Banking regulation and collateral screening in a model of information asymmetry*

Benjamin Hemingway†

April 18, 2019

Abstract

The role of collateral in debt contracts is explored within an environment of aggregate risk and banking regulation. I model a credit market with imperfect information and aggregate uncertainty. Banks have limited liability and do not endogenize the value of insured deposits, creating a problem of over-lending. This inefficiency can be corrected through banking regulation. I consider two regulatory tools, capital requirements and a regulatory stress-test. The stress-test provides a dual role for collateral. First, it can help mitigate the adverse selection problem by acting as a screening device. Second it also helps the bank satisfy any regulatory constraint by reducing the loss given default that the bank suffers in bad states of the world. Banking regulation impacts both the loan terms and the type of equilibrium, if firms do not have sufficient collateral, a pooling equilibrium exists.

*The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of Lithuania.

†CEFER, Bank of Lithuania and Vilnius University (bhemingway@lb.lt).

1 Introduction

Following the Financial Crisis of 2007-2008, regulators and policy makers have increased their focus on ensuring stability in the banking sector. One key tool at the regulator's disposal is stress testing, which has become more widely used by regulators since the financial crisis. The US led the way on stress testing, with one of the first post-crisis stress tests being the US Supervisory Capital Assessment Program (SCAP) which was conducted by the Federal Reserve early in 2009. The SCAP and its successor, the Comprehensive Capital Analysis and Review (CCAR) aim to ensure that the largest US banks have sufficient capital to survive the stress scenario. One important innovation of the SCAP and CCAR is that the results were publicly disclosed on a bank-by-bank basis. This creates a strong incentive, in terms of ensuring market confidence, for the banks to ensure they have sufficient policies and capital in place to ensure they are able to pass the stress test.

There is some evidence to suggest that the use of these stress tests by the Federal Reserve, and other similar stress testing programs used by other banking regulators may have negative consequences. For example, Acharya et al. (2018), focusing on lending to large firms in the US, find that stress-tested banks tend to reduce the quantity of loans supplied to firms and tend to increase borrowing rates. Similarly, Cortés et al. (2018) complement this by documenting similar negative effects of stress-testing on small business loans. Specifically, they provide evidence that stress tests conducted under the CCAR led to a decrease in affected banks' credit supply to small business. An overview of the recent history of stress testing in the financial sector can be found in Dent et al. (2016).

This paper constructs a theoretical framework to highlight the role that regulatory stress-tests can play in the regulation of banks. I propose an adverse selection credit market model with aggregate uncertainty where firms operate a decreasing returns to scale production function and have private information regarding the riskiness of their project. In order to produce, firms obtain loans from banks and are able to mitigate the adverse selection problem by pledging collateral to the banks.

The banking sector features perfect competition and limited liability implying that banks may default following the realization of a bad aggregate state. Banks have two sources of funding, insured deposits and costly equity. In the absence of regulatory intervention, banks do not fully internalize the deposit insurance and will over-supply capital and undervalue collateral.

The model allows for two regulatory interventions which are used to mitigate the over-supply of capital. The first, is a minimum capital requirement that mandates banks to fund a fraction of their loans using costly bank equity as opposed to insured deposits. The second, a regulatory stress-test, constrains the losses banks are allowed to make conditional on the realization of a bad aggregate state. Collateral in this model plays two roles, it resolves the adverse selection problem by acting as a screening device, while also helping the bank satisfy any regulatory constraint by reducing the loss given default that the bank suffers in the bad aggregate state.

The paper has three main theoretical results. First, given the assumptions set out in the paper, the regulatory stress-test increases the required collateral on loans is positive even in the full-information case. Second, I show that in the asymmetric information case, while cross-subsidization cannot take place when there is sufficient collateral available, if the upper-bound on collateral binds both cross-subsidzation and pooling equilibria can exist. Third, I characterize optimal policy and I show that if sufficient collateral is available, the first-best outcome if the regulator mandates loans must be fully collateralized using the regulatory stress-test while if there is insufficient collateral it is optimal for the regulator to set a less stringent stress-test and loans in this case are not fully collateralized.

This paper is directly related to the literature on adverse selection in credit markets. Papers that focus on the use of collateral as a screening device in credit markets featuring adverse selection include papers such as Stiglitz and Weiss (1981) and Lacker (2001). The possibility of pooling and credit rationing equilibria were raised by Acharya et al. (2018)and Clemenz (1993). This paper also complements the empirical literature on the impact of regulatory stress-testing on banks such as Acharya et al. (2018) and Cortés et al. (2018) by providing a theoretical mechanism through which more stringent regulation can impact lending outcomes.

2 Model

2.1 Firms and Technology

Consider a one period credit model. There is a continuum of firms, each have access to a risky technology. Firms receive a known end-of-period endowment W. The timing of the endowment means firms cannot use the endowment to invest in a project but are able to pledge this wealth as collateral. Firms must obtain funding from a bank if they are to invest in their technology. Both firms and banks are risk neutral.

The technology operated by the firm features decreasing returns to scale and generates either a zero cash flow or a positive cash flow. There are two discrete firm types which are indexed by $i \in \{L, H\}$. The fraction of firms of type i is denoted by $\mu_i \in (0, 1)$ and $\sum_i \mu_i = 1$.The distribution of firms in the economy is public information. The probability a firm's project is successful is denoted by p_i with $p_L > p_H$ implying that H-type firms are high risk firms and feature a lower probability of success than low risk (L-type) firms.

The expected cash flow of firm i from investing k_i at the beginning of the period is $E[y_i(k)] = p_i \varphi_i k^\alpha$ where $\alpha \in (0,1)$, p_i is the probability the project is successful and φ_i is a productivity parameter. I assume that the two firm types have the same expected cash flow such that $p_i \varphi_i = \bar{\varphi}$. The firm's risk type i is private information, known only to the firm.

In addition to the firm type, the probability of a project being successful also depends on the realization of an aggregate state $z \in \{z_B, z_G\}$. The aggregate state z_G occurs with probability q and z_B with probability $1-q$. I denote the probability of firm i's project being successful conditional on z as $p_i(z)$. The probability of a project being successful is higher in the 'good' state (z_G) than in the 'bad' state (z_b).

$$p_i(z_G) > p_i(z_B) \quad \forall i \in \{L, H\} \tag{1}$$

The expected probability of firm i's project being successful can be written as follows

$$p_i = q p_i(z_G) + (1-q) p_i(z_B) \tag{2}$$

To simplify the analysis, I assume that the ratio of success probabilities conditional on z_G and z_B is the same across firm types such that

$$\frac{p_i(z_B)}{p_i(z_G)} = \xi \quad \forall i \in \{L, H\} \tag{3}$$

It follows from equation (1) that $\xi \in (0,1)$.

For purposes of exposition I shall refer from now on to type H firms as 'high risk' firms and type L firms as 'low risk' firms. Similarly, I will refer to z_B as the bad aggregate state and z_G as the good aggregate state.

2.2 Banking Sector

Banks are risk neutral and competitive and have access to insured depositors who earn a risk-free return. For simplicity, this return is normalized to 1. Banks have limited liability and default on depositors if their proceeds from lending are not sufficient to repay depositors. If banks default, depositors are compensated by the government. The government does not charge banks an insurance premium but instead funds the deposit insurance by a lump-sum tax on firm profits. Importantly, banks do not endogenize the cost of this insurance. To avoid the case where banks never repay depositors, I assume that in equilibrium banks will only ever default in the bad aggregate state.

They are also able to raise funds through bank capital, though this is costly. Holders of bank capital must be compensated by an expected return ψ that is assumed to be constant and larger than 1. The cost of issuing capital captures in a somewhat reduced form way, the agency costs involved in issuing equity.

2.3 Financial Contracts

Credit contracts between banks and firms of type i are denoted by the triple $\{k_i, R_i, C_i\}$ where $R_i \geq 0$ is the interest rate charged to the firm by the bank, $k_i \geq 0$ is the size of the loan and $C_i \in [0, W]$ is the amount of collateral sacrificed by the firm if it chooses to default on the payment $R_i k_i$. The end-of period endowment forms the upper-bound on the amount of pledgable collateral, while non-negative collateral is ruled out by assumption. Firms have limited liability and pay the payment $R_i k_i$ only if the project is successful. The bank is unable to stake a claim on the firm's end-of-period endowment W unless this is agreed upon between the two parties beforehand.

It is assumed that any contracts made between the bank and the firm cannot be contingent on the aggregate state. To motivate this assumption, consider the idea that the bank only learns of the aggregate state through defaults within its own loan book, if loans do not all default simultaneously, the bank is unable to enforce state-contingent payoffs to those served first. In order to treat all borrowers within a period equally, the bank must choose contracts that are not contingent on the aggregate state. This assumption and its motivation is similar to the sequential service assumption made in the bank run literature as discussed in Allen and Gale (2009).

The expected profit firm i receives from a loan $\{k_i, R_i, C_i\}$ is:

$$U_i^F (k_i, R_i, C_i) = p_i [\varphi_i k_i^\alpha - R_i k_i] - (1 - p_i) C_i + W \tag{4}$$

The bank must also determine how it is to finance itself. Thus it must choose the level of deposits$d_i \in [0, k_i]$ to issue, with any remaining part of the loan financed through the issuance of bank capital. The expected profit of a bank is denoted by

$$\begin{aligned} U_i^B (d_i, k_i, R_i, C_i) = q (p_i (z_G) R_i k_i + (1 - p_i (z_G)) C_i - d_i) - \psi (k_i - d_i) \\ + (1 - q) \max \{p_i (z_B) R_i k_i + (1 - p_i (z_B)) C_i - d_i, 0\} \end{aligned} \tag{5}$$

Some features of the bank profit function are worth pointing out. First, it is assumed that if the bank defaults it will only do so in the bad aggregate state. Also, the price of issuing bank capital adjust so that in expectation, the marginal cost is ψ and as such, banks must make a sufficient return to repay the cost of issuing bank capital. Deposits on the other hand are fully insured and banks do not need to compensate depositors should they default in the bad aggregate state.

2.4 Banking Regulation

In this model, contracts are set before the realization of and are not contingent on the aggregate state. Thus, if the loan is not fully collateralized, the loan is risky and the return is correlated with the aggregate state z. The bank may not have sufficient funds to repay the depositors in the bad aggregate state, in which case the government will be required to tax (lump sum) firms in order to compensate depositors. In addition banks do not endogenize the cost of default when competing for loan contracts and as will be shown later this results in banks providing larger loans than is optimal.

The government may wish to intervene in order to mitigate these issues and can do so in two ways. First, the government can mandate a minimum capital requirement for the banks, requiring a proportion $\underline{e} \in [0, 1]$ of loans to be funded by bank capital and not deposits. This imposes the following constraint on loan contracts

$$d_i \leq (1 - \underline{e})\, k_i \tag{6}$$

Second, the government can require the bank to pass a regulatory stress-test. I model a regulatory stress-test as the requirement that banks raise at least a fraction $\gamma \in [0, 1]$ of its deposits in the bad aggregate state.

$$\sum_i \mu_i \left[p_i\left(z_B\right) R_i k_i + \left(1 - p_i\left(z_B\right)\right) C_i\right] \geq \gamma \sum_i \mu_i d_i \tag{7}$$

This is equivalent to stating that the bank only defaults on a fraction $(1 - \gamma)$ of defaults if the bad aggregate state z_B is realized. The parameter γ captures how strict the stress-test is. Modeling a regulatory stress-test as an inequality constraint can be motivated by appealing to recent empirical studies such as Acharya et al. (2018) and Cortés et al. (2018) who find that banks that fail stress tests adjust their lending in response. Furthermore, if the bank is publicly traded on a stock market, fully disclosing stress-test results as in the US is likely to create a strong incentive for the management of the bank to ensure the regulatory stress test is passed.

2.5 Equilibrium Concept

I focus on Nash equilibria, the study of which in the context of imperfect information contracts has been extensively studied. Competition in the banking sector occurs as follows. Banks announce credit contracts and compete *ex ante* on the contract terms. Entry into the banking sector occurs *ex ante*. Once banks commit to contracts, borrowers then choose which loan to accept. A separating Nash equilibrium is a set of contracts and bank capital structure $\{d_i, k_i, R_i, C_i\}_{i \in \{L,H\}}$ such that each contract earns non-negative profits for the bank, the regulatory constraints equations (6) and (7) are satisfied and there exists no other set of contracts which, when offered in addition to the existing set of contracts which all earn non-negative profits with at least one offering strictly positive profits.

Due to the presence of an upper bound on collateral, W, there remains the possibility that if this constraint binds then pooling equilibria can exist as a Nash equilibrium. The idea that a binding upper bound on collateral can distort contracts was discussed in Besanko and Thakor (1987) and Clemenz (1993). When the upper bound on collateral does not bind, there is a cream-skimming deviation from the pooling contract and the contract cannot exist as a Nash equilibrium.

The possible non-existence of a competitive Nash equilibrium in models with asymmetric information have been well documented in Rothschild and Stiglitz (1976), Wilson (1977) and Riley (1979) among others. This issue is discussed in more detail later in the paper.

3 Competitive Equilibrium

3.1 Full Information Equilibrium when the wealth constraint does not bind

Under full information, the Nash equilibrium credit contract maximizes a firm's expected profit subject to the constraint that the bank earns zero profit in expectation and the regulatory constraints equations (6) and (7) are satisfied for each contract.

There are two key features of the competitive equilibrium under full information that are worth pointing out. First, given perfect competition in the banking sector, banks will always default in the bad aggregate state and will earn zero expected profit in the good aggregate state as defined by the equation below

$$q\left(p_i\left(z_G\right) R_i k_i + \left(1 - p_i\left(z_G\right)\right) C_i - d_i\right) - \psi\left(k_i - d_i\right) \geq 0$$

To see why this is the case, note that for any contract that is not fully collateralized with $R_i k_i > C_i$, the bank will earn strictly higher profit in the good state than in the bad state and thus any contract where the bank does not default in the bad state will provide the bank with positive profit in expectation and will not be a competitive equilibrium.[1]

[1] Technically, if $\gamma \geq 1$, then the loan is fully collateralized and the bank will not default. I rule out this case by restricting the upper-bound of γ that can be implemented by the regulator to be the limit from below $\lim_{\gamma \nearrow 1} \gamma$. This restriction ensures that the regulator cannot prevent the bank defaulting through use of the stress-test.

Second, while equation (7) allows for cross-subsidization of firms, this will not happen in equilibrium and instead the following equation will hold for all i

$$p_i\left(z_B\right)R_ik_i + \left(1 - p_i\left(z_B\right)\right)C_i \geq \gamma d_i$$

This can easily be verified from the equilibrium contract terms. To understand why this is the case, first note that a higher value of γ places a greater constraint on the contracts that can be offered by the bank and will reduce the firm's expected profit from the contract. It follows that any cross-subsidization where $\gamma \sum_i \mu_i d_i = \sum_i \mu_i d_i \gamma_i$ with $\gamma_H \neq \gamma_L$ will not survive as a Nash equilibrium because it is not robust to a rival bank entering and offering the firm's with $\gamma_i > \gamma$ a contract that does not feature cross-subsidization.

The full information contract for firm i solve the following problem

$$\max_{\{k_i, R_i, C_i, d_i\}} p_i\left[\varphi_i k_i^\alpha - R_ik_i\right] - \left(1 - p_i\right)C_i + W \tag{8}$$

subject to

$$q\left(p_i\left(z_G\right)R_ik_i + \left(1 - p_i\left(z_G\right)\right)C_i - d_i\right) \geq \psi\left(k_i - d_i\right) \tag{9}$$

$$p_i\left(z_B\right)R_ik_i + \left(1 - p_i\left(z_B\right)\right)C_i \geq \gamma d_i \tag{10}$$

$$0 \leq d_i \leq \left(1 - \underline{e}\right)k_i \tag{11}$$

$$0 \leq C_i \leq W \tag{12}$$

Before solving this full problem, a useful benchmark is the case where the government does not impose any regulations such that equations (10) and (11) do not bind. In this case the competitive equilibrium is given by

$$k_i^* = \left(\frac{\alpha\bar{\varphi}}{q + \left(1 - q\right)\xi}\right)^{\frac{1}{1-\alpha}}, \ C_i^* = 0, \ R_i^* = \frac{1}{p_i\left(z_G\right)}, \ d_i^* = k_i^*$$

The loan size in the unregulated equilibrium above is larger than the first-best loan size which is $k_i^{FB} = \left(\alpha\bar{\varphi}\right)^{\frac{1}{1-\alpha}}$. This is because the bank is fully funded by insured deposits and does not take into account the default risk it faces and thus lends too much to firms.

Turning now to the problem when the government does impose the regulatory constraints set out in equations (10) and (11), the equilibrium contracts may now feature positive collateral. Intuitively, if the regulatory stress-test is a binding constraint, then in order to satisfy this constraint, the bank can choose to reduce the loan size, reduce the fraction of the loan funded through insured deposits or increase the required collateral. The proposition below summarizes the conditions under which it is optimal to increase the required collateral to satisfy equation (10).

Proposition 1. *If* $\gamma \geq \xi\left(\frac{\underline{e}}{1-\underline{e}}\frac{1}{q}\psi + 1\right)$ *and* $W > 0$ *the full information competitive equilibrium contract for firm* i *will feature strictly positive collateral.*

Proof. See Appendix. □

It follows from the proposition above that the wealth constraint on collateral may now bind. Thus, there are two cases to consider, $C_i < W$ and $C_i = W$.

In the first case, where the wealth constraint does not bind, the competitive equilibrium is given by

$$k_i^{**} = \left(\frac{\alpha\bar{\varphi}}{\underline{e}\psi + (1-\underline{e})\left[q + \gamma(1-q)\right]}\right)^{\frac{1}{1-\alpha}} \tag{13}$$

$$C_i^{**} = \left(\frac{1}{1-\xi}\right)\left[(1-\underline{e})\gamma - \left[\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right]\xi\right]k_i^{**} \tag{14}$$

$$R_i^{**} = \left(\frac{1}{1-\xi}\right)\left[\frac{1}{p_i(z_G)}\left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})(1-\gamma)\right) + (1-\underline{e})\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\xi\right] \tag{15}$$

$$d_i^{**} = (1-\underline{e})\,k_i^{**} \tag{16}$$

The competitive equilibrium set out above will hold whenever $W > C_i^{**}$. In this case, the bank capital requirement, equation (11), binds. Both firm types will receive the same sized loan and will face the same collateral requirement. However, low-risk firms will obtain a loan with a lower interest rate and so the full information equilibrium will not be incentive compatible in the asymmetric information case. The impact of banking regulation can be seen from the fact that the terms of the contract depend on both regulatory parameters γ and $\underline{e}$.

The unregulated competitive equilibrium can be seen as the limit of the above case at the point where $\gamma \to \xi\left(\frac{\underline{e}}{1-\underline{e}}\frac{1}{q}\psi + 1\right)$ and $\underline{e} = 0$. Without loss of generality, I will assume from now on that $\gamma \geq \xi\left(\frac{\underline{e}}{1-\underline{e}}\frac{1}{q}\psi + 1\right)$.

3.2 Contract Frontier

Before moving on to the competitive equilibrium in the case where the wealth constraint binds, I will first characterize the frontier of feasible contracts conditional on the collateral required C_i, which is simply the contract that maximizes the utility of firm i, equation (8) subject to the constraints in equations (9), (10) and (11) conditional on a fixed amount of collateral C_i. This will be useful for characterizing both the full information equilibrium when the wealth constraint binds and the asymmetric information case with separating contracts.

This contract frontier can be split into three regions. First, when the collateral supplied by the firm is not sufficiently high, it is optimal for the bank to reduce the quantity of deposits and increase the amount of bank capital it uses in order to satisfy the stress-test equation (10) rather than reducing the loan size. In this case, the capital requirement set out by equation (11) does not bind. This case is summarized in the proposition below.

Proposition 2. *For fixed collateral C such that $C < \underline{C}$ the optimal contract that satisfies equations* (9), (10) *and* (11) *features the following contract terms*

$$k_i(C) = \left[\alpha\bar{\varphi}\left(\frac{q\gamma + (\psi - q)\xi}{\gamma\psi(q + (1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}}$$

$$d_i(C) = \left[\frac{\frac{1}{q}\psi\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right]k_i(C) + \left(\frac{1-\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right)C$$

$$R_i(C) = \left(\frac{\frac{1}{p_i(z_G)}\frac{1}{q}\psi\gamma - \left[\gamma\left(\frac{1}{p_i(z_G)} - 1\right) + \left(\frac{1}{q}\psi - 1\right)\left(\frac{1}{p_i(z_G)} - \xi\right)\right]\frac{C}{k_i(C)}}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right)$$

where

$$\underline{C} = \left[\left(\frac{1-\underline{e}}{1-\xi}\right)\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\left(\frac{\xi}{1-\xi}\right)\right]\left[\alpha\bar{\varphi}\left(\frac{q\gamma + (\psi - q)\xi}{\gamma\psi(q + (1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}}$$

Proof. See Appendix. □

In the case above, both high- and low-risk firms receive the same loan size at a given level of collateral and banks fund this with the same quantity of insured deposits. The interest rate charged to low-risk firms will be lower than that charged to high-risk firms, and as a result, in the presence of asymmetric information, there will be an incentive for high-risk firms to masquerade as low-risk firms. In this region, the loan size is independent of the collateral amount pledged.

The second region occurs when the collateral supplied by the firm is not sufficiently high, the stress-test equation (10) may no longer become a binding constraint. This case is summarized in the proposition below.

Proposition 3. *For fixed collateral C such that $\bar{C} < C \leq R_i(C)k_i(C)$ the optimal contract that satisfies equations* (9), (10) *and* (11) *features the following contract terms*

$$\underline{C} = \left[\left(\frac{1-\underline{e}}{1-\xi}\right)\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\left(\frac{\xi}{1-\xi}\right)\right]\left[\alpha\bar{\varphi}\left(\frac{q\gamma + (\psi - q)\xi}{\gamma\psi(q + (1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}}$$

$$k_i(C) = \left(\frac{\alpha\bar{\varphi}}{[q + (1-q)\xi]\left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)} \right)^{\frac{1}{1-\alpha}}$$

$$d_i(C) = (1-\underline{e})\, k_i(C)$$

$$R_i(C) = \left(\psi\frac{1}{q}\underline{e} + (1-\underline{e})\right) - \left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)\frac{C}{k_i(C)}$$

where

$$\bar{C} = \left[\left(\frac{1-\underline{e}}{1-\xi}\right)\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\left(\frac{\xi}{1-\xi}\right)\right]\left(\frac{\alpha\bar{\varphi}}{[q + (1-q)\xi]\left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)}\right)^{\frac{1}{1-\alpha}}$$

Proof. See Appendix. □

As in the case above, both high- and low-risk firms receive the same loan size at a given level of collateral and banks fund this with the same quantity of insured deposits. The interest rate charged to low-risk firms will be lower than that charged to high-risk firms, and as a result, in the presence of asymmetric information, there will be an incentive for high-risk firms to masquerade as low-risk firms. In this case, the upper-bound on deposits binds and the loan size is increasing in the collateral amount pledged.

The final region occurs in intermediate ranges of collateral, where $C \in \left[\underline{C}, \bar{C}\right]$. In this case, equations (9), (10) and (11) all bind and the remaining three contract terms, conditional on collateral C, can be found as the solution to the system of these three equations. This case is summarized in the proposition below.

Proposition 4. *For fixed collateral C such that $C \in \left[\underline{C}, \bar{C}\right]$ the optimal contract that satisfies equations* (9), (10) *and* (11) *features the following contract terms*

$$k_i(C) = \left(\frac{1-\xi}{(1-\underline{e})\gamma - \left[\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right]\xi} \right) C$$

$$d_i(C) = (1-\underline{e})\, k_i(C)$$

$$R_i(C) = \left(\frac{1}{1-\xi}\right)\left(\frac{1}{p_i(z_G)}\left[\underline{e}\frac{1}{q}\psi + (1-\underline{e})(1-\gamma)\right] + (1-\underline{e})\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\xi\right)$$

where $\underline{C}$ and $\bar{C}$ are defined in propositions 2 and 3 above.

Proof. See Appendix. □

As in the cases above, both high- and low-risk firms receive the same loan size at a given level of collateral and banks fund this with the same quantity of insured deposits. The interest rate charged to low-risk firms will be lower than that charged to high-risk firms, and as a result, in the presence of asymmetric information, there will be an incentive for high-risk firms to masquerade as low-risk firms. In this case, the upper-bound on deposits binds and the loan size is independent of the collateral pledged.

Having defined the contract frontier conditional on a fixed C, it will be useful to express the contract space graphically along with the indifference curves of the firm. First, note that while the fraction of deposits used to fund the project is important in determining the other contract terms, it does not impact the utility of firms as defined in equation (4). In order to collapse the remaining contract terms into another dimension, I consider the following function

$$\pi_i(k, R) \equiv \varphi_i k^{\alpha} - Rk \tag{17}$$

where $\pi_i(k, R)$ is simply the profit the firm receives conditional on the success of the project. From equation (4) it follows that the indifference curves are linear in (π_i, C)-space with the following marginal rate of substitution:

$$\left.\frac{d\pi_i(k, R)}{d\eta}\right|_{u_i^F} = \frac{1-p_i}{p_i} \tag{18}$$

The marginal rate of substitution between $\pi_i(k, R)$ and C depends on the firm type i. Due to the lower probability their project is successful, high-risk firms have steeper indifference curves and are less willing to trade higher collateral requirements for a higher payoff if successful. As a result, in the presence of incomplete information, low-risk firms are able to separate from high-risk firms through a willingness to accept a contract with higher collateral constraints.

The contract frontier can be represented in (π_i, C)-space by substituting in the optimal contract terms conditional on C into the definition of π_i defined by equation (17). This frontier is denoted by $\Pi_i(C)$ and is defined as follows

$$\Pi_i(C) = \begin{cases} \varphi_i \underline{k}^{\alpha} - \frac{1}{p_i(z_G)}\left(\frac{\frac{1}{q}\psi\gamma}{\gamma+\left(\frac{1}{q}\psi-1\right)\xi}\right)\underline{k} + \left(\frac{\gamma(1-p_i(z_G))+\left(\frac{1}{q}\psi-1\right)(1-p_i(z_B))}{\gamma p_i(z_G)+\left(\frac{1}{q}\psi-1\right)p_i(z_B)}\right)\bar{C} & C < \underline{C} \\ \varphi_i\left(\frac{1-\xi}{(1-\underline{e})\gamma-\left[\underline{e}\frac{1}{q}\psi+(1-\underline{e})\right]\xi}\right)^{\alpha} C^{\alpha} - \left(1+\frac{1}{p_H(z_G)}\left[\frac{\underline{e}\frac{1}{q}\psi+(1-\underline{e})(1-\gamma)}{(1-\underline{e})\gamma-\left[\underline{e}\frac{1}{q}\psi+(1-\underline{e})\right]\xi}\right]\right)C & \underline{C} \leq C \leq \bar{C} \\ \varphi_i \bar{k}^{\alpha} - \frac{1}{p_i(z_G)}\left(\underline{e}\frac{1}{q}\psi+(1-\underline{e})\right)\bar{k} + \left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)C & C > \bar{C} \end{cases} \tag{19}$$

The gradient of $\frac{\partial \Pi_i(C)}{\partial C}$ is weakly increasing in C and always remain within the following interval

$$\frac{\partial \Pi_i(C)}{\partial C} \in \left[\left(\frac{\gamma(1-p_i(z_G))+\left(\frac{1}{q}\psi-1\right)(1-p_i(z_B))}{\gamma p_i(z_G)+\left(\frac{1}{q}\psi-1\right)p_i(z_B)}\right), \left(\frac{1-p_i(z_G)}{p_i(z_G)}\right)\right]$$

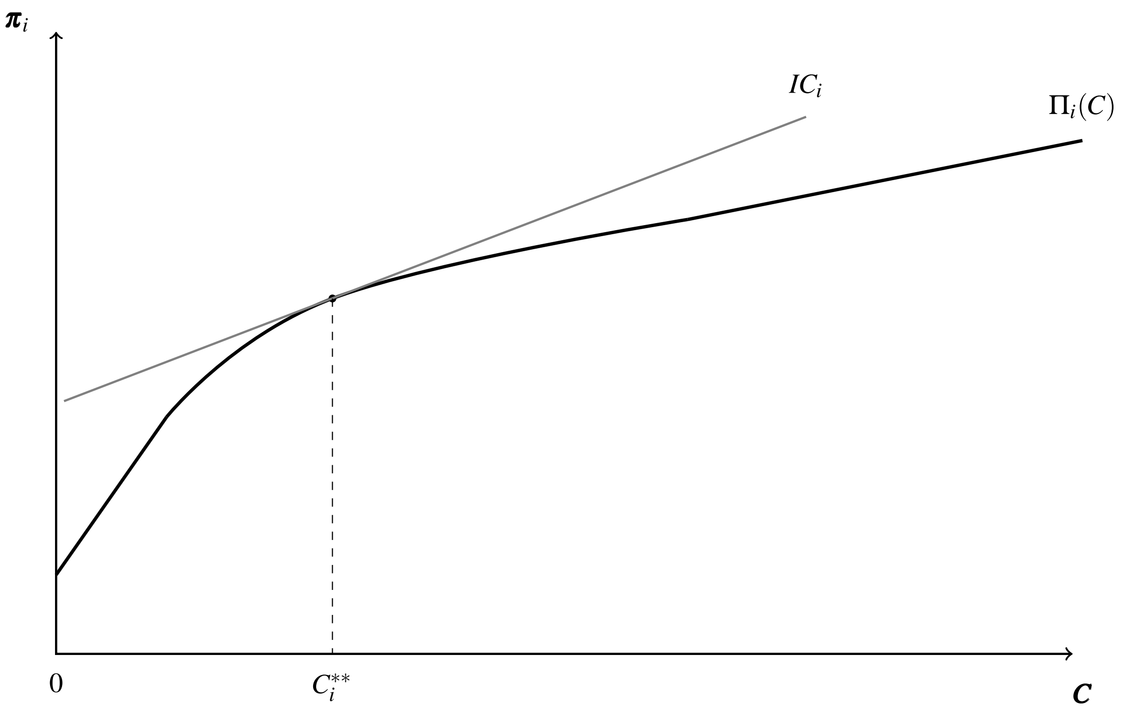


Figure 1: Frontier of feasible contracts for firm i

Furthermore, it can be shown that for any $\gamma < 1$, the gradient of $\Pi_i(C)$ at $C < \underline{C}$ is strictly larger than the slope of the firm's indifference curve (18). The optimal contract where the wealth constraint does not bind, as defined in Proposition 1, lies at a point $C \in \left(\underline{C}, \bar{C}\right)$ and is the point where the slope of the firm's indifference curve is tangent to the contract frontier $\Pi_i(C)$. This is illustrated graphically in figure 1.

Having fully characterized the contract frontier in the previous section, the full information competitive equilibrium when the wealth constraint binds can be found by plugging in the wealth constraint into the equations defined in propositions 2 and 3.The wealth constraint will bind whenever there is insufficient collateral to implement the competitive equilibrium set out in proposition 1 which occurs when $W < C^{**}$. The contract terms will depend on both W and can be defined below.

$$k_i(W) = \begin{cases} \left[\alpha\bar{\varphi}\left(\frac{q\gamma+(\psi-q)\xi}{\gamma\psi(q+(1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}} & W < \underline{C} \\ \left(\frac{1-\xi}{(1-\underline{e})\gamma-\left[\underline{e}\frac{1}{q}\psi+(1-\underline{e})\right]\xi}\right)W & \underline{C} \leq W < C^{**} \end{cases} \tag{20}$$

$$d_i(W) = \begin{cases} \left(\frac{\frac{1}{q}\psi\xi}{\left(\frac{1}{q}\psi-1\right)\xi+\gamma}\right)\left[\alpha\bar{\varphi}\left(\frac{q\gamma+(\psi-q)\xi}{\gamma\psi(q+(1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}} + \left(\frac{1-\xi}{\left(\frac{1}{q}\psi-1\right)\xi+\gamma}\right)W & W < \underline{C} \\ \left(\frac{(1-\underline{e})(1-\xi)}{(1-\underline{e})\gamma-\left[\underline{e}\frac{1}{q}\psi+(1-\underline{e})\right]\xi}\right)W & \underline{C} \leq W < C^{**} \end{cases} \tag{21}$$

$$R_i\left(W\right)=\begin{cases}\left(\dfrac{\frac{1}{p_i(z_G)}\frac{1}{q}\psi\gamma\left[\alpha\bar{\varphi}\left(\frac{q\gamma+(\psi-q)\xi}{\gamma\psi(q+(1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}}-\left[\gamma\left(\frac{1}{p_i(z_G)}-1\right)+\left(\frac{1}{q}\psi-1\right)\left(\frac{1}{p_i(z_G)}-\xi\right)\right]W}{\left(\left(\frac{1}{q}\psi-1\right)\xi+\gamma\right)\left[\alpha\bar{\varphi}\left(\frac{q\gamma+(\psi-q)\xi}{\gamma\psi(q+(1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}}}\right) & W<\underline{C}\\ \left(\frac{1}{1-\xi}\right)\left(\frac{1}{p_i(z_G)}\left[\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\left(1-\gamma\right)\right]+\left(1-\underline{e}\right)\gamma-\left(\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\right)\xi\right) & \underline{C}\leq W<C^{**}\end{cases} \tag{22}$$

Notice that when the wealth constraint binds, the loan size will be strictly below the full information competitive equilibrium k^{**}.

3.3 Separating Equilibrium when the wealth constraint does not bind

If the upper-bound on collateral does not bind, the separating Nash equilibrium will consist of two separating credit contracts: $\{k_i, R_i, C_i, d_i\}_{i\in\{L,H\}}$, where the set of separating contracts solves the following maximization problem:

$$\max_{\{k_i,R_i,C_i,d_i\}_{i\in\{L,H\}}}\sum\mu_i\left(p_i\left[\varphi_i k_i^{\alpha}-R_i k_i\right]-\left(1-p_i\right)C_i+W\right) \tag{23}$$

subject to

$$p_L\left[\varphi_L k_L^{\alpha}-R_L k_L\right]-\left(1-p_L\right)C_L+W\geq p_L\left[\varphi_L k_H^{\alpha}-R_H k_H\right]-\left(1-p_L\right)C_H+W \tag{24}$$

$$p_H\left[\varphi_H k_H^{\alpha}-R_H k_H\right]-\left(1-p_H\right)C_H+W\geq p_H\left[\varphi_H k_L^{\alpha}-R_L k_L\right]-\left(1-p_H\right)C_L+W \tag{25}$$

$$q\left(p_i\left(z_G\right)R_i k_i+\left(1-p_i\left(z_G\right)\right)C_i-d_i\right)\geq\psi\left(k_i-d_i\right)\qquad i\in\{L,H\} \tag{26}$$

$$p_i\left(z_B\right)R_i k_i+\left(1-p_i\left(z_B\right)\right)C_i\geq\gamma d_i\qquad i\in\{L,H\} \tag{27}$$

$$0\leq d_i\leq\left(1-\underline{e}\right)k_i\qquad i\in\{L,H\} \tag{28}$$

$$0\leq C_i\leq W\qquad i\in\{L,H\} \tag{29}$$

The problem is similar to the full information problem but there are now two additional constraints. Equations (24) and (25) are truth telling constraints that ensure both firm types reveal their true type to the bank. In the separating equilibrium where the wealth constraint does not bind, no cross-subsidization of the stress-test constraint is permissible. As before, any increase in γ_i above γ for a firm of type i will result in a the existence of a profitable deviation existing where banks offer that firm a contract featuring the γ without cross subsidization and, if necessary, alter collateral to ensure incentive compatibility is maintained.

In the presence of incomplete information, the banks are unable to condition contracts on the firm type. The relative slope of the firms' indifference curves means low-risk firms are able to separate from high-risk firms by accepting a contract with a larger collateral requirement. From the previous section, it was shown that conditional on any collateral level C, low risk firms are offered a better contract than the high-risk firms. It follows

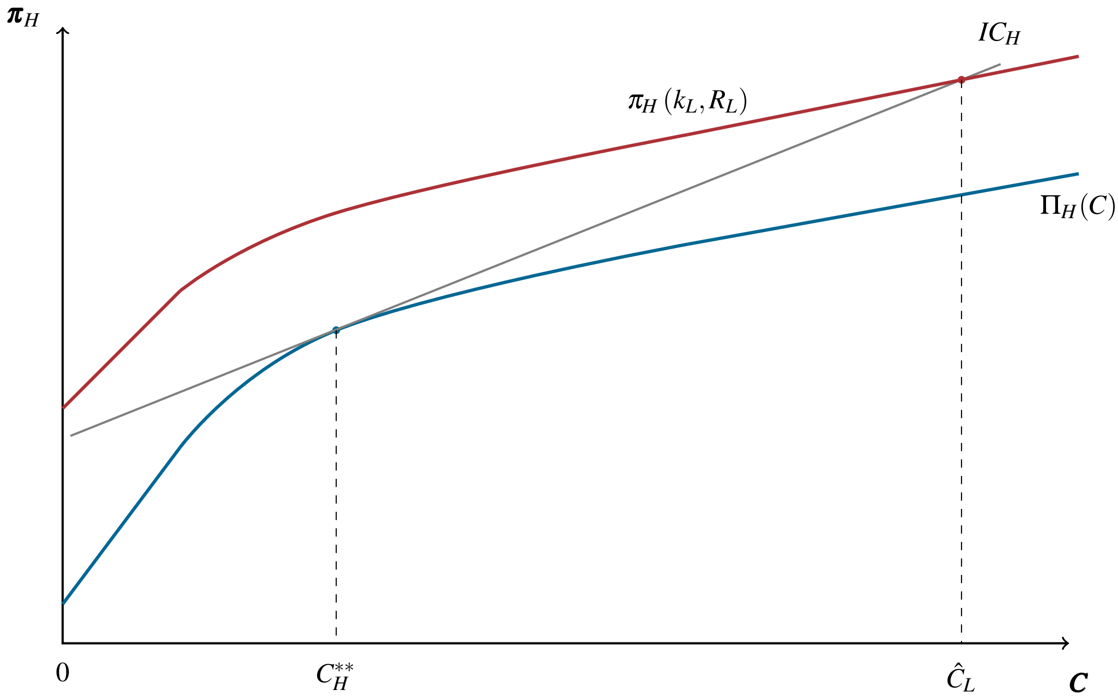


Figure 2: Asymmetric Information Contracts

from this that the low-risk firm's truth-telling constraint, equation (24), will not bind and that the relevant truth-telling constraint is equation (25). Low-risk firms must then pledge a higher quantity of collateral than high-risk firms.

The separating contract, should it exist, can be illustrated graphically by noting that the truth telling-constraint can be rewritten as

$$\pi_H(k_H, R_H) \geq \pi_H(k_L, R_L) + \left(\frac{1-p_H}{p_H}\right)(C_H - C_L)$$

and noting that in the separating equilibrium, high-risk firms will receive the same contract they would in full information. The separating contract for low-risk firms, can be found as the point on the frontier $\pi_H(k_L(C_H), R_L(C_H))$ at which high-risk firms are indifferent between this contract and their full-information contract $\{k_H^{**}, R_H^{**}, C_H^{**}, d_H^{**}\}$. An example of this intersection is illustrated in figure 2.

For brevity, I omit the precise contract terms for low-risk firms in the case where equation (25) binds as their precise formulation will depend on whether C_H is larger than $\bar{C}$ or not. However, from the properties of the contract frontier Π_i set out earlier, in addition to a higher collateral requirement, the separating contract will feature a strictly larger loan size and a weakly lower interest rate relative to the full-information contract.

As pointed out by Rothschild and Stiglitz (1976) and Wilson (1977), a Nash equilibrium is not guaranteed to exist in an economy that features asymmetric information. This occurs when a pooling contract Pareto dominates separating contracts and thus would be preferred to the separating contract by both types of firms. Due to the curvature of the production function, the precise conditions required for the existence of a separating equilibrium cannot be found in closed form. However, in the appendix I derive a sufficient condition for existence of a Nash equilibrium. Even in the case where a Nash equilibrium does not exist, the separating equilibrium discussed in this section will exist as a Riley reactive equilibrium as set out in Riley (1979). Similarly, a pooling equilibrium would exist as a Wilson anticipatory equilibrium as in Wilson (1977).

3.4 Separating Equilibrium when wealth constraint binds

The separating equilibrium discussed above assumed that firms have sufficient wealth to supply the collateral required by the bank and the upper-bound on collateral as specified by equation (29) did not bind. I now discuss the equilibrium contracts under asymmetric information in the case where the wealth constraint does bind.

First, note that if the wealth constraint binds and there is no cross-subsidization, incentive compatibility requires that the k_L and R_L offered to the low-risk firm are such that the following equation holds

$$\bar{\pi}_H\left(k_L, R_L\right) = \pi_H\left(k_H^{**}, R_H^{**}\right) + \left(\frac{1-p_H}{p_H}\right)\left(W - C_H^{**}\right) \tag{30}$$

When the wealth constraint binds, the low-risk firm's contract lies off the frontier of contracts and the firm is indifferent between any pair of contract terms $\{k_L, R_L\}$ which yields the payoff $\bar{\pi}_H\left(k_L, R_L\right)$ specified above. This would allow a bank to earn positive profit at this point. This possibility is illustrated in figure 3 panel (3a).

However, if we allow for a bank to offer a set of contracts such that the stress-test constraint is satisfied for the entire portfolio of bank lending as in equation (7), rather than on a loan-by-loan basis as in equation (27), a set of separating contracts can be found that improve on the set without cross-subsidization. These contracts will be such that equation (27) will be replaced by

$$p_i\left(z_B\right) R_i k_i + \left(1 - p_i\left(z_B\right)\right) C_i \geq \gamma_i d_i \qquad i \in \{L, H\}$$

and

$$\gamma \sum_i \mu_i d_i = \sum_i \mu_i d_i \gamma_i$$

with the latter equation ensuring that the set of contracts satisfies the stress-test defined in equation (7). The separating equilibrium will now feature cross-subsidization such that

$\gamma_L > \gamma_H$ and the incentive compatibility constraint below is satisfied.

$$\pi_H \left(k_L \left(W, \gamma_L\right), R_L \left(W, \gamma_L\right)\right) = \pi_H \left(k_H^{**} \left(\gamma_H\right), R_H^{**} \left(\gamma_H\right)\right) + \left(\frac{1 - p_H}{p_H}\right) \left(W - C_H^{**} \left(\gamma_H\right)\right) \quad (31)$$

Where the contract terms $\{k_L (W, \gamma_L), R_L (W, \gamma_L)\}$ lie on the frontier of feasible contracts given $\gamma_L > \gamma$. This contract is illustrated in figure 3 panel (3b).

To understand why cross-subsidization and specifically $\gamma_L > \gamma_H$ will occur, first note that if the separating contract lies off the contract frontier as in figure 3 panel (3a), the bank will be able to earn positive profit on the contract offered to the low-risk firm. By raising γ_L from γ, the contract frontier at W will fall for low-risk firms but it will also allow γ_H to fall below γ such that the frontier for high-risk firms increases. A shift up in $\Pi_H (C)$ will improve the welfare of not only the high-risk firms, but also the welfare of the low-risk firms as the π they are able to receive at $C_L = W$ increases as $\Pi_H (C)$ increases. This cross-subsidization can increase until the point where the π offered to the low-risk firms lies on the feasibility frontier. This case is illustrated in figure 3 panel (3b).

3.5 Pooling Equilibrium

When low-risk firms pledge their entire wealth as collateral such that the upper-bound on collateral binds, there is also the possibility of pooling contracts existing as a Nash Equilibrium. In a pooling contract both high- and low-risk firms receive the same contract $\{k_P, R_P, C_P, d_P\}$. The expected probability of success conditional on aggregate state z is simply a weighted average of the success probabilities for high- and low-risk firms, weighted by the proportion of that firm type in the economy

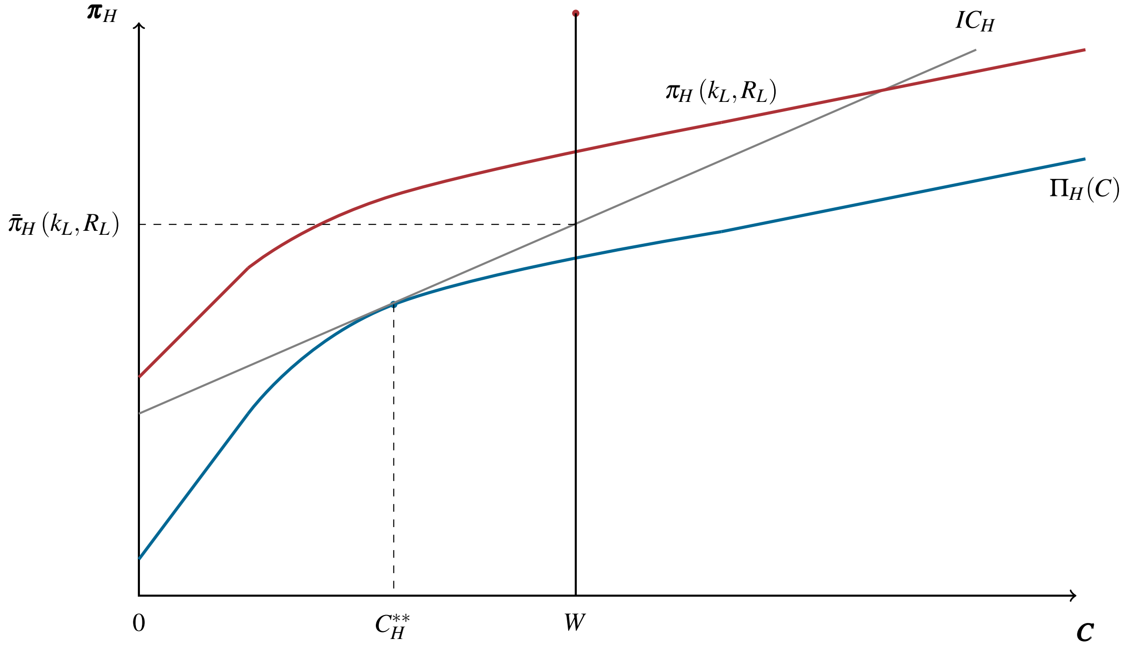
$$p_P (z_j) = \mu_H p_H (z_j) + \mu_L p_L (z_j) \quad \forall j \in \{G, B\}$$

It also follows that the unconditional expected probability of success is also a weighted average of the success probabilities of the high- and low-risk firms

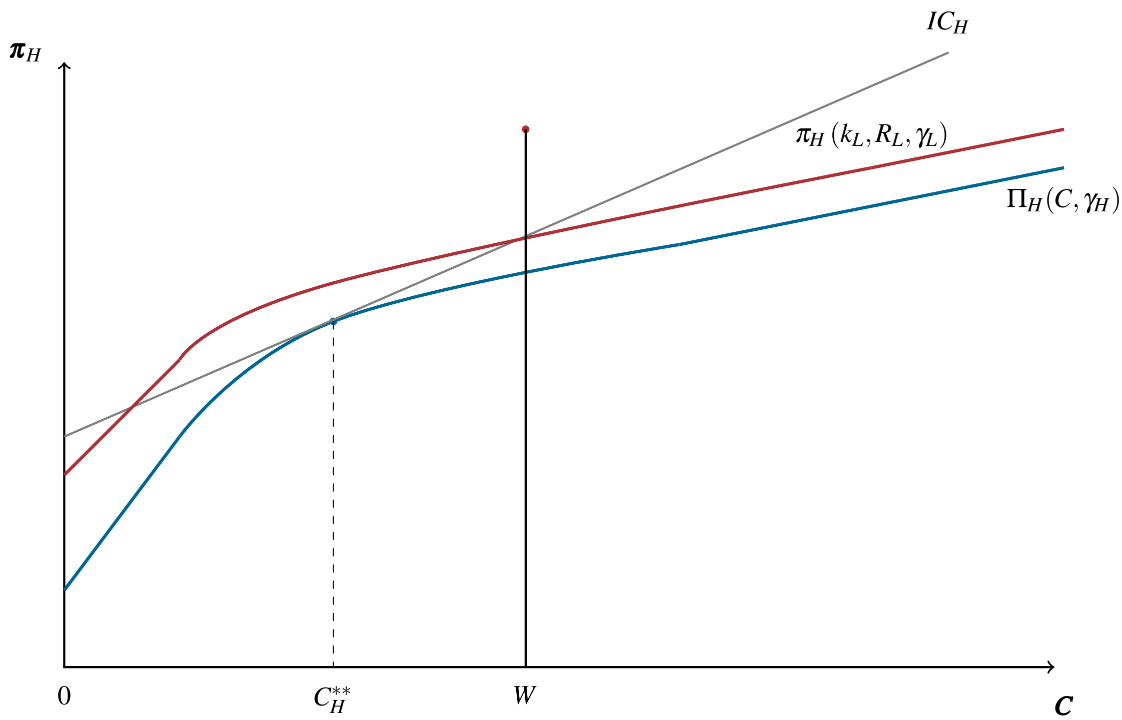
$$p_P = \mu_H p_H + \mu_L p_L$$

The frontier of feasible pooling contracts $\Pi_P (C)$ is simply $\Pi_i (C)$ for the composite type P.

For a pooling contract $\{k_P, R_P, C_P, d_P\}$ to exist as a Nash Equilibrium, there must exist no deviating contract that would satisfy equations (26) and (27) while making the pooling contract either unprofitable or violate the regulatory constraint. That is, there cannot exist a cream-skimming contract that will attract only one type of firm.



(a) Contracts without cross-subsidization



(b) Contracts with cross-subsidization

Figure 3: Contracts with cross-subsidization

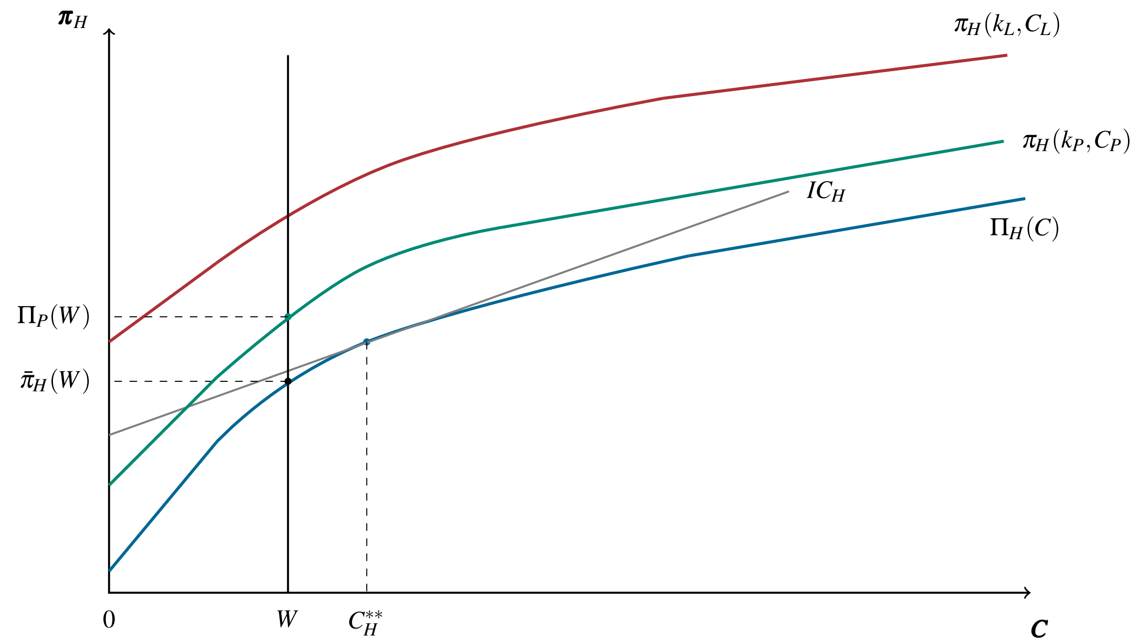


Figure 4: Example of a Pooling Contract

It follows that a necessary condition for the existence of a pooling contract is for both high- and low-risk firms to prefer the pooling contract to the best separating contract available, otherwise the firms would choose the separating contracts over the pooling contracts. Similarly, any pooling contract must lie on the frontier Π_P of pooling contracts, otherwise a better pooling contract could be found that would be preferred by both types of firm.

The existence of a cream-skimming contract depends on the ability to offer a contract with higher collateral than the pooling contract. If the upper-bound on collateral binds at the pooling contract such that $C_P = W$, no contract with higher collateral can be offered to low-risk firms and thus there exists no cream-skimming contract. A sufficient condition for the existence of a pooling equilibrium is for $C_H^{**} \leq W$ such that the full-information competitive equilibrium features a (weakly) binding wealth constraint. Here, the only possible separating condition would involve cross-subsidization. However, the pooling contract will lie weakly above the cross-subsidization contract in this case because the pooling contract involves cross-subsidization of both the profit condition, equation (26) and the stress-test constraint, equation (27), while the cross-subsidization contracts would only allow for pooling in the latter set of constraints. An example of a pooling contract existing as a Nash equilibrium is illustrated in figure 4. The precise terms of the pooling contract, as in the separating case, depend where the contract is located on the frontier.

4 Optimal Policy

4.1 Welfare Function

In this section, I briefly discuss optimal policy of the government in this economy. Until this point, the regulatory parameters, $\underline{e}$ and γ, were taken as given. Now, I will derive the optimal values of these parameters.

The government is assumed to be benevolent and maximizes the welfare of a risk-neutral household that owns both firms and banks. In this case, the objective function for the planner is

$$\bar{U}\left(\{d_i, k_i, R_i, C_i\}_{i\in\{L,H\}}\right) = \sum_i \mu_i \left(U_i^F\left(k_i, R_i, C_i\right) + U_i^B\left(d_i, k_i, R_i, C_i\right)\right) - (1-q)\,\tau$$

where $U_i^F\left(k_i, R_i, C_i\right)$ is the firm expected profit set out in equation (4), $U_i^B\left(d_i, k_i, R_i, C_i\right)$ is the expected profit of a bank contract set out in equation (5) and τ is the lump-sum tax levied on firms in the case of bank default. This tax covers the losses of the depositors and is set such that

$$\tau = \max\left\{\sum_i \mu_i \left(d_i - p_i\left(z_B\right) R_i k_i - \left(1 - p_i\left(z_B\right)\right) C_i\right), 0\right\}$$

Assuming, as in the rest of the paper, that the bank always defaults in the bad aggregate state, the objective function of the planner can be rewritten as

$$\bar{U}\left(\{d_i, k_i, R_i, C_i\}_{i\in\{L,H\}}\right) = \sum_i \mu_i \left(\bar{\varphi} k_i^{\alpha} - d_i - \psi\left(k_i - d_i\right)\right) \tag{32}$$

A useful benchmark to consider is the first-best contract under full-information, if the planner could choose directly the contracts provided to firms. Maximizing the above subject to $0 \leq d_i \leq k_i$ results in the following

$$k_i^{FB} = \left(\alpha\bar{\varphi}\right)^{\frac{1}{1-\alpha}}, \; d_i^{FB} = \left(\alpha\bar{\varphi}\right)^{\frac{1}{1-\alpha}}$$

Thus the first best contracts feature loans of the optimal scale, fully financed by deposits. As the household owns both banks and firms, the planner only cares about aggregate efficiency and does not have any distributional concerns. This implies that the other contract terms, C_i and R_i do not directly enter the planner's problem. A key assumption made in this paper is that the use of collateral is not costly and thus its use has no welfare implications. The use of bank equity on the other hand is assumed to be costly and so the planner would prefer to avoid its use. This turns out to be a key assumption when thinking about optimal policy because it will be optimal for the government to increase the use of collateral, through a more stringent stress-test rather than through the use of costly bank capital.

4.2 Optimal Policy

The government's optimal policy problem consists of choosing γ and $\underline{e}$ to maximize the objective function defined in equation (32) taking into account the contracts that will occur in the competitive equilibrium. In the case of full information with the wealth constraint slack, the relevant functions from the government's perspective are the equations (13) and (16) which define k_i^{**} and d_i^{**}respectively.

The first best can be achieved through setting $k_i^{**} = k_i^{FB}$ and $d_i^{**} = d_i^{FB}$ which imply

$$\left(\frac{\alpha\bar{\varphi}}{\underline{e}\psi + (1-\underline{e})\left[q + \gamma(1-q)\right]}\right)^{\frac{1}{1-\alpha}} = (\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$$

$$(1-\underline{e})\left(\frac{\alpha\bar{\varphi}}{\underline{e}\psi + (1-\underline{e})\left[q + \gamma(1-q)\right]}\right)^{\frac{1}{1-\alpha}} = (\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$$

It should be straightforward to see that the first-best can be achieved and consists of the government setting $\gamma = 1$ and $\underline{e} = 0$. Thus it is optimal for the government to resolve over-lending in the banking sector through a stress-test that mandates loans be fully collateralized. For this to be implementable, the firms must have have sufficient collateral, which given the collateral required in equation (14) means that this is implementable only if

$$W \geq (\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$$

Consider now the case where there is insufficient for loans to be fully collateralized and so $W < (\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$. To simplify the study of this case, I define $W = \eta(\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$ with $\eta < 1$ capturing the fraction of the optimal loan size available for use as collateral. In this case, it can be shown that the first-best set of contracts are still implementable and that the optimal government policy is to set

$$\gamma = \xi + (1-\xi)\eta, \ \underline{e} = 0$$

It may seem surprising that the first-best is still achievable when the wealth constraint binds. However, from the equilibrium loan size set out in equation (20), it can be seen that the equilibrium loan size is decreasing in the available wealth. As, the problem facing the economy is one of over-lending, this does not present the government a challenge in implementing the first best. While the implementation of the first-best through a stress-test is achievable, the implication of this section is that a less stringent stress-test should be implemented if there is less collateral available.

In the presence of asymmetric information, the first-best can again be implemented and in fact, the policy rules are the same as in the case of perfect information. To see why this is the case, note that if the government sets $\gamma = 1$ and $\underline{e} = 0$, then all loans are fully collateralized and as such both firms receive the same contract terms, and thus the truth-telling constraint is trivially satisfied. In the case where wealth is insufficient to allow for full collateralization and $W < (\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$, then a pooling equilibrium occurs and the optimal policy for a pooling equilibrium at $W = \eta\,(\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$ is simply $\gamma = \xi + (1-\xi)\,\eta$ and $\underline{e} = 0$ as above. It is straightforward to confirm that the pooling equilibrium is the Nash equilibrium contract at $W = \eta\,(\alpha\bar{\varphi})^{\frac{1}{1-\alpha}}$, $\gamma = \xi + (1-\xi)\,\eta$ and $\underline{e} = 0$. It should be noted here that that the ability of the government to use macro-prudential policy to resolve the adverse selection problem in this way is a direct result of the specific assumptions made in the model. Two assumptions seem key to this result, first is the assumption that there is no efficiency loss in the use of collateral and the second is that the expected output of the two firm types are constant across types ($p_H\varphi_H = p_L\varphi_L$). Relaxing either of these assumptions will complicate the government's problem significantly in the case of asymmetric information.

5 Conclusion

This paper presented a one-dimensional adverse selection model of firm financing where banks face a regulatory stress-test constraint that restricts the losses they can make in a recession. As the regulatory constraint tightens, banks value collateral not only as a screening device but also as a way of reducing the risk of loans. Through banking regulation, the government can affect both the terms of the loan contracts and the type of equilibrium. First, regulation reduces the size of the loans offered by raising bank capital requirements and by imposing a more stringent stress-test on banks. Second, in response to a more stringent stress-test, banks raise collateral requirements on loans in order to reduce their loss given default. If the collateral requirements becomes sufficiently high, the competitive equilibrium may involve some cross-subsidzation, or a full pooling contract.

The model presents a simple motivation for government intervention in the banking sector. Banks are able to fund their lending using insured deposits but also have limited liability. As banks do not default, they do not endogenize this default risk and as a result, they lend too much to firms. Governments are able to resolve this issue through the use of a stress-test and also bank capital requirements.

The optimal policy for the government is to focus on the use of collateral through the stress-test rather than capital requirements. This is not that surprising given the assumptions made in this paper, namely that bank equity is costly, but collateral is not. However, a result that is of more interest, is that if less collateral is available, it is optimal to reduce the severity of the stress-test. This somewhat counter-intuitive result is because when there is more collateral available to the banking sector, banks lend more and the over-lending problem becomes more severe. In contrast, when there is less collateral available, interest rates are higher, loans are smaller and the deviation from the first best is less pronounced.

This paper also emphasises the interaction between a firm's pledgeable collateral and the impact of banking regulation. The misallocation which occurs in a pooling equilibria can only occur if firms have insufficient pledgeable collateral. If on the other hand, firms have access to sufficient collateral, the tightening of the regulatory constraint may be beneficial in forcing an increased use in collateral and consequently a fall in lending risk.

References

Viral V Acharya, Allen N Berger, and Raluca A Roman. Lending implications of U.S. bank stress tests: Costs or benefits? *Journal of Financial Intermediation*, 34(C):58–90, 2018.

Franklin Allen and Douglas Gale. *Understanding Financial Crises*. Oxford University Press, 2009.

David Besanko and Anjan V Thakor. Collateral and Rationing: Sorting Equilibria in Monopolistic and Competitive Credit Markets. *International Economic Review*, 28(3): 671–689, October 1987.

Gerhard Clemenz. A Note on the Nonexistence of a Rationing Equilibrium in the Besanko-Thakor Model. *International Economic Review*, 34(3):727–737, August 1993.

Kristle Cortés, Yuliya Demyanyk, Lei Li, Elena Loutskina, and Philip E Strahan. Stress tests and small business lending. Working Paper 24365, National Bureau of Economic Research, March 2018.

Kieran Dent, Ben Westwood, and Miguel Segoviano. Stress testing of banks: an introduction. *Bank of England Quarterly Bulletin*, 56(3):130–143, 2016.

Jeffrey Lacker. Collateralized debt as the optimal contract. *Review of Economic Dynamics*, 4(4):842–859, October 2001.

John G Riley. Informational equilibrium. *Econometrica*, 47(2):331–59, March 1979.

Michael Rothschild and Joseph E Stiglitz. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *The Quarterly Journal of Economics*, 90(4):629–649, 1976.

Joseph E Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, June 1981.

Charles Wilson. A model of insurance markets with incomplete information. *Journal of Economic Theory*, 16(2):167–207, December 1977.

Appendix

Proof of Proposition 1

The Lagrangian for the full information problem can be written as

$$\begin{aligned}\mathcal{L}_i &= p_i\left(\varphi_i k_i^{\alpha} - R_i k_i\right) - (1-p_i)\, C_i + W \\ &+ \lambda_B\left[q\left(p_i\left(z_G\right) R_i k_i + \left(1-p_i\left(z_G\right)\right) C_i - d_i\right) - \psi_1\left(k_i - d_i\right)\right] \\ &+ \lambda_S\left[p_i\left(z_B\right) R_i k_i + \left(1-p_i\left(z_B\right)\right) C_i - \gamma d_i\right] \\ &+ \mu_i^- C_i + \mu_i^+\left(W - C_i\right) + \mu_i^d\left(\left(1-\underline{e}\right) k_i - d_i\right)\end{aligned}$$

Then the FOCs are

$$\frac{\partial \mathcal{L}}{\partial R_i} = -p_i k_i + \lambda_B q p_i\left(z_G\right) k_i + \lambda_S p_i\left(z_B\right) k_i$$

$$\frac{\partial \mathcal{L}}{\partial C_i} = -\left(1-p_i\right) + \lambda_B q\left(1-p_i\left(z_G\right)\right) + \lambda_S\left(1-p_i\left(z_B\right)\right) + \mu_i^- - \mu_i^+$$

$$\frac{\partial \mathcal{L}}{\partial k_i} = p_i\left(\alpha\varphi_i k_i^{\alpha-1} - R_i\right) + \lambda_B\left[q p_i\left(z_G\right) R_i - \psi_1\right] + \lambda_S p_i\left(z_B\right) R_i + \left(1-\underline{e}\right)\mu_i^d$$

$$\frac{\partial \mathcal{L}}{\partial d_i} = \lambda_B\left(\psi - q\right) - \lambda_S\gamma - \mu_i^d$$

There is a minimum requirement for γ which can be found such that the stress–test condition is satisfied with zero collateral. First note that if the stress-test condition does not bind, $\lambda_S = 0$ and the optimal contract terms are given by

$$C_i = 0$$

$$R_i k_i = \left(\underline{e}\frac{1}{q}\psi + \left(1-\underline{e}\right)\right) k_i$$

$$d_i = \left(1-\underline{e}\right)\bar{k}$$

Plugging these equations into the stress-test constraint and solving for γ yields

$$\underline{\gamma} = \xi\left(\frac{\underline{e}}{1-\underline{e}}\frac{1}{q}\psi + 1\right)$$

thus it must be the case that for any $\gamma > \underline{\gamma}$ the stress-test constraint binds.

Assuming this holds, and rearranging the FOCs stated above we note that one simple adjustment can be used noting that

$$p_i = \lambda_B q p_i (z_G) + \lambda_S p_i (z_B)$$

We can rewrite $\frac{\partial \mathcal{L}}{\partial k_i}$ as

$$\frac{\partial \mathcal{L}}{\partial k_i} = p_i \alpha \varphi_i k_i^{\alpha-1} - \lambda_B \psi_1 + (1 - \underline{e}) \mu_i^d$$

and $\frac{\partial \mathcal{L}}{\partial C_i}$ as

$$\frac{\partial \mathcal{L}}{\partial C_i} = -1 + \lambda_B q + \lambda_S + \mu_i^- - \mu_i^+$$

and $\frac{\partial \mathcal{L}}{\partial d_i}$ as

$$\lambda_S = \lambda_B \left(\frac{\psi_1 - q}{\gamma} \right) - \frac{1}{\gamma} \mu_i^d$$

which can be combined with $\frac{\partial \mathcal{L}}{\partial R_i}$to yield

$$p_i = \lambda_B q p_i (z_G) + \lambda_B \left(\frac{\psi_1 - q}{\gamma} \right) p_i (z_B) - \frac{1}{\gamma} \mu_i^d p_i (z_B)$$

We can also substitute out λ_S from the third equation to get

$$\frac{\partial \mathcal{L}}{\partial C_i} = -1 + \lambda_B q + \lambda_B \left(\frac{\psi_1 - q}{\gamma} \right) - \frac{1}{\gamma} \mu_i^d + \mu_i^- - \mu_i^+$$

To verify that the constraint on d_i binds in equilibrium we note that if the constraint does not bind we have

$$\mu_i^d = \lambda_B \left[q\gamma \frac{p_i (z_G)}{p_i (z_B)} + (\psi - q) \right] - \gamma \frac{p_i}{p_i (z_B)} \leq 0$$

$$\lambda_B = \frac{1}{q} \left(\frac{p_i - p_i (z_B)}{p_i (z_G) - p_i (z_B)} \right)$$

Note that we can use

$$p_i \equiv q p_i (z_G) + (1 - q) p_i (z_B)$$

To rewrite the above as

$$\lambda_B = 1$$

Then for the multiplier to be non-positive we require

$$\gamma \geq \left(\frac{\psi - q}{1 - q} \right) > 1$$

which by definition cannot hold.

Give the constraints hold at equilibrium, the first order conditions and constraints can be rearranged to yield the contract terms.

Proof of Proposition 2

In this case, both the stress-test condition and the zero profit condition will both bind. Rearranging these equations yields

$$R_i k_i = \frac{1}{p_i\left(z_G\right)} \frac{1}{q} \psi k_i - \frac{1}{p_i\left(z_G\right)} \left(1 - p_i\left(z_G\right)\right) C - \frac{1}{p_i\left(z_G\right)} \left(\frac{1}{q}\psi - 1\right) d_i$$

$$R_i k_i = \frac{1}{p_i\left(z_B\right)} \gamma d_i - \frac{1}{p_i\left(z_B\right)} \left(1 - p_i\left(z_B\right)\right) C$$

Setting these equal yields

$$k_i = q\frac{1}{\psi}\left[\left(\left(\frac{1}{q}\psi - 1\right) + \frac{p_i\left(z_G\right)}{p_i\left(z_B\right)}\gamma\right) d_i - \left(\frac{p_i\left(z_G\right) - p_i\left(z_B\right)}{p_i\left(z_B\right)}\right) C\right]$$

Now we can rewrite firm profit as

$$\begin{aligned}\Pi_i^F &= p_i \varphi_i \left[q\frac{1}{\psi}\left(\left(\frac{1}{q}\psi - 1\right) + \frac{p_i\left(z_G\right)}{p_i\left(z_B\right)}\gamma\right) d_i - q\frac{1}{\psi}\left(\frac{p_i\left(z_G\right) - p_i\left(z_B\right)}{p_i\left(z_B\right)}\right) C\right]^{\alpha} \\ &- \frac{p_i}{p_i\left(z_B\right)}\gamma d_i + q\left(\frac{p_i\left(z_G\right) - p_i\left(z_B\right)}{p_i\left(z_B\right)}\right) C + W\end{aligned}$$

We want to find the optimal deposit level. Which can be found as the point where

$$\left(q p_i\left(z_G\right)\gamma + \left(\psi - q\right) p_i\left(z_B\right)\right) \alpha \varphi_i k_i^{\alpha - 1} = \psi\gamma$$

When the constraint on d_i is slack, the equation for k_i is given by

$$k_i = \left[\alpha\bar{\varphi}\left(\frac{\gamma q + \left(\psi - q\right)\xi}{\gamma\psi\left(q + \left(1 - q\right)\xi\right)}\right)\right]^{\frac{1}{1-\alpha}}$$

and the deposit level is then given by

$$d_i = \frac{\frac{1}{q}\psi\xi\left[\alpha\bar{\varphi}\left(\frac{\gamma q + (\psi - q)\xi}{\gamma\psi(q + (1-q)\xi)}\right)\right]^{\frac{1}{1-\alpha}} + \left(1 - \xi\right) C}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}$$

Now note that we can rewrite this as

$$d_i = \left[\frac{\frac{1}{q}\psi\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right] k_i + \left(\frac{1-\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right) C$$

The remaining contract terms can be found form the remaining constraints.

The cutoff for collateral is found below.

Proof of Proposition 3

We note that when γ is sufficiently low, the stress test is slack. In this case, we have two equations that bind

$$p_i(z_G) R_i k_i + (1 - p_i(z_G)) C + \left(\frac{1}{q}\psi - 1\right) d_i = \frac{1}{q}\psi k_i$$

$$(1 - \underline{e}) k_i \geq d_i$$

The aim now is to substitute out the interest rate in the firm's profit function.

$$\Pi_i^F \equiv p_i(\varphi_i k_i^\alpha - R_i k_i) - (1 - p_i) C + W$$

Then we have (by rearranging the first equation only)

$$R_i k_i = \frac{1}{p_i(z_G)} \frac{1}{q}\psi k_i - \frac{1}{p_i(z_G)}(1 - p_i(z_G)) C - \frac{1}{p_i(z_G)}\left(\frac{1}{q}\psi - 1\right) d_i$$

Thus we can write firm profit as

$$\Pi_i^{ZP} = \left(p_i \varphi_i k_i^\alpha - \frac{p_i}{p_i(z_G)} \frac{1}{q}\psi k_i\right) - (1-q)\left(\frac{p_i(z_G) - p_i(z_B)}{p_i(z_G)}\right) C + \frac{p_i}{p_i(z_G)}\left(\frac{1}{q}\psi - 1\right) d_i + W$$

Now note that it is optimal for the firm to set $\bar{C} = 0$ and to set d_i as high as possible.

Thus we note that in the case where only the zero-profit constraint binds, the upper-bound on deposit financing will also bind.

Then we have

$$\Pi_i^{ZP} = p_i\left(\varphi_i k_i^\alpha - \frac{1}{p_i(z_G)}\left[\underline{e}\frac{1}{q}\psi + (1 - \underline{e})\right] k_i\right) - (1-q)\left(\frac{p_i(z_G) - p_i(z_B)}{p_i(z_G)}\right) C + W$$

and we can find the optimal loan size as the point where

$$\alpha \varphi_i k_i^{\alpha - 1} = \frac{1}{p_i(z_G)}\left[\underline{e}\frac{1}{q}\psi + (1 - \underline{e})\right]$$

or equivalently

$$\alpha\bar{\varphi}k_i^{\alpha-1} = (q + (1-q)\,\xi)\left[\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right]$$

The cutoff for collateral is found below.

Proof of Proposition 4

When the constraint on d_i and the stress-test constraint both bind we can rearrange the constraints to get an equation for k_i in terms of C

$$k_i = \left(\frac{1-\xi}{(1-\underline{e})\,\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\xi}\right)C$$

and the deposit level is

$$d_i = \left(\frac{(1-\underline{e})\,(1-\xi)}{(1-\underline{e})\,\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\xi}\right)C$$

Note that we also have

$$R_i k_i = \left[\left(\frac{\left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)(1-p_i\,(z_B)) - (1-\underline{e})\,\gamma\,(1-p_i\,(z_G))}{(1-\underline{e})\,\gamma p_i\,(z_G) - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)p_i\,(z_B)}\right)\right]C$$

Cutoff values of collateral

(Lower) Cutoff value of collateral The cutoff level of collateral where the constraint on d_i binds is defined as the point where

$$(1-\underline{e})\,k_i = \left[\frac{\frac{1}{q}\psi\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right]k_i + \left(\frac{1-\xi}{\left(\frac{1}{q}\psi - 1\right)\xi + \gamma}\right)\underline{C}$$

Note that the smallest loan size is now

$$\underline{k} = \left[\alpha\bar{\varphi}\left(\frac{q\gamma + (\psi - q)\,\xi}{\gamma\psi\,(q + (1-q)\,\xi)}\right)\right]^{\frac{1}{1-\alpha}}$$

Thus we can write the above as

$$\underline{C} = \left[\left(\frac{1-\underline{e}}{1-\xi}\right)\gamma - \left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\right)\left(\frac{\xi}{1-\xi}\right)\right]\underline{k}$$

(Upper) Cutoff value of collateral The remaining contract terms follow from rearranging the constraints. The cutoff level of collateral where the stress-test constraint no longer binds is defined as the point where

$$\left(\frac{\alpha\varphi_i p_i\left(z_G\right)}{\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)}\right)^{\frac{1}{1-\alpha}}=\left(\frac{1-\xi}{\left(1-\underline{e}\right)\gamma-\left[\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\right]\xi}\right)\bar{C}$$

Thus we can write the above as

$$\bar{C}=\left[\left(\frac{1-\underline{e}}{1-\xi}\right)\gamma-\left(\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\right)\left(\frac{\xi}{1-\xi}\right)\right]\bar{k}$$

where

$$\bar{k}=\left(\frac{\alpha\bar{\varphi}}{\left[q+\left(1-q\right)\xi\right]\left(\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\right)}\right)^{\frac{1}{1-\alpha}}$$

Sufficient Condition for Existence of Separating Equilibria

For a separating equilibrium to exist (without cross-subsidization) it must be preferred to the pooling contract. I assume in this case the wealth constraint also doesn't bind at the high-risk firm's contract. At the separating equilibrium the loan size will be the same as in the high-risk firm's separating contract.

Consider now the difference between $\Pi_H\left(C_H^{**}\right)$ and the profit level at the pooling contract $\pi_H\left(k_P,C_P\right)$. This is simply the difference in loan payments

$$\pi_H\left(k_P,C_P\right)-\Pi_H\left(C_H^{**}\right)=\left(\frac{1}{p_H}-\frac{1}{p_P}\right)\left(\frac{q+\left(1-q\right)\xi}{1-\xi}\right)\left(\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\left(1-\gamma\right)\right)k_H^{**}$$

Now consider the wealth level W^*at which point a low-risk firm would be indifferent between the separating contract at wealth W^* and the pooling contract. This can be shown to be

$$\Pi_L\left(C_H^{**}\right)+\left(\frac{1-p_H}{p_H}\right)\left(W^*-C_H^{**}\right)-\left(\frac{1-p_L}{p_L}\right)\left(W^*-C_H^{**}\right)=\pi_L\left(k_P,C_P\right)$$

as $\pi_H\left(k_P,C_P\right)-\Pi_H\left(C_H^{**}\right)=\pi_L\left(k_P,C_P\right)-\Pi_L\left(C_H^{**}\right)$ we have

$$W^*=C_H^{**}+\left(\frac{\left(1-\mu_H\right)p_L}{\mu_H p_H+\left(1-\mu_H\right)p_L}\right)\left(\frac{q+\left(1-q\right)\xi}{1-\xi}\right)\left(\underline{e}\frac{1}{q}\psi+\left(1-\underline{e}\right)\left(1-\gamma\right)\right)k_H^{**}$$

Thus a necessary condition is that $W\geq W^*$. However, we also need to verify that this separating contract lies below the feasible frontier of low-risk firm contracts, otherwise it will not be implementable. Unfortunately, due to the concavity of the production function and the fact that $k_L(W^*) > k_H^{**}$, we cannot derive an exact condition.

However, we note that if we held k_L fixed at k_H^{**}, we could find a lower-bound for the feasible frontier at W^*, this point is given by

$$\Pi_L\left(W^*\right) > \Pi_L\left(C_L^{**}\right) + \left(\frac{1-p_L\left(z_G\right)}{p_H\left(z_G\right)}\right)\left(W^* - C_H^{**}\right)$$

Thus a sufficient condition for the existence of a separating conditions is that

$$\left(\frac{1}{p_H} - \frac{1}{p_L}\right)\left(\frac{q+(1-q)\,\xi}{1-\xi}\right)\left(\underline{e}\frac{1}{q}\psi + (1-\underline{e})\,(1-\gamma)\right)k_H^{**} \geq \left(\frac{1-p_H}{p_H} - \frac{1-p_L\left(z_G\right)}{p_H\left(z_G\right)}\right)\left(W^* - C_H^{**}\right)$$