MA 4780 Homework 1

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Problem 1.4

Use R to simulate 48 random indepent chi-square distributed values, each with 2 degrees of freedom.

```
set.seed(1)
chiSq <- as.ts(rchisq(48, 2, ncp = 0))
```

Use this random data to generate a time series plot.

```
plot(chiSq,
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random Chi-Squared Distribution")
```

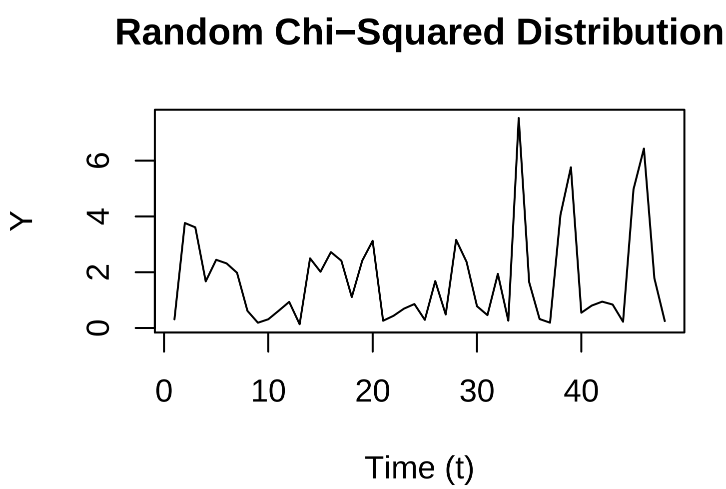

Figure 1: Time series plot of 48 random chi-squared distributed points.

Use the same code to generate two more random plots.

```
set.seed(2)
chiSq <- as.ts(rchisq(48, 2, ncp = 0))
plot(chiSq,
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random Chi Squared Distribution")
```

```
set.seed(3)
chiSq <- as.ts(rchisq(48, 2, ncp = 0))
plot(chiSq,
```

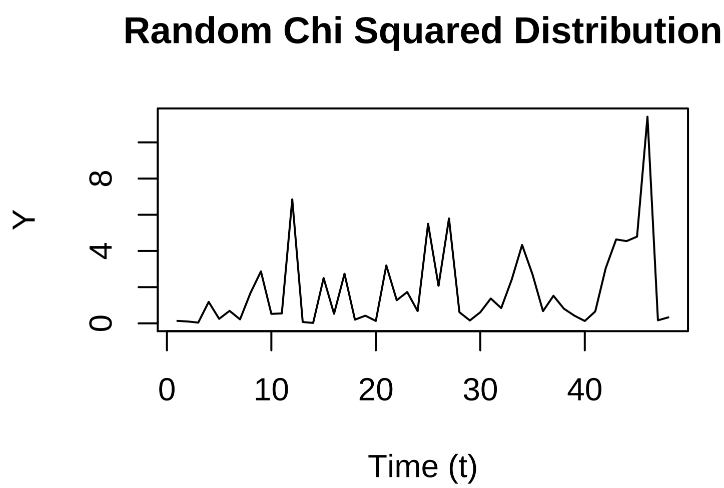

Figure 2: Time series plot of 48 random chi-squared distributed points.

```r
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random Chi Squared Distribution")
```

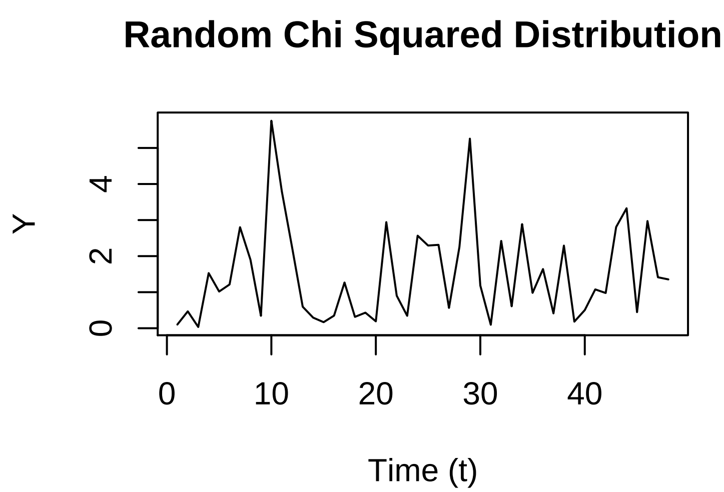

Figure 3: Time series plot of 48 random chi-squared distributed points.

Each plot appears random. When comparingthe plots in Figures 1, 2, 3, there is no common trend or pattern to be followed. Each plot is unique enough to suggest randomness.

Problem 1.5

Use R to simulate 48 random indepent t-distributed values, each with 5 degrees of freedom.

Use this random data to generate a time series plot.

```r
set.seed(4)
t <- as.ts(rt(48, 5, ncp = 0))
```

```r
plot(t,
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random t-Distribution")
```

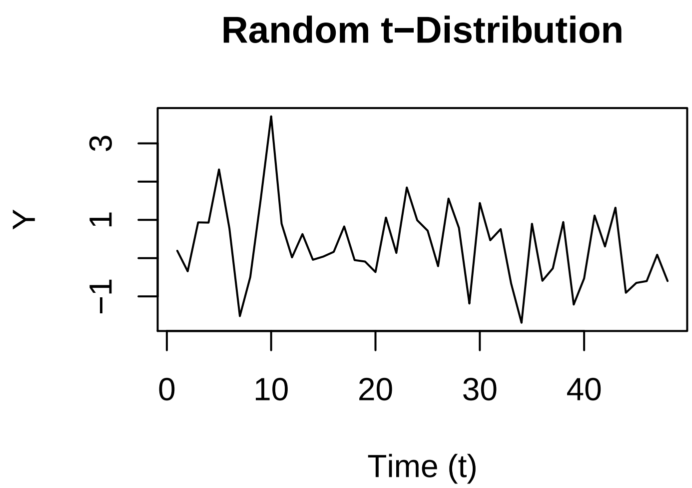

Figure 4: Time series plot of 48 random t-distributed points.

Use the same code to generate two more random plots.

```r
set.seed(5)
t <- as.ts(rt(48, 5, ncp = 0))
plot(t,
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random t-Distribution")
```

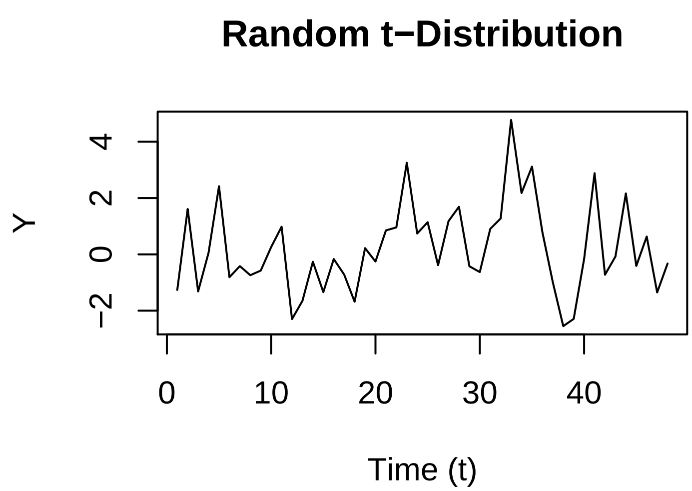

Figure 5: Time series plot of 48 random t-distributed points.

```r
set.seed(6)
t <- as.ts(rt(48, 5, ncp = 0))
plot(t,
     xlab = "Time (t)",
     ylab = "Y",
     main = "Random t-Distribution")
```

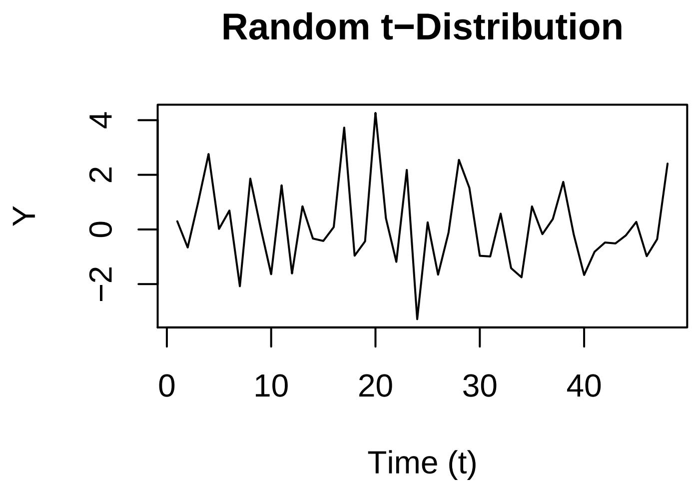

Figure 6: Time series plot of 48 random t-distributed points.

Each plot appears random. When comparingthe plots in Figures 4, 5, 6, there is no common trend or pattern to be followed. Each plot is unique enough to suggest randomness.

Problem 1.6

The following code constructs a time series plot with monthly plotting symbols for the Dubuque temperature series in Exhibit 1.9, on page 7 in the text. The output is seen in Figure 7 on the next next page.

```r
data("tempdub")
plot(tempdub, type='l', ylab='Sales', main = "Dubuque Temperatures over Time")
points(y=tempdub,x=time(tempdub),
       pch=as.vector(season(tempdub)))
```

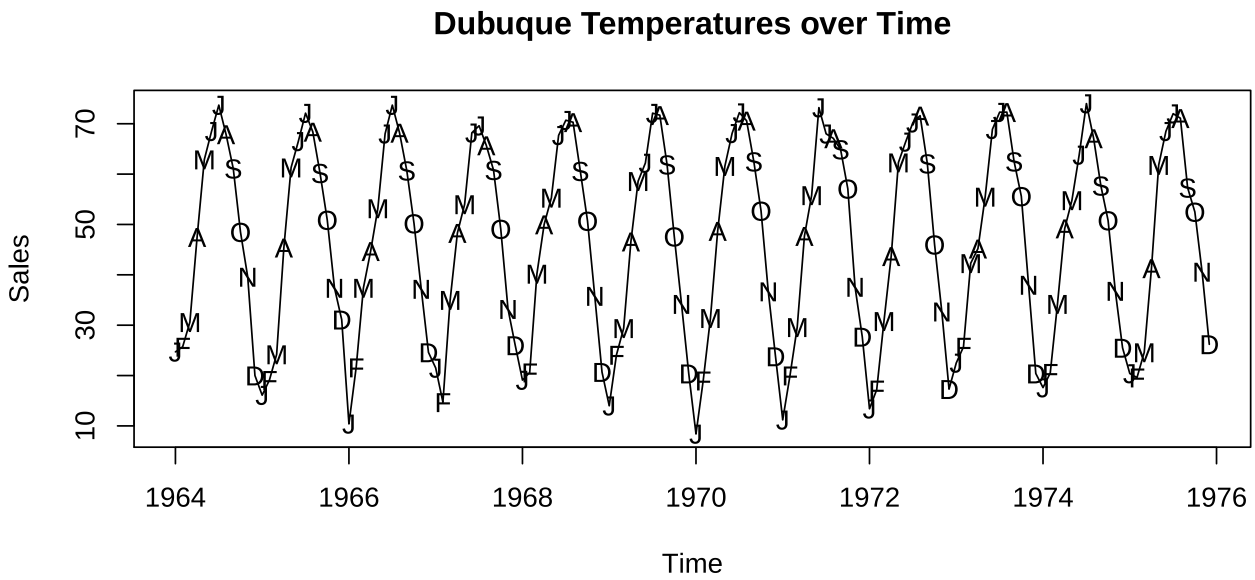

Figure 7: Dubuque temperatures over time with monthly plotting symbols.

Problem 2.9

Part A

Let $Y_t = \beta_0 + \beta_1 t + X_t$.

Because the expected value

$$\begin{aligned} E(Y_t) &= E(\beta_0 + \beta_1 t + X_t) \\ &= E(\beta_0) + E(\beta_1 t) + E(X_t) \\ &= \beta_0 + \beta_1 t \end{aligned}$$

is not constant and depends on t, Y_t is not stationary.

Consider $W_t = Y_t - Y_{t-1}$ where

$$\begin{aligned} E(W_t) &= E(Y_t - Y_{t-1}) \\ &= E(\beta_0 + \beta_1 t) - E(\beta_0 + \beta_1 (t-1)) \\ &= E(\beta_1 t - \beta_1 t + \beta_1) \\ &= \beta_1 \end{aligned}$$

is constant. The autocovariance, where

$$\begin{aligned} cov(W_t, W_s) &= cov(Y_t - Y_{t-1}, Y_s - Y_{s-1}) \\ &= \gamma_{|t-s|} + \gamma_{|(t-1)-(s-1)|} - \gamma_{|t-1-s|} - \gamma_{|s-1-t|} \\ &= \gamma_{|t-s|} + \gamma_{|t-s|} - \gamma_{|t-1-s|} - \gamma_{|s-1-t|} \\ &= 2\gamma_h + \gamma_{h-1} - \gamma_{h+1} \end{aligned}$$

only depends on the lag h. Therefore, W_t is stationary.

Problem 2.10

Part A

Consider $Y_t = \mu_t + \sigma_t X_t$ where

$$\begin{aligned} E(Y_t) &= E(\mu_t) + E(\sigma_t X_t) \\ &= \mu_t + \sigma_t E(X_t) \\ &= \mu_t + 0 \\ &= \mu_t \end{aligned}$$

is constant. The autocovariance, where

$$\begin{aligned} cov(Y_t, Y_s) &= cov(\mu_t + \sigma_t X_t, \mu_s + \sigma_s X_s) \\ &= cov(\sigma_t X_t, \sigma_s X_s) \\ &= \sigma_t \sigma_s cov(X_t, X_s) \\ &= \sigma_t \sigma_s (corr(X_t, X_s \sqrt{Var(X_t) Var(X_s)})) \\ &= \sigma_t \sigma_s (\rho_k \sqrt{(1)(1)}) \\ &= \sigma_t \sigma_s \rho_k \end{aligned}$$

depends on the lag k.

Part B

The autocorrelation function

$$\begin{aligned} corr(Y_t, Y_s) &= \frac{cov(Y_t, Y_s)}{\sqrt{Var(Y_t) Var(Y_s)}} \\ &= \frac{\sigma_t \sigma_s \rho_k}{\sigma_t \sigma_s} \\ &= \rho_k \end{aligned}$$

only depends on the lag k. However, Y_t is *not stationary* because the expected value of Y_t, μ_t, depends on time t.

Part C

Even if the expected value of a function Y_t with a constant mean, μ. The autocovarianc function $cov(Y_t, Y_s) = \sigma_t \sigma_s \rho_k$ still depends on t, therefore making Y_t *not stationary*.

Problem 2.13

Part A

Consider $Y_t = e_t - \theta(e_{t-1})^2$ where

$$\begin{aligned} E(Y_t) &= E(e_t) - \theta E(e_{t-1}^2) \\ &= \theta\sigma^2 \end{aligned}$$

is constant. The autocovariance function, where

$$\begin{aligned} cov(Y_t, Y_s) &= cov(e_t - \theta(e_{t-1})^2, e_s - \theta(e_{t-1})^2) \\ &= E((e_t e_s) - (\theta(e_{t-1})^2 e_s) - (\theta(e_{s-1})^2 e_t) + (\theta^2(e_{t-1})^2(e_{s_1})^2)) - \theta^2\sigma^4 \\ &= E(e_t e_s) - \theta E((e_{t-1})^2 e_s) - \theta E((e_{s-1})^2 e_t) + \theta^2 E((e_{t-1})^2(e_{s-1})^2) - \theta^2\sigma^4 \\ &= 0 - 0 - 0 + \theta^2\sigma^4 - \theta^2\sigma^4 = 0 \end{aligned}$$

depends on the time lag. Therefore, becuase $\gamma = 0$, the autocorrelation function $\rho = 0$.

Part B

Y_t is stationary because the expected value is constand the the auotcovariance function only depnds on the time lag.

Problem 2.14

Part A

Consider $Y_t = \theta_0 + te_t$, where

$$\begin{aligned} E(Y_t) &= E(\theta_0 + te_t) \\ &= \theta_0 \end{aligned}$$

is constant. The autocovariance function

$$\begin{aligned} cov(Y_t, Y_s) &= cov(\theta_0 + te_t, \theta_0 + se_s) \\ &= cov(te_t, se_s) \\ &= tscov(e_t, e_s) \\ &= ts\sigma^2 \end{aligned}$$

depends on t. Therefore, Y_t is *not stationary.*

Part B

Consider $W_t = Y_t - Y_{t-1} = te_t - (t-1)e_{t-1}$, where

$$\begin{aligned} E(W_t) &= E(te_t - (t-1)e_{t-1}) \\ &= tE(e_t) - (t-1)E(e_{t-1}) \\ &= 0 \end{aligned}$$

is constant. The autocovariance function

$$\begin{aligned} cov(W_t, W_s) &= cov(te_t - (t-1)e_{t-1}, se_s - (s-1)e_{s-1}) \\ &= E(te_t Se_s,) - E(se_s(t-1)e_{t-1}) - E(te_t(s-1)e_{s-1}) + E((t-1)e_{t-1}(s-1)e_{s-1}) \\ &= tsE(e_t e_s) - s(t-1)E(e_S e_{t-1}) - t(s-1)E(e_{s-1}e_t) + (t-1)(s-1)E(e_{s-1}e_{t-1}) \\ &= ts\sigma^2 - (st - s)\sigma^2 - (ts - t)\sigma^2 + (ts - s - t + 1)\sigma^2 \\ &= \sigma^2(ts - st + s - ts + t + ts - s - t + 1) \\ &= \sigma^2 \end{aligned}$$

does not depend on the time lag. Therefore W_t is *not stationary.*

Part C

Consider $Y_t = e_t e_{t-1}$, where

$$\begin{aligned} E(Y_t) &= E(e_t e_{t-1}) \\ &= E(e_t)E(e_{t-1}) \\ &= 0 \end{aligned}$$

is constant. The autocovariance function

$$\begin{aligned} cov(Y_t, Y_s) &= cov(e_t e_{t-1}, e_s e_{s-1}) \\ &= E(e_t e_{t-1} e_s e_{s-1}) - \sigma^4 \\ &= \sigma^4 \end{aligned}$$

does not depond the the time lag. Therefore Y_t is *not stationary.*

Problem 2.19

Part A

$$\begin{aligned} Y_t &= \theta_0 + Y_{t-1} + e_t \\ &= \theta_0 + (\theta_0 + Y_{t-2} + e_{t-1}) + e_t \\ &= 2\theta_0 + (\theta_0 + Y_{t-3} + e_{t-2}) + e_t + e_{t-1} \\ &= 3\theta_0 + (\theta_0 + Y_{t-4} + e_{t-3}) + e_t + e_{t-1} + e_{t-2} \end{aligned}$$

for t iterations until we get $Y_t = t\theta + e_t + e_{t-1} + \ldots + e_1$.

Part B

The expected value

$$
\begin{aligned}
E(Y_t) &= E(t\theta_0 + e_t + \ldots + e_1) \\
&= E(t\theta_0) \\
&= t\theta_0
\end{aligned}
$$

depends on t.

Part C

The autocovariance function

$$
\begin{aligned}
cov(t\theta_0 + e_t + e_{t-1} + \ldots + e_1, s\theta_0 + e_s + e_{s-1} + \ldots + e_1) &= var(e_t + e_{t-1} + \ldots + e_1) \\
&= t\theta_t
\end{aligned}
$$

also depends on t. Therefore, Y_t is not stationary.