Simulating electromagnetic wave propagation in ice with finite difference time domain

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Abstract: Radar and seismic methods are used as geophysical techniques in many disciplines within the earth sciences. These techniques tell us something about the internal structure of a material that we can not physically see into. The utility of that knowledge is widespread. For example, geophysical methods are commonly used in the cryosphere to know the internal form of snow and ice masses, thereby giving indirect information about the accumulation and deformation of the mass

1 Introduction

Radar and seismic methods are used as geophysical techniques in many disciplines within the earth sciences. These techniques tell us something about the internal structure of a material that we can not physically see into. The utility of that knowledge is widespread. For example, geophysical methods are commonly used in the cryosphere to know the internal form of snow and ice masses, thereby giving indirect information about the accumulation and deformation of the mass itself.

As a project for ESS 511, Geophysical Continuum Mechanics, I will simulate these equations for electromagnetic wave propagation through ice. Starting in one dimension, I will prescribe some contrasts in the material constants that force a reflection of the wave back toward the surface. Provided that I have time I will move on to a two-dimensional model. Finally, I will discuss the practical applications of this model in glaciology research Christianson et al. 2016 as well as the general use of geophysical techniques in this field.

2 Methods

2.1 Maxwell's Equations

As with any physical process, using a computer model as a simulation can help us better understand the process itself. In this case, we can use Maxwell's curl equations to simulate the propagation of an electromagnetic wave through ice. Following Irving and Knight 2006, the equations are written as.

Frequency Domain

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H} \tag{1}$$

$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + i\omega\epsilon\mathbf{E} \tag{2}$$

Time Domain

$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{H} + J = \frac{\partial D}{\partial t} \tag{4}$$

Constitutive Equations

$$B = \mu H \tag{5}$$

$$D = \epsilon E \tag{6}$$

2.2 Finite Difference Method

Taylor Series

$$f(x_i + \Delta x_i) = f(x_i) + \Delta x_i \left.\frac{\partial f}{\partial x}\right|_{x_i} + \frac{\Delta x_i^2}{2!} \left.\frac{\partial^2 f}{\partial x^2}\right|_{x_i} + \dots \tag{7}$$

Finite Difference

$$\left.\frac{\partial f}{\partial x}\right|_{x_i} = \frac{f(x_i + \Delta x_i) - f(x_i)}{\Delta x_i} + O(\Delta x_i) \tag{8}$$

2.3 Finite Difference Time Domain

2.3.1 Yee Grid

2.3.2 PML

```
for x in range(x): do
    H+=1
end for
```

PML in 2 or 3 dimensions

2.3.3 FDTD Modes

Hx-Ey mode

$$H_i^{n+1} = H_i^n + \frac{\Delta t}{\mu} \left(\frac{E_{j+1/2}^n - E_{j-1/2}^n}{\Delta x_j} \right) \tag{9}$$

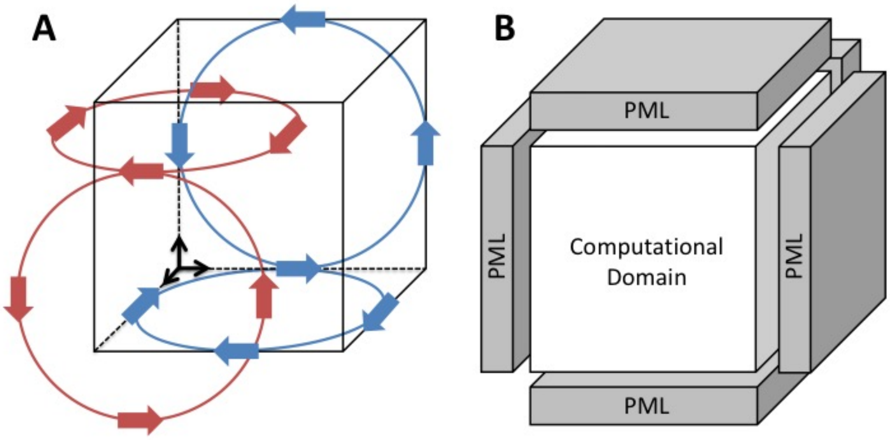

Figure 1: a) Yee Grid b) Perfectly matched layer boundary condition

$$E_i^{n+1/2} = E_i^{n-1/2} + \frac{\Delta t}{\epsilon}\left(\frac{H_{j+1}^n - E_j^n}{\Delta x_j}\right) \tag{10}$$

Ez Mode

3 Results

4 Discussion

5 Conclusions

Data Availability

Model scripts are available for download as a git repository at `https://github.com/benhills/FDTD.git`.

References

[1] Knut Christianson et al. "Basal conditions at the grounding zone of Whillans Ice Stream, West Antarctica, from ice-penetrating radar". In: *Journal of Geophysical Research F: Earth Surface* (2016), pp. 1954–1983. ISSN: 21699011. DOI: 10.1002/2015JF003806.

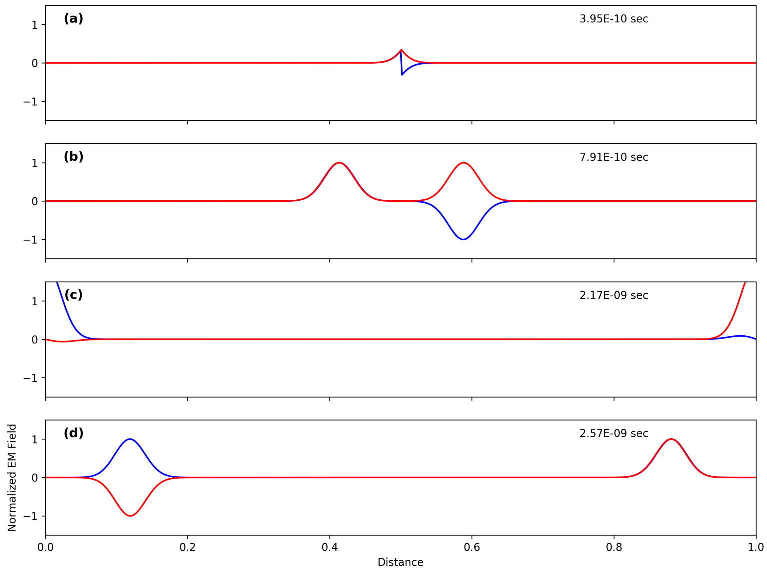

Figure 2: A sequence of plots for a one-dimensional finite difference time domain simulation with Dirichlet boundary conditions.

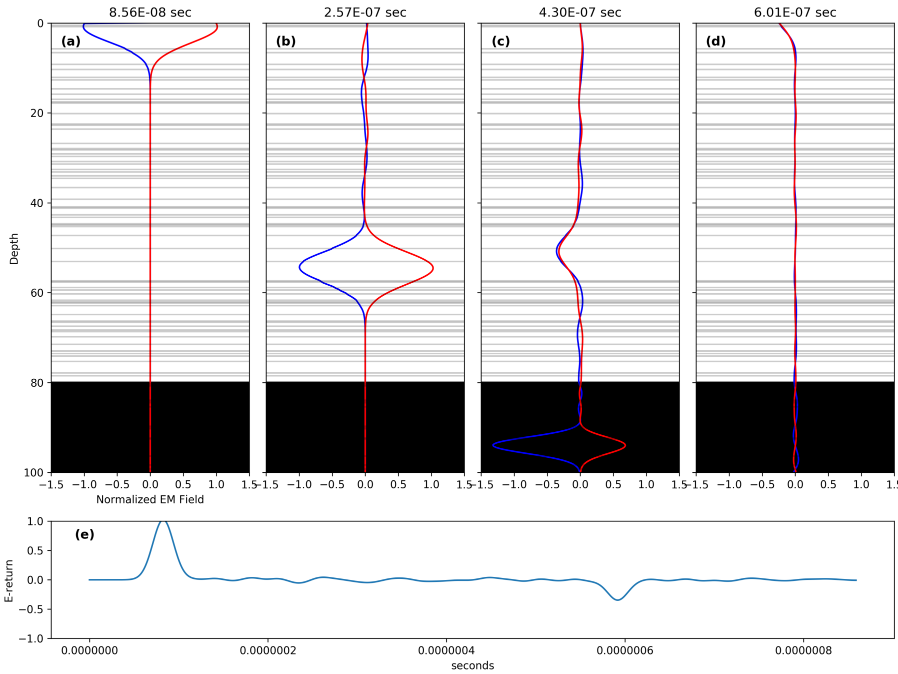

Figure 3: Simulation of EM wave propagation through ice.

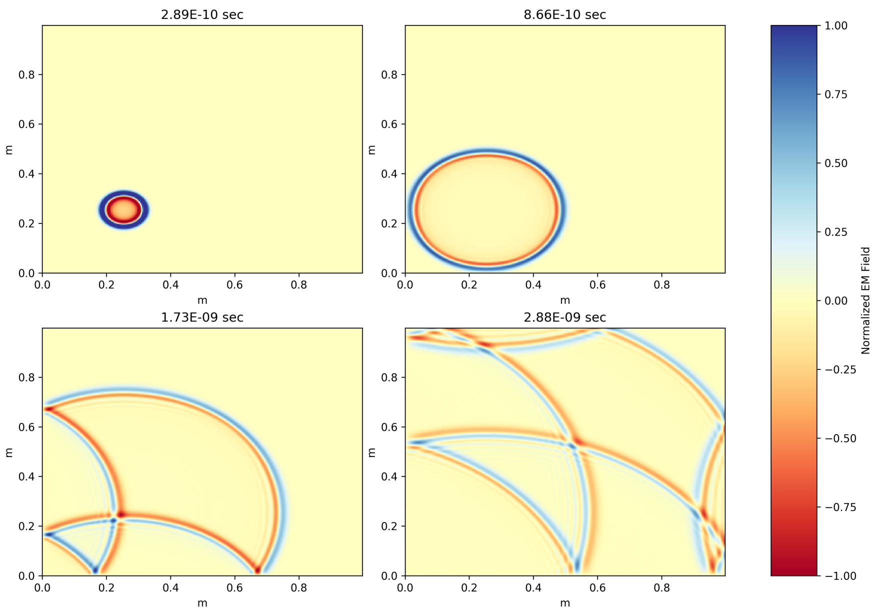

Figure 4: Two-dimensional finite difference time domain.

[2] James Irving and Rosemary Knight. “Numerical modeling of ground-penetrating radar in 2-D using MATLAB”. In: *Computers and Geosciences* 32.9 (2006), pp. 1247–1258. ISSN: 00983004. DOI: 10.1016/j.cageo.2005.11.006.