

# MA 4710 Homework 8

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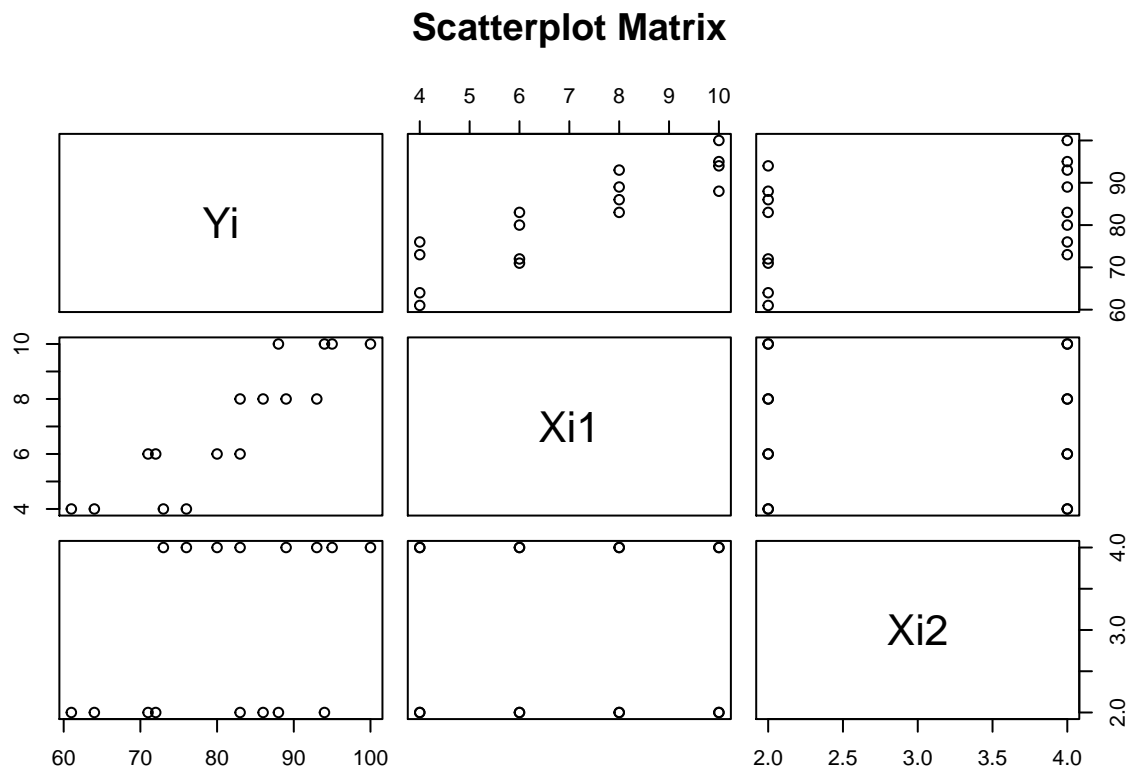
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## Problem 6.5

```
brand <- read.table("~/GitHub/MA-4710/Homework 8/brand.txt",  
                    quote="\"", comment.char="")  
names(brand) <- c("Yi", "Xi1", "Xi2")
```

### Part A

```
pairs(brand, main = "Scatterplot Matrix")
```



```
cor(brand)
```

```
##           Yi           Xi1           Xi2  
## Yi  1.0000000  0.8923929  0.3945807  
## Xi1 0.8923929  1.0000000  0.0000000  
## Xi2 0.3945807  0.0000000  1.0000000
```

According to the correlation matrix, there is no correlation between  $X_{i1}$  and  $X_{i2}$ .

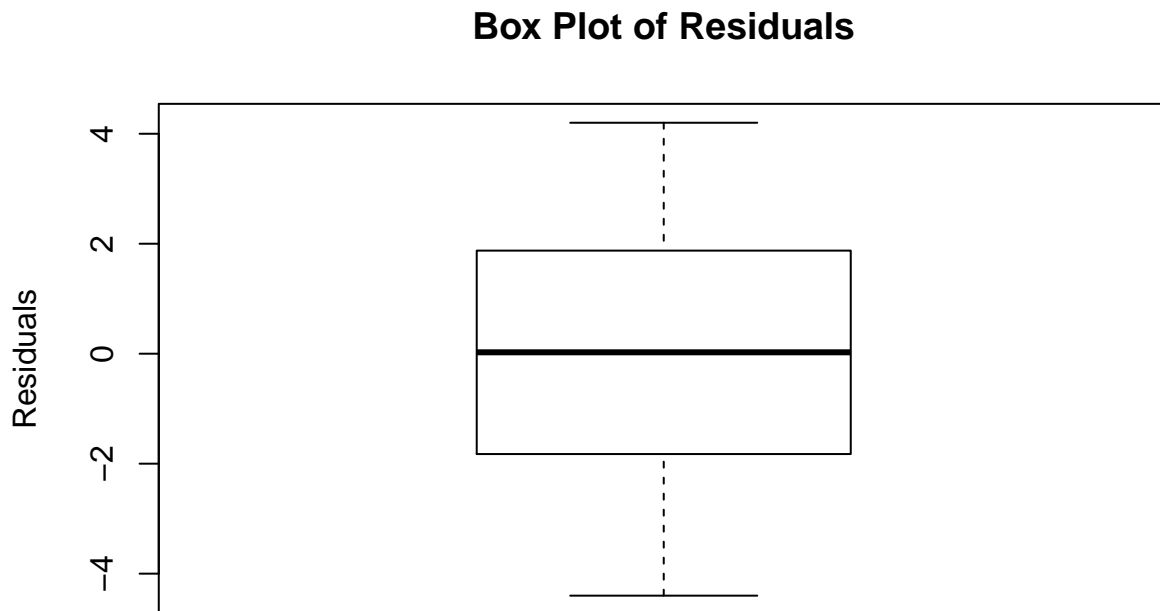
## Part B

```
brand.lm <- lm(Yi ~ Xi1 + Xi2, data=brand)
```

The estimated regression function is  $\hat{Y}_i = 37.65 + 4.425X_{i1} + 4.375X_{i2}$ .  $b_1$  provides the dependency of the moisture content and brand liking. For every increase in brand liking, the moisture content increases by 4.425.

## Part C

```
brand.lm.resid <- resid(brand.lm)
boxplot(brand.lm.resid, main = "Box Plot of Residuals", ylab = "Residuals")
```

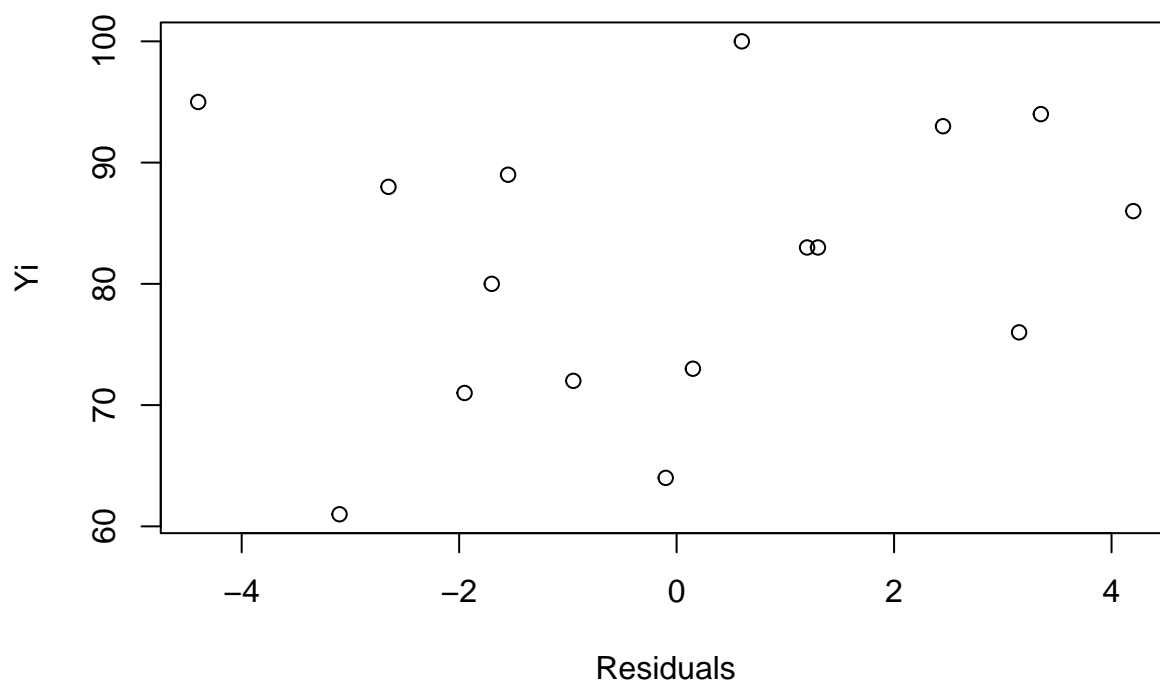


The residuals appear to be symmetrically distributed around zero and have zero mean.

## Part D

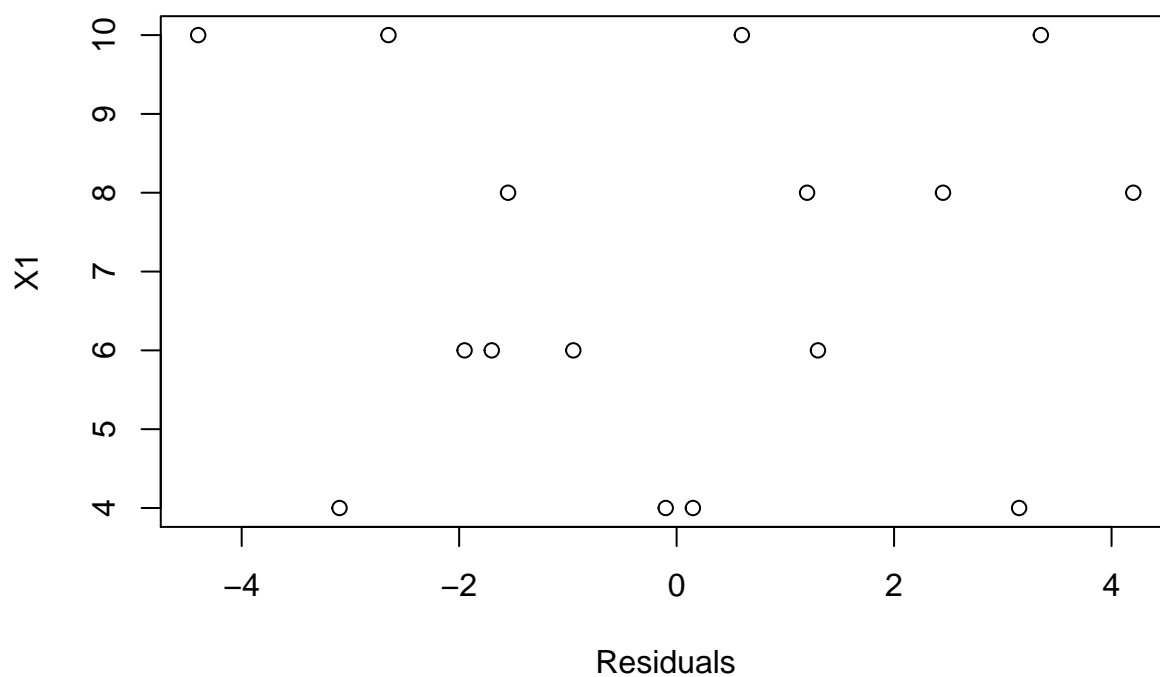
```
# Resid vs Yi
plot(x = brand.lm.resid, y = brand$Yi,
     xlab = "Residuals", ylab = "Yi",
     main = "Residuals vs. Yi")
```

## Residuals vs. $Y_i$

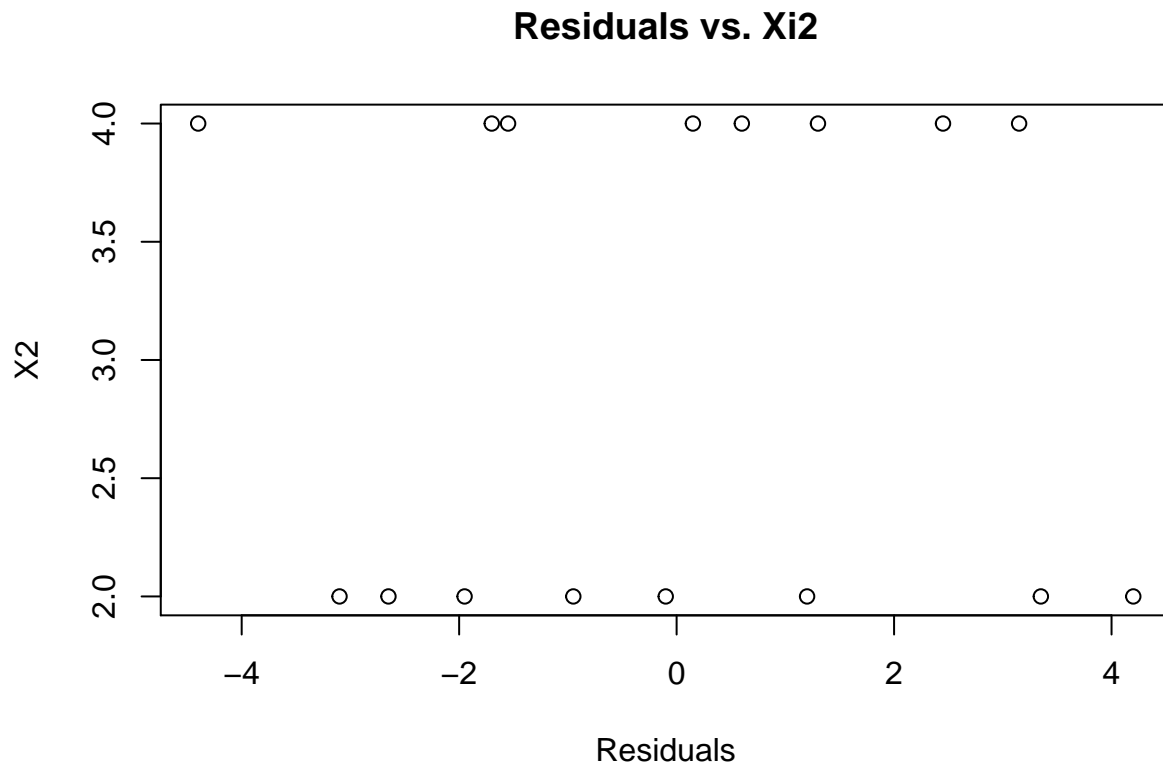


```
# Resid vs Xi1  
plot(x = brand.lm.resid, y = brand$Xi1,  
     xlab = "Residuals", ylab = "Xi1",  
     main = "Residuals vs. Xi1")
```

## Residuals vs. $X_{i1}$

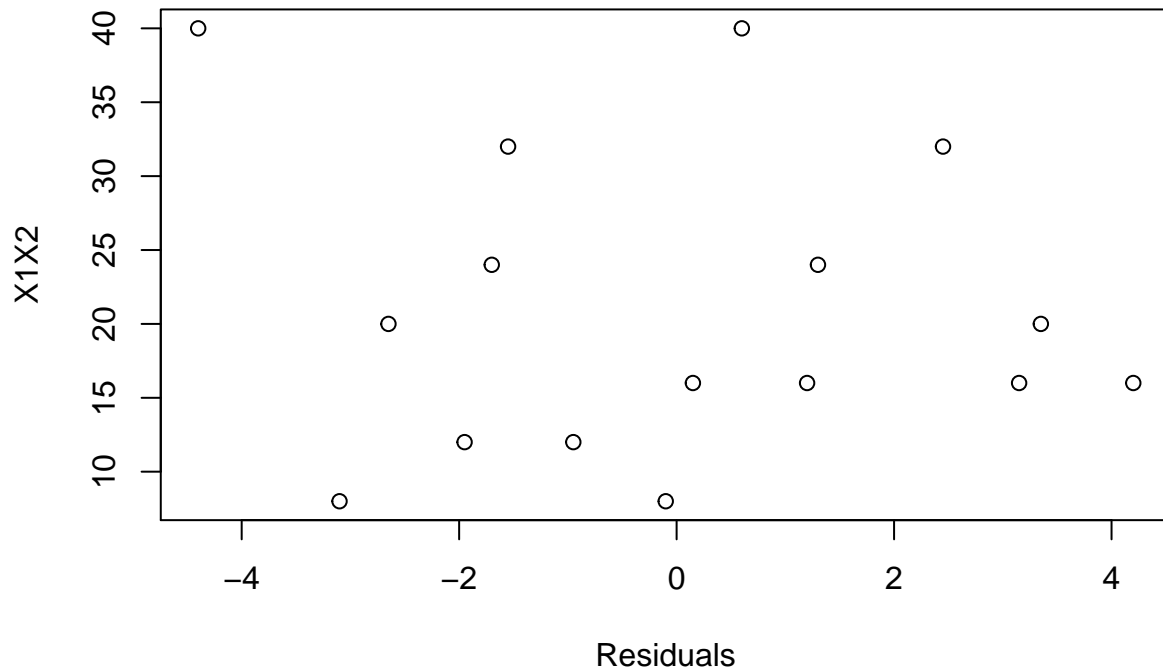


```
# Resid vs. Xi2
plot(x = brand.lm.resid, y = brand$Xi2,
     xlab = "Residuals", ylab = "X2",
     main = "Residuals vs. Xi2")
```



```
# Resid vs. Xi1 * Xi2
plot(x = brand.lm.resid, y = brand$Xi1 * brand$Xi2,
     xlab = "Residuals", ylab = "X1X2",
     main = "Residuals vs. X1X2")
```

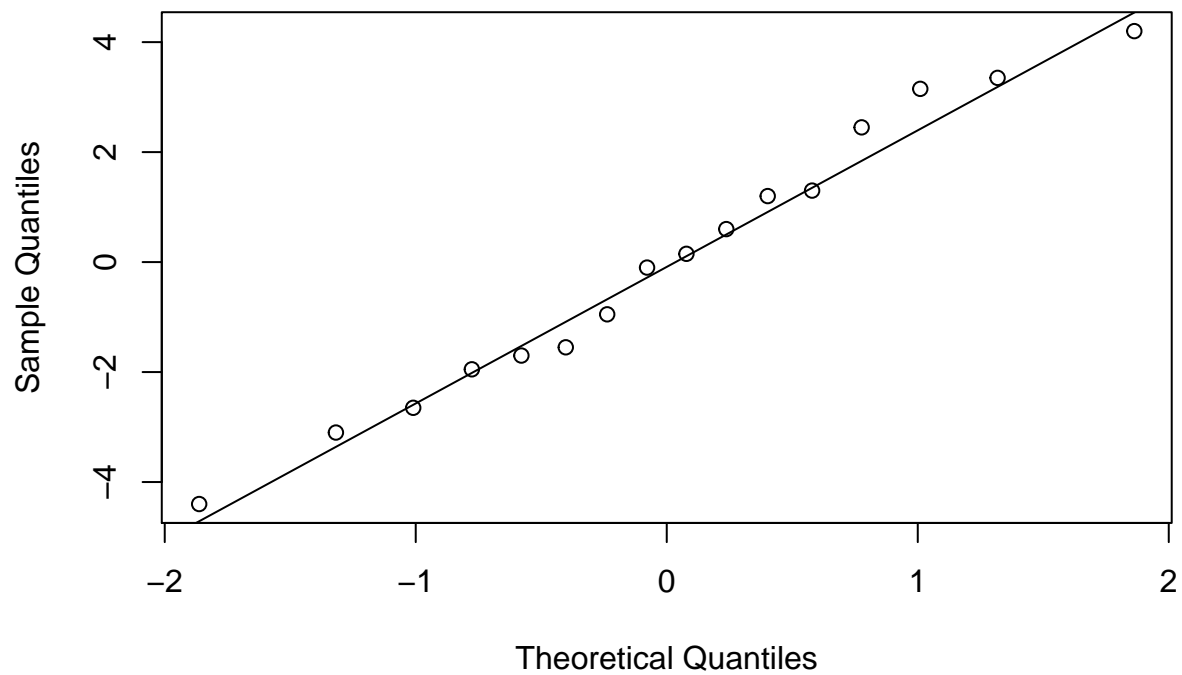
## Residuals vs. X1X2



The residual plots have a uniform spread across all variables.

```
qqnorm(brand.lm.resid)
qqline(brand.lm.resid)
```

## Normal Q-Q Plot



The normal probability plot suggests that the residuals are normally distributed.

## Part E

```
library(lmtest)
```

```
## Loading required package: zoo
##
## Attaching package: 'zoo'
##
## The following objects are masked from 'package:base':
##
##      as.Date, as.Date.numeric
```

```
bptest(Yi ~ Xi1 + Xi2, data = brand)
```

```
##
## studentized Breusch-Pagan test
##
## data: Yi ~ Xi1 + Xi2
## BP = 2.0441, df = 2, p-value = 0.3599
```

$H_0 : v_1 = 0$  vs.  $H_a : v_1 \neq 0$ .

Because the p-value, 0.3598534, is greater than 0.01, we fail to reject  $H_0$ , proving that the error variance is constant.

## Problem 6.6

### Part A

```
summary(brand.lm)
```

```
##
## Call:
## lm(formula = Yi ~ Xi1 + Xi2, data = brand)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.400 -1.762  0.025  1.587  4.200
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.6500     2.9961  12.566 1.20e-08 ***
## Xi1           4.4250     0.3011  14.695 1.78e-09 ***
## Xi2           4.3750     0.6733   6.498 2.01e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.693 on 13 degrees of freedom
## Multiple R-squared:  0.9521, Adjusted R-squared:  0.9447
## F-statistic: 129.1 on 2 and 13 DF, p-value: 2.658e-09
```

$H_0 : \beta_i = 0$  vs.  $H_1 : \beta_i \neq 0$ .

From the `summary` function, we find that the p-value for  $\beta_1$ ,  $1.78 \times 10^{-9}$ , is less than  $\alpha = 0.1$ . Therefore we reject  $H_0$  proving that  $\beta_1 \neq 0$ .

The p-value for  $\beta_2$ ,  $2.01 \times 10^{-5}$ , is less than  $\alpha = 0.1$ . Therefore we reject  $H_0$  proving that  $\beta_2 \neq 0$ .

## Part B

The p-values from of the test in Part A are  $1.20 \times 10^{-8}$ ,  $1.78 \times 10^{-9}$ ,  $2.01 \times 10^{-5}$  for  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ , respectively.

## Part C

```
coef <- summary(brand.lm)$coefficients      # Statement Confidence Level
alpha <- 0.01                               # alpha : significance level

B <- qt(1-alpha/(2),brand.lm$df.residual)
BCI <- cbind(coef[,2]-B*coef[,3],coef[,2]+B*coef[,3])
colnames(BCI) <- c("Lower Bound","Upper Bound")
BCI[2:3,]
```

```
##      Lower Bound Upper Bound
## Xi1   -43.96473    44.56697
## Xi2   -18.89928    20.24593
```

The bounds of the Bonferroni procedure for  $X_{i1}$  are (-43.9647332, 44.5669726). The bounds of the Bonferroni procedure for  $X_{i2}$  are (-18.8992802, 20.2459285).

## Problem 6.7

The coefficient of multiple determinaiton  $R^2$  from the `summary` function is 0.952059