The Impact of CBDC on Bank Deposits and the Interbank Market*

Benjamin Hemingway[†]

December 2022
Click for latest version

Abstract

This paper investigates how the introduction of a central bank digital currency (CBDC) impacts the banking sector. The deposit market is modeled as a Salop circle and deposits are subject to liquidity shocks. Absent a CBDC the interbank market can redistribute liquidity between banks. However, the central bank does not take part in the interbank market and CBDC leads to greater reliance of the banking sector on central bank standing facilities. The model shows adjusting the remuneration rate of CBDC has little pass-through to the deposit rate set by banks and may have implications for transmission of monetary policy.

Keywords: central bank digital currency, banking, money, interbank Market

JEL codes: E42, E52, E58, G21

*I would like to thank Marius Jurgilius, Jack Meaning and Sigitas Siaudinis as well as the participants of the CEBRA Annual Conference 2022 and the members of the Eurosystem Monetary Policy Committee Expert Group on CBDC. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of Lithuania or the Eurosystem.

†CEFER, Bank of Lithuania and Vilnius University. Totoriu g. 4, LT-01121 Vilnius, Lithuania +37069938140 bhemingway@lb.lt

1 Introduction

An increasing number of central banks are entertaining the possibility of issuing a Central Bank Digital Currency (CBDC) as a modern, digital version of physical currency. From the perspective of a retail depositor, a CBDC is likely to have technical features that make it a closer substitute to bank deposits than physical currency. Thus CBDC is likely be a greater source of competition for banks in the deposit market. This is the starting position this paper adopts. I adopt a rather narrow view of CBDC and analyze the possible implications of introducing a retail CBDC on the structure of the deposit market. In particular, this paper models CBDC as a liability of the central bank that is held by retail depositors and is an imperfect substitute for deposits issued by private banks. The model of the deposit market is based on the spatial competition model of Salop (1979) and features imperfect substitutability between deposits issued by a finite number of banks. Deposits are subject to liquidity shocks and there exists an interbank market that allows banks to transfer liquidity between themselves. I assume that the central bank does not participate in the interbank market. Should a liquidity imbalance arise between the CBDC and the banking sector, banks would have to increase their use of central bank standing facilities.

The model makes several predictions with important policy implications. First, if the banks do not face liquidity risk from deposit financing then in the short-run the introduction of a CBDC results in an increase in interest rates on bank deposits and a fall in the market shares of banks in the deposit market. This leads to a fall in bank profitability and so the model predicts that in the long-run the number of banks active in the deposit market falls following the introduction of CBDC. As the banking sector becomes more concentrated in the long-run, the interest rate on bank deposits may not be increasing in the remuneration rate of CBDC as a more concentrated banking sector places downward pressure on the bank deposit rate. This paper also highlights the importance of the liquidity risk channel for monetary policy transmission more generally. Absent liquidity risk, the bank deposit rate increases one-for-one following an increase in the policy rate, even following the introduction of a CBDC. However, if banks face liquidity risk in the deposit market introducing a CBDC will impact monetary policy transmission as there will be imperfect pass-through of the policy rate to the deposit rate. Furthermore, the impact of monetary policy will now impact the structure of the deposit market and thus monetary policy will impact the deposit rate to differing degrees in the short-run and long-run.

This paper is complementary to the growing literature on the policy implications of CBDC. A large literature focuses on issues of financial stability, in particular both Böser and Gersbach (2020) and Fernández-Villaverde et al. (2021) consider the increased risk of bank runs that may occur if bank depositors had access to a CBDC that they could transfer their deposits to in times of financial stress. Both Brunnermeier and Niepelt (2019) and Niepelt (2020) discuss equivalence results where appropriate transfers from the central bank to the financial system is able to neutralize the impact of introducing a CBDC and mitigate the risk of a CBDC induced bank run. This paper also introduces liquidity risk of deposits, the focus is not on bank runs but on the costs imposed on banks when they obtain liquidity from a central bank lending facility.

This paper is also related to the literature on how CBDC should be remunerated. Agur et al. (2022) consider the welfare trade-off for the central bank in choosing a non interest-bearing versus an interest-bearing CBDC. Barrdear and Kumhof (2022) find that a counter-cyclical remuneration rate rule for CBDC can contribute toward stabilizing the business cycle. Similarly Bordo (2021) finds that an interest-bearing CBDC may improve the transmission mechanism of monetary policy. On the other hand, Chiu and Davoodalhosseini (2021) find a non interest-bearing CBDC increases bank intermediation and thus welfare while an interest-bearing CBDC results in bank disintermediation and lower welfare. Williamson (2022) studies various implementations of CBDC and shows how an interest-bearing CBDC can increase welfare by competing with private means of payment. This paper casts doubt on the use of the CBDC remuneration rate in the monetary toolkit. Instead, the paper sets out a model that predicts imperfect pass-through of the CBDC remuneration rate to the bank deposit rat and proposes a channel through which it also affects the structure of the banking sector.

Another related strand of literature focuses on the implications of CBDC for monetary policy. A summary of the possible monetary policy implications of CBDC can be found in Bindseil (2019). For example Keister and Sanches (2019) suggests that while CBDC can promote efficient exchange, it may also raise funding costs. Meaning et al. (2021) provide a detailed discussion on the monetary transmission mechanism in general as well as other possible policy implications. Burlon et al. (2022) study the welfare implications of a CBDC and attempt to characterize the welfare-maximizing CBDC policy rules. Kumhof and Noone (2021) discuss the remuneration of CBDC in detail and its possible use for monetary policy. Kumhof and Noone (2021) propose a two-tier remuneration system while Barrdear and Kumhof (2022) propose both a quantity rule and a price rule for CBDC. This paper does not consider the impact of a quantity rule but does consider how in the presence of deposit liquidity risk a CBDC may impact the transmission of monetary policy through the bank deposit rate.

This paper is most closely related to the literature on the impact of CBDC on the banking sector. In a macroeconomic framework, Bacchetta and Perazzi (2021) assume a constant elasticity of substitution between a CBDC and a continuum of monopolistically competitive banks. While Andolfatto (2021) analyzes the case of a single monopoly bank where CBDC and bank deposits are perfect substitutes but there is a fixed cost for depositors to switch between the two. Chiu et al. (2019) study a model of Cournot oligopoly with a finite number of banks where banks compete in the quantity rather than the remuneration of deposits. CBDC is assumed to be a perfect substitute for bank deposits and so imposes a minimum remuneration rate on bank deposits.

This paper is also related to the literature on spatial models of imperfect competition as the deposit market is based on the classic paper by Salop (1979). Models of spatial competition have been widely used to study deposit markets. For example, Chiappori et al. (1995) study the regulation of deposit rates using a Salop circle model of both loans and deposit markets while Matutes and Vives (1996) study the impact of deposit insurance in a model of Spatial competition in the deposit market. Along similar lines Repullo (2004) investigates the effect of capital requirements on bank behavior when imperfect competition in the deposit market is modeled using a Salop circle. Empirical support for spatial models of the deposit market is provided by Park and Pennacchi (2008) and Ho and Ishii (2011) among others. The structure of competition following the introduction of a CBDC is closely related to Salop Circle models with a center such as Bouckaert (2000) and Madden and Pezzino (2011).

Finally, this paper is also related to the literature on interbank markets. In particular, the theoretical treatment of the interbank market in this paper is closest to that of Hauck and Neyer (2014) and Bucher et al. (2020) who both study the operation of an interbank market within the framework of the euro area.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 goes into further detail on how the banking sector is modeled. In Section 4 the equilibrium is presented. Section 5 analyzes the impact of the CBDC remuneration rate on the bank deposit rate. Section 6 provides a discussion of the model's implications for monetary policy transmission and Section 7 concludes.

2 Model

I consider a three period model of the retail deposit market. The economy consists of three types of agents; risk-neutral banks, a central bank, and a continuum of depositors. There are three discrete periods, $t = 1, 2, 3$.

In the first period $t = 1$, $N \geq 2$ banks enter the market, paying a fixed cost $F > 0$. Banks have access to a technology which yields an exogenously given return R_L on liquidity.

Banks must obtain an exogenously given quantity of liquidity $L > 1$ in order to operate this technology. Banks obtain liquidity in period $t = 2$, either from the central bank or from depositors. Bank i obtains liquidity B_i from the central bank at time $t = 2$ through open market operations at an interest rate R_{RO}. The market for retail deposits is subject to liquidity risk that is realized in $t = 3$.

At the end of $t = 3$ banks must return to a liquidity neutral position. In order to do so, banks may borrow or lend liquidity either through the interbank market or the central bank standing facilities. The central bank offers a deposit facility with an interest rate R_{DF} and a lending facility with an interest rate R_{LF}. The central bank charges penalty rates on these standing facilities such that $R_{DF} < R_{RO} < R_{LF}$. Banks are able to trade liquidity between themselves in an interbank market. Trade in the interbank market takes place at a state-dependent interbank rate R^s_{IB}, where the superscript s denotes the state. The interest rates on the central bank standing facilities define a corridor that sets an upper- and lower-bound on the interbank rate.

The retail deposit market is modeled as a Salop circle as in Salop (1979). There is a continuum of depositors located around a circle with unit mass. Banks locate equidistant from each other around the circle. A depositor located at a distance $x \geq 0$ away from the bank must pay a linear transportation cost $t_B x \geq 0$ in order to deposit their funds. Banks compete in prices *à la* Bertrand. The interest rate paid on deposits by bank i denoted as $R_{D,i}$.

The central bank may also choose in $t = 1$ to enter the deposit market through issuing a CBDC. Should the central bank issue a CBDC, it chooses a remuneration rate R_{CB} and locates itself at the center of the Salop circle. Each depositor is assumed to pay a fixed transportation cost to obtain CBDC with this transport cost drawn uniformly from the interval $t_{CB} \in [0, t_B]$. The structure of CBDC transport costs serves two purposes. First, it captures the idea that CBDC may have specific features that differentiate it from retail deposits. Examples given in the literature include privacy concerns or preferences over additional security of deposited funds. Second, it allows for competition between neighbor banks and CBDC to occur simultaneously. This would not be the case if CBDC transport costs were identical among depositors.

The entry of CBDC into the deposit market is known to all participants and banks choose whether to enter the deposit market and set their deposit rates with full knowledge of whether they will be competing against a CBDC. I assume that the CBDC is only held by households and that banks cannot deposit into CBDC. This assumption allows the central bank to set R_{CB} larger than the interest rate on the deposit facility R_{DF}. In cases where $R_{CB} \leq R_{DF}$ this assumption is rendered unnecessary as from the bank's perspective, the return on CBDC is weakly dominated by the central bank's deposit facility.

Deposits are subject to liquidity shocks which are realized at $t = 3$. With probability λ, a fraction $\xi \in [0, 1]$ of bank deposits relocate to other locations, evenly distributed around the circle, while CBDC depositors do not relocate. While with probability $1 - \lambda$, CBDC depositors are subject to the same liquidity shocks as banks and a fraction ξ of all depositors relocate around the circle.

The liquidity shocks are such that absent CBDC, the law of large numbers ensures that each bank receives a liquidity inflow equal to its liquidity outflow. The presence of CBDC introduces additional liquidity risk into the banking sector. While these liquidity shocks are similar in spirit to those in papers such as Fernández-Villaverde et al. (2021) that focus on the possibility of CBDC generated bank runs, here there is no risk of bank runs. Instead, liquidity risk generates additional costs of deposits for banks. If CBDC depositors are not subject to the liquidity shock, the aggregate liquidity of the banking sector falls and banks will need to increase their use of the central bank standing facilities.

To summarize the model timing once more: in the first period, $t = 1$, both commercial banks and the central bank decide on whether to enter the deposit market. Should the central bank decide to enter the deposit market by issuing a CBDC, it will also set the remuneration rate R_{CB} in $t = 1$. In the second period, $t = 2$, commercial banks compete in the deposit market by setting a deposit rate $R_{D,i}$ and obtain liquidity B_i from the central bank. In the third period, $t = 3$, the liquidity shock is realized and commercial banks use the central bank standing facilities as well as the interbank market in order to obtain a liquidity neutral position. I solve the game for its Subgame Perfect Nash Equilibrium in pure strategies using backward induction.

3 Banking Sector

3.1 Bank Liquidity

I begin the analysis of the banking sector with the final period, $t = 3$. At this point in time the $N \geq 2$ banks, indexed by i, have made their decisions regarding their funding structure. The bank's funding structure consists of a quantity of deposits $q_{B,i}$ and central bank liquidity B_i.

The bank's choice of liquidity B_i and deposits $q_{B,i}$ implies that before the realization of the liquidity shocks the banks have the following ex ante liquidity deficit

$$\epsilon_i \equiv L - B_i - q_{B,i}. \tag{1}$$

With probability $1-\lambda$, a fraction ξ of all depositors relocate to locations evenly distributed around the Salop circle. Here banks face the same liquidity inflows as liquidity outflows and their ex post liquidity deficit is simply equal to their ex ante liquidity deficit

$$\epsilon_i^0 = \epsilon_i. \tag{2}$$

With probability λ, a fraction ξ of bank depositors relocate while CBDC depositors do not. Now bank i receives a liquidity outflow equal to $q_{B,i}\xi$, while each bank receives an inflow of liquidity equal to $(1 - q_{CB})\, q_{B,i}\xi$. Thus each bank will receive a net outflow of liquidity and has an ex post liquidity deficit equal to

$$\epsilon_i^+ = \epsilon_i + q_{CB}q_{B,i}\xi. \tag{3}$$

Banks must return to a liquidity neutral position by the end of $t = 3$. How much liquidity they must trade in order to achieve this depends on their ex post liquidity deficit ϵ_i^s, where $s \in \{0, +\}$ denotes the realization of the liquidity shock. If $\epsilon_i^s > 0$, banks will need to obtain additional liquidity either through the interbank market or through the central bank liquidity facility, while if $\epsilon_i^s < 0$ banks will reduce their liquidity either by lending in the interbank market or by depositing liquidity at the central bank deposit facility.

The interest rate in the interbank market depends on the aggregate liquidity deficit of the banking system, $\sum_i \epsilon_i^s$. This in turn depends on the realization of the liquidity shock. The interest rates on the central bank standing facilities act as upper- and lower-bounds on the interbank rate and thus the interbank rate is

$$R_{IB}^s \begin{cases} = R_{LF} & \text{if } \sum_i \epsilon_i^s > 0 \\ = R_{DF} & \text{if } \sum_i \epsilon_i^s < 0 \\ \in [R_{DF}, R_{LF}] & \text{otherwise.} \end{cases} \tag{4}$$

The expected liquidity cost of deposits is simply the expected cost of returning to a liquidity neutral position

$$E\left[C_{D,i}\right] = (1 - \lambda)\, R_{IB}^0 \epsilon_i^0 + \lambda R_{IB}^+ \epsilon_i^+. \tag{5}$$

As the central bank standing facilities are more costly for banks to access compared to the interbank market, this generates a greater liquidity cost of deposits that will be increasing in the size of the liquidity shock.

3.2 Bank's Problem

I first consider the bank's problem in the intermediate period, $t = 2$, taking the number of banks N as given. In this period, the bank decides on its funding structure by obtaining liquidity from the central bank and sets the interest rate it offers to depositors $R_{D,i}$. Banks are risk neutral and maximize expected profits. The profit function of bank i is

$$\pi_i = \max_{B_i, R_{D,i}} \left\{ R_L L - R_{RO} B_i - R_{D,i} q_{B,i} - E\left[C_{D,i}\right] - F \right\}, \tag{6}$$

where $E\left[C_{D,i}\right]$ is defined in equation (5) and $F > 0$ is the fixed cost that banks are assume to pay in order to enter the deposit market. Banks compete for depositors in prices à la Bertrand, taking as given both the deposit rates set by other banks and the funding structure of other banks.

In the case where the central bank does not implement a CBDC, competition between banks in the deposit market is identical to Salop's circle model. If bank i offers a deposit rate equal to $R_{D,i}$, and the other banks offer a deposit rate equal to $R_{D,-i}$ then a depositor that is located at a distance x from bank i, where $x \in \left[0, \frac{1}{N}\right]$, will choose to deposit their funds at bank i rather than the neighboring bank so long as

$$R_{D,i} - t_B x \geq R_{D,-i} - t_B \left(\frac{1}{N} - x \right), \tag{7}$$

where t_B is the linear transport cost that is incurred by depositors. Bank i thus faces the following demand function

$$q_{B,i} = \frac{1}{N} + \frac{1}{t_B} \left(R_{D,i} - R_{D,-1} \right). \tag{8}$$

In the case where the central bank does implement a CBDC, bank i faces competition not just from the two banks that neighbor it but also from the CBDC. I assume that the central bank sets a fixed interest rate R_{CB} and that depositors incur a transport cost t_{CB} if they deposit funds in the CBDC. The transport costs associated with CBDC are assumed to be drawn randomly from a uniform distribution over the interval $[0, t_B]$. Thus a depositor located at distance x from bank i would prefer to deposit funds in bank i rather than in the CBDC so long as

$$R_{D,i} - t_B x \geq R_{CB} - t_{CB}. \tag{9}$$

Following the introduction of CBDC, in order for a depositor to deposit funds in bank i they must satisfy both equation (7) and equation (9) and must prefer bank i to the

CBDC as well as all other banks.

To calculate the demand function bank i faces in the presence of CBDC I first define the point $x^*_{B,i} \in \left[0, \frac{1}{2N}\right]$ as the point where a depositor is indifferent between bank i and the neighboring bank $-i$. The equation for $x^*_{B,i}$ is as in the standard Salop model and is given by

$$x^*_{B,i} = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_{D,i} - R_{D,-i}\right)\right). \tag{10}$$

From equation (9) it follows that if the remuneration rate of CBDC, R_{CB}, is sufficiently low, no depositors will prefer CBDC to bank deposits and thus the model will collapse to a Salop model of bank deposits. In order for CBDC to have a positive market share, it is required that $R_{CB} \geq \underline{R}_{CB,i}$ for at least one i, where

$$\underline{R}_{CB,i} \equiv R_{D,i} - t_B x^*_{B,i}. \tag{11}$$

Similarly, if the remuneration of CBDC is sufficiently high banks will no longer compete directly with each other and instead would only compete with the CBDC. While banks will not be completely driven out of the deposit market at this point, banks will instead operate on a local monopoly on a smaller section of the market with CBDC acting as a random outside option. As the central bank is unlikely to want to avoid banks competing directly with each other, I assume that the central bank imposes an upper-bound on the remuneration rate such that $R_{CB} < \bar{R}_{CB,i}$ for all i where

$$\bar{R}_{CB,i} \equiv R_{D,i} + t_B\left(1 - x^*_{B,i}\right) \tag{12}$$

Given these limits on R_{CB}, there exists an $\hat{x}_{B,i} \leq x^*_{B,i}$ such that the following condition holds

$$\hat{x}_{B,i} \equiv \frac{1}{t_B}\left(R_{D,i} - R_{CB}\right), \tag{13}$$

where all depositors located at a distance $\hat{x}_{B,i}$ or closer to bank i will prefer to hold deposits at bank i over CBDC.

For the interval $x \in \left[\hat{x}_{B,i}, x^*_{B,i}\right] \cap \left[0, x^*_{B,i}\right]$ there exists a function $t^*_i(x) \in (0, t_B)$ such that the following condition holds

$$t^*_i(x) = t_B x + R_{CB} - R_{D,i}, \tag{14}$$

where the function $t^*_i(x)$ defines that smallest value of t_{CB} that depositors that are located at a distance x away from bank i must have in order to prefer depositing in bank i rather than depositing in the CBDC.

Given the above definitions and assumptions, the demand function that bank i faces is

$$q_{B,i} = \begin{cases} 2x^*_{B,i} & \text{if } \hat{x}_{B,i} \geq x^*_{B,i} \\ 2\left(\int_{\hat{x}_{B,i}}^{x^*_{B,i}} \left(\frac{t_B - t^*(x)}{t_B}\right) dx + \hat{x}_{B,i}\right) & \text{if } x^*_{B,i} > \hat{x}_{B,i} \geq 0 \\ 2\int_0^{x^*_{B,i}} \left(\frac{t_B - t^*(x)}{t_B}\right) dx & \text{otherwise.} \end{cases} \tag{15}$$

In the case where where $\hat{x}_{B,i} \geq 0$ bank i receives deposits from all depositors located at a distance under $\hat{x}_{B,i}$ and a fraction $\left(\frac{t_B - t^*(x)}{t_B}\right)$ of depositors located at a distance $x \in \left[\hat{x}_{B,i}, x^*_{B,i}\right]$, while if $\hat{x}_{B,i} < 0$, banks receive a fraction of depositors located over the distance $x \in \left[0, x^*_{B,i}\right]$.

It will be easier to work with the definite integral form of equation (15), which can be written as

$$q_{B,i} = \begin{cases} 2x^*_{B,i} & \text{if } \hat{x}_{B,i} \geq x^*_{B,i} \\ 2x^*_{B,i} - \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2 & \text{if } x^*_{B,i} > \hat{x}_{B,i} \geq 0 \\ 2x^*_{B,i} - \left(x^{*2}_{B,i} - 2\hat{x}_{B,i}x^*_{B,i}\right) & \text{otherwise.} \end{cases} \tag{16}$$

One useful benchmark to consider is the case where $\hat{x}_{B,i} = x^*_{B,i}$, which occurs when R_{CB} is sufficiently low that the banks do not lose market share to CBDC. In this case equation (16) collapses to $q_{B,i} = 2x^*_{B,i}$ which, noting that in a symmetric equilibrium with $R_{D,i} = R_{D,-i}$, yields a market share for bank i of $1/N$ as in the standard Salop model.

As depositors are assumed not to have an outside option, there will be full coverage in the deposit market and all deposits will be deposited either at a retail bank or at the central bank. As a result, the market share of CBDC can be written as

$$q_{CB} = 1 - \sum_i q_{B,i}. \tag{17}$$

3.3 Bank Entry

In $t = 2$, the number of banks competing in the deposit market can be taken to be fixed. I now consider the $t = 1$ entry problem for banks, given banks have to pay a fixed cost $F > 0$ in order to enter the deposit market. Banks will choose to enter so long as the expected profits defined by equation (6) are weakly positive. The number of banks that enter in $t = 1$, denoted by N, will adjust until expected profits are driven to zero.

4 Equilibrium

I focus on a symmetric equilibrium where in $t = 2$ all banks make the same decision regarding their funding structure, B_i, and set the same deposit rate, $R_{D,i}$. As banks set the same rate on deposits, they will obtain the same market share of deposits, $q_{B,i}$. I distinguish between a short-run equilibrium, where the number of banks N is fixed, and a long-run equilibrium where $N \geq 2$ adjusts according to a free entry condition such that all banks make zero profits.

4.1 Interbank Market and Bank Funding Structure

I begin by characterizing the equilibrium funding structure of the bank and the equilibrium interest rates in the interbank market. In choosing their funding structure banks take the interest rates in the interbank market, the policy rate and the interest rates on standing facilities as given. In addition, banks are able to perfectly anticipate the market share of deposits they obtain conditional on the deposit rate they set. Obtaining one additional unit of liquidity from the central bank in $t = 2$ has a marginal cost of R_{RO} but also reduces by one unit the bank's ex ante liquidity deficit ϵ_i. Thus in equilibrium, the bank will adjust B_i such that the marginal cost of increasing B_i equals the expected marginal cost of increasing its ex ante liquidity deficit ϵ_i and thus

$$\frac{\partial E\left[C_{D,i}\right]}{\partial \epsilon_i} = R_{RO}. \tag{18}$$

In equilibrium, the interbank rates are such that the bank funding decision in $t = 2$ is consistent. Given a bank's choice of B_i and its market share $q_{B,i}$, a bank's ex post liquidity deficit ϵ_i^s is conditional on the realization of the liquidity shock $s \in \{0, +\}$. In equilibrium the interbank rate conditional on the liquidity shock s can be found from equation (4). The equilibrium interbank rate and the bank's equilibrium funding structure is summarized in proposition 1.

Proposition 1. *If* $\lambda \leq \left(\frac{R_{RO}-R_{DF}}{R_{LF}-R_{DF}}\right)$ *then banks obtain liquidity from the central bank in* $t = 2$ *such that*
i) $B_i = L - q_{B,i}$
ii) $R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda}\right)\left(R_{LF} - R_{RO}\right)$ *and*
iii) $R_{IB}^+ = R_{LF}$.

Otherwise if $\lambda > \left(\frac{R_{RO}-R_{DF}}{R_{LF}-R_{DF}}\right)$ *then banks obtain liquidity from the central bank in* $t = 2$ *such that*

i) $B_i = L - (1 - q_{CB}\xi)\, q_{B,i}$
ii) $R_{IB}^0 = R_{DF}$ *and*
iii) $R_{IB}^+ = R_{DF} + \frac{1}{\lambda}(R_{RO} - R_{DF})$.

Proof. See Appendix. □

One implication of proposition 1 is that the interest rates in the interbank market depends on λ, the probability the bank is hit by a net outflow of liquidity. A low λ implies $\epsilon_i^0 = 0$ and banks would have a neutral liquidity position if they are not hit by an outflow of liquidity. It also follows that $\epsilon_i^+ > 0$ and thus banks would have a liquidity deficit should they be hit by an outflow of liquidity. As λ increases, the probability that the banks are hit by a liquidity outflow increases. The interbank rate is weakly decreasing in λ as when the probability of being hit by a liquidity shock increases, banks have a greater incentive to accumulate liquidity in $t = 2$ and thus banks have a greater supply of liquidity in $t = 3$. With a sufficiently high λ, banks accumulate sufficient liquidity such that $\epsilon_i^+ = 0$ and banks have a neutral liquidity position if they are hit by an outflow of liquidity. This implies that $\epsilon_i^0 < 0$ and the banks will have a liquidity surplus if they are not hit by an outflow of liquidity.

By combining the bank's equilibrium funding decision with equation (5), the bank's expected cost of deposits can be written as

$$E\left[C_{D,i}\right] = \epsilon_i R_{RO} + \lambda R_{IB}^+ q_{CB} q_{B,i} \xi. \tag{19}$$

An important property of equation (19) is that if $\lambda > 0$ the expected cost of deposits is strictly increasing in the market share of the central bank. This is an important feedback mechanism of the model. An increase in the market share of CBDC raises the liquidity risk of deposits and thus raises the expected cost of deposits for banks. To simplify the analysis, I assume in the remainder of the paper that $\lambda \leq \left(\frac{R_{RO} - R_{DF}}{R_{LF} - R_{DF}}\right)$ and banks obtain sufficient liquidity to have a neutral liquidity position in the case where they are not hit by an outflow of liquidity.

4.2 Deposit Market Equilibrium

I turn now to the equilibrium in the deposit market. In $t = 2$ the bank sets a deposit rate $R_{D,i}$ that in combination with its funding decision set out in Proposition 1 maximizes the bank's expected profit.

In general, the equilibrium deposit rate can be found by differentiating the bank's profit function given by equation (6) with respect to the deposit rate chosen by the bank, $R_{D,i}$ yielding

$$-q_{B,i} - \frac{\partial q_{B,i}}{\partial R_{D,i}} \left(R_{D,i} + \frac{\partial E\left[C_{D,i}\right]}{\partial q_{B,i}} \right) - \frac{\partial q_{CB}}{\partial R_{D,i}} \frac{\partial E\left[C_{D,i}\right]}{\partial q_{CB}} = 0. \tag{20}$$

I first focus on the case where there is no market share of CBDC, $q_{CB} = 0$. As discussed earlier, from equation (11), it follows that for a sufficiently low CBDC remuneration rate such that $R_{CB} \leq \underline{R}_{CB}$, no depositor prefers CBDC to bank deposits. In a symmetric equilibrium, all banks receive an equal market share. As depositors are assumed to have no outside option, absent CBDC, they will always deposit their funds in a bank and thus bank i's market share is simply $q_{B,i} = \frac{1}{N}$.

In the case where $q_{CB} = 0$, the deposit rate first-order condition described in equation (20) can be simplified leading to a closed-form solution for the deposit rate as in a textbook Salop circle model. The short-run equilibrium without CBDC is fully characterized by proposition 2.

Proposition 2. *For any $N \geq 2$, if $R_{CB} \leq \underline{R}_{CB}$ where $\underline{R}_{CB} = R_{RO} - \frac{3}{2} t_B \frac{1}{N}$ then there exists a unique symmetric short-run equilibrium with $q_{CB} = 0$ and banks choose $R_{D,i}$ such that*
i) $R_{D,i} = R_{RO} - \frac{1}{N} t_B \ \forall i$ and
ii) $q_{B,i} = \frac{1}{N} \ \forall i$.

Proof. See Appendix. □

With $q_{CB} = 0$, the bank's equilibrium funding decision combined with equation (19) implies that $E\left[C_{D,i}\right] = 0$ and thus the expected liquidity cost of deposits is zero. This is a direct consequence of the structure of the liquidity shock. In an economy without CBDC banks face net inflows of liquidity that exactly offset the net outflows of liquidity regardless of the realization of s. Thus banks are able to accumulate sufficient liquidity that they will not need to make use of the central bank's standing facilities.

The profit bank i makes in the case where $R_{CB} \leq \underline{R}_{CB}$ can be found by substituting the equilibrium deposit rate and market share into equation (6), yielding

$$\pi_{B,i} = \left(R_L - R_{RO}\right) L - F + t_B \frac{1}{N^2}. \tag{21}$$

A necessary requirement for $N \geq 2$ banks to enter the market in an equilibrium with $q_{CB} = 0$ is that the fixed cost of entry is such that $F - \left(R_L - R_{RO}\right) L \in \left[0, \frac{1}{4} t_B\right]$. Then, in the long-run the number of banks adjusts subject to a free entry condition such that banks make zero profit in expectation. Should R_{CB} be sufficiently low, then a symmetric long-run equilibrium exists and is fully characterized by proposition 3.

Proposition 3. *If* $N^* \equiv t_B^{\frac{1}{2}} \left(F - (R_L - R_{RO})\, L\right)^{-\frac{1}{2}} \geq 2$ *and* $R_{CB} \leq \underline{R}_{CB}$ *where* $\underline{R}_{CB} = R_{RO} - \frac{3}{2} t_B \frac{1}{N^*}$ *then there exists a unique symmetric long-run equilibrium with* $q_{CB} = 0$ *where* N^**banks enter in* $t = 1$ *and banks choose* $R_{D,i}$ *such that*
i) $\pi_{B,i} = 0\ \forall i$,
ii) $R_{D,i} = R_{RO} - \frac{1}{N^*} t_B\ \forall i$ *and*
iii) $q_{B,i} = \frac{1}{N^*}\ \forall i$.

Proof. See Appendix. □

The cutoff CBDC remuneration rate $\underline{R}_{CB}$ is increasing in N, thus the more concentrated the banking sector, the lower the threshold for CBDC to obtain a positive market share. With fewer banks active in the deposit market, banks offer lower deposit rates and thus CBDC poses greater competitive pressure to banks at a given R_{CB}.

I now consider the bank's choice of deposit rate when $R_{CB} > \underline{R}_{CB}$ and thus the CBDC remuneration rate is sufficiently high that it poses meaningful competition to banks. With $q_{CB} > 0$, the market share of each bank in a symmetric equilibrium is no longer equal to $1/N$ and instead depends on the deposit rate offered by the banks. As a consequence, the short-run deposit rate is now determined by a system of two equations; the first-order condition for the deposit rate, equation (20), and the definition of $\hat{x}_{B,i}$ set out by equation (13). The long-run equilibrium will also require that the free entry condition of banks holds.

In a symmetric short-run equilibrium, it follows from equation (10) that $x^*_{B,i} = 1/N$. When $R_{CB} > \underline{R}_{CB}$, through combining equations (13) and (20) the short-run equilibrium can be found as the $\hat{x}_{B,i}$ that solves the following equation

$$\Gamma_{\hat{x}} \equiv t_B \hat{x}_{B,i} + R_{CB} - R_{RO} + \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} + \left(\frac{\partial q_{CB}}{\partial R_{D,i}} \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} + q_{CB}\right) \lambda R_{IB}^{+} \xi = 0, \tag{22}$$

where $q_{B,i}$ and q_{CB} are functions of $x^*_{B,i}$ and $\hat{x}_{B,i}$ given by equations (16) and (17) respectively.

The bank takes the deposit rates set by the other banks as well as the CBDC remuneration rate as given. It chooses its deposit rate $R_{D,i}$ taking into account the effect a change in the deposit rate it has on both its own market share $q_{B,i}$ and on the market share of

CBDC q_{CB}. From equation (15) the impact of an increase in $R_{D,i}$ on the bank's market share is

$$\frac{\partial q_{B,i}}{\partial R_{D,i}} = \begin{cases} \frac{1}{t_B} & \text{if } \hat{x}_{B,i} \geq x^*_{B,i} \\ \frac{1}{t_B}\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right) & \text{if } x^*_{B,i} > \hat{x}_{B,i} \geq 0 \\ \frac{1}{t_B}\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right) & \text{otherwise.} \end{cases} \tag{23}$$

Equation (22) also depends on the impact of an increase of $R_{D,i}$ on the market share of CBDC, holding the deposit rates of other banks fixed. To obtain this, consider the definition of q_{CB} set out by equation (17). As all depositors are assumed to deposit their funds somewhere, the market share of CBDC is simply the mass of depositors that choose not to deposit funds at any bank. As an increase in $R_{D,i}$ affects the market share not only of bank i but also of the neighboring banks, the impact of $R_{D,i}$ on the market share of CBDC can be calculated from

$$\frac{\partial q_{CB}}{\partial R_{D,i}} = -\frac{\partial q_{B,i}}{\partial R_{D,i}} - \frac{\partial q_{B,i+1}}{\partial R_{D,i}} - \frac{\partial q_{B,i-1}}{\partial R_{D,i}}, \tag{24}$$

where $q_{B,i+1}$ and $q_{B,i-1}$ denote the market shares of the two banks neighboring bank i. This can be rewritten as

$$\frac{\partial q_{CB}}{\partial R_{D,i}} = \begin{cases} 0 & \text{if } \hat{x}_{B,i} \geq x^*_{B,i} \\ -2\frac{1}{t_B}\left(x^*_{B,i} - \hat{x}_{B,i}\right) & \text{if } x^*_{B,i} > \hat{x}_{B,i} \geq 0 \\ -2\frac{1}{t_B}x^*_{B,i} & \text{otherwise.} \end{cases} \tag{25}$$

From equation (22), if $R_{CB} = \underline{R}_{CB}$ then $\hat{x}_{B,i} = x^*_{B,i}$ and the equilibrium is such that $q_{CB} = 0$ as set out by Propositions 2 and 3. In cases where $R_{CB} > \underline{R}_{CB}$, should a symmetric short-run equilibrium exist, the properties of equation (22) ensure that $\hat{x}_{B,i} < x^*_{B,i}$ and thus $q_{CB} > 0$. From equation (12) there is an upper-bound on the CBDC remuneration rate denoted by $\bar{R}_{CB}$ above which banks will not directly compete with each other and instead would operate a local monopoly competing only against the CBDC. In the intermediate range $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ a unique symmetric short-run equilibrium is guaranteed to exist in the case where $\lambda = 0$ and the banks face no liquidity cost of deposit. This result is summarized in the following proposition.

Proposition 4. *For any $N \geq 2$, if $\lambda = 0$ there exists some interval of the CBDC remuneration rate $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ where $\underline{R}_{CB} \equiv R_{RO} - \frac{3}{2}t_B\frac{1}{N}$ and $\bar{R}_{CB} = R_{RO} + t_B\left(1 - \frac{3}{4N}\right)$ such there exists a unique symmetric short-run equilibrium with $q_{CB} > 0$ where*
i) $\Gamma_{\hat{x}} = 0$,
ii) $x^*_{B,i} = \frac{1}{2N}$,

iii) $\hat{x}_{B,i} \in \left[-\left(1-\frac{1}{2N}\right), \frac{1}{2N}\right)$
iv) $R_{D,i} = t_B \hat{x}_{B,i} + R_{CB}$ $\forall i$ *and*
v) $q_{B,i}$ *and* q_{CB} *are given by equations (16) and (17) respectively.*

Proof. See Appendix. □

It is unsurprising that the market share of CBDC is strictly increasing in R_{CB} over the interval $\left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ as a higher remuneration rate of CBDC leads to more depositors choosing CBDC over bank deposits. In the long-run, the number of banks entering the market in $t = 1$ adjusts such that expected bank profits are zero.

In the case where $q_{CB} > 0$, the profit a bank makes by setting the deposit rate at the profit maximizing level can be written as a function of $\hat{x}_{B,i}$ and $x^*_{B,i}$

$$\pi_{B,i} \equiv (R_L - R_{RO}) L - F + \left(1 + \frac{\partial q_{CB}}{\partial R_{D,i}} \lambda R^+_{IB} \xi\right) \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q^2_{B,i} = 0. \tag{26}$$

The long-run equilibrium in the deposit market can be summarized as the $\left\{x^*_{B,i}, \hat{x}_{B,i}\right\}$ pair that satisfies equations (22) and (26).

With $\lambda = 0$ it is shown in the Appendix that bank profits are strictly increasing in $\hat{x}_{B,i}$ over the interval and thus$\left[-\left(1-\frac{1}{2N}\right), \frac{1}{2N}\right)$ decreasing in R_{CB} over the interval $\left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ while bank profit is strictly increasing in $x^*_{B,i}$ and thus as the number of bank entrants falls, an individual bank's profit increases. As I focus on the equilibrium where $N \geq 2$ banks compete against each other and a CBDC, the remuneration rate on CBDC must be sufficiently low to allow two banks to make non-negative profits. I denote this threshold value by R^*_{CB}, the formal definition of which is set out in the following Lemma.

Lemma 1. *If* $\lambda = 0$ *and* $t_B^{\frac{1}{2}} \left(F - (R_L - R_{RO}) L\right)^{-\frac{1}{2}} > 2$ *there exists some* $R_{RO} - \frac{3}{4} t_B < R^*_{CB} \leq R_{RO} + \frac{5}{8} t_B$ *such that if* $N = 2$ *banks enter in* $t = 1$*, both banks make weakly positive profit,* $\pi_{B,i} \geq 0$ *and that one of these inequalities holds strictly.*

Proof. See Appendix. □

If $\lambda = 0$, the bank faces no liquidity shocks and a unique symmetric long-run equilibrium with $N^* \geq 2$ banks and $q_{CB} > 0$ exists so long as the remuneration rate of CBDC is such that $R_{CB} \in (\underline{R}_{CB}, R^*_{CB}]$. This result is set out in detail in the proposition below.

Proposition 5. *If* $\lambda = 0$, $t_B^{\frac{1}{2}} \left(F - (R_L - R_{RO}) L\right)^{-\frac{1}{2}} > 2$ *and* $R_{CB} \in (\underline{R}_{CB}, R^*_{CB}]$ *then there exists a unique symmetric long-run equilibrium with* $q_{CB} > 0$ *where* $N^* \geq 2$ *banks enter in* $t = 1$ *and banks choose* $R_{D,i}$ *such that*

i) $\pi_{B,i} = 0 \; \forall i$,
ii) $\Gamma_{\hat{x}} = 0$,
iii) $\hat{x}_{B,i} \in \left[-\left(1 - \frac{1}{2N}\right), \frac{1}{2N}\right)$
iv) $R_{D,i} = t_B \hat{x}_{B,i} + R_{CB} \; \forall i$ *and*
v) $q_{B,i}$ *and* q_{CB} *are given by equations (16) and (17) respectively.*

Proof. See Appendix. □

In the case where $\lambda > 0$ and banks face liquidity risk from CBDC, the existence of a short-run equilibrium with $q_{CB} > 0$ cannot be guaranteed to hold over the entire interval $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ as it could if $\lambda = 0$. Instead, equilibrium existence can only be guaranteed for some R_{CB} above but sufficiently close to $\underline{R}_{CB}$. To understand why this is the case, first note that if $\lambda = 0$, a key property of equation (22) that helps guarantee uniqueness is that it is strictly increasing in $\hat{x}_{B,i}$ over the interval $\hat{x}_{B,i} \in \left[-\left(1 - x^*_{B,i}\right), x^*_{B,i}\right)$. If $\lambda > 0$, equation (22) is guaranteed to be strictly increasing in $\hat{x}_{B,i}$ for $\hat{x}_{B,i}$ sufficiently close to $x^*_{B,i}$. If the liquidity cost term, $\lambda R^+_{IB}\xi$, is sufficiently large, equation (22) may be decreasing in $\hat{x}_{B,i}$ at sufficiently small values of $\hat{x}_{B,i}$. Intuitively, when $\lambda > 0$, the liquidity cost generates a disincentive for banks to use deposits as a source of liquidity. This disincentive is increasing in the market share of CBDC. If $\hat{x}_{B,i}$ is sufficiently high the market share of CBDC, and hence the liquidity cost of deposits will be low and banks will set a deposit rate in order to compete for market share in deposits. If on the other hand $\hat{x}_{B,i}$ is sufficiently low, the market share of CBDC will be relatively large and banks face a high liquidity cost of deposits. In this situation, banks may choose to compete less for bank deposits and obtain more liquidity directly from the central bank.

Similarly, if $\lambda > 0$, the existence of a long-run equilibrium with $q_{CB} > 0$ cannot be guaranteed over as large an interval of R_{CB} as in the case where $\lambda = 0$. Existence of a long-run equilibrium can only be guaranteed for some R_{CB} above but sufficiently close to $\underline{R}_{CB}$. Analogous to the discussion regarding short-run equilibrium existence, if $\lambda = 0$ equilibrium bank profits given by equation (26) are strictly increasing in $x^*_{B,i}$ and strictly increasing in $\hat{x}_{B,i}$. Thus bank profit decreases as the number of banks increases and decreases in the market share of CBDC. If $\lambda > 0$, this property can only be guaranteed for $\hat{x}_{B,i}$ sufficiently close to $x^*_{B,i}$, that is when the market share of CBDC is sufficiently small. Similarly to the above discussion, if $\lambda > 0$ and the liquidity cost term, $\lambda R^+_{IB}\xi$, is sufficiently large, equation (26) need not be strictly increasing in $x^*_{B,i}$ or $\hat{x}_{B,i}$. The reason behind this is that decreasing the number of active banks leads to an increase in the market share of the banks that remain, which from equation (5) also leads to an increase in the liquidity cost of deposits. Should both $\lambda R^+_{IB}\xi$ and q_{CB} be sufficiently high, bank profit may become higher when more banks enter, lowering each individual bank's market share as well as the liquidity cost of deposits.

As highlighted earlier, in the case where the short-run or long-run equilibrium do not exist, it is possible for local monopolies to form. If it is not profitable for banks to operate a local monopoly, then banks will leave the deposit market altogether, setting $q_{B,i} = 0$ and funding their liquidity solely through central bank liquidity. I consider either case a breakdown in the functioning of the deposit market. A key takeaway from above is that if $\lambda > 0$ banks face additional liquidity risk following the introduction of CBDC and this increases the likelihood that the deposit market may suffer a breakdown should the central bank set too high a CBDC remuneration rate.

Finally, while the central bank's balance sheet is not explicitly modeled, proposition 1 highlights that in equilibrium, each bank increases their holdings of central bank liquidity (B_i) as their market share falls. Summing over all N banks and using the definition of q_{CB} given in equation 17 yields the following equation for the aggregate liquidity borrowed from the central bank by the banking sector

$$\sum_i B_i = NL - (1 - q_{CB}) \,. \tag{27}$$

This shows that as the CBDC market share increases, the aggregate banking sector holds more central bank liquidity and thus the introduction of a CBDC will see the central bank balance sheet increase both its liabilities (q_{CB}) and its assets ($\sum_i B_i$).

5 Impact of CBDC on deposit rates

I now present the impact of a change in the CBDC remuneration rate, R_{CB}, on the equilibrium deposit rate $R_{D,i}$ offered by the banks. The impact of the CBDC remuneration rate will depend on whether the bank faces liquidity costs in its use of deposits. In particular, I consider the impact of a change in R_{CB} when $\lambda = 0$ versus when $\lambda > 0$. The modeling framework I use also allows me to distinguish between the short-run impact of a change in R_{CB}, where the number of banks is fixed, with the long-run impact where the number of banks adjusts according to the free entry condition. The results presented in this section are especially important in regards to whether the remuneration rate of a CBDC can be used as an additional tool in the central bank's toolbox as has been discussed among others in Meaning et al. (2021).

In the case where $R_{CB} \leq \underline{R}_{CB}$ and $q_{CB} = 0$, CBDC has no market share and the deposit rate is given by proposition 2. An increase in the CBDC remuneration rate will have no impact on the bank deposit rate. This holds regardless of whether or not the bank faces liquidity risk from its deposits.

If R_{CB} is sufficiently high that in equilibrium $q_{CB} > 0$ then the impact of an increase in R_{CB} on the deposit rate will depend on the size of the liquidity risk of deposits facing the bank. The short-run impact of an increase in R_{CB} on the deposit rate can be found through use of the Implicit Function Theorem and by rearranging equation (13) and differentiating with respect to R_{CB} which yields

$$\frac{\partial R_{D,i}}{\partial R_{CB}} = 1 - t_B \left. \frac{d\hat{x}_{B,i}}{dR_{CB}} \right|_{\Gamma_{\hat{x}}=0} . \tag{28}$$

The long-run impact of an increase in R_{CB} can be calculated in the same way, applying the Implicit Function Theorem to the equations (22) and (26). In the case where $\lambda = 0$ and $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$, both banks and CBDC will have a positive market share. The properties of the response of $R_{D,i}$ to an increase in R_{CB} in this case are summarized in the proposition below for both the short-run and the long-run.

Proposition 6. *If $\lambda = 0$ and $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ then in the short-run equilibrium described by Proposition 4, $R_{D,i}$ will be strictly increasing in R_{CB} but at a rate less than 1.*
If $\lambda = 0$ and $R_{CB} \in (\underline{R}_{CB}, R^*_{CB}]$ then in the long-run equilibrium, the derivative of $R_{D,i}$ with respect to R_{CB} will always be lower than in the short-run equilibrium, holding R_{CB} fixed, and may no longer be strictly positive.

Proof. See Appendix. □

To better illustrate the response of the bank deposit rate $R_{D,i}$ as R_{CB} changes I present a numerical example. The model is solved numerically assuming some fixed parameters. For the interest rates I assume that the policy rate R_{RO} is set at 25 basis points, the interest rate on the liquidity facility R_{LF} is set at 50 basis points and the interest rate on the deposit facility is set to zero. I set the linear transport cost $t_B = 0.01$ and the fraction of depositors that move subject to the liquidity shock $\xi = 0.1$. The probability of the liquidity shock hitting I allow to take two values, $\lambda \in \{0, 0.05\}$ to illustrate the two cases with and without deposit liquidity. I set the number of banks to $N = 5$ and set R_L and L such that the free entry condition holds with $N = 5$ and $R_{CB} = 0$. Thus in the numerical examples I assume the economy starts in a long-run equilibrium with $q_{CB} = 0$ and $N = 5$.

When $\lambda = 0$, the response of $R_{D,i}$ to a change in R_{CB} is plotted in figure 1 for some example set of parameters. For $R_{CB} \leq \underline{R}_{CB}$, there is no change in $R_{D,i}$ as R_{CB} changes

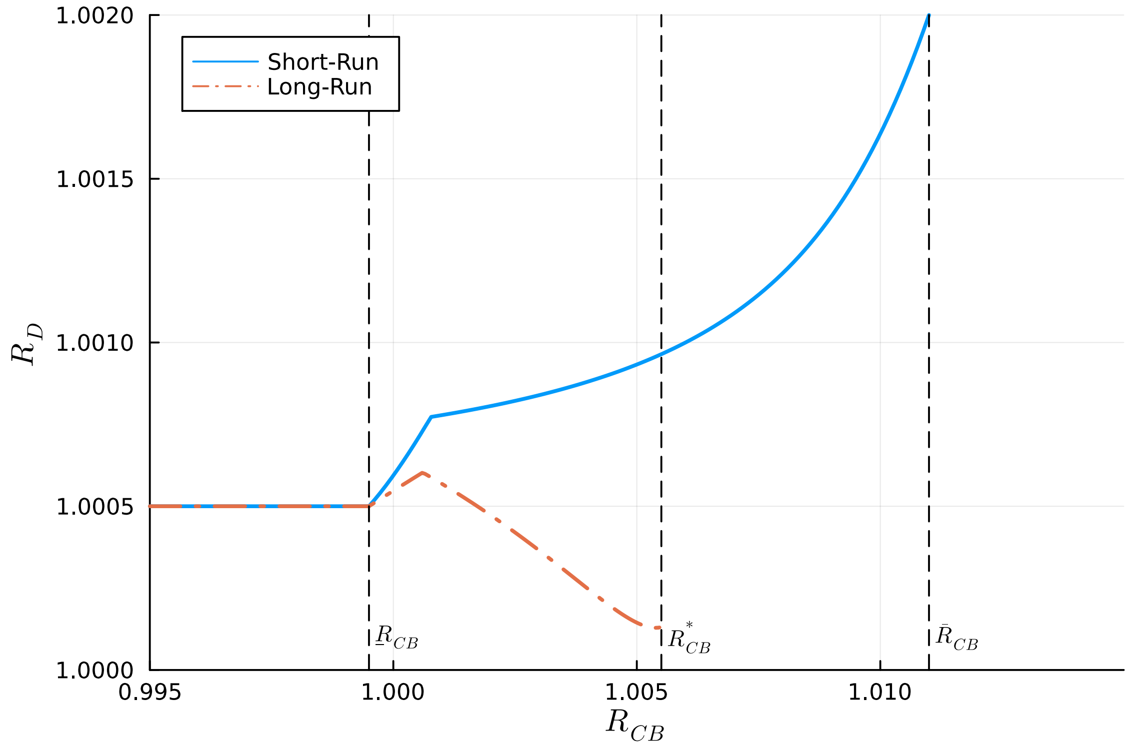

Figure 1: Impact of R_{CB} on deposit rate ($\lambda = 0$)

because the market share of CBDC is zero and at these low remuneration rates CBDC does not pose meaningful competition to bank deposits. As R_{CB} increases above $\underline{R}_{CB}$, $R_{D,i}$ is strictly increasing in the short-run. In the long-run, $R_{D,i}$ is increasing in R_{CB} only for R_{CB} above but sufficiently close to R_{CB}. As R_{CB} further increases, $R_{D,i}$ actually becomes decreasing in R_{CB} in the long-run. It should be noted that $R_{D,i}$ being decreasing in the long-run is a property of the numerical example only and need not hold for other parameterizations. What does always hold, as stated in Proposition 6 is that the long-run response of $R_{D,i}$ to an increase in R_{CB} will always be lower than in the short-run. The reason for this is that in the short-run, with the number of banks fixed, an increase in R_{CB} will lead to a fall in the market share of the banks with banks raising deposit rates in order to mitigate this loss of market share. The fall in market share and increase in cost of deposits leads to a fall in bank profits in the short-run which results in a consolidation of the banking sector in the long-run. As the number of banks falls in the long-run in response to an increase in R_{CB}, banks face less competition from their neighboring banks and thus may be able to reduce deposit rates even in the face of greater competition from CBDC.

Figure 1 also highlights two kinks that exist in the response of $R_{D,i}$ to R_{CB}. The first of these occurs at the point where $\hat{x}_{B,i} = x^*_{B,i}$, that is the point at which CBDC starts putting competitive pressure on bank deposits. The second kink occurs at the point where $\hat{x}_{B,i} = 0$ which is the point at which some proportion of depositors will choose

CBDC over bank deposit no matter how far away they are located from a bank. These kinks correspond to the intervals over which the piecewise continuous function for $q_{B,i}$ is defined by equation (16).

With $\lambda > 0$, the first part of Proposition 6 no longer holds. In the short-run, $R_{D,i}$ may no longer be monotonically increasing for $R_{CB} > \underline{R}_{CB}$. This result is summarized in the following proposition.

Proposition 7. *If $\lambda > 0$ and $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ and where there exists a short-run equilibrium, $R_{D,i}$ will be strictly increasing in R_{CB} only if $\lambda R_{IB}^{+}\xi$ is sufficiently small. If $\lambda R_{IB}^{+}\xi$ is sufficiently large, $R_{D,i}$ will be decreasing in R_{CB} for R_{CB} sufficiently high.*

Proof. See Appendix. □

Setting $\lambda = 0.05$, the response of $R_{D,i}$ to a change in R_{CB} is plotted in figure 2. Apart from λ, all other parameters remain the same as in figure 1. Figure 2 illustrates the case where the expected liquidity cost of deposits, $\lambda R_{IB}^{+}\xi$, is sufficiently large that even in the short-run, the deposit rate is decreasing in R_{CB} at sufficiently high levels of R_{CB}. In the previous case, with $\lambda = 0$, an increase in R_{CB} placed additional competitive pressure on the banking sector in the short-run and thus forced banks to raise the deposit rates they offer in order to compete for market share. In the case with $\lambda > 0$, increasing R_{CB} now has an additional effect which is to increase the cost of deposits for banks. This can be seen from equation (5) where the expected liquidity cost of deposits is increasing in the market share of CBDC and decreasing in the bank's own market share $q_{B,i}$. Thus if R_{CB} is sufficiently large, and the market share of CBDC is sufficiently large, banks may choose to lower their deposit rate in order to lower their own market share and thus lower the liquidity cost of deposits.

The results in this section have important policy implications. In particular regarding the use of the CBDC remuneration rate as an additional tool in the central bank's toolkit. Even in the most benign scenario where there is no liquidity risk and without considering the long-run impact on the banking sector, the pass-through of the CBDC remuneration rate to the bank deposit rate is imperfect. In this scenario banks do raise their deposit rates in response to increased competition from CBDC but as competition is imperfect they do so less than one-for-one. This section also highlights that if the central bank chose to use the CBDC remuneration rate as a policy tool, there may be long-run consequences on the banking sector which would serve to dampen the pass-through to the bank deposit rate. Finally, in the case where there is risk of liquidity flowing from bank deposits to a CBDC, the additional cost this imposes on banks further weakens the pass-through of the CBDC remuneration rate to the bank deposit rate.

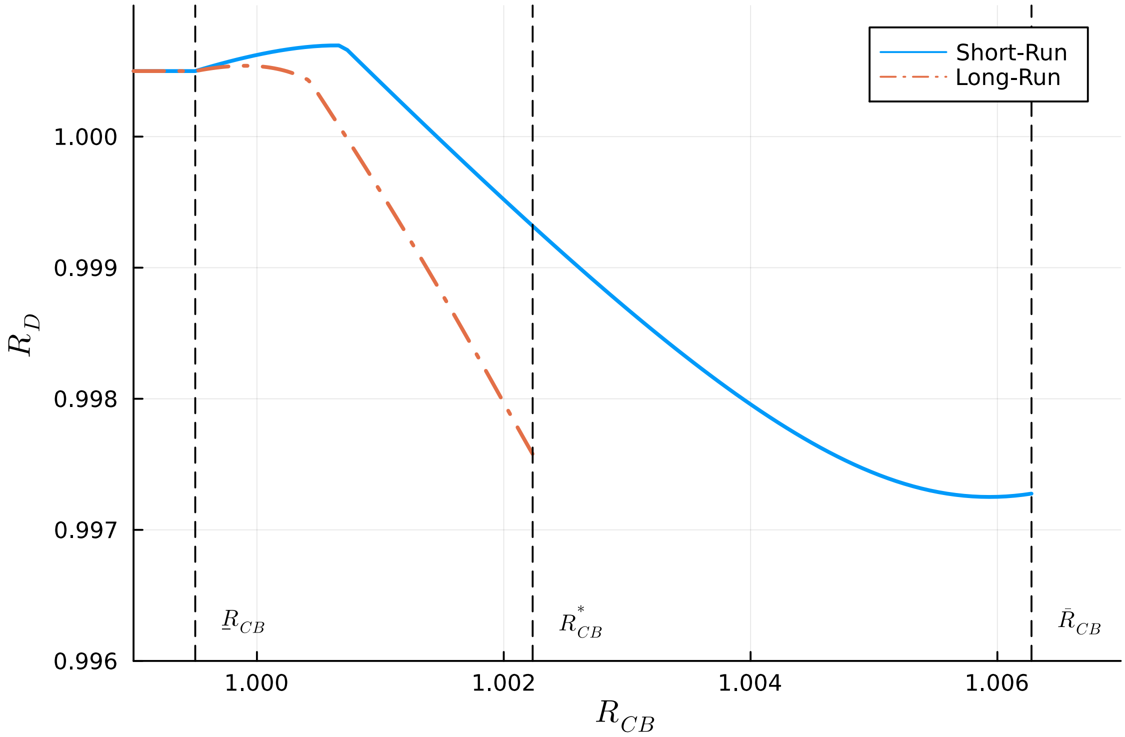

Figure 2: Impact of R_{CB} on deposit rate ($\lambda = 0.05$)

6 Implications for Monetary Policy Transmission

In this section, I consider the implication of CBDC for the transmission of monetary policy within the context of the model. To this end, I add some additional structure to the model in the following way. First, I assume that the spreads on the central bank standing facilities are held fixed and that the interest rate on the liquidity facility and on the deposit facility are of the form

$$R_{LF} = R_{RO} + \Delta_{LF}, \tag{29}$$

and

$$R_{DF} = R_{RO} - \Delta_{DF}, \tag{30}$$

with $\Delta_{LF} > 0$ and $\Delta_{DF} > 0$. Second, I assume that the interest rate on bank loans is equal to the policy rate plus a fixed mark-up such that

$$R_L = R_{RO} + \Delta_L, \tag{31}$$

with $\Delta_L > 0$. Finally, I assume that the central bank sets the remuneration rate of CBDC such that it is a fixed distance from the policy rate such that

$$R_{CB} = R_{RO} + \Delta_{CB}. \tag{32}$$

Here Δ_{CB} could be positive or negative depending on the central bank's CBDC policy. It should also be noted that this is just one possible remuneration policy that central banks could choose for CBDC. However, the remuneration policy considered here can be interpreted as the most neutral implementation of CBDC remuneration in the model. Other remuneration policies can be obtained through combining a change in the policy rate with a change in R_{CB}.

First, in the case where $q_{CB} = 0$, it follows from Proposition 2 and Proposition 3 that the deposit rate increases one-for-one with the policy rate. Turning now to the case where $q_{CB} > 0$, given the above assumptions on interest rates, the two key equations that determine the short-run and long-run equilibrium in the case where $q_{CB} > 0$, equations (22) and (26) can be rewritten respectively as

$$\tilde{\Gamma}_{\hat{x}} \equiv t_B \hat{x}_{B,i} + \Delta_{CB} + \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} + \left(\frac{\partial q_{CB}}{\partial R_{D,i}}\left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} + q_{CB}\right)\lambda\left(R_{RO} + \Delta_{IB}^{+}\right)\xi = 0, \tag{33}$$

and

$$\tilde{\pi}_{B,i} \equiv \Delta_L L - F + \left(1 + \frac{\partial q_{CB}}{\partial R_{D,i}}\lambda\left(R_{RO} + \Delta_{IB}^{+}\right)\xi\right)\left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i}^2 = 0, \tag{34}$$

where

$$\Delta_{IB}^{+} = \begin{cases} \Delta_{LF} & \text{if } R_{IB}^{+} = R_{LF} \\ -\Delta_{DF} & \text{if } R_{IB}^{+} = R_{DF}. \end{cases}$$

Finally, the deposit rate can be written in terms of R_{RO} as

$$R_{D,i} = t_B \hat{x}_{B,i} + R_{RO} + \Delta_{CB}. \tag{35}$$

The equilibrium with $q_{CB} > 0$ can be found as the solution to equations (33) and (34). In the case where $\lambda = 0$, both equations (33) and (34) do not respond to a change in the policy rate R_{RO} and thus the equilibrium value of $\hat{x}_{B,i}$ is also unchanged. Thus the deposit rate will increase one-for-one with the policy rate.

When $q_{CB} > 0$ and $\lambda > 0$, equations (33) and (34) now are impacted by changes in the policy rate R_{RO}. This in turn will impact the equilibrium value of $\hat{x}_{B,i}$ and thus the market share of CBDC will also be impacted by the policy rate. Thus the deposit rate will no longer increase one-for-one with the policy rate. Furthermore, as both equations are

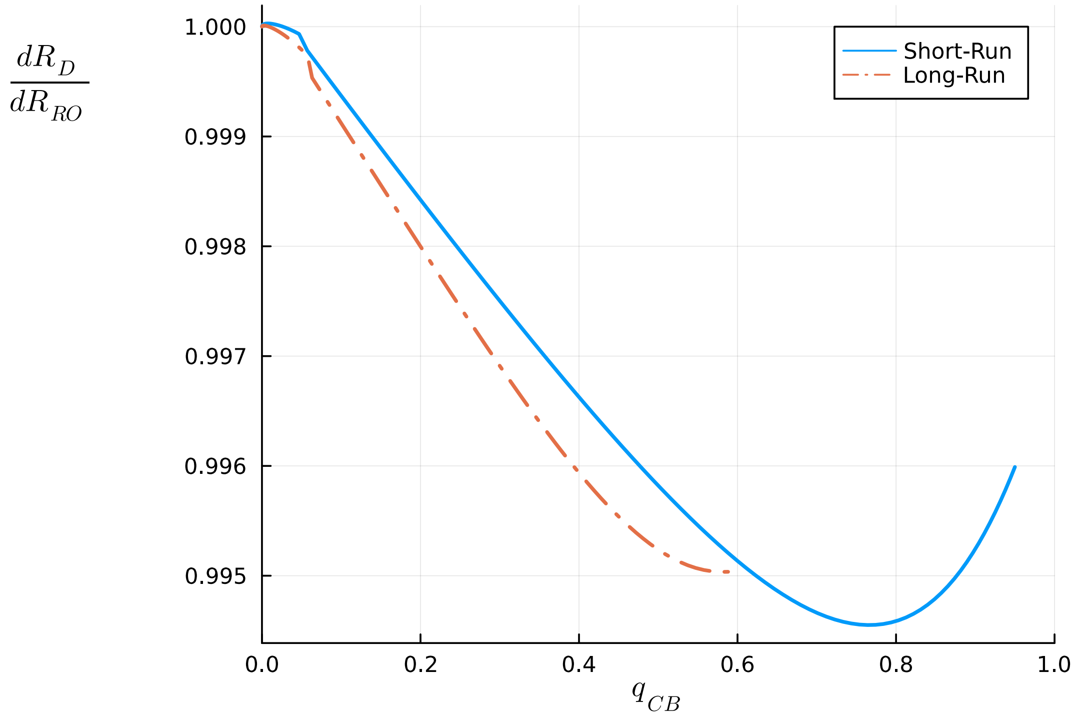

Figure 3: Impact of CBDC on Monetary Policy Transmission ($\lambda = 0.05$)

impacted by a change in the policy rate, the pass-through of the policy rate to the deposit rate will be different in the short-run and long-run. Figure 3 provides a numerical example of the policy rate pass-through to the deposit rate, $\frac{dR_{D,i}}{dR_{RO}}$ in the case where $\lambda = 0.05$. The parameterization remains the same as that used earlier in figure 2.

It is instructive to consider what generates the non-linear response of $\frac{dR_{D,i}}{dR_{RO}}$ to the increase in q_{CB}, especially in the short-run. From equation 33 it is clear that an increase in R_{RO} would lead to a change in $\hat{x}_{B,i}$ and hence each bank's share of the deposit market. Thus in the presence of liquidity shocks, the increase in the policy rate will have a similar impact on the deposit market as an increase in R_{CB}. The main mechanism occurs as an increase in R_{RO} will, if q_{CB} is sufficiently large, lead to a fall in $\hat{x}_{B,i}$ and hence through equation 35, a lower increase in the bank deposit rate, with $\frac{dR_{D,i}}{dR_{RO}} < 1$. Analogously to the result stated in proposition 7, the fall in $\hat{x}_{B,i}$ for a given increase in R_{RO} will vary with q_{CB}, hence the non-linear response of $\frac{dR_{D,i}}{dR_{RO}}$.

This section highlights a possible risk the introduction of CBDC poses for the transmission of monetary policy to the economy. In the model, monetary transmission happens solely through pass-through of the policy rate to the deposit rate offered by banks. In the case without deposit liquidity risk, a CBDC can be introduced without impacting the normal workings of monetary policy. However, if the banks face a liquidity risk in obtaining liquidity from retail deposits, this cost will be increasing in the deposit rate and this in turn will affect the transmission of monetary policy through the deposit rate. This occurs because the cost of this liquidity risk that banks face depends on the cost of obtaining additional liquidity through the central bank lending facility. The cost of obtaining this liquidity increases with the policy rate.

7 Conclusion

As the policy debate surrounding the potential introduction of a retail CBDC grows, so does the need for further analysis of its potential implications. This paper focuses on the impact of CBDC on the structure of the market for retail bank deposits and on bank liquidity. In this paper, CBDC is modeled as a source of direct competition for bank deposits. Competition in the deposit market is modeled using a Salop circle model and thus there is imperfect substitutability between deposits of different banks and the CBDC. This framework allows us to distinguish between the short-run impact of CBDC, where the number of banks is fixed from the long-run impact where the number of banks may adjust. In addition, the model suggests a liquidity risk channel through which CBDC may further raise the costs of banks operating in the deposit market.

Absent liquidity risk, the model suggests that in the short-run the introduction of CBDC will result in an increase in interest rates on bank deposits. This leads to a reduction of the market shares of banks in the deposit market. Banks substitute these deposits by obtaining additional liquidity from the central bank through open market operations and bank profitability falls. In the long-run, the model suggests that the introduction of CBDC will reduce the number of banks active in the deposit market and lead to greater concentration in the banking sector. In the long-run, the pass-through of the CBDC remuneration rate to the bank deposit rate is lower than in the short-run and the deposit rate may even be decreasing in the CBDC remuneration rate. This effect is amplified if banks face liquidity risk in holding deposits and in this case the bank deposit rate may be decreasing in the CBDC remuneration even in the short-run. Thus the paper casts doubt on the use of the remuneration rate of CBDC as an additional tool in the monetary policy toolkit of central banks.

The paper also highlights the importance of the liquidity risk channel for monetary policy transmission more generally. Absent liquidity risk, the bank deposit rate increases one-for-one following an increase in the policy rate, even following the introduction of a CBDC. However, if banks face liquidity risk in the deposit market introducing a CBDC will now impact monetary policy transmission through the bank deposit rate. Furthermore, the impact of monetary policy will now impact the structure of the deposit market and thus monetary policy will have a different impact in the short-run and long-run.

While this paper makes no claims regarding the welfare implications of the introduction of CBDC, it would be prudent for policymakers to take into account the welfare implications of a more concentrated banking sector that may follow the introduction of a CBDC as well as possible implications for the transmission of monetary policy.

References

Itai Agur, Anil Ari, and Giovanni Dell'Ariccia. Designing central bank digital currencies. *Journal of Monetary Economics*, 125:62–79, 2022.

David Andolfatto. Assessing the Impact of Central Bank Digital Currency on Private Banks. *The Economic Journal*, 131(634):525–540, March 2021.

Philippe Bacchetta and Elena Perazzi. CBDC as Imperfect Substitute for Bank Deposits: A Macroeconomic Perspective. Swiss Finance Institute Research Paper Series 21-81, Swiss Finance Institute, December 2021.

John Barrdear and Michael Kumhof. The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, 142(104148), 2022.

Ulrich Bindseil. Central bank digital currency: Financial system implications and control. *International Journal of Political Economy*, 48(4):303–335, 2019.

Michael D Bordo. Central bank digital currency in historical perspective: Another crossroad in monetary history. Working Paper 29171, National Bureau of Economic Research, August 2021.

Florian Böser and Hans Gersbach. Monetary policy with a central bank digital currency: The short and the long term. CEPR Discussion Papers 15322, C.E.P.R. Discussion Papers, September 2020.

Jan Bouckaert. Monopolistic competition with a mail order business. *Economics Letters*, 66(3):303–310, March 2000.

Markus K. Brunnermeier and Dirk Niepelt. On the equivalence of private and public money. *Journal of Monetary Economics*, 106:27–41, October 2019.

Monika Bucher, Achim Hauck, and Ulrike Neyer. Interbank market friction-induced holdings of precautionary liquidity: Implications for bank loan supply and monetary policy implementation. *Economic Theory*, 70(1):165–222, July 2020.

Lorenzo Burlon, Carlos Montes-Galdón, Manuel A. Muñoz, and Frank Smets. The optimal quantity of CBDC in a bank-based economy. Working Paper Series 2689, European Central Bank, July 2022.

Pierre-André Chiappori, David Perez-Castrillo, and Thierry Verdier. Spatial competition in the banking system: Localization, cross subsidies and the regulation of deposit rates. *European Economic Review*, 39(5):889–918, 1995.

Jonathan Chiu and Mohammad Davoodalhosseini. Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design. Staff Working Papers 21-63, Bank of Canada, December 2021.

Jonathan Chiu, Mohammad Davoodalhosseini, Janet Hua Jiang, and Yu Zhu. Bank market power and central bank digital currency: Theory and quantitative assessment. Staff Working Papers 19-20, Bank of Canada, May 2019.

Jesús Fernández-Villaverde, Daniel Sanches, Linda Schilling, and Harald Uhlig. Central bank digital currency: Central banking for all? *Review of Economic Dynamics*, December 2021.

Achim Hauck and Ulrike Neyer. A model of the Eurosystem's operational framework and the euro overnight interbank market. *European Journal of Political Economy*, 34: S65–S82, June 2014.

Katherine Ho and Joy Ishii. Location and competition in retail banking. *International Journal of Industrial Organization*, 29(5):537–546, 2011.

Todd Keister and Daniel R. Sanches. Should central banks issue digital currency? Working Papers 19-26, Federal Reserve Bank of Philadelphia, June 2019.

Michael Kumhof and Clare Noone. Central bank digital currencies – Design principles for financial stability. *Economic Analysis and Policy*, 71(C):553–572, 2021.

Paul Madden and Mario Pezzino. Oligopoly on a Salop Circle with Centre. *The B.E. Journal of Economic Analysis & Policy*, 11(1), January 2011.

Carmen Matutes and Xavier Vives. Competition for Deposits, Fragility, and Insurance. *Journal of Financial Intermediation*, 5(2):184–216, April 1996.

Jack Meaning, Ben Dyson, James Barker, and Emily Clayton. Broadening Narrow Money: Monetary Policy with a Central Bank Digital Currency. *International Journal of Central Banking*, 17(2):1–42, 2021.

Dirk Niepelt. Reserves for all? Central bank digital currency, deposits, and their (non)-equivalence. *International Journal of Central Banking*, 16(3):211–238, 2020.

Kwangwoo Park and George Pennacchi. Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation. *The Review of Financial Studies*, 22(1): 1–40, 05 2008.

Rafael Repullo. Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156–182, 2004. ISSN 1042-9573. Bank Capital Adequacy Regulation under the New Basel Accord.

Steven C. Salop. Monopolistic Competition with Outside Goods. *The Bell Journal of Economics*, 10(1):141, 1979.

Stephen Williamson. Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861, 2022.

Appendix

Proof of Proposition 1

By combining (5) and (6) the single bank's problem can be written as follows

$$\pi_i = \max_{B_i, R_{D,i}} \left\{ R_L L - R_{RO} B_i - R_{D,i} q_{B,i} - (1-\lambda) R_{IB}^0 \epsilon_i^0 - \lambda R_{IB}^+ \epsilon_i^+ - F \right\} \tag{A.36}$$

where ϵ_i^0 and ϵ_i^+ are defined by equations (2) and (3) respectively.

Differentiating with respect to B_i yields the following first-order condition

$$-R_{RO} + (1-\lambda) R_{IB}^0 + \lambda R_{IB}^+ = 0. \tag{A.37}$$

Given $\epsilon_i^0 < \epsilon_i^+$ it follows from equation (4) and that $R_{RO} \in (R_{DF}, R_{LF})$ that $R_{IB}^0 < R_{IB}^+$. There are two possible equilibria cases that are possible. Furthermore, as obtaining too much or too little liquidity from the central bank in $t = 2$ is costly, the following inequality constraints must hold, $\epsilon_i^0 \leq 0$ and $\epsilon_i^+ \geq 0$, with one of these inequality constraints holding with equality.

Thus there are two cases to consider. First, if $\epsilon_i^0 = 0$, then banks will hold exactly enough liquidity such that if they are in a neutral liquidity position if they do not receive a net outflow of liquidity. In this case, $R_{IB}^+ = R_{LF}$ and $R_{IB}^0 \in [R_{DF}, R_{LF})$. From equation (A.37) it follows that the value of R_{IB}^0 that ensures the first condition holds is

$$R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda} \right) (R_{LF} - R_{RO}), \tag{A.38}$$

and that for $R_{IB}^0 \geq R_{DF}$ it must be the case that

$$\lambda \leq \left(\frac{R_{RO} - R_{DF}}{R_{LF} - R_{DF}} \right). \tag{A.39}$$

Finally, for $\epsilon_i^0 = 0$ it follows from equation (2) that

$$B_i = L - q_{B,i}. \tag{A.40}$$

Thus the first part of the proposition has been obtained.

The second case to consider occurs if $\epsilon_i^+ = 0$ where banks hold exactly enough liquidity that they do not require any additional liquidity should they suffer a net outflow of liquidity. In this case, $R_{IB}^{0} = R_{DF}$ and $R_{IB}^{+} \in (R_{DF}, R_{LF})$. From equation (A.37) it follows that the value of R_{IB}^{+} that ensures the first condition holds is

$$R_{IB}^{-} = R_{DF} + \frac{1}{\lambda}\left(R_{RO} - R_{DF}\right), \tag{A.41}$$

and that for $R_{IB}^{+} < R_{LF}$ it follows that

$$\lambda > \left(\frac{R_{RO} - R_{DF}}{R_{LF} - R_{DF}}\right). \tag{A.42}$$

Finally, for $\epsilon_i^{+} = 0$ it follows from equation (3) that

$$B_i = L - \left(1 - q_{CB}\xi\right) q_{B,i}. \tag{A.43}$$

Thus the whole proposition has now been obtained.

Proof of Proposition 2

First, differentiating the bank's profit function given in equation (A.36) with respect to $R_{D,i}$ and combining with equation (A.37) gives the following first-order condition for the bank given by equation (20).

In the case where $q_{CB} = 0$, the demand function that the bank faces is given by equation (8) and thus

$$\frac{\partial q_{B,i}}{\partial R_{D,i}} = \frac{1}{t_B}, \tag{A.44}$$

while from equation (19) if $q_{CB} = 0$ then

$$\frac{\partial E\left[C_{D,i}\right]}{\partial q_{B,i}} = -R_{RO}, \tag{A.45}$$

and

$$\frac{\partial E\left[C_{D,i}\right]}{\partial q_{CB}} = 0. \tag{A.46}$$

In the short-run equilibrium with $q_{CB} = 0$, all banks have an equal market share and as there is full coverage, $q_{B,i} = {}^{1}\!/\!{}_{N}$. Combining the above with the first-order condition yields the following equation for the deposit rate in the short-run

$$R_{D,i} = R_{RO} - t_B \frac{1}{N}. \tag{A.47}$$

Finally, from equation (10), the distance from bank i where a depositor is indifferent between holding a bank i deposit and CBDC is $x^*_{B,i} = \frac{1}{2N}$. Thus, the largest possible remuneration rate on CBDC such that all depositors prefer bank deposits to CBDC, given by equation (11) can be written as

$$\underline{R}_{CB} = R_{RO} - \frac{3}{2} t_B \frac{1}{N}. \tag{A.48}$$

Proof of Proposition 3

In the long-run, N adjusts such that banks enter and make zero expected profits. The bank's profit function is given by equation (6). Substituting out the expected liquidity of deposits given by equation (19), the deposit rate given by equation (A.47) and the optimal funding decision as set out by proposition 1 yields the following equation for profit in the case where $q_{CB} = 0$

$$\pi_{B,i} = (R_L - R_{RO}) L - F + t_B \frac{1}{N^2}. \tag{A.49}$$

Denote the number of firms that drive bank profit to zero as N^*. Assuming that $F - (R_L - R_{RO}) L > 0$, a positive N^* exists and can be written as

$$N^* = t_B^{\frac{1}{2}} \left(F - (R_L - R_{RO}) L\right)^{-\frac{1}{2}}. \tag{A.50}$$

Thus, if $N^* \geq 2$, and $R_{CB} \leq R_{RO} - \frac{3}{2} t_B \frac{1}{N^*}$ there exists a long-run symmetric equilibrium with $q_{CB} = 0$ where banks compete until profits are driven to zero. The rest of the long-run equilibrium follows from proposition 2.

Proof of Proposition 4

First, note that in the case of $\lambda = 0$, equation (22) can be written as

$$\Gamma_{\hat{x}} \equiv t_B \hat{x}_{B,i} + R_{CB} - R_{RO} + \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} = 0, \tag{A.51}$$

where

$$\left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} = \begin{cases} t_B \left(\frac{2x^*_{B,i} - \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2}{1 + x^*_{B,i} - \hat{x}_{B,i}}\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ t_B \left(\frac{2x^*_{B,i} - \left(x^{*2}_{B,i} - 2\hat{x}_{B,i} x^*_{B,i}\right)}{1 + x^*_{B,i} + \hat{x}_{B,i}}\right) & \text{if } \hat{x}_{B,i} < 0. \end{cases} \tag{A.52}$$

Then note that in the limiting case when $R_{CB} \to \underline{R}_{CB} \equiv R_{RO} - \frac{3}{2} t_B \frac{1}{N}$, the solution to equation (A.51) is summarized by $\hat{x}_{B,i} = x^*_{B,i} = \frac{1}{2N}$ and the deposit rate follows immediately from equation (13). Next, note that by combining the largest possible value of R_{CB} set out by equation (12) with equation (13) yields the following lower-bound on $\hat{x}_{B,i}$

$$\hat{x}_{B,i} \geq -\left(1 - x^*_{B,i}\right) = -\left(1 - \frac{1}{2N}\right). \tag{A.53}$$

The solution to equation (A.51) at $\hat{x}_{B,i} = -\left(1 - \frac{1}{2N}\right)$ yields the following definition of $\bar{R}_{CB}$

$$\bar{R}_{CB} = R_{RO} + t_B \left(1 - \frac{3}{4N}\right). \tag{A.54}$$

For a unique short-run equilibrium for any $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ it is sufficient to show that $\Gamma_{\hat{x}}$ is monotonically increasing in $\hat{x}_{B,i}$. As the derivative is simply

$$\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} = \begin{cases} t_B \left(1 + \frac{2\left(2x^*_{B,i} - \hat{x}_{B,i}\right) + \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2}{\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right)^2}\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ t_B \left(1 + \frac{3x^{*2}_{B,i}}{\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right)^2}\right) & \text{if } \hat{x}_{B,i} < 0, \end{cases} \tag{A.55}$$

it is straightforward to note that $\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} > 0$ and thus $\Gamma_{\hat{x}}$ is monotonically increasing in $\hat{x}_{B,i}$.

Given there exists an equilibrium value of $\hat{x}_{B,i}$ such that equation(A.51) is satisfied and in the short-run $x^*_{B,i} = \frac{1}{2N}$, it follows that the deposit rate is simply $R_{D,i} = t_B \hat{x}_{B,i} + R_{CB}$ and $q_{B,i}$ and q_{CB} are given by equations (16) and (17) respectively. Also note that for any $\hat{x}_{B,i} < x^*_{B,i}$, $q_{CB} > 0$.

Proof of Lemma 1

First note that with $\lambda = 0$, the free entry condition given by equation (26) can be written as

$$\pi_{B,i} \equiv \left(R_L - R_{RO}\right) L - F + \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q^2_{B,i} = 0. \tag{A.56}$$

The derivative of equation (A.56) with respect to $x^*_{B,i}$ is

$$\frac{\partial \pi_{B,i}}{\partial x^*_{B,i}} = \begin{cases} t_B q_{B,i} \left(\frac{4 - 3\left(x^*_{B,i} - \hat{x}_{B,i}\right)^2 - 2x^*_{B,i}}{\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right)^2}\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ t_B q_{B,i} \left(\frac{\left(3 - 4\left(x^{*2}_{B,i} - 2\hat{x}_{B,i} - \hat{x}^2_{B,i}\right)\right) + \left(1 - q_{B,i}\right)}{\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right)^2}\right) & \text{if } \hat{x}_{B,i} < 0, \end{cases}$$

It is clear that this derivative will be positive for any $\hat{x}_{B,i} \geq 0$. For the case where $\hat{x}_{B,i} < 0$, and noting that the lower-bound on $\hat{x}_{B,i}$ is $-\left(1 - x^*_{B,i}\right)$, it follows that the term $\left(x^{*2}_{B,i} - 2\hat{x}_{B,i} - \hat{x}^2_{B,i}\right)$ is decreasing in $\hat{x}_{B,i}$ and thus the largest value this term can take be is $x^{*2}_{B,i}$. Thus for any $N \geq 2$ with $\lambda = 0$ it follows that the derivative of equation $\frac{\partial \pi_{B,i}}{\partial x^*_{B,i}} > 0$ and thus $\frac{\partial \pi_{B,i}}{\partial N} < 0$.

Now note that the derivative of equation with respect to $\hat{x}_{B,i}$ is

$$\frac{\partial \pi_{B,i}}{\partial \hat{x}_{B,i}} = \begin{cases} t_B \left(\frac{q_{B,i}\left(3\left(x^*_{B,i} - \hat{x}_{B,i}\right)^2 + 6x^*_{B,i} - 4\hat{x}_{B,i}\right)}{\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right)^2} \right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ t_B \left(\frac{x^*_{B,i} q_{B,i}\left(2\hat{x}_{B,i} + x^*_{B,i} + 2\right)}{\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right)^2} \right) & \text{if } \hat{x}_{B,i} < 0. \end{cases}$$

Thus for any $N \geq 2$ with $\lambda = 0$, it follows that $\frac{\partial \pi_{B,i}}{\partial \hat{x}_{B,i}} > 0$ and thus $\frac{\partial \pi_{B,i}}{\partial R_{CB}} < 0$.

If $t_B^{\frac{1}{2}} \left(F - \left(R_L - R_{RO}\right) L\right)^{-\frac{1}{2}} > 2$ it follows from Proposition 3 that at $\hat{x}_{B,i} = x^*_{B,i}$ with $N = 2$ bank profit will be strictly positive. As bank profit will be maximized at $N = 2$, if at $N = 2$ and $R_{CB} = R_{RO} + \frac{5}{8} t_B$ bank profit is positive, then there will exist an equilibrium over the entire range $R_{CB} \in \left[R_{RO} - \frac{3}{4} t_B, R_{RO} + \frac{5}{8} t_B\right]$. If on the other hand at $N = 2$ the banks make negative profit, then there exists some $R^*_{CB} \in \left(R_{RO} - \frac{3}{4} t_B, R_{RO} + \frac{5}{8} t_B\right)$ such that with $N = 2$ bank profit is zero.

Proof of Proposition 5

First note that assuming $t_B^{\frac{1}{2}} \left(F - \left(R_L - R_{RO}\right) L\right)^{-\frac{1}{2}} > 2$, there exists an equilibrium at $\hat{x}_{B,i} = x^*_{B,i}$ with $N > 2$ which corresponds to the long-run equilibrium described in Proposition 3. Next, by Lemma 1 that there exists a $\hat{x}_{B,i} < x^*_{B,i}$ where there exists an equilibrium such that $R_{CB} = R^*_{CB}$ with $N^* \geq 2$. To show the existence of a short-run equilibrium for any $R_{CB} \in (\underline{R}_{CB}, R^*_{CB}]$ it is sufficient to show that the equilibrium $\hat{x}_{B,i}$ is monotonically decreasing in R_{CB}. This can be shown by applying the Implicit Function Theorem to the system of equilibrium equations, equation (A.51) and (A.56).

In particular, note that we require

$$\frac{\partial \hat{x}_{B,i}}{\partial R_{CB}} = - \frac{\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial R_{CB}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial R_{CB}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix}}{\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix}} < 0, \tag{A.57}$$

where

$$\frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} = \begin{cases} t_B \left(\frac{\left(1-2x^*_{B,i}\right)+\left(1-\left(x^*_{B,i}-\hat{x}_{B,i}\right)^2\right)}{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^2} \right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ t_B \left(\frac{2+\left(4\hat{x}_{B,i}-2x^*_{B,i}\right)+\left(2\hat{x}^2_{B,i}-2\hat{x}_{B,i}x^*_{B,i}-x^*_{B,i}{}^2\right)}{\left(1+x^*_{B,i}+\hat{x}_{B,i}\right)^2} \right) & \text{if } \hat{x}_{B,i} < 0, \end{cases} \tag{A.58}$$

$$\frac{\partial \Gamma_{\hat{x}}}{\partial R_{CB}} = 1, \tag{A.59}$$

$$\frac{\partial \pi_{i,B}}{\partial R_{CB}} = 0. \tag{A.60}$$

and the other elements of equation (A.57) are given in the proof of Lemma 1 above. First note that

$$\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial R_{CB}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial R_{CB}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix} = \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}}, \tag{A.61}$$

and thus it is sufficient to show that

$$\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix} = \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} - \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} > 0. \tag{A.62}$$

To simplify this note that the above equation can be rewritten as

$$\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix} = \left[\frac{\partial q_{B,i}}{\partial R_{D,i}} \right]^{-1} q_{B,i} \left(\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} \frac{\partial q_{i,B}}{\partial x^*_{B,i}} - \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \frac{\partial q_{i,B}}{\partial \hat{x}_{B,i}} \right). \tag{A.63}$$

and given $\left[\frac{\partial q_{B,i}}{\partial R_{D,i}} \right]^{-1} q_{B,i} > 0$ a sufficient condition for the determinant to be positive is that

$$\left(\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} \frac{\partial q_{i,B}}{\partial x^*_{B,i}} - \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \frac{\partial q_{i,B}}{\partial \hat{x}_{B,i}} \right) > 0. \tag{A.64}$$

First note that in the case where $\hat{x}_{B,i} \geq 0$

$$\begin{aligned} \det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix} &= t_B \left(1 + \frac{2\left(2x^*_{B,i} - \hat{x}_{B,i}\right) + \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2}{\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right)^2} \right) 2\left(1 - x^*_{B,i} + \hat{x}_B\right) \\ &- t_B \left(\frac{\left(1 - 2x^*_{B,i}\right) + \left(1 - \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2\right)}{\left(1 + x^*_{B,i} - \hat{x}_{B,i}\right)^2} \right) 2\left(x^*_{B,i} - \hat{x}_B\right) \end{aligned} \tag{A.65}$$

which can be rewritten as

$$\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} \frac{\partial q_{i,B}}{\partial x^*_{B,i}} - \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \frac{\partial q_{i,B}}{\partial \hat{x}_{B,i}} = \frac{2t_B}{(1+\tilde{x})^2} \left[(1+\tilde{x})^2 (1-\tilde{x}) + q_{i,B} \right], \tag{A.66}$$

where $\tilde{x} \equiv x^*_{B,i} - \hat{x}_{B,i}$ and for $N \geq 2$, $\tilde{x} \in \left[0, \frac{1}{4}\right]$ and this is strictly positive over the entire range of $\tilde{x}$.

Next note that in the case where $\hat{x}_{B,i} \in \left[-\left(1 - x^*_{B,i}\right), 0\right)$

$$\det \begin{pmatrix} \frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial \Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial \pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial \pi_{i,B}}{\partial x^*_{B,i}} \end{pmatrix} = \frac{2t_B\left(1 + x^*_{B,i}\right)^2\left(1 - x^*_{B,i} + \hat{x}_B\right)}{\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right)^2} + \frac{2t_B\left[x^{*2}_{B,i}\left(4 - 3x^*_{B,i}\right) - \hat{x}_{B,i}x^*_{B,i}\left(2 - 4x^*_{B,i}\right)\right]}{\left(1 + x^*_{B,i} + \hat{x}_{B,i}\right)^2}, \tag{A.67}$$

which again is strictly positive given $x^*_{B,i} \in \left[0, \frac{1}{4}\right]$ and we have shown that the determinant of the system of equations is always strictly positive.

Proof of Proposition 6

In the short-run with $\lambda = 0$ and $R_{CB} \in \left(\underline{R}_{CB}, \bar{R}_{CB}\right]$ there exists a short-run equilibrium described by Proposition 4. As described by the derivative of $R_{D,i}$ with respect to R_{CB} can be written as

$$\left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma_{\hat{x}}=0} = 1 - t_B \left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma_{\hat{x}}=0}. \tag{A.68}$$

Through the Implicit Function Theorem, the latter term can be solved as

$$\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma_{\hat{x}}=0} = -\frac{\partial \Gamma_{\hat{x}}/\partial R_{CB}}{\partial \Gamma_{\hat{x}}/\partial \hat{x}_{B,i}} \tag{A.69}$$

which can be written as

$$\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma_{\hat{x}}=0} = \begin{cases} -\frac{1}{t_B}\left(\frac{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^2}{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^2+2\left(2x^*_{B,i}-\hat{x}_{B,i}\right)+\left(x^*_{B,i}-\hat{x}_{B,i}\right)^2}\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ -\frac{1}{t_B}\left(\frac{\left(1+x^*_{B,i}+\hat{x}_{B,i}\right)^2}{\left(1+x^*_{B,i}+\hat{x}_{B,i}\right)^2+3x^{*2}_{B,i}}\right) & \text{if } \hat{x}_{B,i} < 0, \end{cases} \tag{A.70}$$

where we note that $-\frac{1}{t_B} < \left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma_{\hat{x}}=0} < 0$ and thus combining this with equation (28) yields the following property

$$0 < \frac{\partial R_{D,i}}{\partial R_{CB}} < 1. \tag{A.71}$$

Thus in the short-run, with $\lambda = 0$ and increase in R_{CB} will increase $R_{D,i}$ but by less than the increase in R_{CB}.

In the long-run with $\lambda = 0$ and $R_{CB} \in (\underline{R}_{CB}, R^*_{CB}]$ there exists a long-run equilibrium described by Proposition 5. In a similar way to above, the derivative of $R_{D,i}$ with respect to R_{CB} can be written as

$$\left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma=0} = 1 - t_B \left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma=0}, \tag{A.72}$$

where through the Implicit Function Theorem,

$$\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma=0} = -\frac{\det\begin{pmatrix}\frac{\partial\Gamma_{\hat{x}}}{\partial R_{CB}} & \frac{\partial\Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial\pi_{i,B}}{\partial R_{CB}} & \frac{\partial\pi_{i,B}}{\partial x^*_{B,i}}\end{pmatrix}}{\det\begin{pmatrix}\frac{\partial\Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial\Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial\pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial\pi_{i,B}}{\partial x^*_{B,i}}\end{pmatrix}}. \tag{A.73}$$

Using equations (A.61) the above equation can be written as

$$\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma=0} = -\left(\frac{\partial\Gamma_{\hat{x}}}{\partial\hat{x}_{B,i}} - \frac{\partial\Gamma_{\hat{x}}}{\partial x^*_{B,i}}\frac{\partial\pi_{i,B}}{\partial\hat{x}_{B,i}}\left[\frac{\partial\pi_{i,B}}{\partial x^*_{B,i}}\right]^{-1}\right)^{-1}. \tag{A.74}$$

It follows from the property that $\det\begin{pmatrix}\frac{\partial\Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} & \frac{\partial\Gamma_{\hat{x}}}{\partial x^*_{B,i}} \\ \frac{\partial\pi_{i,B}}{\partial \hat{x}_{B,i}} & \frac{\partial\pi_{i,B}}{\partial x^*_{B,i}}\end{pmatrix} > 0$, which was shown in the previous section of this Appendix, and that $\frac{\partial\Gamma_{\hat{x}}}{\partial x^*_{B,i}} > 0$ and $\frac{\partial\pi_{i,B}}{\partial\hat{x}_{B,i}} > 0$ that

$$\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma=0} < \left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma_{\hat{x}}=0}, \tag{A.75}$$

and thus

$$\left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma_{\hat{x}}=0} > \left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma=0}. \tag{A.76}$$

Furthermore, $\left.\frac{d\hat{x}_{B,i}}{dR_{CB}}\right|_{\Gamma=0}$ may now be smaller than $-\frac{1}{t_B}$, thus $\left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma=0}$ may no longer be strictly positive.

Proof of Proposition 7

First, note that with $\lambda > 0$, equation (22) can be written as

$$\Gamma_{\hat{x}} \equiv t_B\hat{x}_{B,i} + R_{CB} - R_{RO} + \left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i} + \Upsilon_{\hat{x}}\lambda R^+_{IB}\xi = 0, \tag{A.77}$$

where

$$\Upsilon_{\hat{x}} = \begin{cases} \frac{1}{2x^*_{B,i}}\left(x^*_{B,i} - \hat{x}_{B,i}\right)^2 - 2\left(\frac{x^*_{B,i}-\hat{x}_{B,i}}{1+x^*_{B,i}-\hat{x}_{B,i}}\right)\left(2x^*_{B,i} - \left(x^*_{B,i} - \hat{x}_{B,i}\right)^2\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ \frac{1}{2}x^*_{B,i} - \hat{x}_{B,i} - 2x^*_{B,i}\frac{\left(2x^*_{B,i}-x^{*2}_{B,i}+2\hat{x}_{B,i}x^*_{B,i}\right)}{1+x^*_{B,i}+\hat{x}_{B,i}} & \text{if } \hat{x}_{B,i} < 0. \end{cases} \tag{A.78}$$

In the case of $\lambda > 0$, as $R_{CB} \to \underline{R}_{CB}$, in the equilibrium $\hat{x}_{B,i} \to x^*_{B,i}$ and $\Upsilon_{\hat{x}} \to 0$, thus the derivative properties from the previous section of the Appendix will hold. In general, the derivative in the short-run will depend on

$$\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} = \frac{\partial}{\partial \hat{x}_{B,i}}\left\{\left[\frac{\partial q_{B,i}}{\partial R_{D,i}}\right]^{-1} q_{B,i}\right\} + \frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}}\lambda R^+_{IB}\xi. \tag{A.79}$$

The first term is greater than t_B and it can be noted from before that

$$\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}} > t_B \iff \left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma_{\hat{x}}=0} > 0. \tag{A.80}$$

Thus if $\frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}}\lambda R^+_{IB}\xi$ is positive or if it is negative but sufficiently small in absolute magnitude, then the short-run response of $R_{D,i}$ to an increase in R_{CB} will remain positive. However, this will depend in a large part on the properties of $\frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}}$ as well as the magnitude of $\lambda R^+_{IB}\xi$.

First, note that

$$\frac{\partial \Upsilon_{\hat{x}}}{\partial x^*_{B,i}} = \begin{cases} \frac{1}{2}\left(8\left(x^*_{B,i} - \hat{x}_{B,i}\right) - 11 - \left(\frac{\hat{x}_{B,i}}{x^*_{B,i}}\right)^2 + \frac{12-8\hat{x}_{B,i}}{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^2}\right) & \text{if } x^*_{B,i} \geq \hat{x}_{B,i} \geq 0 \\ \frac{4+\hat{x}_{B,i}\left(\hat{x}_{B,i}\left(6\hat{x}_{B,i}+14\right)+12\right)}{\left(1+x^*_{B,i}+\hat{x}_{B,i}\right)^2} - 4\hat{x}_{B,i} - 3.5 & \text{if } \hat{x}_{B,i} < 0, \end{cases} \tag{A.81}$$

where it should be noted that

$$\lim_{\hat{x}_{B,i}\to 0^+} \frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}} < 0 \tag{A.82}$$

and

$$\frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}} < 0 \ \forall \hat{x}_{B,i} < 0. \tag{A.83}$$

Finally, note that

$$\frac{\partial^2 \Upsilon_{\hat{x}}}{\partial \hat{x}^2_{B,i}} = \begin{cases} 4 + \frac{1}{x^*_{B,i}} + 4\frac{1-2x^*_{B,i}}{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^3} > 0 & \text{if } \hat{x}_B \geq 0 \\ \frac{4x^*_{B,i}\left(3x^{*2}_{B,i}+2x^*_{B,i}+1\right)}{\left(1+x^*_{B,i}-\hat{x}_{B,i}\right)^3} > 0 & \text{otherwise.} \end{cases} \tag{A.84}$$

Given $\frac{\partial \Upsilon_{\hat{x}}}{\partial \hat{x}_{B,i}}$ is increasing in $\hat{x}_{B,i}$ and negative for sufficiently small values of $\hat{x}_{B,i}$, if $\lambda R^{+}_{IB}\xi$ is sufficiently large, then at high levels of R_{CB}, when $\hat{x}_{B,i}$ is small, $\frac{\partial \Gamma_{\hat{x}}}{\partial \hat{x}_{B,i}}$ will be less than t_B and thus $\left.\frac{\partial R_{D,i}}{\partial R_{CB}}\right|_{\Gamma_{\hat{x}}=0} < 0$.