress testing in the financial sector can be found in Dent et al. (2016).

This paper seeks to contribute to the analysis of implementing more stringent banking regulation such as regulatory stress tests by offering a theoretical model that assesses the interaction between banking regulation and loan terms in a traditional model of loan contracts. I propose an adverse selection credit market model with aggregate uncertainty where firms have private information regarding the riskiness of their project. Firms operate a decreasing returns to scale production technology and fund their project by obtaining a bank loan. Banks have limited liability and are able to default on insured depositors. As banks do not internalize the social cost of default, they lend more than is socially optimal. This inefficiency can be corrected through banking regulation. I abstract from the implementation of banking regulation and assume that the government can impose a constraint on the level of systemic risk directly through a limit on the losses banks make conditional on their default. The regulatory constraint in my model can be interpreted as a condition that banks must be able to meet some minimum threshold following a regulatory stress-test.

The model allows banks flexibility in satisfying the regulatory requirements, they may reduce the size of the loans they offer, increase interest rates or reduce their loss given default through demanding more collateral from borrowers. Thus collateral now has two roles; as in traditional adverse selection models collateral can be used as a screening device but in addition it may also help the bank satisfy regulatory requirements by reducing the loss given default of a loan.

This paper highlights a novel channel through which banking regulation may distort equilibrium lending; the interaction of incomplete information and banking regulation. The paper has three main theoretical results which are contrary to standard adverse selection models. First, I set out conditions under which collateral is demanded of both high and low risk firms, even in the absence of asymmetric information. Second, if banking regulation is sufficiently strict, there may not exist an adverse selection problem, as the difference in loan size at the full information contract is sufficient to separate the two firm types. Finally, I show that if there is insufficient pledgable collateral, the two firm types can receive the same contract in equilibrium, that is to say a pooling Nash equilibrium can exist.

This paper is directly related to the literature on adverse selection in credit markets. Papers that focus on the use of collateral as a screening device in credit markets featuring adverse selection include papers such as Stiglitz and Weiss (1981), Bester (1985a) and Lacker (2001). The addition of variable loan size to signaling models has also been studied previously by Bester (1985b) and Milde and Riley (1988). The existence of credit rationing equilibria, though not pooling equilibria, when there is insufficient collateral was raised by Besanko and Thakor (1987) and Clemenz (1993). This paper also complements the empirical literature on the impact of regulatory stress testing on bank lending such as Acharya et al. (2018) and Cortés et al. (2018) by providing a theoretical mechanism through which more stringent regulation can impact lending outcomes. A related paper is Estrella (2004) who considers the impact of regulatory restrictions on a bank's value at risk on the probability of bank failure in a dynamic setting. His emphasis is on the portfolio choice of a bank choosing between safe and risky assets. In this paper, I emphasize the impact of banking regulation on the terms of loan contracts.

The paper is organized as follows. Section 2 presents the model. Section 3 derives the main results on the loan contracts in a competitive equilibrium. Section 4 discusses the optimal policy decision of the regulator and section 5 concludes.

# 2 Model

## 2.1 Firms and Technology

Consider a credit market with a continuum of risk-neutral firms. Each firm has access to a project such that an investment of $k$ will yield a cash-flow of $\varphi k^\alpha$ if successful and zero if it fails. The curvature parameter $\alpha \in (0, 1)$ is such that the cash-flow of a successful project features decreasing returns to scale and the productivity parameter $\varphi > 0$ is common to all firms. There exists two types of firms indexed by $i \in \{L, H\}$ that differ in the success probability of their projects. The probability a firm's project is successful is denoted by $p_i$ with $0 < p_H < p_L < 1$ implying that $H$-type firms are high risk and feature a lower probability of success than low risk ($L$-type) firms. The fraction of firms of type $i$ is denoted by $\mu_i \in (0,1)$ with $\sum_i \mu_i = 1$. The distribution of firms in the economy is public information.

Firms receive a known end-of-period endowment $W > 0$. The timing of the endowment means firms cannot use the endowment to invest in a project but instead must obtain a loan. Banks make loan offers to firms that consist of a loan size $k_i \geq 0$, an interest rate $R_i \geq 0$ and an amount of pledged collateral $C_i \in [0, W]$. The collateral is the amount of the firm's endowment sacrificed by the firm if it defaults on the loan payment $R_i k_i$.

In addition to the firm type, the probability of a project being successful also depends on the realization of an aggregate state $z \in \{z_B, z_G\}$. The aggregate state $z_G$ occurs with probability $q \in (0,1)$ and $z_B$ with probability $1-q$. I denote the probability of firm $i$'s project being successful conditional on $z$ as $p_i(z)$. The probability of a project being successful is higher in the 'good' state ($z_G$) than in the 'bad' state ($z_b$) for both firm types such that

$$0 < p_i(z_B) < p_i(z_G) < 1 \quad \forall i \in \{L, H\}. \tag{1}$$

It follows from above that the expected probability of firm $i$'s project being successful can be written as follows

$$p_i = q p_i(z_G) + (1-q) p_i(z_B). \tag{2}$$

To simplify the analysis, I assume that the ratio of success probabilities conditional on $z_G$ and $z_B$ is the same across firm types such that

$$\frac{p_i(z_B)}{p_i(z_G)} = \xi \quad \forall i \in \{L, H\}, \tag{3}$$

where it follows from equation (1) that $\xi \in (0,1)$. It is assumed that the aggregate state is not known at the beginning of the period and thus loan contracts made between the bank and the firm cannot be made contingent on the realization of $z$.

The expected utility firm $i$ receives from a loan contract $(k_i, R_i, C_i)$ is

$$U_i(k_i, R_i, C_i) = p_i [\varphi k_i^\alpha - R_i k_i] - (1-p_i) C_i + W. \tag{4}$$

To simplify the later analysis, I define the payoff the firm receives from a successful project as $\pi(k, R) = \varphi k_i^\alpha - R_i k_i$. The firm's marginal rate of substitution between the payoff from a successful project $\pi$ and the collateral pledged is

$$\left.\frac{d\pi}{dC}\right|_{U_i} = \frac{1-p_i}{p_i}. \tag{5}$$

As the marginal cost of collateral is lower for low-risk firms than high-risk firms, banks will be able to use collateral to screen between unobservable firm types.

## 2.2 Banking Sector and Regulation

There exists a large number of risk-neutral banks that fund loan contracts through deposits. Deposits are fully insured by the government and depositors earn a risk-free return which for simplicity is normalized to 1. Banks have limited liability and default on depositors if the proceeds from lending are less than what banks owe their depositors. If banks default, depositors are compensated by the government. The government does not charge banks an insurance premium but instead funds the deposit insurance by a lump-sum tax on firm profits.[1] A contract $(k_i, R_i, C_i)$ that is accepted by firms of type-$i$ will earn expected profit per firm before fixed costs of

$$\begin{aligned} V_i(k_i, R_i, C_i) &= q\left(p_i(z_G) R_i k_i + \delta(1 - p_i(z_G)) C_i - k_i\right) \\ &\quad + (1-q)\max\left\{p_i(z_B) R_i k_i + \delta(1 - p_i(z_B)) C_i - k_i, 0\right\}, \end{aligned} \tag{6}$$

where $\delta \in (0,1)$ is a discount parameter on collateral implying that the use of collateral in a loan contract is costly. The parameter $\delta$ can be thought of as a reduced form way of capturing the agency and liquidation costs of transferring collateral to the banks. Competition in the banking sector will drive profits towards zero but I assume that banks do not default following the realization of the good aggregate state $z_G$. However, due to limited liability banks may default following the realization of $z_B$. I further assume that if a loan contract is accepted by at least one firm, it is accepted by a representative mass of firms, such that the law of large numbers holds. This assumption ensures that bank default would only occur due to aggregate risk and not due to the idiosyncratic firm risk.

If banks default following the realization of $z_B$, the government levies a lump-sum tax $\tau$ on firms in order to make depositors whole again. Due to the presence of this deposit insurance banks do not fully endogenize the cost of default. In order to address the resulting externality I assume that the government can impose the following restriction on the riskiness of bank borrowing

$$p_i(z_B) R_i k_i + \delta(1 - p_i(z_B)) C_i \geq \gamma k_i, \tag{7}$$

where $\gamma \in (0,1)$ is a parameter chosen by the government that determines how strict the regulatory regime is. This regulatory constraint (7) is equivalent to stating that the bank only defaults on a fraction $(1-\gamma)$ of deposits if the bad aggregate state $z_B$ is realized.

The regulatory constraint set out by equation (7) is equivalent to requiring banks to pass a regulatory stress-test, with the parameter $\gamma$ capturing how strict this stress-test is. Recent

[1] In this paper I do not consider the optimality of deposit insurance or who should pay for it. While this is an obvious limitation, the key message of this paper should remain unchanged so long as banks do not fully endogenize the cost of default and thus there is a role for banking regulation to address this externality.

empirical studies such as Acharya et al. (2018) and Cortés et al. (2018) find that banks that fail stress tests adjust their lending in response. Furthermore, if the bank is publicly traded on a stock market, fully disclosing stress-test results as in the US is likely to create a strong incentive for the management of the bank to ensure the regulatory stress test is passed.

I focus on subgame perfect Nash equilibria of the following three-stage variant of the Rothschild and Stiglitz (1976) screening game. In the first stage, the government chooses the regulatory parameter $\gamma$. In the second stage, banks offer a single loan contract to firms. In the third stage, firms choose a single loan contract among those on offer. In this paper I discuss the existence of both separating and pooling Nash equilibria. A Nash equilibrium is a set of contracts $\{(k_i, R_i, C_i)\}_{i\in\{L,H\}}$ such that i) each contract earns non-negative profits for the bank, ii) the regulatory constraint defined by equation (7) is satisfied and iii) there exists no other set of contracts which, when offered in addition to the existing set of contracts which all earn non-negative profits with at least one offering strictly positive profits. I will consider both separating and pooling equilibria, a Nash equilibrium is separating if $(k_L, R_L, C_L) \neq (k_H, R_H, C_H)$ and is a pooling equilibrium otherwise.

# 3 Competitive Equilibrium

## 3.1 Equilibrium with identical customers

I first examine the case where there is a single type of firm and as a result banks have perfect information regarding the quality of the firms they are lending to. For an economy that consists of a single firm of type $i$, a competitive equilibrium must feature a contract that satisfies the regulatory constraint and ensures that banks make non-negative profits. A preliminary step in describing the equilibrium is to characterize the set of contracts that satisfy these two constraints in $(\pi, C)$-space. I will refer to this set as the *feasible set.*

First, I consider the case where the regulatory constraint is slack. In this case, the only constraint on the set of feasible contracts offered is that banks must make non-zero profits. Then the maximum payoff $\pi$ that can be promised to firm $i$ for a given pledge in collateral $C$ that satisfies the following constraint binds

$$p_i(z_G) R_i k_i + \delta (1 - p_i(z_G)) C_i - k_i \geq 0, \tag{8}$$

which is simply the constraint that banks make non-negative profits if $z_G$ is realized given that they default if $z_B$ is realized. If banks did not default following $z_B$ then that would imply that they would make non-negative profits following the realization of $z_B$, but then a competing bank could offer a lower interest rate $R_i$ and thus a higher $\pi$ such that equation (8) is satisfied while defaulting on any losses they make in $z_B$. By maximizing equation (4) subject to equation (8) we can find the payoff maximizing loan size as

$$\bar{k}_i = (\alpha\varphi p_i(z_G))^{\frac{1}{1-\alpha}}. \tag{9}$$

Assuming (8) binds, this can be substituted this into equation (7) to yield the following inequality

$$C_i \geq \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)k_i. \tag{10}$$

An immediate corollary of equation (10) is that the regulatory constraint will be satisfied for all $C_i \geq 0$ whenever $\gamma \leq \xi$. This establishes a lower-bound for $\gamma$ below which the regulatory constraint will have no effect on the equilibrium. Furthermore, even if regulation is sufficiently high such that $\gamma > \xi$, if the collateral specified in the contract is sufficiently high, then the regulatory constraint will not bind. Specifically, there exists a cutoff level of collateral $\bar{C}_i$ such that for any $C_i > \bar{C}_i$ there is sufficient collateral to ensure that the regulatory constraint will be slack when banks offer a loan size of $\bar{k}_i$. This cutoff is defined by the following equation

$$\bar{C}_i = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\bar{k}_i. \tag{11}$$

Next, I consider the possibility that the regulatory constraint binds but where equation (8) is slack. By maximizing equation (4) subject to equation(7) we can find the payoff maximizing loan size as

$$\underline{k}_i = (\alpha\varphi p_i(z_B))^{\frac{1}{1-\alpha}}. \tag{12}$$

It follows from the above discussion that if $\gamma > \xi$ and collateral is sufficiently low, banks offer a loan size equal to $\underline{k}_i$ and make strictly positive profits to ensure that the regulatory constraint binds. Specifically, there exists a cutoff level of collateral $\underline{C}_i$ such that for any $C_i < \underline{C}_i$ there is insufficient collateral available for competition to drive bank profits to zero while ensuring that the regulatory constraint will be satisfied. This cutoff is defined by the following equation

$$\underline{C}_i = \frac{1}{\delta}\left(\frac{\gamma-\xi}{1-\xi}\right)\underline{k}_i. \tag{13}$$

To understand this somewhat puzzling case, note that there are three dimensions along which contracts can be adjusted in order to meet the regulatory requirement; the loan size, the quantity of collateral and the interest rate. When the quantity of collateral is sufficiently low, firms would trade off a higher interest rate for a larger loan and thus allowing banks to make positive profits on an accepted contract.

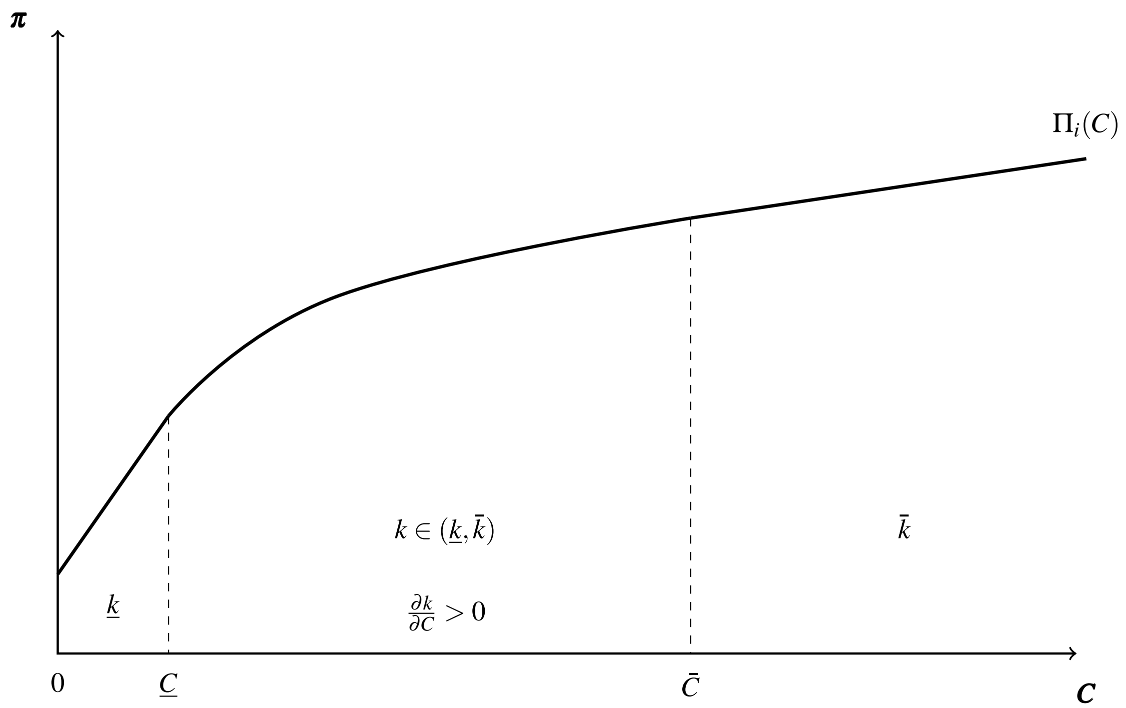


Figure 1: Set of feasible contracts for firm $i$

By assumption $p_i(z_B) < p_i(z_G)$, thus it follows that $\underline{k}_i < \bar{k}_i$ and $\underline{C}_i < \bar{C}_i$. Thus when $\gamma > \xi$ there exists a range of collateral $C \in \left(\underline{C}_i, \bar{C}_i\right)$, where in order to maximize the payoff to firms, both equations (7) and (8) bind. The loan size can be found by rearranging the two binding constraints to get $k_i = \delta\left(\frac{1-\xi}{\gamma-\xi}\right)C_i$ such that this loan size is increasing in collateral and lies between the upper- and lower-bounds for the loan size$\underline{k}_i$ and $\bar{k}_i$.

The boundary of the feasible set of contracts on the interior of $\mathbb{R}_+^2$ can be summarized by the function $\Pi_i(C)$ that denotes the maximum payoff that can be offered to firms as a function of collateral . This function is characterized as follows

$$\Pi_i(C) = \begin{cases} \varphi\underline{k}^\alpha - \gamma\left(\frac{1}{p_i(z_B)}\right)\underline{k} + \delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)C & \text{when } C < \underline{C}_i \text{ and } \gamma > \xi \\ \varphi\left[\delta\left(\frac{1-\xi}{\gamma-\xi}\right)C\right]^\alpha - \delta\left[\frac{1}{p_i(z_G)}\left(\frac{1-\gamma}{\gamma-\xi}\right)+1\right]C & \text{when } \underline{C}_i \le C \le \bar{C}_i \text{ and } \gamma > \xi \\ \varphi\bar{k}^\alpha - \frac{1}{p_i(z_G)}\bar{k} + \delta\left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)C & \text{otherwise.} \end{cases} \tag{14}$$

The set of feasible contracts is illustrated graphically for the case where $\gamma > \xi$ in figure 1.

Equation (14) denotes the largest possible payoff $\pi$ that can be offered to firm $i$ conditional on a collateral level $C$. As firm utility is strictly increasing in $\pi$, the competitive equilibrium is simply the point on equation (14) that maximizes firm utility.

In the case where $\gamma \le \xi$, the regulatory constraint is always slack and the competitive equilibrium with identical customers is analogous to that in the Rothschild and Stiglitz (1976) case. The function $\Pi_i(C)$ is linear in $C$ and with a gradient of $\delta\left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)$, which

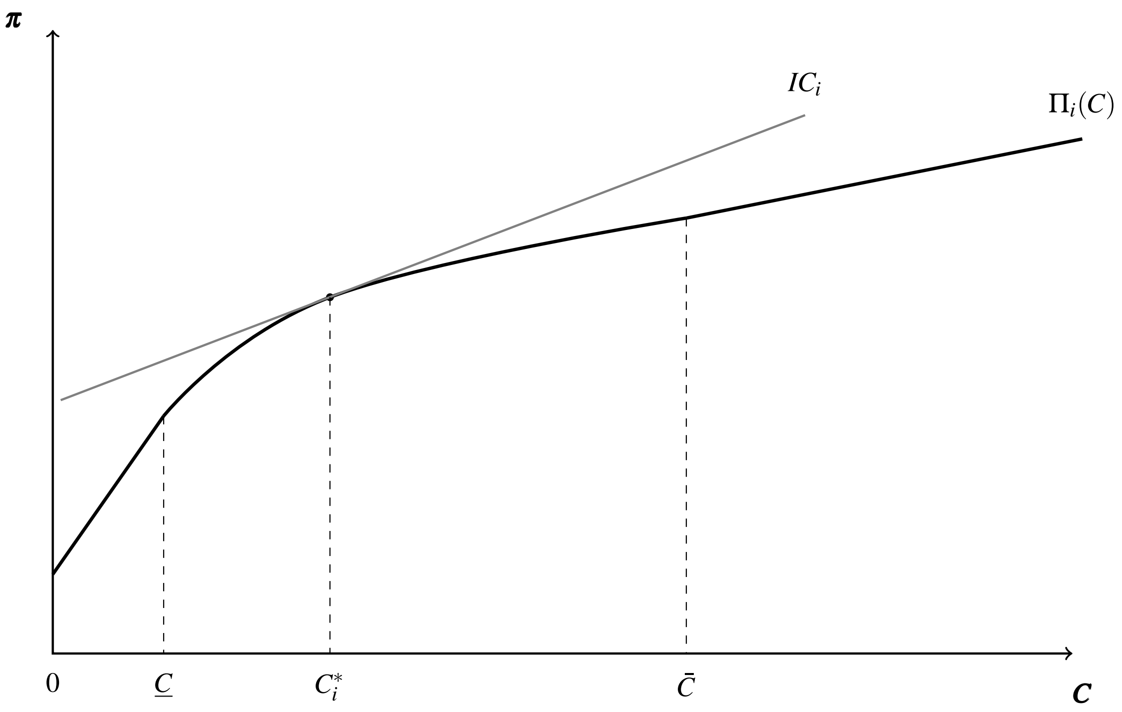


Figure 2: Competitive equilibrium with single firm type

is strictly lower than the gradient of firm $i$'s indifference curves set out in equation (5) and thus the equilibrium contract will feature $C_i = 0$.

When $\gamma > \xi$, equation (14) is weakly concave in $C$ and the gradient at $C_i = 0$ is $\delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)$. A sufficient condition for the competitive equilibrium to feature $C_i > 0$ is that $\delta\left(\frac{1-p_i(z_B)}{p_i(z_B)}\right)$ is strictly larger than the marginal rate of substitution of firm $i$, which holds whenever

$$\delta > \left(\frac{p_i\left(z_B\right)}{1-p_i\left(z_B\right)}\right)\left(\frac{1-p_i}{p_i}\right). \tag{15}$$

If the cost of collateral is sufficiently low, then the equilibrium will feature positive collateral; the inequality set out in equation 15 will be satisfied as $\delta \to 1$. If equation 15 is satisfied, the competitive equilibrium will be the point of tangency between equation (14) and firm $i$'s indifference curves. This is illustrated graphically in figure 2 where $C_i^*$ denotes the collateral level at the competitive equilibrium.

This result is set out more formally in the following proposition.

**Proposition 1.** *If $\gamma > \xi$, $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$ and $W$ is sufficiently high, the competitive equilibrium contract for an economy featuring a single type of firm will feature strictly positive collateral $C_i^* > 0$ and a loan size $k_i^* < \bar{k}$ where $C_i^*$ and $k_i^*$ are given by the* *following equations*

$$k_i^* = \left( \frac{\alpha p_i \varphi}{[q + (1-q)\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)} \right)^{\frac{1}{1-\alpha}}, \tag{16}$$

$$C_i^* = \frac{1}{\delta}\left(\frac{\gamma - \xi}{1-\xi}\right) k_i^*, \tag{17}$$

*and the payoff that the firm receives from the competitive equilibrium contract is*

$$\pi_i^* = \left(\frac{1-\alpha}{\alpha}\right)\left[(q + (1-q)\gamma) + \left(\frac{1-\eta}{\eta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)\right] k_i^* \tag{18}$$

*Proof.* See Appendix. □

In words, proposition 1 states that if the regulatory constraint is sufficiently strict and the cost of pledging collateral is sufficiently low, the competitive equilibrium will feature positive collateral even in the case of a single firm type. The reason for this is that pledging collateral, while costly, increases the loan size that firms are able to receive. Another key property of the equilibrium is that the loan size $k_i^*$ is decreasing in $\gamma$ and thus the government is able to affect the size of firm loans through the regulatory constraint.

In order to focus on the novel aspects of this model, from now on I assume first that $\gamma > \xi$ such that the regulatory constraint is a relevant consideration for agents and second that equation (15) is satisfied so that firms are willing to use collateral to ensure that their credit contract satisfies any regulatory constraint.

## 3.2 Separating equilibrium with asymmetric information

I now return to the case of asymmetric information where there are two firm types $i \in \{H, L\}$ and a firm's type is known only to itself. Banks are unable to condition contracts on the firm type. Instead, in a separating equilibrium, there are two distinct contracts $\{(k_i, R_i, C_i)\}_{i\in\{L,H\}}$ chosen by firms in equilibrium, with each contract being chosen by a single firm type. In order for this to occur, firms must self-select into the contract intended for them and thus the following two incentive compatibility constraints must hold in equilibrium

$$p_H\left[\varphi k_H^\alpha - R_H k_H\right] - (1-p_H)\,C_H \geq p_H\left[\varphi k_L^\alpha - R_L k_L\right] - (1-p_H)\,C_L, \tag{19}$$

and

$$p_L\left[\varphi k_L^\alpha - R_L k_L\right] - (1-p_L)\,C_L \geq p_L\left[\varphi k_H^\alpha - R_H k_H\right] - (1-p_L)\,C_H. \tag{20}$$

To see which of these constraints is likely to bind, first note that if there is sufficient collateral the separating contract will lie on the boundary of the feasible set. Furthermore, for any loan that is not fully collateralized, $\Pi_L(C) > \Pi_H(C)$ and thus at a given $C$ the separating payoff available to low-risk firms will be higher than that of high-risk firms. This can be shown by noting that if the amount of collateral is held fixed, low-risk firms can receive the same loan size as high-risk firms at a lower interest rate without violating either the zero profit condition or the regulatory constraint. Thus $\Pi_H$ must lie on the interior of the feasible contract set for $L$-type firms. It follows immediately from this that the relevant incentive compatibility constraint to consider is equation (19). Low-risk firms will never prefer a contract intended for high-risk firms and equation (20) will always be satisfied in equilibrium.

In contrast to a more standard adverse selection model, equation (19) may not always bind. To see this, consider the following equation which can be found by substituting in the full information contracts $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ into equation (19) and rearranging

$$\Gamma(\gamma) = (1-\alpha)\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{\alpha}{1-\alpha}} + \alpha\left(\frac{(q+(1-q)\gamma)+\left[\left(\frac{1-\delta}{\delta}\right)(1-p_L)+\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\right]\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right) - 1. \tag{21}$$

When $\Gamma(\gamma) \geq 0$, the incentive compatibility constraint is slack and the contracts $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ constitute the Nash equilibrium with imperfect information. On the other hand, if $\Gamma(\gamma) < 0$, the incentive compatibility constraint will bind and low-risk firms must pledge a higher quantity of collateral in order to separate from high-risk firms.

The limit of $\Gamma(\gamma)$ as $\gamma \to \xi$ is negative, thus if the regulatory constraint was removed, the incentive compatibility constraint would always bind. In this case, the full information contracts do not require any collateral to be pledged and, as $\Pi_L(0) > \Pi_H(0)$, it follows that equation (19) will bind. The limit of $\Gamma(\gamma)$ as $\gamma \to 1$ is

$$\lim_{\gamma\to 1}\Gamma(\gamma) = (1-\alpha)\left[\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1+\left(\frac{1-\delta}{\delta}\right)(1-p_L)}{1+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right)^{\frac{\alpha}{1-\alpha}} - 1\right] + \alpha\left(\frac{\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)}{1+\left(\frac{1-\delta}{\delta}\right)(1-p_L)}\right). \tag{22}$$

The sign of equation (22) depends on the parameters of the model but is positive as $\alpha \to 1$ indicating that as the returns of the firm projects increases, a sufficiently strict regulatory constraint will result in the incentive compatibility constraint being slack. This occurs because as $\gamma$ increases, $\pi$ falls and $C$ increases to ensure that the regulatory constraint is met. As the marginal rates of substitution between $\pi$ and $C$ differ across firms, at high levels of $\gamma$, firms may separate purely based on their optimal trade-off between $\pi$ and $C$. While this means that sufficiently strict banking regulation may resolve the adverse selection problem, it may not do so in a socially optimal way due to collateral being costly. The proposition below summarizes this discussion more formally.

**Proposition 2.** *If $\gamma \geq \xi$, $\delta \geq \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ and $W \geq C_L^*$, a necessary and sufficient condition for the incentive compatibility constraint of high-risk firms to bind for any value of $\gamma \in [\xi, 1)$ is that $\lim_{\gamma \to 1} \Gamma(\gamma) < 0$. Otherwise there exists a cutoff $\gamma^* \in [\xi, 1)$ such that for any $\gamma > \gamma^*$ the incentive compatibility constraints will not bind and the contracts $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$ constitute the Nash equilibrium.*

*Proof.* See Appendix. □

Should equation (19) bind the separating contracts can then be illustrated graphically. In the separating equilibrium high-risk firms receive the same contract as they would in a single-type equilibrium and that the separating contract for low-risk firms, can be found as the point on the frontier $\Pi_L(C)$ at which high-risk firms are indifferent between this contract and their separating contract $(k_H^*, R_H^*, C_H^*)$. I denote the separating contract offered to low-risk firms by $\left(\hat{k}_L, \hat{R}_L, \hat{C}_L\right)$. As this contract must lie on the boundary of the feasible set, it follows from rearranging equation (19) that $C_H^*$ and $\hat{C}_L$ have the following relationship

$$\Pi_L\left(\hat{C}_L\right) = \Pi_H\left(C_H^*\right) + \left(\frac{1-p_H}{p_H}\right)\left(\hat{C}_L - C_H^*\right). \tag{23}$$

An example of a separating contract is illustrated in figure 3.

For brevity, I omit the precise contract terms for low-risk firms in the case where equation (19) binds as their formulation depends on whether $\hat{C}_L$ is larger than $\bar{C}_L$ or not. However, from the properties of the boundary of the feasible set, $\Pi_i$, set out earlier, it follows that the separating contract will feature a strictly larger loan size relative to the full-information contract, that is $\hat{k}_L > k_L^*$.

As pointed out by Rothschild and Stiglitz (1976) and Wilson (1977), a Nash equilibrium is not guaranteed to exist in an economy that features asymmetric information. This occurs when a pooling contract Pareto dominates separating contracts and thus would be preferred to the separating contract by both types of firms. Due to the curvature of the production function, the precise conditions required for the existence of a separating equilibrium cannot be found in closed form. However, even in the case where a Nash

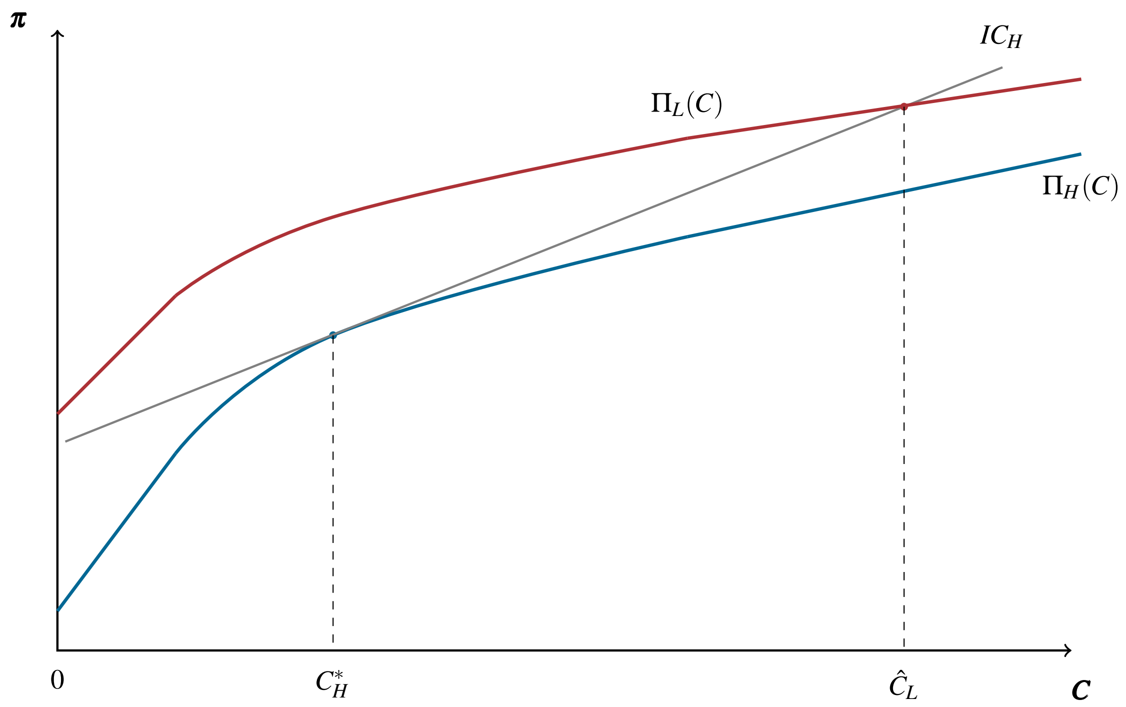


Figure 3: Asymmetric Information Contracts

equilibrium does not exist, the separating equilibrium discussed in this section will exist as a Riley reactive equilibrium as set out in Riley (1979). Similarly, a pooling equilibrium would exist as a Wilson anticipatory equilibrium as in Wilson (1977).

### 3.3 Equilibrium when the wealth constraint binds

The analysis of the previous section assumed that available collateral $W$ was sufficient so that the required collateral for separation could be supplied. I now discuss the equilibrium contracts under asymmetric information in the case where $W$ is sufficiently low that the separating contract discussed earlier cannot be implemented.

First, consider the case where $W$ is sufficiently large that $C_H^* < W$ but not so large that the low-risk firms can provide the level of collateral required for screening and thus $\hat{C}_L > W$. Then incentive compatibility requires that the $k_L$ and $R_L$ offered to the low-risk firm are such that the payoff low-risk firms receive is

$$\tilde{\pi}_L = \Pi_H\left(C_H^*\right) + \left(\frac{1-p_H}{p_H}\right)\left(C_H^* - W\right). \tag{24}$$

In a separating equilibrium when the wealth constraint binds, the low-risk firm's contract lies off the boundary of the feasible set of contracts and thus $\tilde{\pi}_L < \Pi_L\left(W\right)$. The firm is indifferent between any pair of contract terms $(k_L, R_L)$ which yields the payoff $\tilde{\pi}_L$ specified above and banks will choose the combination of $k_L$ and $R_L$ that maximizes their profit subject to the regulatory constraint and supplying the firm a payoff of $\tilde{\pi}_L$. This possibility is illustrated in figure 4a.

When the upper-bound on collateral binds there is the possibility that a pooling Nash Equilibrium exists. In a pooling equilibrium both high- and low-risk firms accept the same contract $(k_P, R_P, C_P)$. The pooling contract can be found in an analogous way to the equilibrium contract with a single firm type where the single firm type is composed of both high-risk and low-risk firms. The expected probability of success for this composition conditional on aggregate state $z$ is simply a weighted average of the success probabilities for high- and low-risk firms, weighted by the proportion of that firm type in the economy

$$p_P(z_j) = \mu_H p_H(z_j) + \mu_L p_L(z_j) \quad \forall j \in \{G, B\}.$$

Similarly, the unconditional expected probability of success is also a weighted average of the success probabilities of the high- and low-risk firms

$$p_P = \mu_H p_H + \mu_L p_L.$$

It follows that the boundary of the set of feasible pooling contracts, $\Pi_P(C)$, is simply $\Pi_i(C)$ for the composite type $P$.

For a pooling contract $(k_P, R_P, C_P)$ to exist as a Nash Equilibrium, there must exist no deviating contract that would satisfy equations (7) and (8) which would resulting in the pooling contract becoming either unprofitable or violate the regulatory constraint. That is, there cannot exist a cream-skimming contract that will attract only low-risk firms.

A necessary condition for the existence of a pooling contract is for both high- and low-risk firms to prefer the pooling contract to the best separating contract available, otherwise firms would choose a separating contract over the pooling contract. Similarly, any pooling contract must lie on the boundary $\Pi_P$ of pooling contracts, otherwise a better pooling contract could be found that would be preferred by at least one firm type.

A further necessary condition is that the upper-bound on collateral binds at the pooling contract such that $C_P = W$. In this case, no contract with higher collateral can be offered to low-risk firms and thus there exists no cream-skimming contract. A sufficient condition for the existence of a pooling equilibrium is that $C_H^* \leq W$ such that the separating contract for high-risk firms features a (weakly) binding wealth constraint. Then the pooling contract will lie strictly above the high-risk separating contract and no cream-skimming deviation exists. An example of a pooling contract existing as a Nash equilibrium is illustrated in figure 4b. The precise terms of the pooling contract, as in the separating case, depend on where the boundary of feasible contracts intersects $C_P = W$. A sufficient condition for the existence of a pooling contract is $W \leq C_H^*$. This is summarized in the

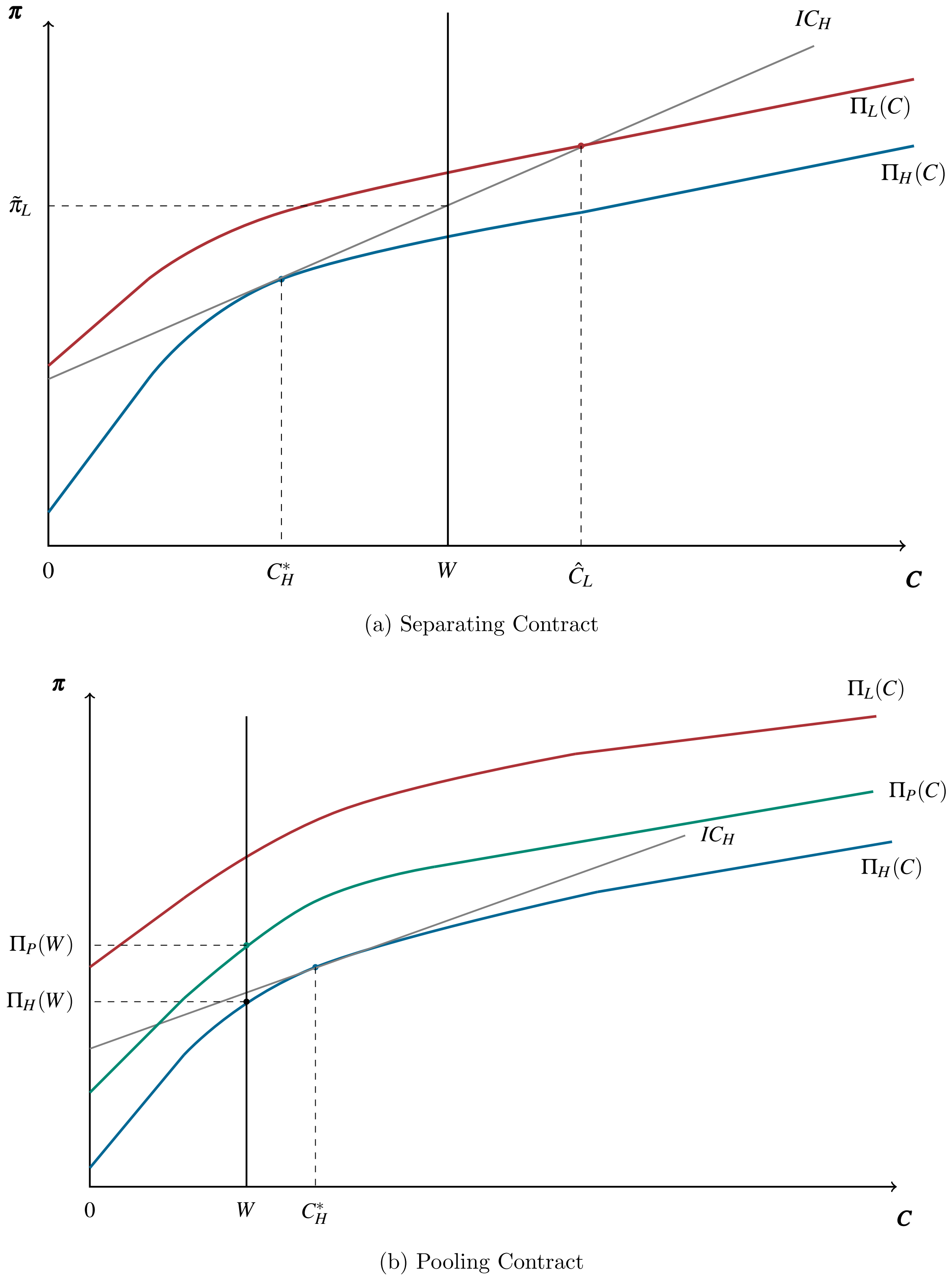


Figure 4: Contracts with binding wealth constraint

proposition below.

**Proposition 3.** *If $\gamma > \xi$, $\delta > \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ and $W \leq C_H^*$ then a Nash equilibrium will consist of a single pooling contract $(k_P, R_P, C_P)$ offered to both firms and where $C_P = W$ and $\pi = \Pi_P(W)$.*

*Proof.* It follows from proposition 1, that if $\gamma > \xi$ and $\delta > \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ that the high-risk firm's collateral in equilibrium will ideally be$C_H^* > 0$ and is defined by equation (17). Furthermore, it follows immediately from equation (14) that $\Pi_P(C) > \Pi_H(C)\ \forall C \leq C_H^*$. Thus if $W \leq C_H^*$ then $\Pi_P(W) > \Pi_H(W)$ and high-risk firms prefer a pooling contract that lies on the frontier $\Pi_P(W)$ over the best possible separating contract. It follows from the relative slope of the firm indifference curves that the only possible separating contract must offer low-risk firms both a higher payoff and higher collateral, but this would violate the restriction that $C_L \leq W$. □

In the classic Rothschild and Stiglitz (1976) screening game, a cream-skimming contract will always exist so long as $W > 0$. This is because no collateral is pledged by the pooling contract and thus there will always exist a deviating contract that features higher collateral and a higher payoff that would allow low-risk firms to separate from high-risk firms. The introduction of a regulatory constraint in my model means that collateral may be non-zero in the pooling contract and thus $W > 0$ is no longer a sufficient condition to ensure that a cream-skimming deviation exists.

# 4 Optimal Policy

## 4.1 Overview

Until this point, the regulatory parameter, $\gamma$, was taken as given. In this section, I consider the optimal policy decision of the government that can only affect the economy through the regulatory constraint described in equation (7).

For the sake of both brevity and simplicity, I restrict this section to the discussion of the separating equilibrium. In particular, I assume that i) $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$ so that both firms will pledge collateral; ii) there is sufficient wealth that any contract can be implemented; iii) parameters are such that a Nash equilibrium always exists; and iv) $\lim_{\gamma\to 1}\Gamma(\gamma) < 0$ such that the incentive constraint always binds.

As discussed earlier if $\gamma \leq \xi$ the regulatory constraint will no longer bind and the model will collapse to a standard adverse selection model. Thus, without loss of generality, I restrict the government's decision to choosing a parameter $\gamma \in [\xi, 1)$.

I now turn to the objective function of the government, which is assumed to be benevolent and maximizes the welfare of a risk-neutral household that owns both firms and banks. In order to provide deposit insurance, the bank levies a lump-sum tax $\tau$ on households. The objective function for the planner is thus

$$\mathcal{U} = \sum_i \mu_i \left(U_i\left(k_i, R_i, C_i\right) + V_i\left(k_i, R_i, C_i\right)\right) - (1-q)\,\tau, \tag{25}$$

where $U_i\left(k_i, R_i, C_i\right)$ is the firm expected profit set out in equation (4) and $V_i\left(k_i, R_i, C_i\right)$ is the expected profit of a bank contract set out in equation (6). The lump-sum tax $\tau$ is set to exactly cover the losses of the depositors in expectation and is set such that

$$\tau = \max\left\{\sum_i \mu_i\left(k_i - p_i\left(z_B\right) R_i k_i - \delta\left(1 - p_i\left(z_B\right)\right) C_i\right), 0\right\}. \tag{26}$$

Substituting the equation for $\tau$ into the government's objective function yields the following equation

$$\mathcal{U} = \sum_i \mu_i\left(p_i \varphi k_i^{\alpha} - (1-\delta)\left(1-p_i\right) C_i - k_i + W\right). \tag{27}$$

A useful benchmark to consider is the first-best contract under full-information, if the planner could choose directly the contracts provided to firms. Maximizing the above results in the following

$$k_i^{FB} = \left(\alpha p_i \varphi\right)^{\frac{1}{1-\alpha}} \text{ and } C_i^{FB} = 0.$$

Comparing this to the competitive equilibrium under full information as set out in proposition 1, the collateral level will be higher than optimal whenever $\gamma > \xi$ while the loan size will be higher than optimal whenever $\gamma < \xi + \frac{(1-q)(1-\xi)}{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)}$. The over-lending problem occurs because, due to the presence of deposit insurance, banks do not fully endogenize the cost of default. Instead, risk-shifting takes place and banks maximize profits only in states where they do not default. The government can reduce this over-lending problem by raising $\gamma$ but does so at the cost of imposing higher collateral requirements on firms. As $\delta < 1$, collateral is assumed to be costly and thus the government faces a trade-off between reducing excessive lending and increasing the dead-weight loss from collateral usage.

## 4.2 Optimal Policy with a single firm type

To begin, I start by discussing the properties of the optimal policy decision in an economy with a single firm type. In this case, rewriting equation (27) yields the following optimization problem for the government

$$\mathcal{U}_i = \max_{\gamma} \left\{ \left( p_i \varphi \left( k_i^* \right)^{\alpha} - (1-\delta)(1-p_i) C_i^* - k_i^* + W \right) \right\}, \tag{28}$$

where $k_i^*$ and $C_i^*$ are defined in proposition 2.

The partial derivative of equation (28) with respect to the policy parameter $\gamma$ is

$$\frac{\partial \mathcal{U}_i}{\partial \gamma} = \left( p_i \varphi \alpha k_i^{\alpha-1} - 1 \right) \frac{dk_i^*}{d\gamma} - (1-\delta)(1-p_i) \frac{dC_i^*}{d\gamma}. \tag{29}$$

At the first-best loan size, the marginal product of the project with respect to the loan size equals the interest rate and thus $p_i \varphi \alpha k_i^{\alpha-1} = 1$. As the derivative of $k_i^*$ with respect to $\gamma$ is negative, equation (29) states that at the optimal policy, the loan size will only be at the first best if at this point $\frac{dC_i^*}{d\gamma} = 0$. In the appendix, I show that this will only occur if $(q + (1-q)\xi) = \alpha$. While it may seem surprising that it could be optimal for the government to set the loan size lower than first-best when there is an over-borrowing problem, it follows from the fact that while the collateral to loan ratio is increasing in $\gamma$, the absolute quantity of collateral $C_i^*$ may not be increasing in $\gamma$. Thus if, around the first-best loan size an increase in $\gamma$ will reduce the quantity of costly collateral used, then it will be optimal to further increase regulation at the expense of a lower loan size than in the first-best.

## 4.3 Optimal Policy with two firm types

I now return to the case of two firm types and where banks are unable to observe the firm type. The government then seeks to maximize equation (27) subject to the incentive compatibility constraint set out in equation (19) and the contract terms that firms receive in equilibrium. In a separating equilibrium the high-risk firm will receive $(k_H^*, R_H^*, C_H^*)$ as described in proposition 2. The low-risk firm receives a separating contract denoted by $\left(\hat{k}_L, \hat{R}_L, \hat{C}_L\right)$ where $\hat{R}_L$ and $\hat{C}_L$ can be found from equations (8) and (19) and $\hat{k}_L$ is given by the following equation

$$\hat{k}_L = \begin{cases} \delta \left( \frac{1-\xi}{\gamma-\xi} \right) \hat{C}_L & \text{if } \hat{C}_L < \bar{C}_L \\ \left( \alpha \varphi p_L \left( z_G \right) \right)^{\frac{1}{1-\alpha}} & \text{otherwise.} \end{cases} \tag{30}$$

The derivative of the government's optimization problem is

$$\begin{aligned}\frac{\partial \mathcal{U}}{\partial \gamma} &= \mu_H \left(p_H \varphi \alpha \left(k_H^*\right)^{\alpha-1} - 1\right) \frac{dk_H^*}{d\gamma} - \mu_H \left(1-\delta\right)\left(1-p_H\right)\frac{dC_H^*}{d\gamma} \\ &\quad - \mu_L \left[\left(1 - p_L \varphi \alpha k_L^{\alpha-1}\right)\frac{\partial \hat{k}_L}{\partial \hat{C}_L} + \left(1-\delta\right)\left(1-p_L\right)\right]\frac{d\hat{C}_L}{d\gamma}. \end{aligned} \tag{31}$$

To simplify the analysis, I substitute out the derivatives of the high-risk firm contract and use equation (31) to define the following function

$$\begin{aligned}\phi \equiv\ & \mu_H \left(\frac{1}{1-\alpha}\right)\left(1-q\right)\left(1 + \left(\frac{1-\delta}{\delta}\right)\left(1-p_H\right)\right)\frac{k_H}{p_H \varphi \alpha \left[k_H\right]^{\alpha-1}} \\ & - \mu_H \left(\frac{1}{1-\alpha}\right)\left(\left(1-q\right) + \left(1-\alpha\right)\left(\frac{1-\delta}{\delta}\right)\left(1-p_H\right)\left(\frac{1}{1-\xi}\right)\right) k_H \\ & - \mu_L \left[\left(1 - p_L \varphi \alpha k_L^{\alpha-1}\right)\frac{\partial \hat{k}_L}{\partial \hat{C}_L} + \left(1-\delta\right)\left(1-p_L\right)\right]\frac{d\hat{C}_L}{d\gamma}. \end{aligned} \tag{32}$$

The government's optimal policy solution is then to find a value of $\gamma$ such that $\phi = 0$. There are now two separate cases to consider, depending on whether $\hat{C}_L$ is less than $\bar{C}_L$ or not.

In the case where $\hat{C}_L \geq \bar{C}_L$, the size of the loan given to low-risk firms will not be affected by changes in collateral and so $\frac{\partial \hat{k}_L}{\partial \hat{C}_L} = 0$. Furthermore, any increase in $\gamma$ will result in a larger collateral requirement for low-risk firms as $\frac{dC_L}{d\gamma} > 0$. From inspecting equation (32) it is clear that the optimal value of $\gamma$ will be strictly lower than would be set optimally if the economy consisted of only high-risk firms. Intuitively, this is because increasing $\gamma$ will only affect the loan size of high-risk firms but comes with an additional cost of raising the amount of costly collateral that low-risk firms must pledge. The optimal level of $\gamma$ can be solved for analytically however, I leave this for the appendix.

In the case where $\hat{C}_L < \bar{C}_L$, the loan size is now increasing in the amount of collateral provided and $\frac{\partial \hat{k}_L}{\partial \hat{C}_L} > 0$ while an increase in $\gamma$ still increases the amount of collateral that is required by $\hat{C}_L$. In the appendix it is shown that the $\frac{dC_L}{d\gamma}$ is now strictly larger than it would be in a case where $\hat{C}_L \geq \bar{C}_L$. From equation (32), this implies that a move from a point where $\hat{C}_L \geq \bar{C}_L$ to a point where $\hat{C}_L < \bar{C}_L$ will lower $\phi$ and thus there is at most one point where $\phi = 0$ ensuring that there is a single value of $\gamma$ that maximizes the government's objective function.

# 5 Conclusion

This paper analyzed credit market equilibrium under private information when banks face a regulatory constraint that restricts the losses they can make in a recession. As in standard signaling models, borrowers are able to signal their type through both loan size and collateral in order to receive a lower loan interest rate. The addition of a regulatory constraint adds an additional consideration for banks as higher collateral requirements will also reduce the loss given default.

I highlight the interaction between the signaling problem and banking regulation. In particular several results differ significantly from more standard signaling models. First, collateral may be demanded of both high- and low-risk firms, even in the absence of asymmetric information. Secondly, if banking regulation is sufficiently strict, there may not exist an adverse selection problem. Additionally, if borrowers have sufficiently low pledgable collateral, a pooling equilibrium may exist as a Nash equilibrium. This last result highlights how regulation may distort bank lending in ways that can have negative distributional effects, through disrupting the ability of bank to screen borrowers.

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# Appendix

## Proof of Proposition 1

In this proof we assume that $W$ is sufficiently high that any quantity of collateral can be implemented in equilibrium. The Lagrangian that solves for the competitive equilibrium contract for an single firm type is

$$\begin{aligned}\mathcal{L}_i = {} & p_i\left(\varphi k_i^\alpha - R_i k_i\right) - \left(1-p_i\right) C_i + W \\ & + q\lambda_B\left[p_i\left(z_G\right) R_i k_i + \delta\left(1-p_i\left(z_G\right)\right) C_i - k_i\right] \\ & + \lambda_S\left[p_i\left(z_B\right) R_i k_i + \delta\left(1-p_i\left(z_B\right)\right) C_i - \gamma k_i\right] \\ & + \lambda_C^- C_i, \end{aligned} \tag{A.33}$$

where $\lambda_S$, $\lambda_B$ and $\lambda_C^-$ are the multipliers on equations (7), (8) and the non-negativity constraint on collateral. The first order conditions are

$$p_i = \lambda_B q p_i\left(z_G\right) + \lambda_S p_i\left(z_B\right), \tag{A.34}$$

$$\left(1-p_i\right) = \lambda_B q\delta\left(1-p_i\left(z_G\right)\right) + \lambda_S\delta\left(1-p_i\left(z_B\right)\right) + \lambda_C^-, \tag{A.35}$$

$$p_i\left(\alpha\varphi k_i^{\alpha-1} - R_i\right) + \lambda_B q\left[p_i\left(z_G\right) R_i - 1\right] + \lambda_S\left[p_i\left(z_B\right) R_i - \gamma\right] = 0. \tag{A.36}$$

First note that if the stress-test condition does not bind, $\lambda_S = 0$ and the contract terms that solve the first order conditions are given by $C_i = 0$,$R_i = 1$ and $k_i = \left(\alpha p_i\left(z_G\right)\right)^{\frac{1}{1-\alpha}}$. Plugging these equations into equation (7), the regulatory constraint will be satisfied only if $\gamma \leq \xi$.

Next, we verify when $\lambda_B$ will be strictly positive. To do this, suppose instead that $\lambda_B = 0$, then equation (A.34) implies that $\lambda_S = \frac{p_i}{p_i(z_B)}$. Substituting this into equation (A.35) and rearranging yields

$$\left(1-p_i\right) = \delta p_i\left(\frac{1-p_i\left(z_B\right)}{p_i\left(z_B\right)}\right) + \lambda_C^-. \tag{A.37}$$

This yields a contradiction whenever $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$. Thus it follows that if $\gamma > \xi$ and $\delta > \left(\frac{p_i(z_B)}{1-p_i(z_B)}\right)\left(\frac{1-p_i}{p_i}\right)$ then $\lambda_S > 0$, $\lambda_B > 0$ and $\lambda_C^- = 0$. Thus equations (7) and (8) will bind in equilibrium and the equilibrium will feature a strictly positive amount of collateral. Solving the system of first order conditions then yields the equilibrium loan size and collateral size set out in equations (16) and (17). The interest rate charged to the firm follows from substituting equations (16) and (17) into equation (8) and the payoff $\pi_i^*$ follows from substituting the contract terms into $\pi\left(k,R\right) = \varphi k_i^\alpha - R_i k_i$.

## Proof of Proposition 2

First, if $\gamma \geq \xi$, $\delta \geq \left(\frac{p_H(z_B)}{1-p_H(z_B)}\right)\left(\frac{1-p_H}{p_H}\right)$ and $W \geq C_L^*$ then the equilibrium contracts if the incentive compatibility constraint does not bind are given by $\{(k_i^*, R_i^*, C_i^*)\}_{i=\in\{L,H\}}$. Then equation (21) can be derived by substituting these contract terms into equation (19) and rearranging. If for some value of $\gamma$ equation (21) is weakly positive, then the incentive compatibility will not bind at $\gamma$, otherwise, it will. As discussed in the text, the limit of $\Gamma(\gamma)$ as $\gamma \to \xi$ is negative and the incentive compatibility constraint will always bind for $\gamma \leq \xi$. On the other hand, from equation (22) it is clear that the incentive compatibility constraint may become slack as $\gamma \to 1$.

The rest off the proposition follows so long as $\frac{\partial\Gamma(\gamma)}{\partial\gamma} > 0$ as in this case, either $\lim_{\gamma\to1}\Gamma(\gamma) \geq 0$ and there exists a threshold $\gamma^* \in [\xi, 1)$ such that for any $\gamma > \gamma^*$ $\Gamma(\gamma) \geq 0$ or for all $\gamma \in [\xi, 1)$ $\Gamma(\gamma) < 0$.

To show that $\frac{\partial\Gamma(\gamma)}{\partial\gamma} > 0$ note that the derivative of equation (21) with respect to $\gamma$ can be written as

$$\begin{aligned}\frac{\partial\Gamma(\gamma)}{\partial\gamma} &= \alpha\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}(p_L - p_H)\left(\frac{q+(1-q)\xi}{1-\xi}\right)\\ &\times\left(\frac{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}}\\ &+\alpha\left(\frac{(q+(1-q)\xi)\frac{1}{\delta}\left(\frac{p_L}{p_H}-1\right)\left(\frac{1}{1-\xi}\right)}{\left[[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)\right]^2}\right)\\ &\times\left[1-p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}}\right] \qquad \text{(A.38)}\end{aligned}$$

A sufficient condition for $\frac{\partial\Gamma(\gamma)}{\partial\gamma} > 0$ is

$$p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right)}{[q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{\gamma-\xi}{1-\xi}\right)}\right)^{\frac{1}{1-\alpha}} \leq 1. \qquad \text{(A.39)}$$

To show this, note that from the definition of $k_i^*$ the following is true $\alpha\varphi p_i(k_i^*)^{\alpha-1} = [q+(1-q)\gamma]+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{\gamma-\xi}{1-\xi}\right)$. Thus equation (A.39) can be rewritten as

$$p_H\left(\frac{p_H}{p_L}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha\varphi p_L(k_L^*)^{\alpha-1}}{\alpha\varphi p_H(k_H^*)^{\alpha-1}}\right)^{\frac{1}{1-\alpha}} \leq 1, \qquad \text{(A.40)}$$

which simplifies to

$$\left(\frac{p_L}{p_H}\right)^{\frac{1}{1-\alpha}}\frac{k_H^*}{k_L^*}\leq 1, \tag{A.41}$$

which will always be satisfied as $k_H^* < k_L^*$ and $p_L > p_H$.

## Analysis of optimal policy with single firm type

Equation (28) yields a first order condition for the government's optimal policy problem when there is a single firm of type $i$. From equations(16) and (17) the derivatives of the equilibrium contracts with respect to the parameter $\gamma$ are as follows

$$\frac{dk_i^*}{d\gamma}=-\left(\frac{1}{1-\alpha}\right)\left(\frac{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}\right)k_i^*, \tag{A.42}$$

$$\frac{dC_i^*}{d\gamma}=\frac{1}{\delta}\left(\frac{1}{1-\xi}\right)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\frac{1}{\alpha}\left(q+(1-q)\,\xi\right)-p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}\right)k_i^*. \tag{A.43}$$

Substituting these derivatives into equation (28) yields the following

$$\begin{aligned}\frac{\partial\mathcal{U}_i}{\partial\gamma}=&-\left(\left(\frac{1-q}{1-\alpha}\right)+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right)k_i^*\\&+\left(\frac{1-q}{1-\alpha}\right)\left(1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\right)\frac{k_i^*}{p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}}.\end{aligned} \tag{A.44}$$

Thus the value of $\gamma$ that sets the government's first order condition to zero is such that the marginal product of the project is given by the following equation

$$p_i\varphi\alpha\left[k_i^*\right]^{\alpha-1}=\left(\frac{1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)}{1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\alpha}{1-q}\right)\left(\frac{1}{1-\xi}\right)}\right). \tag{A.45}$$

As the marginal product at the first best loan size is 1, it follows that the first best loan size is achieved only if this will only occur if $(q+(1-q)\,\xi)=\alpha$. Substituting equation (17) into equation (A.45) and rearranging we can solve for the optimal $\gamma$ as

$$\gamma=\xi+\left(\frac{(1-q)(1-\xi)+\left[1-(1-\alpha)\left(\frac{q+(1-q)\xi}{(1-q)(1-\xi)}\right)\right]\left(\frac{1-\delta}{\delta}\right)(1-p_i)}{\left[(1-q)+\left(\frac{1-\eta}{\eta}\right)(1-p_i)\left(\frac{1}{1-\xi}\right)\right]\left[1+\left(\frac{1-\delta}{\delta}\right)(1-p_i)\left(\frac{1-\alpha}{1-q}\right)\left(\frac{1}{1-\xi}\right)\right]}\right). \tag{A.46}$$

It follows from the above that a sufficient condition for the government to impose a binding regulatory constraint such that $\gamma > \xi$ is the following

$$\left(\frac{1-\delta}{\delta}\right)\left[1-(1-\alpha)\left(\frac{q+(1-q)\,\xi}{(1-q)\,(1-\xi)}\right)\right] > 0. \tag{A.47}$$

## Analysis of optimal policy with two firm types

Equation (31)is the first order condition for the government's optimal policy problem when there are two firm types. As the high-risk firm will receive $(k_H^*, R_H^*, C_H^*)$, the derivatives $\frac{dk_H^*}{d\gamma}$ and $\frac{dC_H^*}{d\gamma}$ are given by equations (A.42) and (A.43) respectively. The loan size for the low risk firm depends on the value of $\hat{C}_L$ relative to $\bar{C}_L$ and the derivative of $\hat{k}_L$ with respect to $\hat{C}_L$ is

$$\frac{\partial \hat{k}_L}{\partial \hat{C}_L} = \begin{cases} \delta\left(\frac{1-\xi}{\gamma-\xi}\right) & \text{if } \hat{C}_L < \bar{C}_L \\ 0 & \text{otherwise.} \end{cases} \tag{A.48}$$

The value of $\hat{C}_L$ can be found from combining equations [zp] and [IC] with the value of $\hat{k}_L$ defined in equation (30) such that $\hat{C}_L$ must satisfy the following equation

$$\hat{C}_L = \frac{1}{\delta}\left(\frac{\frac{p_H}{p_L}\left(p_L\varphi\left(\hat{k}_L\right)^{\alpha-1} - (q+(1-q)\,\xi)\right)\hat{k}_L - \pi_H^*}{1-\frac{p_H}{p_L}(q+(1-q)\,\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right). \tag{A.49}$$

The derivative $\frac{dC_H^*}{d\gamma}$ can then be found by totally differentiating the above equation with respect to $\gamma$.

In the first case where $\hat{C}_L \geq \bar{C}_L$, the derivative is simply

$$\frac{d\hat{C}_L}{d\gamma} = \frac{1}{\delta}\left(\frac{(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{1}{1-\xi}\right)}{1-\left(\frac{p_H}{p_L}\right)(q+(1-q)\,\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)}\right)k_H^*, \tag{A.50}$$

while in the case where $\hat{C}_L < \bar{C}_L$, the derivative becomes

$$\frac{d\hat{C}_L}{d\gamma} = \frac{1}{\delta}\left(\frac{\left[(1-q)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)\left(\frac{1}{1-\xi}\right)\right]k_H^*+\iota_1}{1-\left(\frac{p_H}{p_L}\right)(q+(1-q)\,\xi)+\left(\frac{1-\delta}{\delta}\right)(1-p_H)-\iota_2}\right), \tag{A.51}$$

where

$$\iota_1 \equiv \left(\frac{1}{\gamma-\xi}\right)\left(\frac{p_H}{p_L}\right)\left(\alpha p_L\varphi\left(\hat{k}_L\right)^{\alpha-1} - (q+(1-q)\,\xi)\right)\hat{k}_L, \tag{A.52}$$

$$\iota_2 \equiv \left(\frac{p_H}{p_L}\right)\left(\alpha p_L\varphi\left(\hat{k}_L\right)^{\alpha-1} - (q+(1-q)\,\xi)\right)\left(\frac{1-\xi}{\gamma-\xi}\right). \tag{A.53}$$

From equation (30) it follows that for any $\gamma \in [\xi, 1)$ that $\alpha p_L \varphi \left(\hat{k}_L\right)^{\alpha-1} \geq (q + (1-q)\,\xi)$ and thus $\iota_1 > 0$ and $\iota_2 > 0$. Furthermore, as the incentive compatibility constraint is assumed to bind, it follows that

$$\alpha p_L \varphi \left(\hat{k}_L\right)^{\alpha-1} \leq [q + (1-q)\,\gamma] + \left(\frac{1-\delta}{\delta}\right)(1-p_L)\left(\frac{\gamma-\xi}{1-\xi}\right), \tag{A.54}$$

and thus

$$1 - \left(\frac{p_H}{p_L}\right)(q + (1-q)\,\xi) + \left(\frac{1-\delta}{\delta}\right)(1-p_H) - \iota_2 > 0. \tag{A.55}$$

These two properties are enough to ensure that $\frac{dC_L}{d\gamma} > 0$.