MA 4710 Homework 6

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Problem 4.5

Load the data into `R` and rename the variables.

```
filePath <- "~/GitHub/MA-4710/Homework 6/CH01PR22.txt"
CH01PR22 <- read.table(filePath, quote="\"", comment.char="")
names(CH01PR22)[1] <- "hardness"
names(CH01PR22)[2] <- "time"
```

Part A

Create the linear model using the `lm` function and find the coefficients with the `summary` function.

```
plastic.lm <- lm(hardness ~ time, data = CH01PR22)
plastic.coef <- summary(plastic.lm)$coefficients
alpha <- 0.1
```

Compute the Bonferroni join confidence intervals for β_0 and β_1, using a 90 percent family confidence coefficient.

```
B <- qt(1-alpha/(2*2),plastic.lm$df.residual)
BCI <- cbind(plastic.coef[,1]-B*plastic.coef[,2],
             plastic.coef[,1]+B*plastic.coef[,2])
colnames(BCI) <- c("Lower Bound","Upper Bound")
```

The confidence interval for the intercept β_0 is (162.9012502, 174.2987498). We conclude that the intercept β_0 has a 90 percent chance of being between 162.9012502 and 174.2987498.

The confidence interval for the slope β_1 is (1.8404996, 2.2282504). We conclude that the slope β_1 has a 90 percent chance of being between 1.8404996 and 2.2282504.

Problem 4.9

Part A

Using the data from Problem 4.5, create the 90 percent confidence interval Bonferroni bands at 20, 30, and 40 hours.

```
CI <- predict(plastic.lm,newdata=data.frame(time=c(20,30,40)),se.fit=TRUE)
g <- 3
B <- qt(1-alpha/(2*g),plastic.lm$df.residual)
BBand <- cbind( CI$fit - B * CI$se.fit, CI$fit + B * CI$se.fit )
BBand
```

```
##       [,1]     [,2]
## 1 206.7277 211.8473
## 2 227.6762 231.5863
## 3 246.7824 253.1676
```

The confidence interval at 20 hours is (206.7277428, 211.8472572). We conclude that at 20 hours, the estimate has a 90 chance of being between 206.7277428 and 211.8472572.

The confidence interval at 30 hours is (227.6762032, 231.5862968). We conclude that at 30 hours, the estimate has a 90 chance of being between 227.6762032 and 231.5862968.

The confidence interval at 40 hours is (246.7824219, 253.1675781). We conclude that at 40 hours, the estimate has a 90 chance of being between 246.7824219 and 253.1675781.

Part C

Create the the 90 percent confidence interval Working-Hotelling bands at 30 and 40.

```
CI <- predict(plastic.lm,newdata=data.frame(time=c(30,40)),se.fit=TRUE)

W <- sqrt(2*qf(0.90,length(plastic.lm$coefficients),plastic.lm$df.residual))
WHBand <- cbind( CI$fit - W * CI$se.fit, CI$fit + W * CI$se.fit )
```

The confidence interval at 30 hours is (227.6966397, 231.5658603). We conclude that at 30 hours, the estimate has a 90 chance of being between 227.6966397 and 231.5658603.

The confidence interval at 40 hours is (246.8157946, 253.1342054). We conclude that at 40 hours, the estimate has a 90 chance of being between 246.8157946 and 253.1342054.

Create the the 90 percent confidence interval Bonferroni bands at 30 and 40.

```
g <- 2
B <- qt(1-alpha/(2*g),plastic.lm$df.residual)
BBand <- cbind( CI$fit - B * CI$se.fit, CI$fit + B * CI$se.fit )
BBand
```

```
##       [,1]     [,2]
## 1 227.8544 231.4081
## 2 247.0733 252.8767
```

The confidence interval at 30 hours is (227.8543526, 231.4081474). We conclude that at 30 hours, the estimate has a 90 chance of being between 227.8543526 and 231.4081474.

The confidence interval at 40 hours is (247.0733386, 252.8766614). We conclude that at 40 hours, the estimate has a 90 chance of being between 247.0733386 and 252.8766614.

Based on the above confidence intervals, the Bonferroni model is the most efficient because it has the smallest confience bands.

Problem 5.1

Define the matrices A, B, and C in `R`.

```r
A <- matrix(c(1,2,3,4,6,8),3,2)
B <- matrix(c(1,1,2,3,4,5),3,2)
C <- matrix(c(3,5,8,4,1,0),2,3)
```

Perform the matrix arithmetic in `R` to compute $A+B$, $A-B$, AC, AB', and $B'A$.

Note that the function `t(A)` in `R` is equivalent to A^T and `A %*% B` is equivalent to AB.

```r
A+B #A+B
```

```
##      [,1] [,2]
## [1,]    2    7
## [2,]    3   10
## [3,]    5   13
```

```r
A-B #A-B
```

```
##      [,1] [,2]
## [1,]    0    1
## [2,]    1    2
## [3,]    1    3
```

```r
A %*% C #AC
```

```
##      [,1] [,2] [,3]
## [1,]   23   24    1
## [2,]   36   40    2
## [3,]   49   56    3
```

```r
A %*% t(B) #AB'
```

```
##      [,1] [,2] [,3]
## [1,]   13   17   22
## [2,]   20   26   34
## [3,]   27   35   46
```

```r
t(B) %*% A #B'A
```

```
##      [,1] [,2]
## [1,]    9   26
## [2,]   26   76
```

Problem 5.5

Load the data into `R` and rename the variables.

```r
filePath <- "~/GitHub/MA-4710/Homework 6/CH05PR05.txt"
CH05PR05 <- read.table(filePath, quote="\"", comment.char="")
names(CH05PR05)[1] <- "city"
names(CH05PR05)[2] <- "loans"
```

Put the X and Y values into their respective matrices. **Y** is a 6×1 matrix. **X** is a 6×2 matrix. The first column of **X** is all 1's and the second column is the values of X.

```
Y <- matrix(CH05PR05$loans, 6,1)
X <- matrix(c(1,1,1,1,1,1,CH05PR05$city), 6,  2)
```

Perform the matrix arithmetic in `R` to compute $Y'Y$, XX', and $X'Y$.

```
t(Y) %*% Y #Y'Y
```

```
##      [,1]
## [1,]   55
```

```
X %*% t(X) #XX'
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]  257   81  161  241  209  353
## [2,]   81   26   51   76   66  111
## [3,]  161   51  101  151  131  221
## [4,]  241   76  151  226  196  331
## [5,]  209   66  131  196  170  287
## [6,]  353  111  221  331  287  485
```

```
t(X) %*% Y #X'Y
```

```
##      [,1]
## [1,]   17
## [2,]  261
```

Problem 5.13

Use the same data from Problem 5.5: `CH05PR05`.

Perform the matrix arithmetic in `R` to compute $(X'X)^{-1}$.

```
(t(X) %*% X)^(-1) #(X'X)^-1
```

```
##            [,1]         [,2]
## [1,] 0.16666667 0.0123456790
## [2,] 0.01234568 0.0007942812
```