Exploration of the Van Atta Array

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1 Overview

1.1 What is a Van Atta Array

A Van Atta Array is a special type of phased array whereby it can reflect electromagnetic radiation received in one direction back out into the direction it came from [1, 2, 3]. This is powerful for many reasons; however, a major motivation to use this array is from back-and-forth (tag) communication with a base station [3]. The Van Atta Array is especially well positioned for this use case as it can be set in a passive arrangement whereby the physical properties of the array (such as transmission lines between elements) give rise to the aforementioned reflective behavior without the large energy costs incurred by an active phased array [3, 2]. In fact, a gain based on the number of elements in the array is observed due to the constructive interference between them re-radiating in the same direction [1].

1.2 Scope for this Exploration

For this part of the project, the goal is to devise a model in MATLAB where one can explore the various parameters and their effects on the re-radiated field from the array. The parameters explored will be, as become clear in 2, number of elements in the array (see 3.1), spacing between elements (see 3.2), and impinging angle (see 3.3).

2 Mathematical Theory

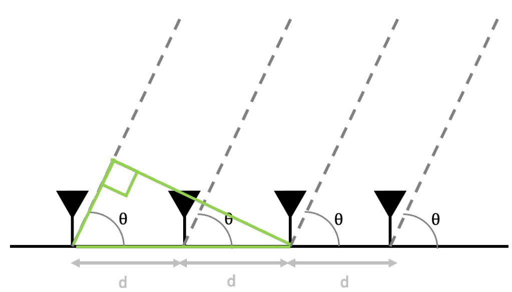

Figure 1: Diagram of a generic uniform linear array with variable number of elements.

2.1 Phased Arrays

In a phased array, we expect each element to receive a offset phase relative to the first element in the array. This is directly related to the relative path length difference between each element [4]. As seen in Figure 1, the impinging angle (θ) and the uniform distance between elements gives us the necessary information to calculate the relative path length difference for the nth element as:

$$S_n = n * d * \cos\theta \tag{2.1}$$

This can then be used to find the relative phase of the received radiation of a particular element n by adjusting scaling with the wave number $k = 2\pi/\lambda$, yielding the following equation:

$$\widetilde{\phi}_n = \phi_n + \frac{2\pi}{\lambda} * S_n \tag{2.2}$$

The purpose of a phased array is then to add a phase factor ($\Delta\phi_n$) to steer the radiation in a certain direction [4] yielding:

$$\widetilde{\phi}_n = \phi_n + \frac{2\pi}{\lambda} * S_n - \Delta\phi_n \tag{2.3}$$

where ϕ_n is the initial phase difference incurred in the element which can be due to transmission lines or explicitly set by the designer to convey information.

When you then sum over all the elements (N) with their phases, you can calculate the reflected field at some angle of observation θ_{obs} through the difference:

$$|E_{reflected}(\theta_{obs})| = |E_{incoming}| * \sum_{n=0}^{N} e^{j*\widetilde{\phi}_n} \tag{2.4}$$

2.2 Van Atta Array

For the Van Atta Array, the geometry of the array itself makes it so there is not overall phase shift (except for any initial ϕ_n) upon reflection. Therefore, be inspection we can see that:

$$\widetilde{\phi}_n = \phi_n = \phi_n + \frac{2\pi}{\lambda} * S_n + \Delta\phi_n \tag{2.5}$$

which through rearranging the equation yields:

$$\Delta\phi_n = -\frac{2\pi}{\lambda} * n * d * \cos\theta_i \tag{2.6}$$

where θ_i is the impinging angle of the incident radiation. This then leads to the following electric field at an observation angle θ_{obs}:

$$|E_{reflected}(\theta_{obs})| = |E_{incoming}| * \sum_{n=0}^{N} e^{j*\frac{2\pi}{\lambda}*\cos\theta_{obs}} * e^{-j*\frac{2\pi}{\lambda}*\cos\theta_i} \tag{2.7}$$

which makes it clear that an observer standing at the transmission site (i.e. $\theta_{obs} = \theta_i$) sees:

$$|E_{reflected}(\theta_{obs} = \theta_i)| = N * |E_{incoming}| \tag{2.8}$$

3 Model

Several MATLAB scripts and and an interactive MATLAB-app were developed according to the findings in 2 and are available on GitHub and the scripts are attached in the appendix.

3.1 Dependence on Number of Elements

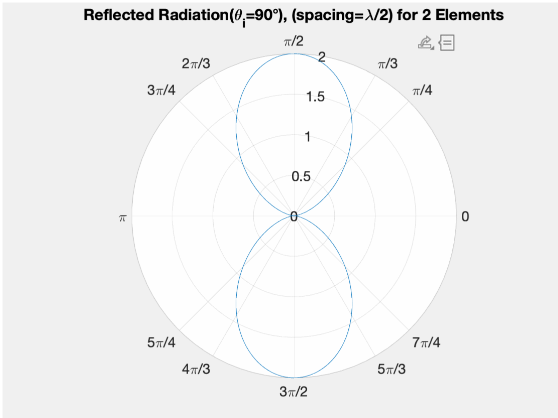

Figure 2: Reflected radiation with 2 array elements.

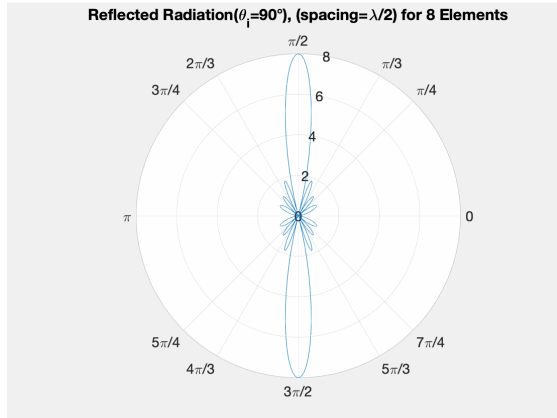

Figure 3: Reflected radiation with 8 array elements.

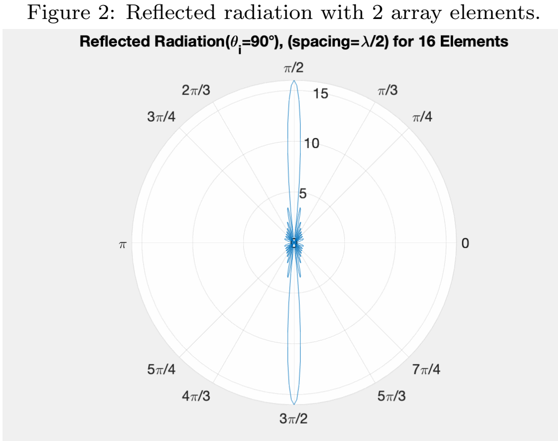

Figure 4: Reflected radiation with 16 array elements.

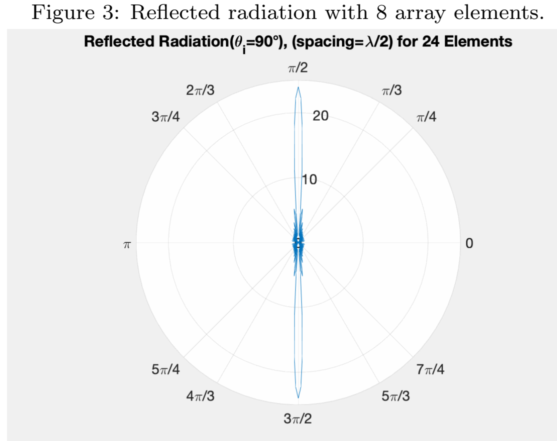

Figure 5: Reflected radiation with 24 array elements.

As seen in figures 2, 3, 4, 5, it appears that there is a direct correlation between the number of elements and the thickness of the main lobe. The more elements, the thinner and more directed the lobe becomes. The other aspect of note is that the magnitude of the reflection appears to be stronger, the more elements present; it appears to be increasing linearly with the number of elements as it should in theory.

3.2 Dependence on Element Spacing

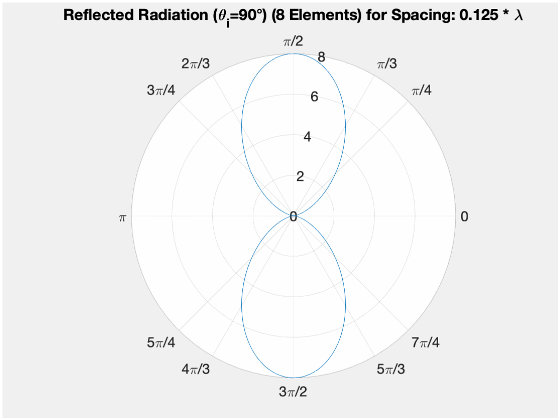

Figure 6: Reflected radiation with element spacing of 0.125λ.

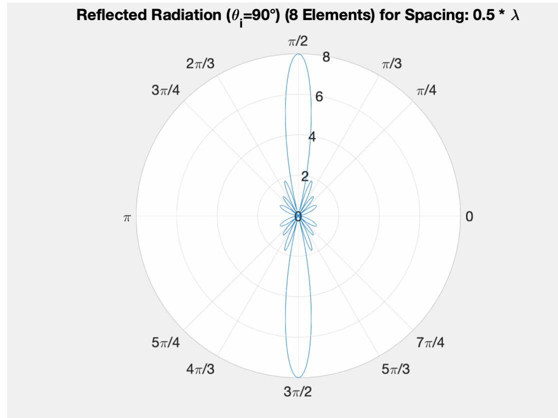

Figure 7: Reflected radiation with element spacing of 0.5λ.

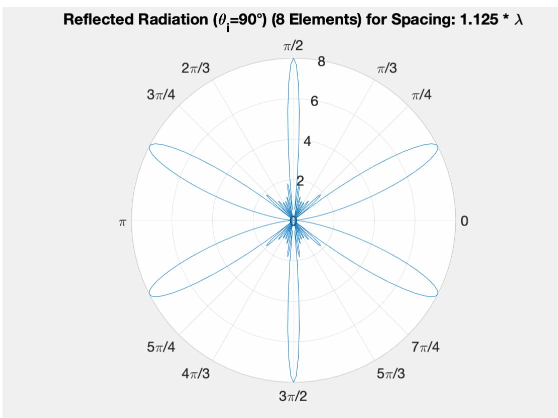

Figure 8: Reflected radiation with element spacing of 1.125λ.

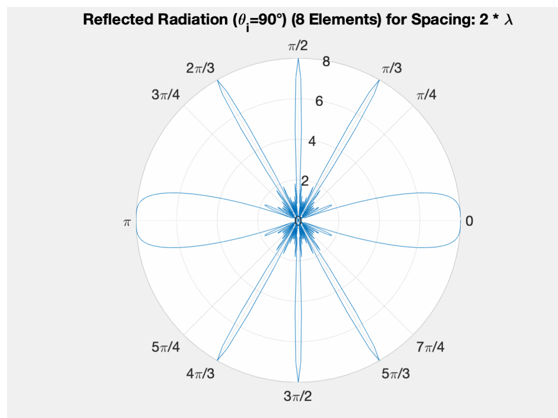

Figure 9: Reflected radiation with element spacing of 2λ.

As seen in figures 6, 7, 8, 9, two major trends appear. The first is that, as the spacing increases, the lobes appear to narrow. Second, as the spacing increases, it appears that more side-lobes are created. This appears to be cyclical with a new lobe appearing and then thinning over a certain period.

3.3 Dependence on Impinging Angle

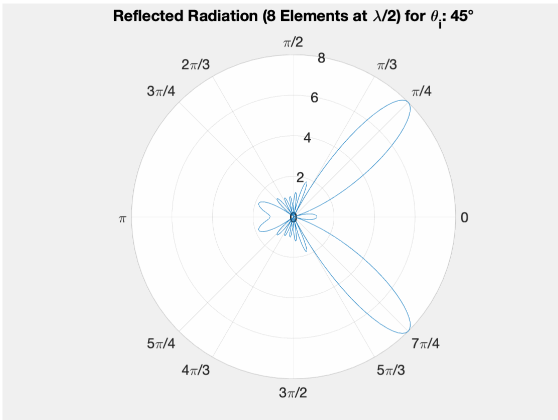

Figure 10: Reflected radiation with transmitter at 45°.

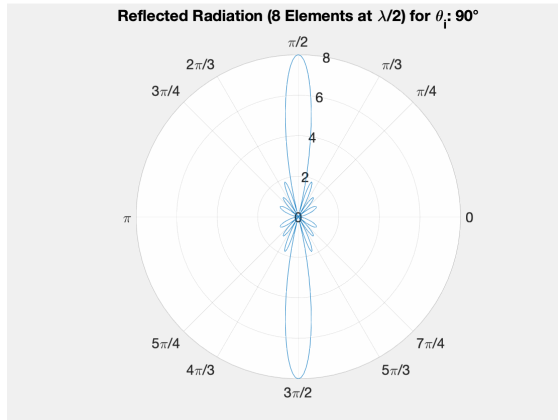

Figure 11: Reflected radiation with transmitter at 90°.

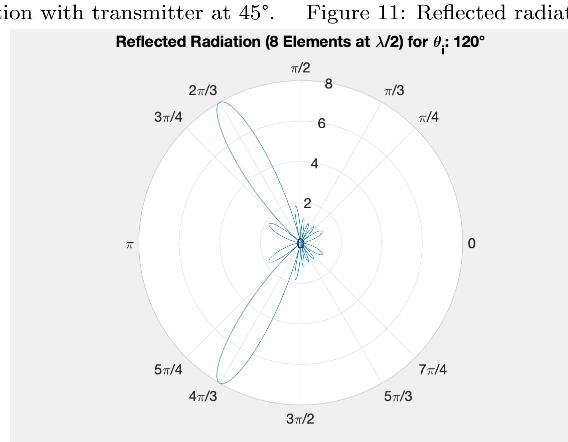

Figure 12: Reflected radiation with transmitter at 120°.

As is seen in Figures 10, 11, 12, the reflected radiation is strongest in the direction of origin of the impinging wave. At a very small change in observation angle difference yields in a much smaller received reflection. This seems more pronounced when the radiation is transmitted perpendicular to the array.

References

[1] E. Sharp and M. Diab. "Van Atta reflector array". In: *IRE Transactions on Antennas and Propagation* 8.4 (1960), pp. 436–438. DOI: 10.1109/tap.1960.1144877.

[2] Shyh-Jung Chung and Tan-Hsiung Ho. "Van Atta Array Reflector". In: *Encyclopedia of RF and Microwave Engineering* (2005). DOI: 10.1002/0471654507.eme528.

[3] Mohammad Hossein Mazaheri, Alex Chen, and Omid Abari. "mmTag". In: *Proceedings of the 2021 ACM SIGCOMM 2021 Conference* (2021). DOI: 10.1145/3452296.3472917.

[4] Doug Tougaw. "Chapter 32 - Antenna Arrays". In: *Applied Electromagnetic Field Theory*. Valparaiso University, 2018.

A Van Atta Calculation

```
% ------------------------ %
% Van Atta Model Script    %
% Name   : vanatta.m       %
% Author : Benjamin Herber %
% Date   : Fall 2021       %
% ------------------------ %
clear
clc
%% Constants
LIGHTSPEED = physconst('Lightspeed');

DEFAULT_THETA_IMPINGING = pi/2;
DEFAULT_NUM_ELEMENTS = 8;
DEFAULT_WAVELEN = LIGHTSPEED/2.4E9;
DEFAULT_SPACING = DEFAULT_WAVELEN/2.0;
DEFAULT_PHASES = zeros(DEFAULT_NUM_ELEMENTS,1);
find_reflectedE = @priv_reflected_E;

%% Find phase change

function phase = find_phase(theta, wavelen, spacing, n)
    phase = ((2.0*pi)/wavelen)*spacing*n*cos(theta);
end

%% Find reflection

function resE = priv_reflected_E(incoming_E, num_elements, wavelen, spacing, theta_impinging, theta_obs)
    tmp = 0;
    for idx = 1:num_elements
        tmp = tmp + exp(1i*(find_phase(theta_obs, wavelen, spacing, (idx-1))-find_phase(theta_impinging,
        ↪  wavelen, spacing, (idx-1))));
    end

    resE = incoming_E*tmp;
end
```