MA 4780 Homework 5

Benjamin Hendrick

April 10, 2016

Exercise 7.3

Exercise 7.15

Simulate the AR(1) model.

```
set.seed(1)
ar1 <- arima.sim(list(order = c(1,0,0), ar = c(-0.7)), n = 100)
```

Part A

Use the `arima` function with method `ML` to find the maximum likelihood estimator of ϕ.

```
ar1.mle <- arima(ar1,order=c(1,0,0),method="ML")
```

The MLE for ϕ is -0.7079663.

Part B

Using the sample size $n = 100$, we can run the `arima` function with method `ML` many times.

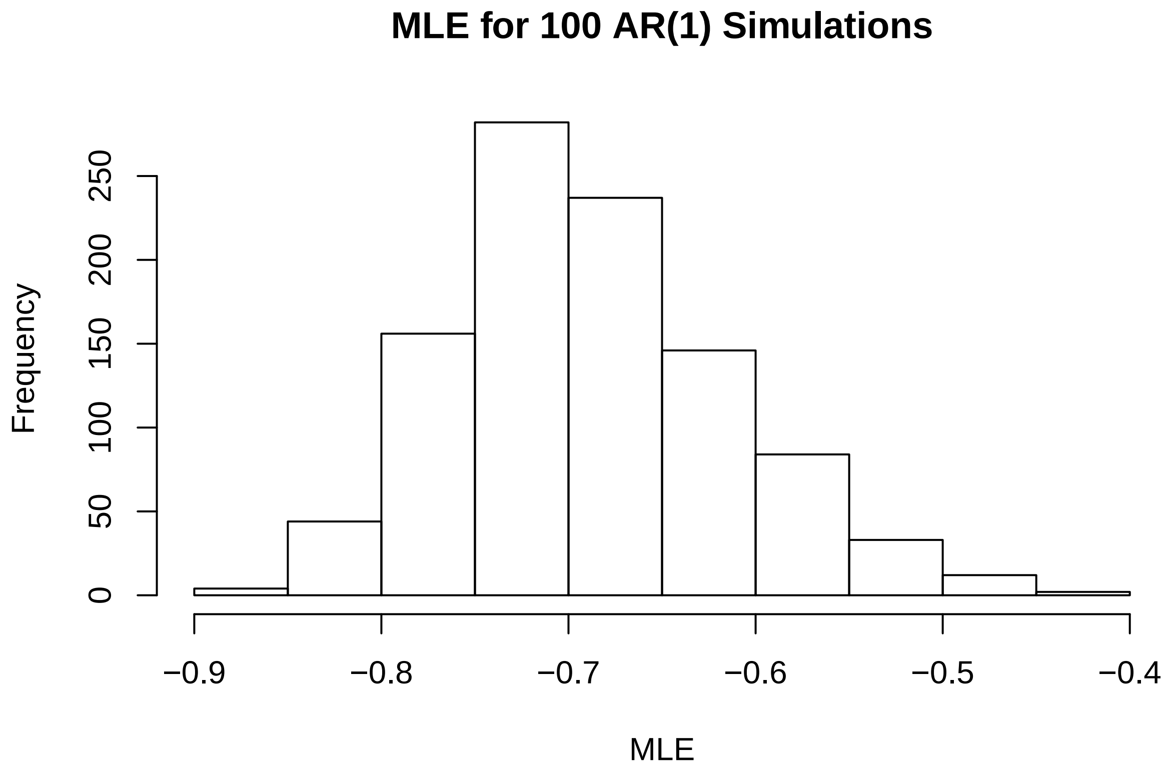
```
mle.list <- c()
for(i in 1:1000){
  ar1 <- arima.sim(list(order = c(1,0,0), ar = c(-0.7)), n = 100)
  ar1.mle <- arima(ar1,order=c(1,0,0),method="ML")
  mle.list <- c(mle.list,ar1.mle$coef[1])
}
```

Part C

The center of the sampling distribution is $\mu =$ -0.6877054.

The histogram of the sampling distribution is:

```
hist(x = mle.list,
     main = "MLE for 100 AR(1) Simulations",
     xlab = "MLE")
```



Part D

The estimators are unbiased and normally distributed because the sample size is so large. The histogram in Part C implied normality and unbias.

Part E

The variance of the sampling distribution is $\sigma^2 = 0.0055825$.

By 7.4.9 on Page 161 in the text, the variance should approximately be $\frac{1-\phi^2}{n} = \frac{1-0.7^2}{100} = 0.0051$.

The two variances are extremely close. The only differ by less than 0.0004.

Exercise 7.21

Part A

Simulate the ARMA(1,1)

```
set.seed(2)
#arma11 <-arima.sim(list(order = c(1,0,1), ar = c(-0.7)), ma = c(-0.6), n = 48)
```

Exercise 7.27

Part A

```
data("oil.price")
oil.ar1.mle <- arima(oil.price, order = c(1,0,0), method = "ML") # AR(1)
oil.ar4.mle <- arima(oil.price, order = c(4,0,0), method = "ML") # AR(4)
```

```
## Warning in stats:::arima(x = x, order = order, seasonal = seasonal, xreg =
## xreg, : possible convergence problem: optim gave code = 1
```

The AIC for the AR(1) model is 1061.0536653. The AIC for the AR(4) model is 1051.869129. Between the two, the AR(4) model has the smallest AIC. It would be a better model than the AR(1).

Part B

```
set.seed(23456)
oil.ma1.mle <- arima(oil.price, order = c(0,0,1), method = "ML")
```

The AIC for the MA(1) model is 1539.8060968. This is much larger than the AIC for the AR(4) model, suggesting that the MA(1) model is worse than the AR(4) model.

Exercise 7.29

```
data(robot)
```

Part A

```
robot.ar1 <- arima(robot, order = c(1,0,0))
```

The parameter(s) of the AR(1) model for the `robot` data are:

- $\phi = 0.3074$
- $\sigma^2 = 6.482 \times 10^{-6}$

Part B

```
robot.ima11 <- arima(robot, order = c(0,1,1))
```

The parameter(s) of the IMA(1,1) model for the `robot` data are:

- $\theta = -0.8713$
- $\sigma^2 = 6.069 \times 10^{-6}$

Part C

The AIC for the AR(1) model in Part A is -2947.0776048. THe AIC for the IMA(1,1) model in Part B is -2959.9010353.

Exercise 8.3

For an AR(2) model, it can be shown that

$$Var(\hat{r}_1) \approx \frac{\phi_2^2}{n}$$

and

$$Var(\hat{r}_2) \approx \frac{\phi_2^2 + \phi_1^2(1+\phi_2)^2}{n}$$

and

$$Var(\hat{r}_k) \approx \frac{1}{n} \text{ for } k \geq 3$$

By these rules:

$$Var(\hat{r}_1) \approx \frac{1.1^2}{200} = 0.00605$$
$$Var(\hat{r}_2) \approx \frac{(-0.8)^2 + 1.1^2(1+(-0.8))^2}{200} = 0.003442$$
$$Var(\hat{r}_3) \approx \frac{1}{200} = 0.005$$

The 95 percent confidence intervals for $\hat{r}_1$, $\hat{r}_2$ and $\hat{r}_3$ are

$$\pm 2\sqrt{Var(\hat{r}_k)} \text{ for } k = 1, 2, 3, \ldots$$

Test for individual support:

$$\pm 2\sqrt{Var(\hat{r}_1)} = \pm 2\sqrt{0.00605} = (-0.1555635, 0.1555635)$$

Becasue 0.13 is in the confidence interval, $\hat{r}_1$ supports AR(2).

$$\pm 2\sqrt{Var(\hat{r}_2)} = \pm 2\sqrt{0.003442} = (-0.1173371, 0.1173371)$$

Becasue 0.13 is not in the confidence interval, $\hat{r}_2$ does support AR(2).

$$\pm 2\sqrt{Var(\hat{r}_3)} = \pm 2\sqrt{0.005} = (-0.1414214, 0.1414214)$$

Becasue 0.12 is not in the confidence interval, $\hat{r}_3$ supports AR(2).

Perform a Ljung-Box test to determine joint support.

Find Q, where $Q = n(\hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2)$

$$Q = 200(0.13^2 + 0.13^2 + 0.12^2) = 9.64$$

Find $Q*$, where $Q* = n(n+2)\left(\frac{\hat{r}_1^2}{n-1} + \frac{\hat{r}_2^2}{n-2} + \frac{\hat{r}_3^2}{n-3}\right)$

$$Q* = 200(202)\left(\frac{0.13^2}{199} + \frac{0.13^2}{198} + \frac{0.12^2}{197}\right) = 9.832334$$

Becasue $Q* > Q$, the residual autocorrelations joingly support AR(2).

Exercise 8.6

```
set.seed(34567)
ar2.sim <- arima.sim(list(order=c(2,0,0),ar=c(1.5, -0.75)),n=48)
```

Part A

```
ar2.fit <- arima(ar2.sim, order = c(2,0,0))
plot(rstandard(ar2.fit), type = "o",
     main = "Residuals or AR(2) Model",
     ylab = "Standard Residuals")
```