Linear Time Computation of Discrete Morse Functions Over Two-Manifolds

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This preprint, though the results are (more or less) finalized, only contains the abstract until the full paper is in a respectable state. The full paper is scheduled to be finished 1/10/2023, and a preprint will be released then.

Abstract

Discrete Morse theory provides a way of studying simplicial complexes akin to studying flows over smooth surfaces. Discrete Morse functions assign a value to each cell, and then pair cells based on homology-preserving gradients. The unpaired cells are either represent an essential homology class of the underlying topological space, or are an artifact of the function itself (e.g., a local minimum of the function). We consider two optimization problems: (1) MinMM, finding a function over a given complex K that minimizes the number of critical cells; (2) ExtMM, extending a function over the vertices of a complex to a discrete Morse function compatible with the input function that minimizes the number of critical cells. While it has been shown that MinMM is NP-hard and W[P]-Hard to approximate, we provide a linear time algorithm for the restricted case where the input is a triangulation of a two-manifold. This improves prior algorithms with $\Theta(dn^3)$ complexity on a d-dimensional simplicial complex with n simplices. We give an implementation of this algorithm to demonstrate its improvements in practice. We show how a previously published algorithm solves (2). Finally, we present a heuristic that uses (2) to solve (1), and has reasonable performance on realistic data, even in higher dimensions.

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References

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