MA4710 Homework 7

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1 Problem 5.15

Consider the simultaneous equations:

$$\begin{aligned} 5y_1 + 2y_2 &= 8 \\ 23y_1 + 7y_2 &= 28 \end{aligned}$$

1.1 Part A

Write these equations in matrix notation.

$$\begin{bmatrix} 5 & 2 \\ 23 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 28 \end{bmatrix}$$

1.2 Part B

Using matrix methods, find the solution for y_1 and y_2.

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} 5 & 2 \\ 23 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ 28 \end{bmatrix} \\ &= \begin{bmatrix} -7/11 & 2/11 \\ 23/11 & -5/11 \end{bmatrix} \begin{bmatrix} 8 \\ 28 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 4 \end{bmatrix} \end{aligned}$$

2 Problem 5.17

Consider the following functions of the random variables Y_1, Y_2, and Y_3

$$\begin{aligned}W_1 &= Y_1 + Y_2 + Y_3\\ W_2 &= Y_1 - Y_2\\ W_3 &= Y_1 - Y_2 - Y_3\end{aligned}$$

2.1 Part A

State the above in matrix notation.

$$\begin{bmatrix}W_1\\W_2\\W_3\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1 & -1 & -1\end{bmatrix}\begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix}$$

2.2 Part B

Find the expectation of the random vector $\boldsymbol{W}$.

Let $\mathbf{W} = \begin{bmatrix}W_1\\W_2\\W_3\end{bmatrix}$.

The expectation of $\mathbf{W}$ is $E(\mathbf{W}) = \begin{bmatrix}E(W_1)\\E(W_2)\\E(W_3)\end{bmatrix} = \begin{bmatrix}E(Y_1)+E(Y_2)+E(Y_3)\\ E(Y_1)-E(Y_2)\\ E(Y_1)-E(Y_2)-E(Y_3)\end{bmatrix}$

2.3 Part C

Find the variance-covariance matrix of $\boldsymbol{W}$.

By Equation 5.46 in the text, $\sigma^2(\mathbf{W}) = \sigma^2(\mathbf{AY}) = \mathbf{A}\sigma^2(\mathbf{Y})\mathbf{A}'$.

Let $\mathbf{A} = \begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 1 & -1 & -1\end{bmatrix}$, $\mathbf{Y} = \begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix}$, $\sigma^2(\mathbf{Y}) = \begin{bmatrix}\sigma^2(Y_1) & \sigma^2(Y_1,Y_2) & \sigma^2(Y_1,Y_3)\\ \sigma^2(Y_1,Y_2) & \sigma^2(Y_2) & \sigma^2(Y_2,Y_3)\\ \sigma^2(Y_1,Y_3) & \sigma^2(Y_2,Y_3) & \sigma^2(Y_3)\end{bmatrix}$

Then

$$\begin{aligned}
\sigma^2(\mathbf{W}) &= \sigma^2(\mathbf{AY}) = \mathbf{A}\sigma^2(\mathbf{Y})\mathbf{A}' \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \sigma^2(Y_1) & \sigma^2(Y_1, Y_2) & \sigma^2(Y_1, Y_3) \\ \sigma^2(Y_1, Y_2) & \sigma^2(Y_2) & \sigma^2(Y_2, Y_3) \\ \sigma^2(Y_1, Y_3) & \sigma^2(Y_2, Y_3) & \sigma^2(Y_3) \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\
&= \begin{bmatrix} \sigma^2(W_1) & \sigma^2(W_1, W_2) & \sigma^2(W_1, W_3) \\ \sigma^2(W_1, W_2) & \sigma^2(W_2) & \sigma^2(W_2, W_3) \\ \sigma^2(W_1, W_3) & \sigma^2(W_2, W_3) & \sigma^2(W_3) \end{bmatrix}
\end{aligned}$$

Where

$$\begin{aligned}
\sigma^2(W_1) &= \sigma^2(Y_1) + 2\sigma^2(Y_1, Y_2) + 2\sigma^2(Y_2) + 2\sigma^2(Y_1, Y_3) + 2\sigma^2(Y_2, Y_3) \\
\sigma^2(W_1, W_2) &= \sigma^2(Y_1) - \sigma^2(Y_2) + \sigma^2(Y_1, Y_3) - \sigma^2(Y_2, Y_3) \\
\sigma^2(W_1, W_3) &= \sigma^2(Y_1) - 2\sigma^2(Y_2) - 2\sigma^2(Y_2, Y_3) \\
\sigma^2(W_2) &= \sigma^2(Y_1) - \sigma^2(Y_2) + \sigma^2(Y_1, Y_3) - \sigma^2(Y_2, Y_3) \\
\sigma^2(W_2, W_3) &= \sigma^2(Y_1) - 2\sigma^2(Y_1, Y_2) + \sigma^2(Y_2) \\
\sigma^3(W_3) &= \sigma^2(Y_1) - 2\sigma^2(Y_1, Y_2) + 2\sigma^2(Y_2) - 2\sigma^2(Y_1, Y_3) + 2\sigma^2(Y_2, Y_3)
\end{aligned}$$

asdf