MA 4870 Homework 3

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Exercise 4.2

The following `R` function, `sketchMA2`, sketches autocorrelation functions for MA(2) processes.

```
sketchMA2 <- function(theta1, theta2) {
  y <- ARMAacf(ma = c(-theta1,-theta2), lag.max = 20)
  plot(y, x = 0:20,
       type = "h",
       ylim = c(-1,1),
       xlab = "k",
       ylab = "Autocorrelation",
       main = paste("Population ACF of an MA(2) model with coefficiens ",
                    theta1, " and ", theta2))
  abline(h=0)
}
```

Part A

Sketch the autocorrelation function for the MA(2) process where $\theta_1 = 0.5$ and $\theta_2 = 0.4$. The plot is shown in Figure 1.

```
sketchMA2(0.5,0.4)
```

Part B

Sketch the autocorrelation function for the MA(2) process where $\theta_1 = 1.2$ and $\theta_2 = -0.7$. The plot is shown in Figure 2.

```
sketchMA2(1.2,-0.7)
```

Part C

Sketch the autocorrelation function for the MA(2) process where $\theta_1 = -1$ and $\theta_2 = -0.6$. The plot is shown in Figure 3.

```
sketchMA2(-1,-0.6)
```

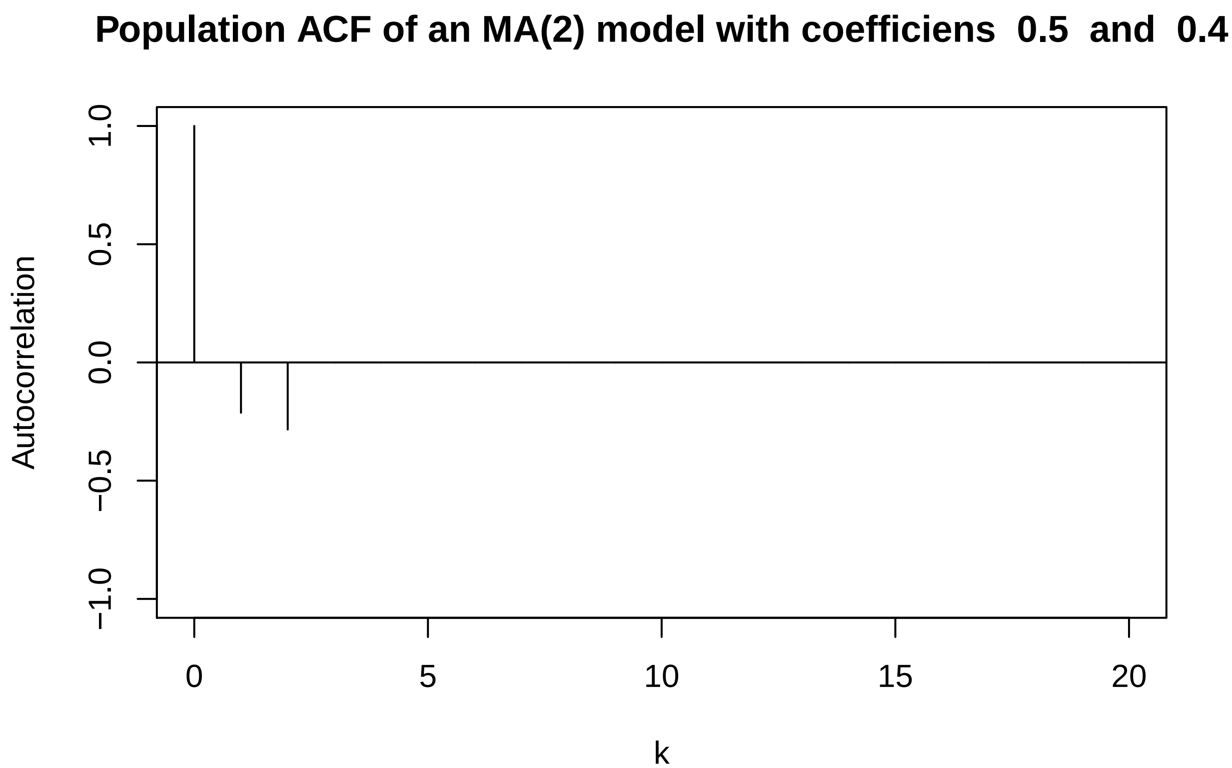

Figure 1: Population ACF of an MA(2) model with coefficiens 0.5 and 0.4

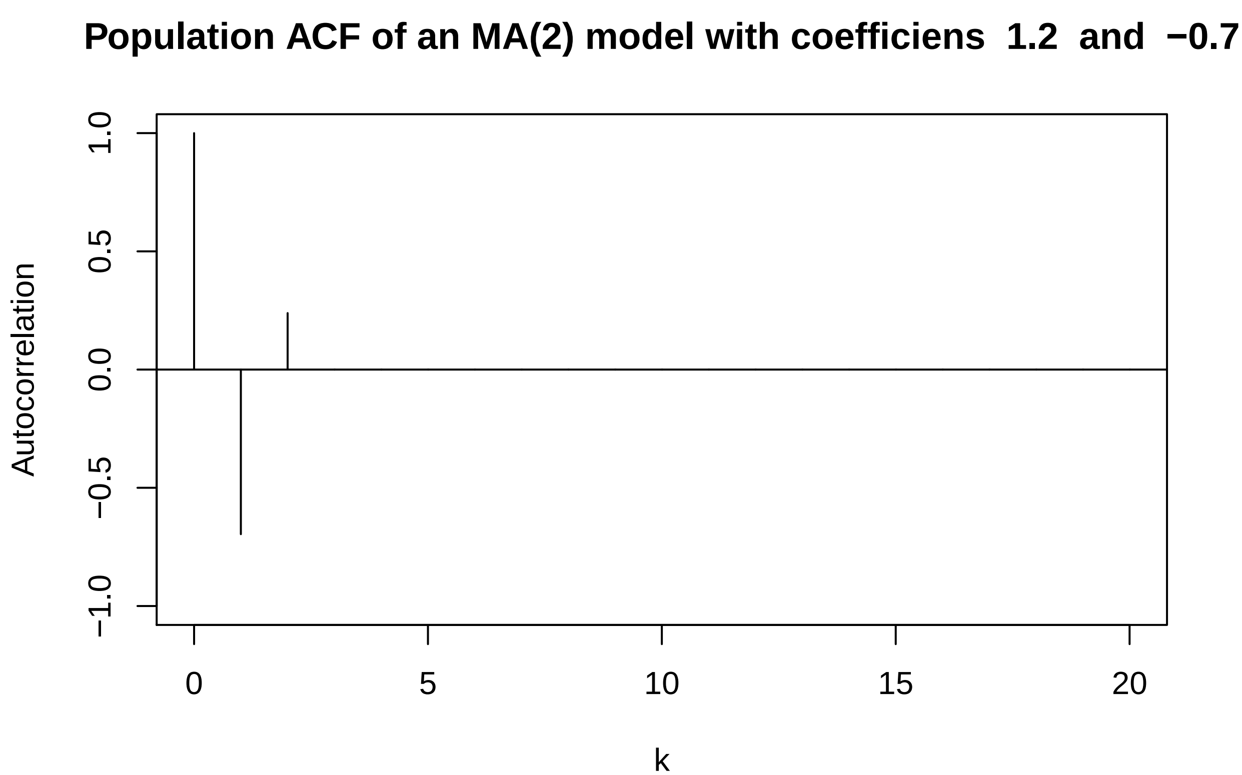

Figure 2: Population ACF of an MA(2) model with coefficiens 1.2 and -0.7

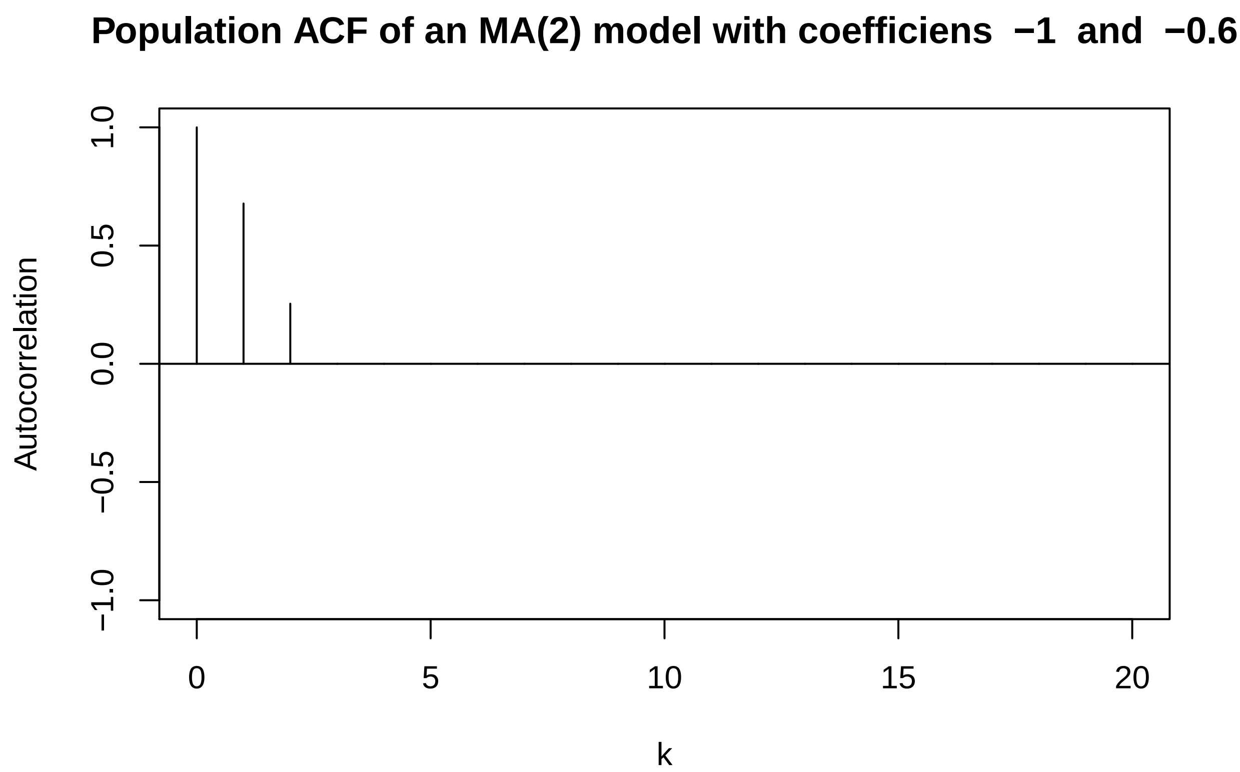

Figure 3: Population ACF of an MA(2) model with coefficiens -1 and -0.6

Exercise 4.5

The following R function, `sketchAR1`, sketches autocorrelation function for AR(1) processes over a specified number of lags.

```
sketchAR1 <- function(phi1, lags) {
  y <- ARMAacf(ar = phi1, lag.max = lags)
  plot(y, x = 0:lags,
       type = "h",
       ylim = c(-1,1),
       xlab = "k",
       ylab = "Autocorrelation",
       main = paste("Population ACF of an AR(2) model with coefficient ", phi1))
  abline(h=0)
}
```

Part A

Sketch the autocorrelation function for the AR(1) process where $\theta = 0.6$. The plot is shown in Figure 4.

```
sketchAR1(0.6, 10)
```

Part B

Sketch the autocorrelation function for the AR(1) process where $\theta = -0.6$. The plot is shown in Figure 5.

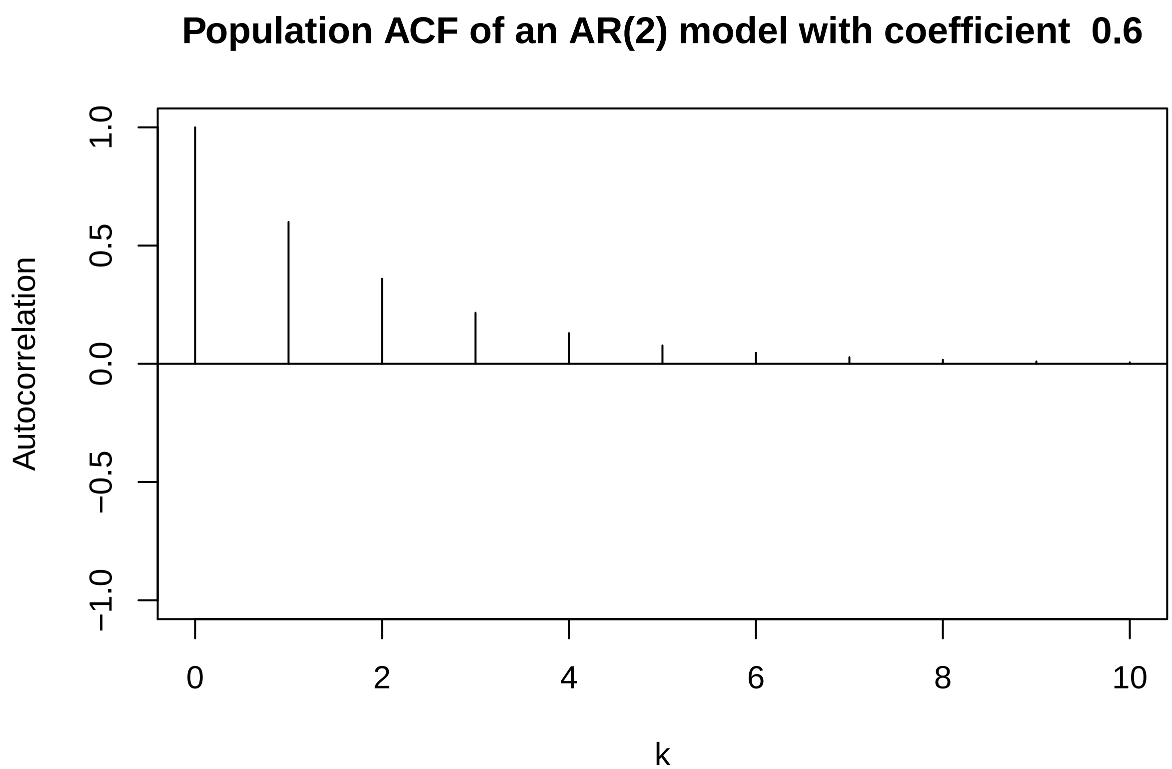

Figure 4: Population ACF of an AR(1) model with coefficient 0.6 over 10 lags.

```
sketchAR1(-0.6, 10)
```

Part C

Sketch the autocorrelation function for the AR(1) process where $\theta = 0.95$. The plot is shown in Figure 6.

```
sketchAR1(95, 20)
```

Part D

Sketch the autocorrelation function for the AR(1) process where $\theta = 0.3$. The plot is shown in Figure 7.

```
sketchAR1(0.3, 5)
```

Exercise 4.9

The following `R` function, `recursiveAR2`, uses the recursive formula of Equation 4.3.13 in the text and sketchs the autocorrelation funciton of AR(2) processes.

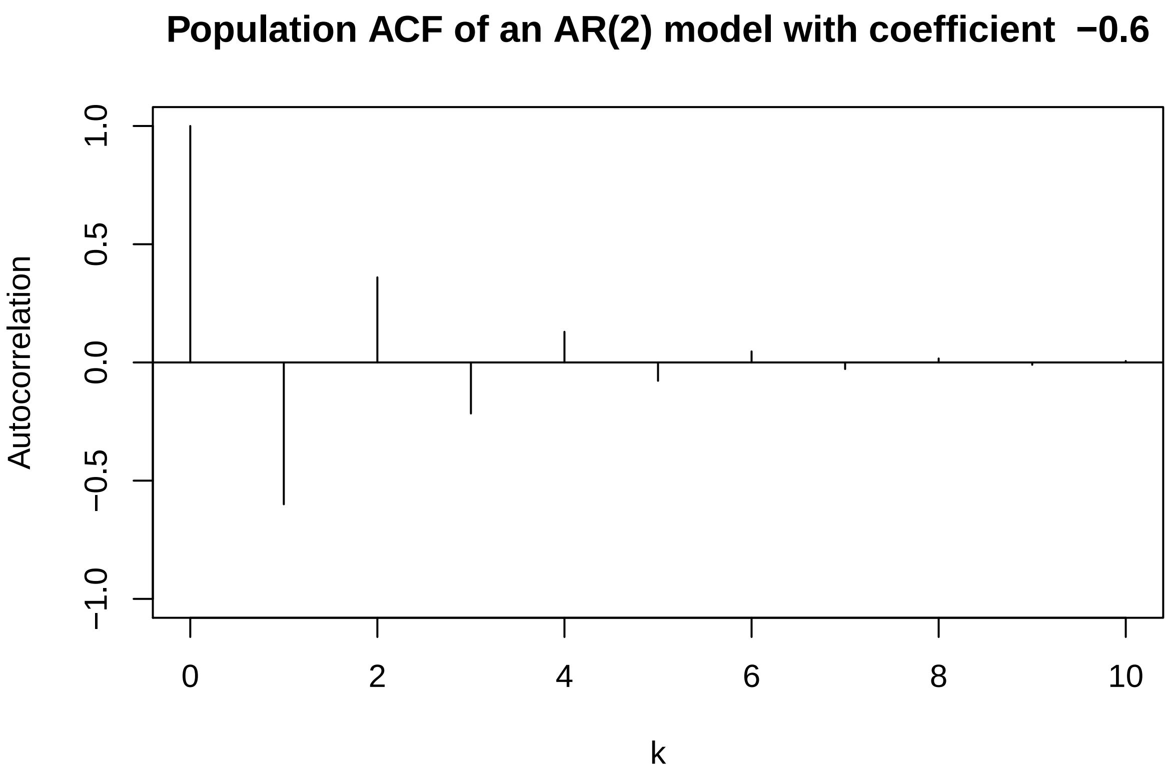

Figure 5: Population ACF of an AR(1) model with coefficient 0.6 over 10 lags.

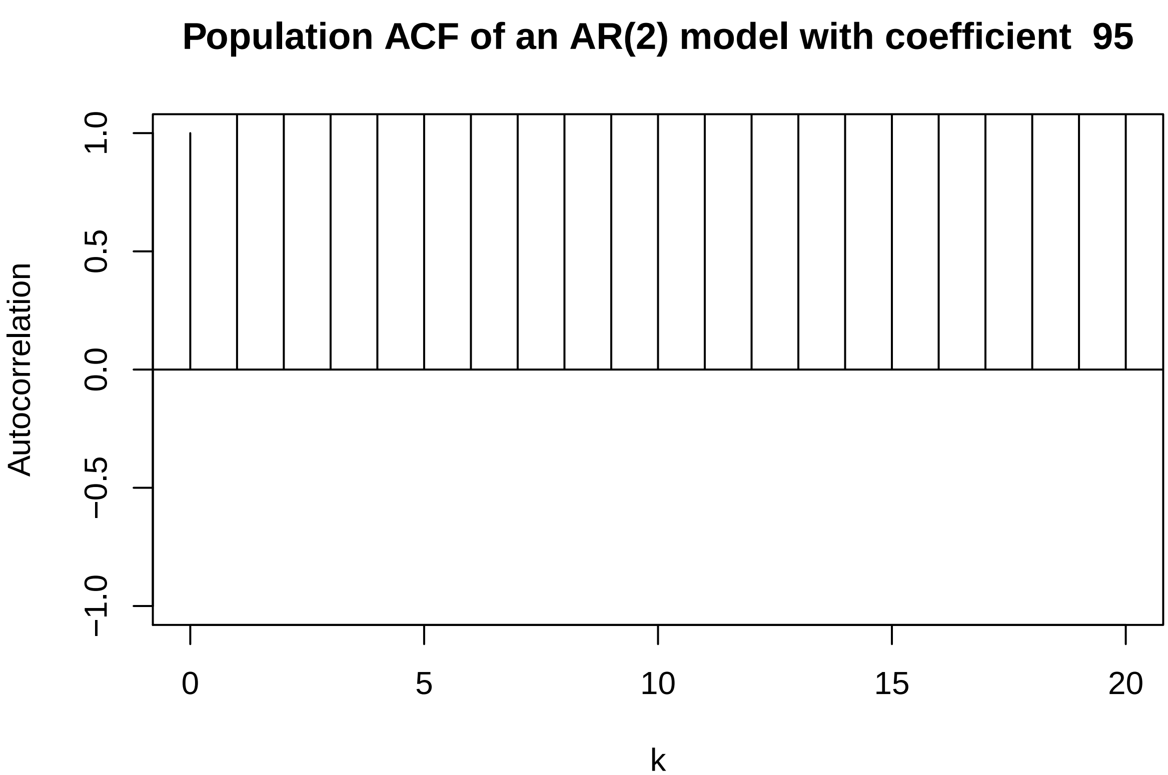

Figure 6: Population ACF of an AR(1) model with coefficient 0.95 over 20 lags.

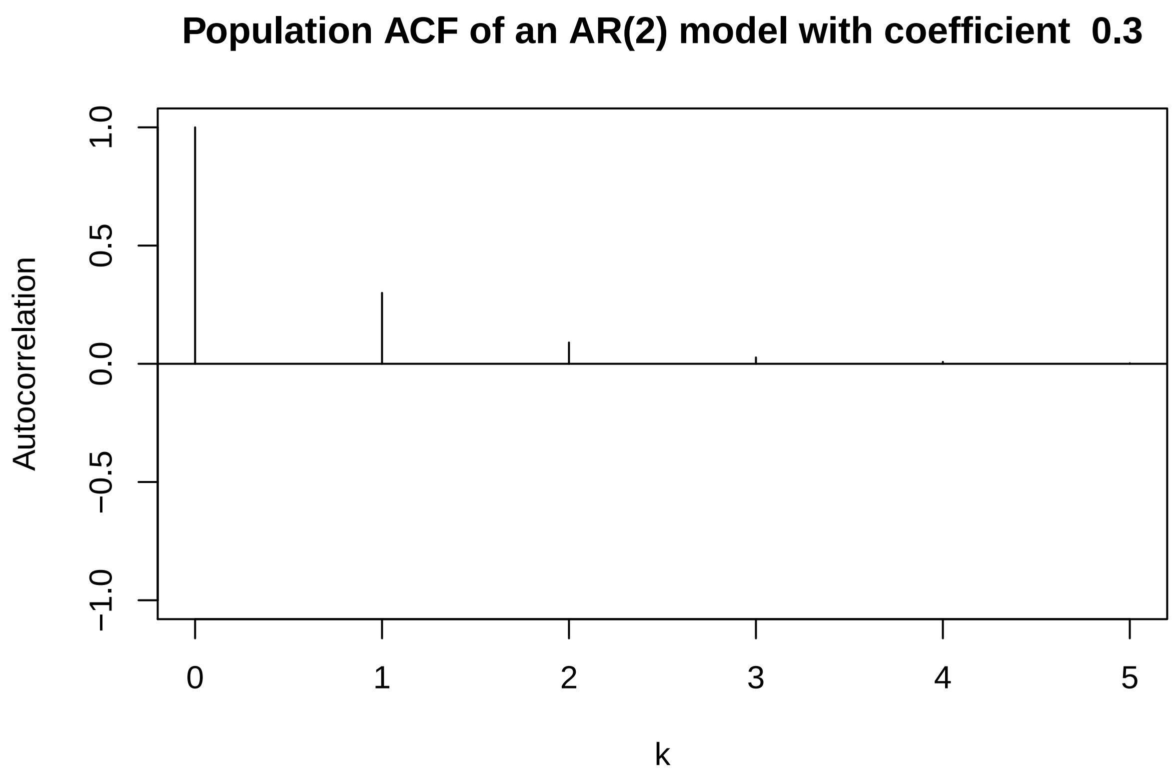

Figure 7: Population ACF of an AR(1) model with coefficient 0.3 over 5 lags.

```
recursiveAR2 <- function(phi1, phi2, lags) {
  max.lag = lags
  rho = rep(0,max.lag)
  rho[1] = phi1/(1-phi2)
  rho[2] = (phi2*(1-phi2)+phi1^2)/(1-phi2)
  for (k in 3:max.lag) {
    rho[k] = phi1*rho[k-1]+phi2*rho[k-2]
  }
  plot(y=c(1,rho), x=0:max.lag, type='h', ylab='ACF', xlab='Lag', ylim=c(-1,+1),
       main = paste("Population ACF of an AR(2) model with coefficients ", phi1, " and ", phi2))
  abline(h=0)

  results <- list()
  results$roots <- polyroot(c(1,phi1,phi2))
  results$class <- class(polyroot(c(1,phi1,phi2)))
  results$R <- sqrt(-phi2+0i)
  results$theta <- cospi(-(phi1 * sqrt(-phi2+01))/(2 *phi2))
  return(results)
}
```

Part A

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = 0.6$ and $\phi_2 = 0.3$. The plot is found in Figure 8.

```r
a <- recursiveAR2(0.6,0.3,20)
```

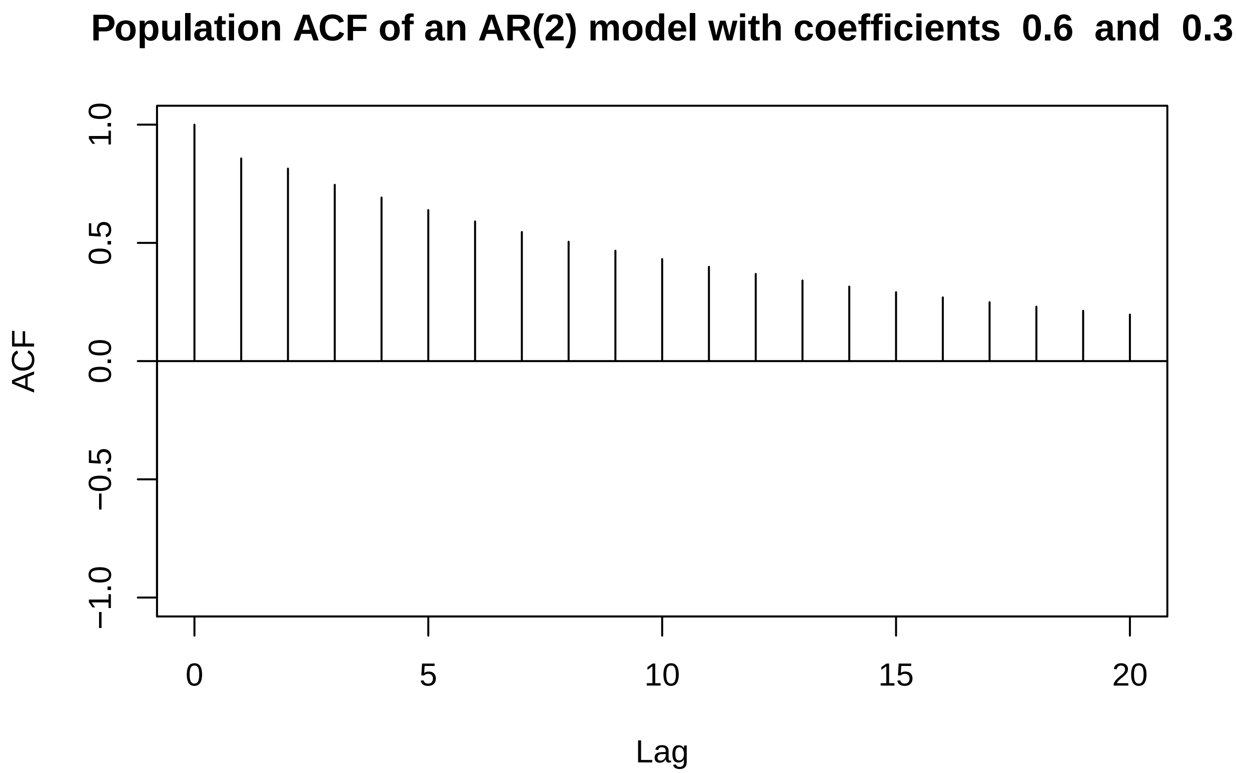

Figure 8: Population ACF of an AR(2) model with coefficients 0.6 and 0.3

The roots for the process are (-1+1.52752523165195i, -1-1.52752523165195i). The roots are complex. The damping factor R equals 0+0.547722557505166i. The frequency θ equals -0.8712036.

Part B

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = -0.4$ and $\phi_2 = 0.5$. The plot is found in Figure 9.

```r
b <- recursiveAR2(-0.4,0.5,20)
```

The roots for the process are (0.4+1.35646599662505i, 0.4-1.35646599662505i). The roots are complex. The damping factor R equals 0+0.707106781186548i. The frequency θ equals 0.6305175.

Part C

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = 1.2$ and $\phi_2 = -0.7$. The plot is found in Figure 10.

```r
c <- recursiveAR2(1.2,-0.7,20)
```

The roots for the process are (-0.613661448712429+0i, 2.32794716299814+0i). The roots are complex. The damping factor R equals 0.836660026534076+0i. The frequency θ equals -0.9325511.

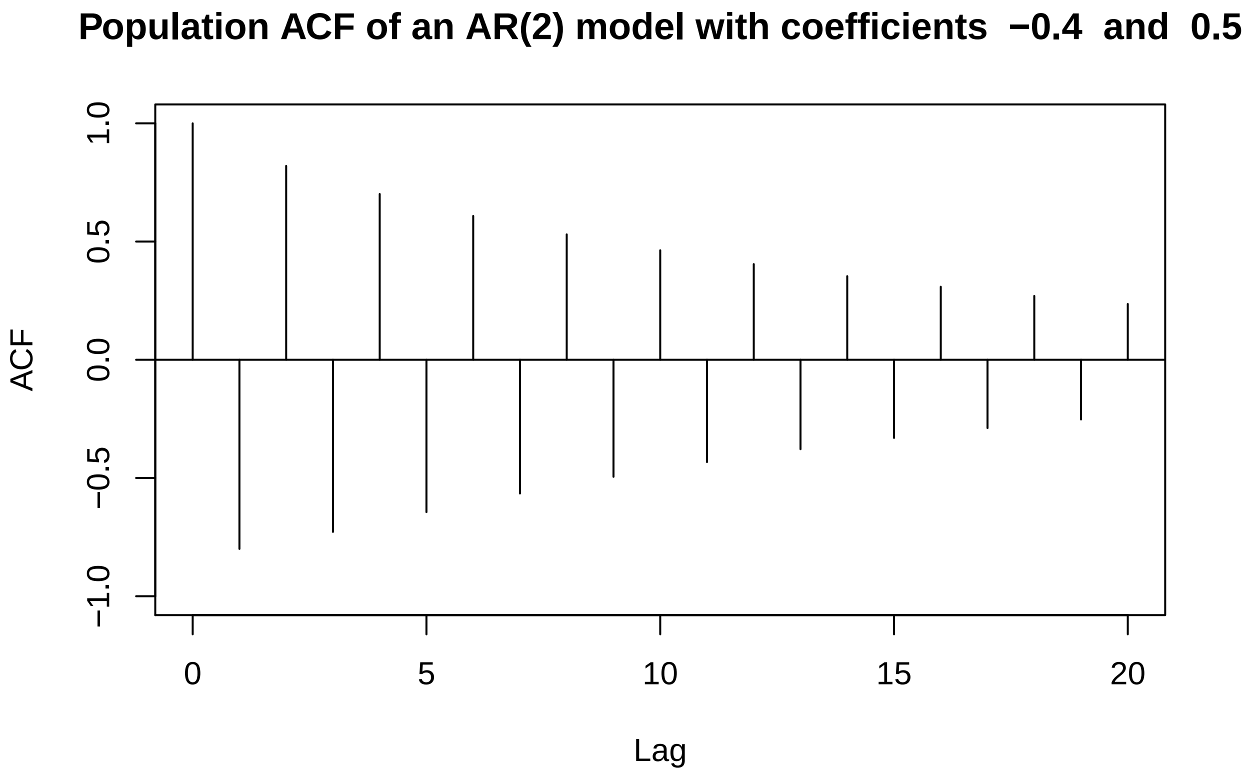

Figure 9: Population ACF of an AR(2) model with coefficients -0.4 and 0.5

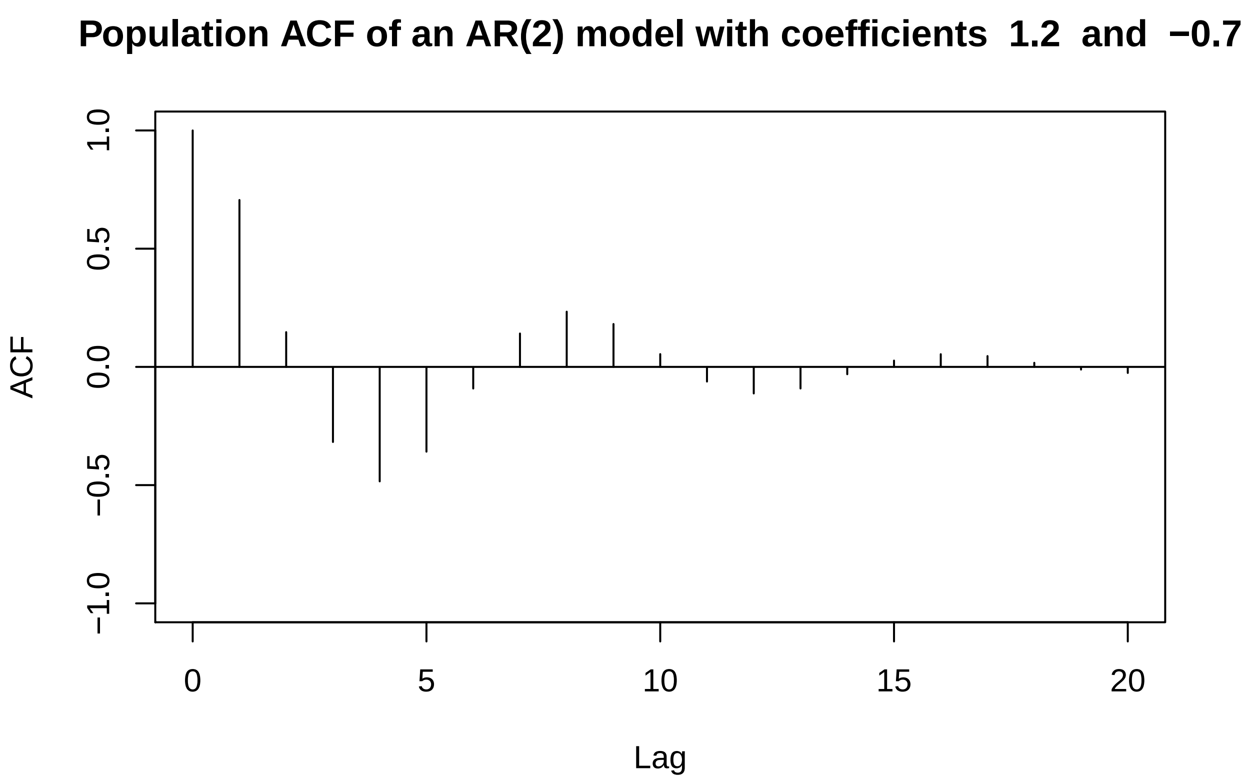

Figure 10: Population ACF of an AR(2) model with coefficients 1.2 and -0.7

Part D

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = -1$ and $\phi_2 = -0.6$. The plot is found in Figure 11.

```
d <- recursiveAR2(-1,-0.6,20)
```

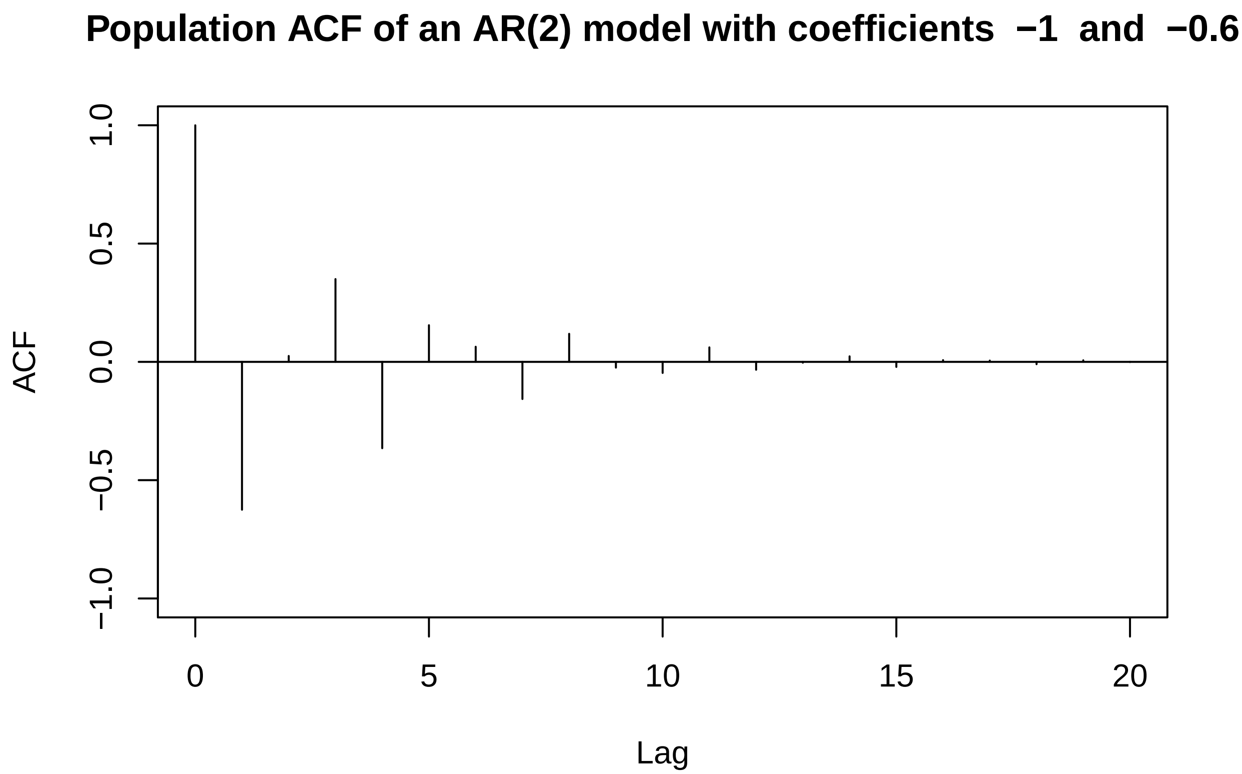

Figure 11: Population ACF of an AR(2) model with coefficients -1 and -0.6

The roots for the process are (0.703257409548815-0i, -2.36992407621548+0i). The roots are complex. The damping factor R equals 0.774596669241483+0i. The frequency θ equals -0.9855955.

Part E

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = 0.5$ and $\phi_2 = -0.9$. The plot is found in Figure 12.

```
e <- recursiveAR2(0.5,-0.9,20)
```

The roots for the process are (1.36785649279714+0i, -0.812300937241588+0i). The roots are complex. The damping factor R equals 0.948683298050514+0i. The frequency θ equals 0.3596672.

Part F

Sketch the auto correlation plot the recursive AR(2) process with $\phi_1 = -0.5$ and $\phi_2 = -0.6$. The plot is found in Figure 13.

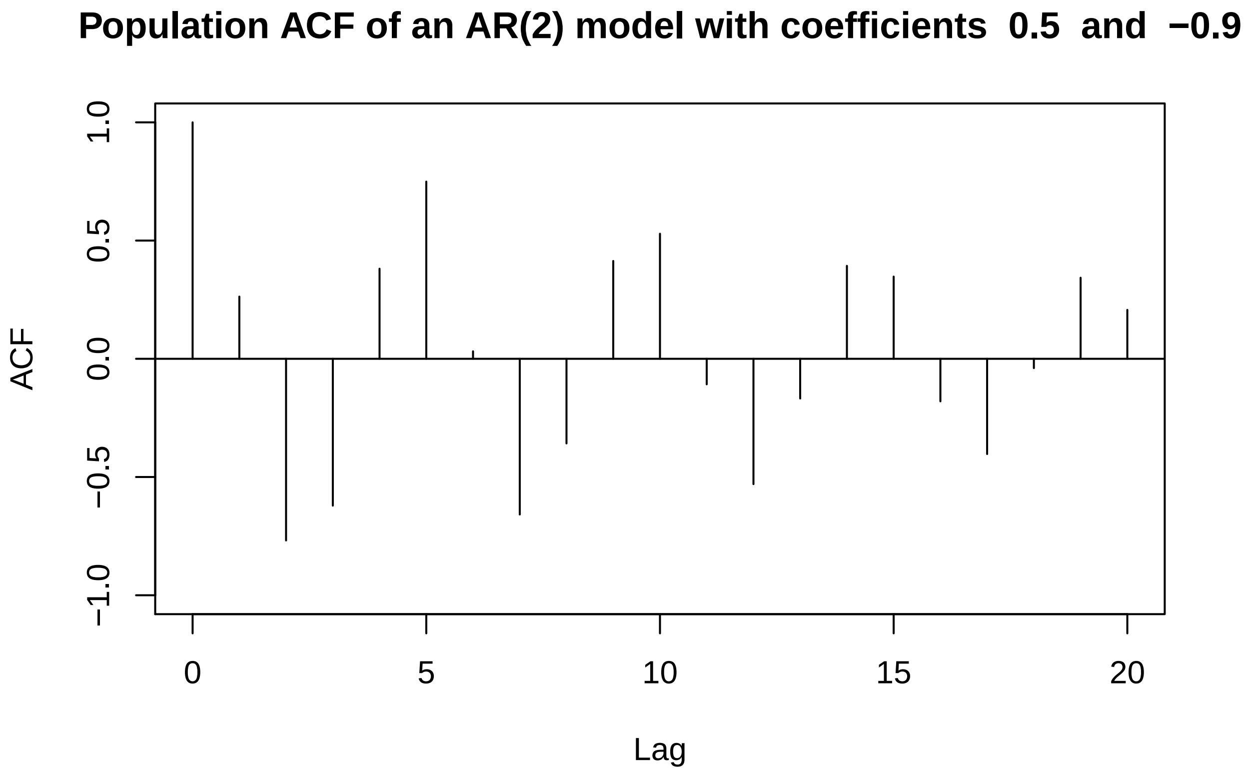

Figure 12: Population ACF of an AR(2) model with coefficients 0.5 and -0.9

```
f <- recursiveAR2(-0.5,-0.6,20)
```

The roots for the process are (0.939901716341642-0i, -1.77323504967498+0i). The roots are complex. The damping factor R equals 0.774596669241483+0i. The frequency θ equals -0.0848662.

Exercise 4.12

Part A

Use the formula into Equation 4.2.3 from the text. When $\theta_1 = \theta_2 = \frac{1}{6}$,

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2} = \frac{-1/6 + 1/6(1/6)}{1 + 1/6^2 + 1/6^2} = -0.1315789$$

and

$$\rho_2 = \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2} = \frac{-1/6}{1 + 1/6^2 + 1/6^2} = -0.1578947$$

When $\theta_1 = -1$ and $\theta_2 = 6$,

$$\rho_1 = \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2} = \frac{1 - 1(6)}{1 + (-1)^2 + 6^2} = -0.1315789$$

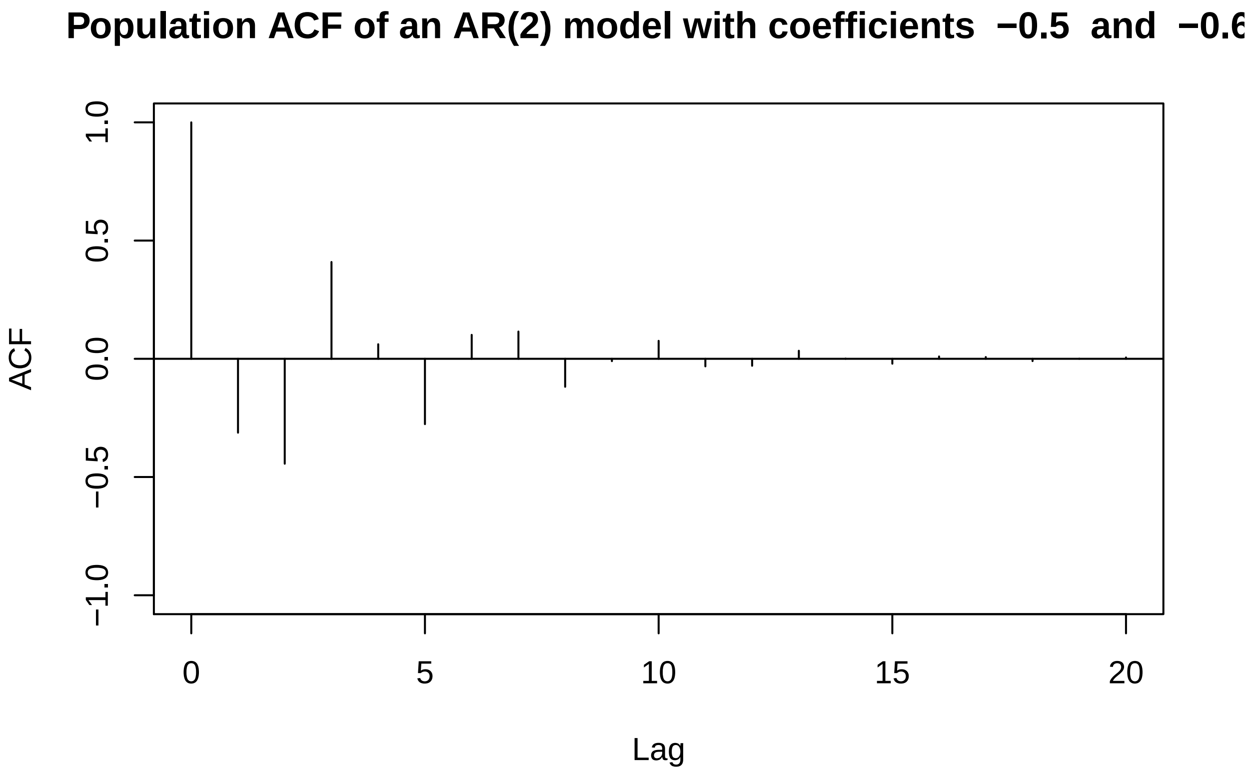

Figure 13: Population ACF of an AR(2) model with coefficients -0.5 and -0.6

and

$$\rho_2 = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2} = \frac{-6}{1+(-1)^2+6^2} = -0.1578947$$

Therefore, the autocorrelation functions are the same because ρ_1 and ρ_2 are the same for each set of θ_1 and θ_2.

Part B

One of the roots is $1 - \frac{1}{6}x - \frac{1}{6}x^2 = -\frac{1}{6}(x+3)(x-2)$ and the other is $1 + x - 6x^2 = -6(x+\frac{1}{3})(x-\frac{1}{2})$. The roots of the two polynomials are reciprocals of each other.

Exercise 4.15

Suppose $\{Y_t\}$ is stationary. $Var(Y_t) = \sigma_e^2/(1-\phi^2)$. Therefore, if $|\phi| = 1$ is impossible. This is a contridiction and is therefore proved.

Exercise 4.16

Part A

$$Y_t = 3Y_{t-1} + e_t = 3(3Y_{t-2} + e_{t-1}) + e_t \Rightarrow 3^j Y_t = -\sum_{j=1}^{\infty} e_{t+j} \Rightarrow Y_t = -\sum_{j=1}^{\infty} (\frac{1}{3})^j e_{t+j}$$

Part B

$$\mu = E(Y_t) = E(e_t + \ldots) = 0$$

$$Cov(Y_t, Y_{t-1}) = \frac{(1/3)\sigma_e^2}{1 - (1/3)^2}$$

The autocovariance is found using the steps in equaion 4.1.2 and 4.1.3 in th text.

Part C

The model is unsatisfactory because Y_t at time t depends on future error terms.

Exercise 4.18

Part A

$$E(W_t) = E(Y_t + c\phi^t) = 0 + c\phi^t = \phi^t$$

Part B

$Y_t = c\phi^t = \phi[Y_{t-1} + c\phi^{t-1}] + e_t$ is valid since the terms $c\phi^t$ cancel on both sides.

Part C

The solution is not stationary because $E(W_t) = c\phi^t$ depends on t.

Exercise 4.19

There are a couple things to take note of. (1) The coefficients decrease exponentially in magnitude at rate 0.5 while alternating signs. (2) The coefficients are nearly died out by θ_6.

Therefore, an AR(1) process with $\phi = -0.5$ would nearly be the same process.

Exercise 4.20

Thre are a few things to take note of. (2) The coefficients decrease exponentially in magnitude ate rate 0.5 while alternating signs. (2) The coefficients are nearly died out by θ_7. Equation 4.4.6 in the text suggests that an ARMA(1,1) with $\phi = -0.5$ and $\theta = 0.5$ would nearly be the same.