The Mathematics of Gossip

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Abstract

This paper will conduct steady-state analysis and numerical simulations on a modified Susceptible/Infectious/Recovered(SIR) model to investigate the spread of rumors, lies, and misinformation that take place throughout the world today. Maple and MATLAB will be used present a numerical simulations using a Runge-Kutta method and the stability of the dynamical system. The models will be adapted SIR models used to simulate the transmission and spread of the Ebola virus in 2014. Different parameters will be tested such as: susceptibility of "infection" depending on the source of the information, forgetting and remembering rumors, and the probability that an infected host can realize the rumor is false.

1 Introduction

Rumors have had an important role throughout history. Rumors, similar to propaganda, can be viewed as information that is not objective to influence an audience and further an agenda. This is done often by presenting facts selectively to encourage a particular perception, or using loaded language to produce an emotional rather than a rational response to the information that is presented [4]. Due to this, a rumor can spread incredibly fast throughout a network of people and can have dire consequences. The purpose of a rumor may be for slandering others, manipulating situations, causing panic, and so on [1]. A notable event in history caused by a rumor was the Paris Riots in 1750 that caused mass social panic resulting in deadly clashes between Parisians and police.

1.1 The Importance of Rumors

With the advancement of the internet and social media, information can spread worldwide in seconds, which can result in physical and economic loss. In 2013, the Associated Press' Twitter account was hacked and sent out a fake tweet stating there was an explosion at the White House that injured President Barack Obama. As the tweet was from a reputable source, it caused the S&P 500 to lose over $134 billion within five minutes, causing instability on the world financial market. Once the rumor was proven false the market partly recovered [3].

The spread of rumors has been described as "infection of the mind" and their spread is analogous to the spread of an infectious disease, such as Ebola. This analogy is made due to information being passed on from carrier to carrier through a network of contacts [6]. Like a disease, the spreading of rumors can be modeled by differential equations and then numerically simulated for different situations. Studying the function provides information on how to suppress rumors from spreading or successfully spread rumors for malicious intent.

2 The SIR and ISR Models

In the typical SIR model for diseases, the entire population is separated into three groups: S is the number of *susceptible* individuals, I is the number of *infected* individuals, and R is the number of *removed* individuals. The removed individuals are a subset of the infected and susceptible individuals, where they cannot become infected and transmit the disease to others. This could be due to an immunity or have perished from the disease. The standard SIR model for disease spreading is

$$\frac{dS}{dt} = -\lambda I(t)S(t)$$
$$\frac{dI}{dt} = \lambda I(t)S(t) - \alpha I(t)$$
$$\frac{dR}{dt} = \alpha I(t),$$

where λ is the parameter of infectivity and α is the recovery rate.

For the spreading of rumors, the three groups are: *ignorants* (susceptible individuals), *spreaders* (similar to infected), and *stiflers* (similar to recovered). However, unlike the *recovered* population in the standard SIR equation above, the *stiflers* have a role to play in the rumor spreading. We define the spreading of rumors to have the following properties:

1. If a spreader contacts an ignorant, then the ignorant has a probability λ of becoming a spreader.
2. If a spreader makes contact with another spreader or stifler, then the initial spreader has probability α of becoming a stifler.
3. When one or more individuals are informed of a rumor, the spreading process is initialized.
4. When there are no more spreaders in the population, the rumor is terminated.

The following equations are for a rumor spreading model, which will be called an ISR model.

$$\dot{I} = \frac{dI}{dt} = -\lambda\,\kappa\,I(t)S(t) \tag{1}$$
$$\dot{S} = \frac{dS}{dt} = \lambda\,\kappa\,I(t)S(t) - \alpha\,\kappa\,S(t)\,[S(t) + R(t)] \tag{2}$$
$$\dot{R} = \frac{dR}{dt} = \alpha\,\kappa\,S(t)\,[S(t) + R(t)] \tag{3}$$

The variable κ is the degree of network reach. For example, the Associated Press has over 12.4 million followers on Twitter. The rumor sent from their hacked Twitter account an extremely high κ due to being a reputable sources with an easy access to millions of people. We assume that the total population N is constant, therefore $I + S + R = N$. When observing the equations found in the ISR model, it can be seen that $\frac{dS}{dt}$ is a linear combination of $\frac{dR}{dt} - \frac{dI}{dt}$ of equation 1 and 2.

2.1 Stability Analysis of the ISR Model

As stated above, the given ISR model can be seen as:

$$\dot{I} = -\lambda\,\kappa\,IS$$
$$\dot{S} = \lambda\,\kappa\,IS - \alpha\,\kappa\,S\,(S + R)$$
$$\dot{R} = \alpha\,\kappa\,S\,(S + R)$$

with the initial conditions

$$I(0) \geq 0, \quad S(0) \geq 0, \quad R(0) \geq 0.$$

To find the equilibrium states of the system, we first set $\dot{I}$, $\dot{S}$, and $\dot{R}$ equal to zero and solve the system of equations.

$$\begin{aligned}
\dot{I} &= -\lambda\, \kappa\, IS = 0 \\
\dot{S} &= \lambda\, \kappa\, IS - \alpha\, \kappa\, S\, [S + R] = 0 \\
\dot{R} &= \alpha\, \kappa\, S\, [S + R] = 0
\end{aligned}$$

Using Maple, the above equations can be easily solved. The first equilibrium point is $\{I = I, R = R, S = 0\}$ and the second is when $\{I = 0, R = -S, S = S\}$. To check the validity of the stability points, we find the eigenvalues of the Jacobian matrix with equilibrium situations. The following matrices are the Jacobian matrix for equations 1-3:

$$J(I,S,R) = \begin{bmatrix} \frac{\partial \dot{I}}{\partial I} & \frac{\partial \dot{I}}{\partial S} & \frac{\partial \dot{I}}{\partial R} \\ \frac{\partial \dot{S}}{\partial I} & \frac{\partial \dot{S}}{\partial S} & \frac{\partial \dot{S}}{\partial R} \\ \frac{\partial \dot{R}}{\partial I} & \frac{\partial \dot{R}}{\partial S} & \frac{\partial \dot{R}}{\partial R} \end{bmatrix} = \begin{bmatrix} -\kappa\, S\lambda & -\lambda\, \kappa\, I & 0 \\ \kappa\, S\lambda & \lambda\, \kappa\, I - \alpha\, \kappa\, (S + R) - \alpha\, \kappa\, S & -\alpha\, \kappa\, S \\ 0 & \alpha\, \kappa\, (S + R) + \alpha\, \kappa\, S & \alpha\, \kappa\, S \end{bmatrix}$$

$$J(I,R,0) = \begin{bmatrix} 0 & -\lambda\, \kappa\, I & 0 \\ 0 & \lambda\, \kappa\, I - \alpha\, \kappa\, R & 0 \\ 0 & \alpha\, \kappa\, R & 0 \end{bmatrix}, \qquad J(0,-S,S) = \begin{bmatrix} -\kappa\, S\lambda & 0 & 0 \\ \kappa\, S\lambda & -\alpha\, \kappa\, S & -\alpha\, \kappa\, S \\ 0 & \alpha\, \kappa\, S & \alpha\, \kappa\, S \end{bmatrix}$$

The eigenvalues of each equilibrium Jacobian matrix are:

<u>Equilibrium 1</u>: $0, 0, I\kappa\lambda - R\alpha\kappa$

<u>Equilibrium 2</u>:$-\kappa S\lambda, 0, 0$

A key aspect of dynamical system theory is the following theorem which allows the ability to determine the stability of fixed points.

Theorem 1. *An equilibrium point* $\mathbf{x}$ *of the differential equation 1 is stable if all the eigenvalues of* $\mathbf{J}$*, the Jacobian evaluated at* $\mathbf{x}$*, have negative real parts. The equilibrium point is unstable if at least one of the eigenvalues has a positive real part.*

The eigenvalues of the 2nd equilibrium matrix are $(-\kappa S\lambda,\ 0,\ 0)$. According the to theorem above, if the eigenvalues are all negative real parts, the equilibrium point is stable. For all values of $\kappa, S,$ orλ, the eigenvalue is going to be negative, which implies stability. The first zero eigenvalue corresponds to the fact that its a 2nd order dynamical system and the other zero eigenvalue is related to the stable center manifold[5] that is the straight line $I + R = 1$ on the (I, R) plane.

$$\frac{dS}{dt} = \lambda\, \kappa\, I(t)S(t) - \alpha\, \kappa\, S(t)\, [S(t) + R(t)] > 0$$

$$\begin{aligned}
\lambda IS &> \alpha S[S + R] \\
S &> \frac{\lambda I}{\alpha} - R
\end{aligned}$$

Furthermore, the spread of a rumor only occurs when $\frac{dS}{dt} > 0$. The rumor will continue to spread until the amount of spreaders is less than $\frac{\lambda I}{\alpha} - R$. Once that threshold is reached, the number of spreaders will quickly decrease to zero and the model will reach equilibrium. In the following sections, the ISR model is numerically simulated and points of equilibrium are found depending on the initial conditions of λ and α.

2.2 Numerical Analysis of ISR Model

In this section, numerical simulations are conducted using MATLAB using a Runge-Kutta method to observe the varying effects that the values of λ and α have on the spreading of a rumor. In the following situations, unless noted otherwise, the inital conditions are $I(0) = 0.99$, $S(0) = 0.01$, $R(0) = 0.0$, and $\kappa = 0.6$. The only effect of changing κ resulted in the change of time needed for the system to reach equilibrium. As κ increases, the time needed for the rumor to propagate decreases.

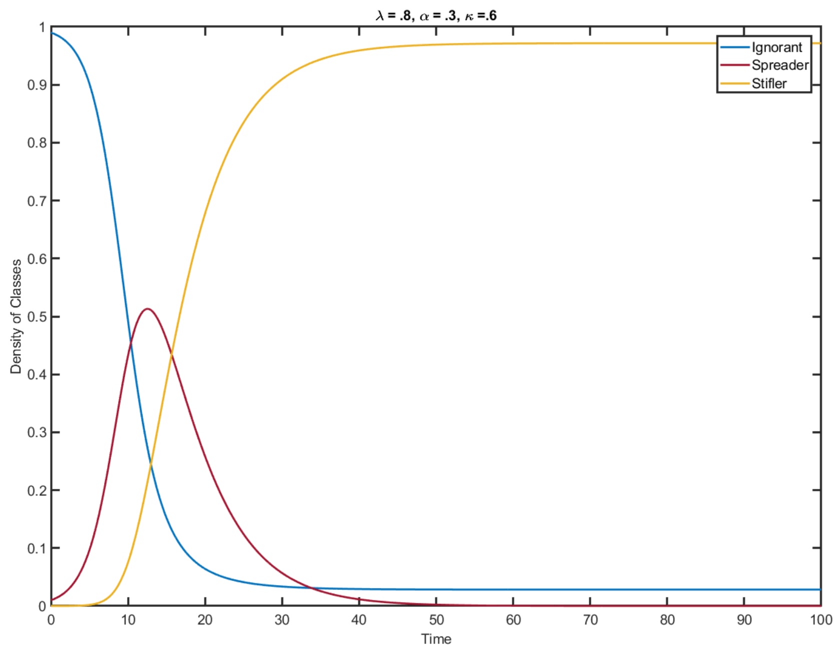

Figure 1: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .8$ and $\alpha = 0.3$

Fig. 1 above shows a typical ISR model, where blue represents the *ignorants*, red represents the *spreaders*, and gold represents the *stiflers* who have overcome the rumor. For the following situation, we can see a sharp increase in the number of spreaders as spreaders reaches a peak and thereafter declines. Finally, the number of spreaders is zero and this leads to the termination of the rumor spreading. For the whole process, the number of ignorants always decreases while the number of stiflers always increases until they reach a balance.

Fig. 2 below shows the densities of ignorants, spreaders, and stiflers over time under different refusing rates $\lambda = 0.2 - 0.8$ while $\alpha = 0.3$ remains constant. The peak value of $S(t)$ shows the highest density of people spreading the rumor and this can be used to measure the maximum rumor influence.

2.2.1 $\lambda \approx \alpha$

Fig. 3 below shows the relationship when $\lambda \approx \alpha$. In this case, the rumor still heavily affects the population. In order to suppress the rumor, α would need to be much larger than λ.

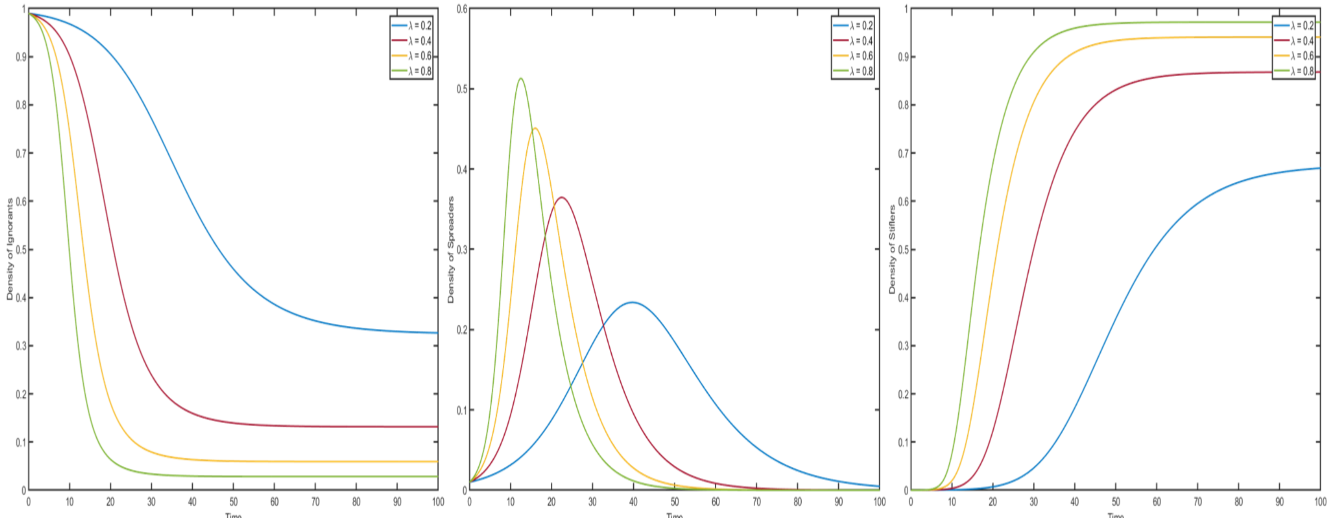

Figure 2: Density of ignorants, spreaders, and stiflers over time under different refusing rates λ

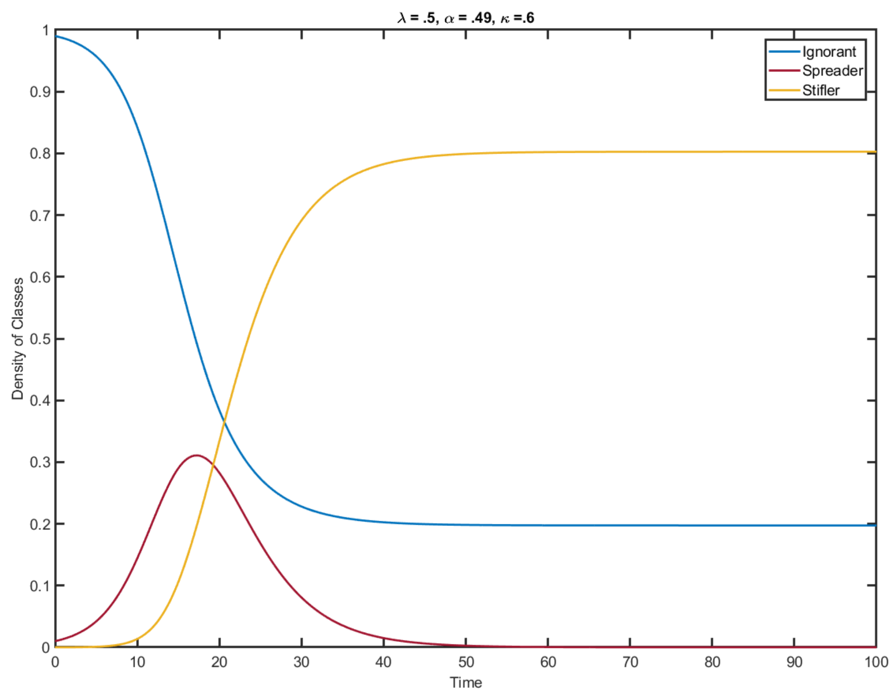

Figure 3: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .5$ and $\alpha = 0.49$

2.2.2 $\lambda \ggg \alpha$

Fig. 4 and 5 show the densities of classes when λ is larger than α. The larger the λ, especially the when $\lambda \ggg \alpha$, the larger the maximum influence of the rumor and the longer it takes for the system to reach equilibrium.

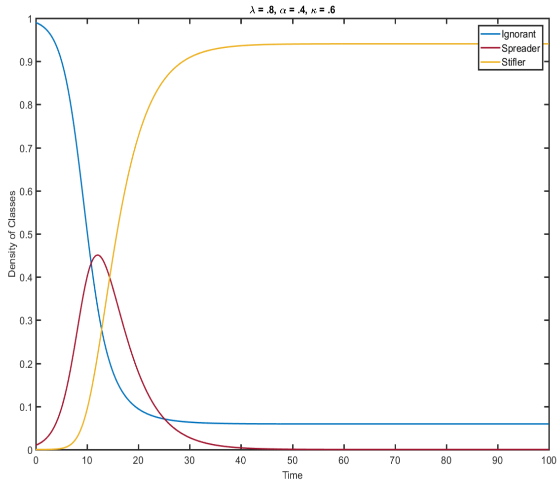

Figure 4: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .8$ and $\alpha = 0.4$

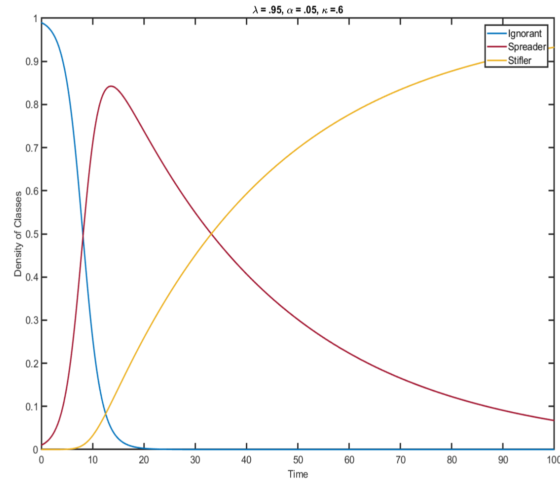

Figure 5: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .95$ and $\alpha = 0.05$

2.2.3 $\lambda \lll \alpha$

In Fig. 6 and 7, λ is less than α. If more people have the ability to see through the rumor, the maximum rumor influence becomes smaller and the rumor termination time comes earlier. In Fig. 6 the population reaches asymptotic equilibrium where approximately 55% of the population had heard the rumor before it was suppressed or forgotten. Fig. 7 shows when the stifling rate is much greater than the spreading rate and the rumor barely manages to spread before being suppressed.

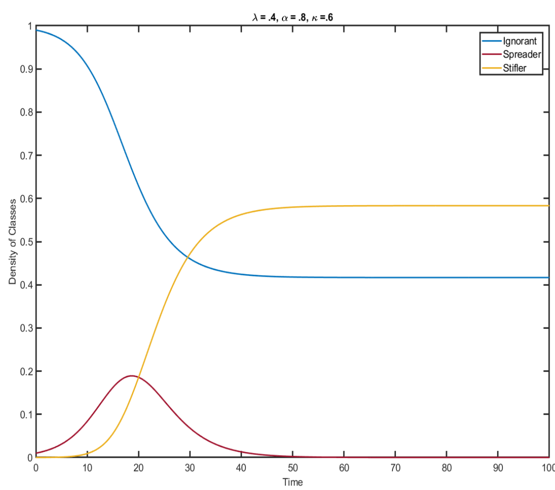

Figure 6: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .4$ and $\alpha = 0.8$

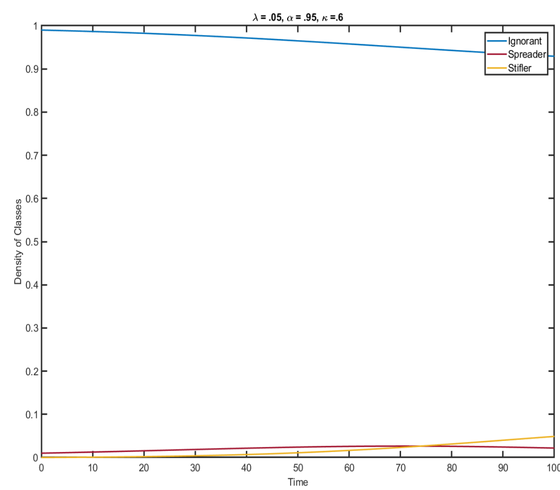

Figure 7: Densities of ignorants, spreaders, and stiflers over time with $\lambda = .05$ and $\alpha = 0.95$

3 Problems with ISR Model

There are some issues with the ISR model's functionality. When an ignorant encounters a spreader, the above model fails to represent the situation where an ignorant does not believe the rumor to be true and immediately becomes a stifler. This could be due to the ignorant applying logical reasoning of the situation or mistrusting the credibility of the spreader if they are a known liar, or simply not having interest in the rumor. Education of a particular topic can be easily seen as a "vaccination" against a rumor. With knowledge gained from education, there is a possibility to transition directly from an ignorant to a stifler when encountering a rumor.

Another issue with the ISR model is the possibility of not spreading the rumor while also not being a stifler. In this case, there would be a new subclass of the "Infected." This can be seen as similar to a disease that has infected a person, but they are not yet infectious to those around them. Because of this new subclass, which will be called *Hibernators*, we introduce a new ISHR Model

4 ISHR Model

The ISHR model, created by L. Zhao [1], attempts to add new parameters to fix the common problems of the ISR model. The ISR model is extended to include the possibility of an ignorant directly becoming a stifler after hearing the rumor, as well as a new class of "infected" called *hibernators* are introduced to have the possibility of forgetting and remembering rumors. The new class of hibernators allows for the repeatability of a rumor re-emerging over time. Fig.8 below shows the interaction possibilities of the four classes of the ISHR model.

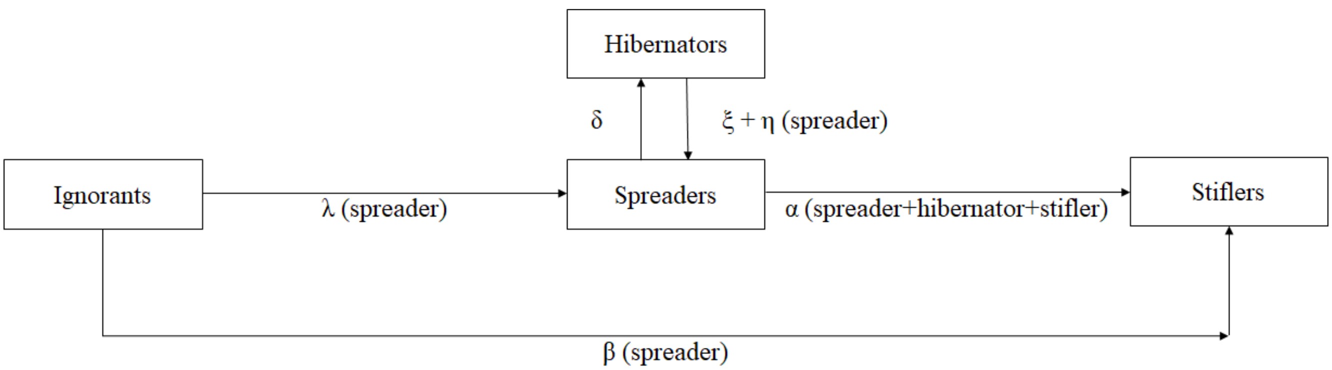

Figure 8: Flowchart demonstrating the ISHR rumor spreading process

The ISHR model can be seen below:

$$\frac{dI}{dt} = (-\lambda + \beta)\kappa I(t)S(t) \tag{4}$$

$$\frac{dS}{dt} = \lambda\kappa I(t)S(t) - \alpha\kappa S(t)[(S(t) + H(t) + R(t)] - \delta S(t) + \xi H(t) + \eta\kappa H(t)S(t) \tag{5}$$

$$\frac{dH}{dt} = \delta S(t) - \xi H(t) - \eta\kappa H(t)S(t) \tag{6}$$

$$\frac{dR}{dt} = \beta\kappa I(t)S(t) + \alpha\kappa S(t)[(S(t) + H(t) + R(t)] \tag{7}$$

As the ISHR model is an extension of the ISR model, in certain conditions the original model can be found. When $\beta = 0, \delta = 0, \xi = 0, \eta = 0$, the ISHR model becomes the classical ISR rumor spreading model. The rules of the ISHR model can be summarized below:

1. If a spreader contacts an ignorant, then the ignorant has a probability λ of becoming a spreader.
2. When a spreader contacts an ignorant, the ignorant has a probability β of becoming a stifler.

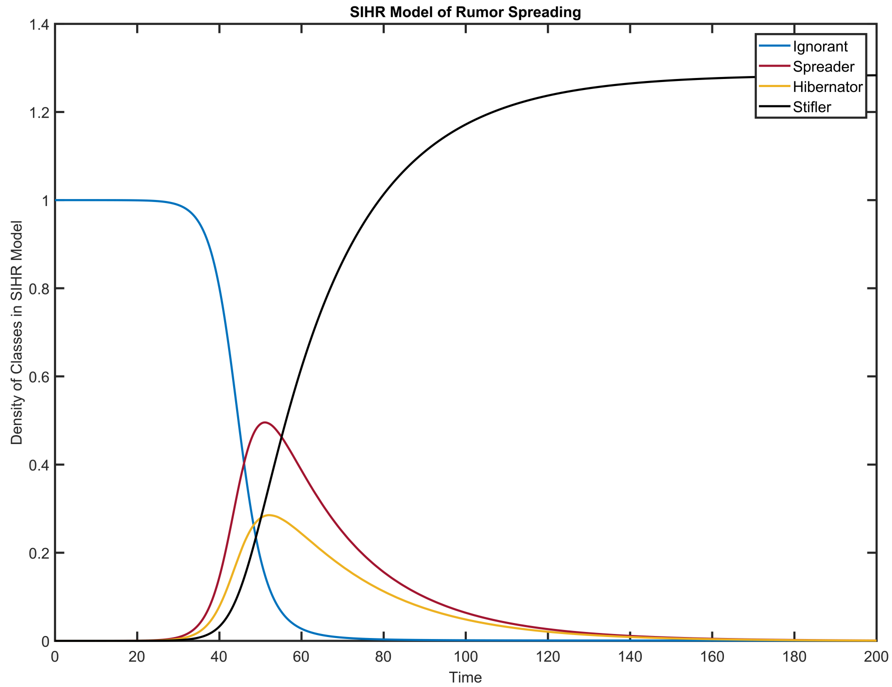

Figure 9: Densities of ignorants, spreaders, hibernators, and stiflers over time with $\lambda = 0.8$, $\beta = 0.2$, $\alpha = 0.3$, $\delta = 0.6$, $\xi = \eta = 0.5$

3. A spreader has the probability of becoming a hibernator at a rate δ, which is the forgetting rate of the rumor. A hibernator is a spreader that is not actively spreading the rumor, but has not stifled the rumor. Hibernators can spontaneously become a spreader again at a rate ξ, which is the remembering rate. When a hibernator contacts a spreader, the hibernator becomes a spreader with probability η, which is the wakened remembering rate.

4. If a spreader makes contact with another spreader, hibernator, or stifler, then the initial spreader has probability α of becoming a stifler.

The data from the ISHR model does not act as expected. The model behaves as expected for T = [0,60], but the rapidly increasing stifler population does not act correctly. We should expect the stiflers to plateau at 100% as $I + S + H + R = 1$. However, in Fig. 9, the stiflers rise to 128.4% before it starts to plateau. The system of equations should not allow the total population to grow above 100%. In order study why the model was behaving differently than expected, steady-state analysis was conducted on the new model.

4.1 Steady-State Analysis of ISHR

To find the equilibrium states of the system, we first set $\dot{I}$, $\dot{S}$, and $\dot{R}$ equal to zero and solve the system of equations. Then as before, solve for the Jacobian matrix and find the eigenvalues of the equilibrium matrices.

$$
\frac{dI}{dt} = 0
$$
$$
\frac{dS}{dt} = 0
$$
$$
\frac{dH}{dt} = 0
$$
$$
\frac{dR}{dt} = 0
$$

The equilibrium solutions are the following:

$$
\begin{aligned} I &= I \\ S &= 0 \\ H &= 0 \\ R &= R \end{aligned}
\qquad\qquad
\begin{aligned} I &= 0 \\ S &= S \\ H &= \frac{\delta S}{(S\eta\kappa + \xi)} \\ R &= \frac{-S(S\eta\kappa + \delta + \xi)}{(S\eta\kappa + \xi)} \end{aligned}
$$

$$
J(I,S,H,R) = \begin{bmatrix} (-\lambda+\beta)\,\kappa\,S & (-\lambda+\beta)\,\kappa\,I & 0 & 0 \\ \lambda\,\kappa\,S & \lambda\,\kappa\,I - \alpha\,\kappa\,(S+H+R) - \alpha\,\kappa\,S - \delta + \eta\,\kappa\,H & -\alpha\,\kappa\,S + S\eta\,\kappa + \xi & -\alpha\,\kappa\,S \\ 0 & -\eta\,\kappa\,H + \delta & -S\eta\,\kappa - \xi & 0 \\ \beta\,\kappa\,S & \beta\,\kappa\,I + \alpha\,\kappa\,(S+H+R) + \alpha\,\kappa\,S & \alpha\,\kappa\,S & \alpha\,\kappa\,S \end{bmatrix}
$$

$$
J(I,0,0,R) = \begin{bmatrix} 0 & (-\lambda+\beta)\,\kappa\,I & 0 & 0 \\ 0 & \lambda\,\kappa\,I - \alpha\,\kappa\,R - \delta & \xi & 0 \\ 0 & \delta & -\xi & 0 \\ 0 & \beta\,\kappa\,I + \alpha\,\kappa\,R & 0 & 0 \end{bmatrix}
$$

$J(0, S, \frac{\delta S}{(S\eta\kappa+\xi)}, \frac{-S(S\eta\kappa+\delta+\xi)}{(S\eta\kappa+\xi)}) =$

$$
\begin{bmatrix} (-\lambda+\beta)\,\kappa\,S & 0 & 0 & 0 \\ S\kappa\,\lambda & -\alpha\,\kappa\left(S + \frac{\delta\,S}{S\eta\,\kappa+\xi} - \frac{S(S\eta\,\kappa+\delta+\xi)}{S\eta\,\kappa+\xi}\right) - S\alpha\,\kappa - \delta + \frac{\eta\,\kappa\,\delta\,S}{S\eta\,\kappa+\xi} & -S\alpha\,\kappa + S\eta\,\kappa + \xi & -S\alpha\,\kappa \\ 0 & -\frac{\eta\,\kappa\,\delta\,S}{S\eta\,\kappa+\xi} + \delta & -S\eta\,\kappa - \xi & 0 \\ \beta\,\kappa\,S & \alpha\,\kappa\left(S + \frac{\delta\,S}{S\eta\,\kappa+\xi} - \frac{S(S\eta\,\kappa+\delta+\xi)}{S\eta\,\kappa+\xi}\right) + S\alpha\,\kappa & S\alpha\,\kappa & S\alpha\,\kappa \end{bmatrix}
$$

The eigenvalues of equilibrium 1 were

1. $\lambda_1 = 0$
2. $\lambda_2 = 0$

3. $\lambda_3 = 1/2\,\lambda\,\kappa\,I - 1/2\,R\alpha\,\kappa - \delta/2 - \xi/2 - 1/2$
$\sqrt{I^2\kappa^2\lambda^2 - 2\,IR\alpha\,\kappa^2\lambda + R^2\alpha^2\kappa^2 - 2\,I\delta\,\kappa\,\lambda + 2\,\lambda\,\kappa\,I\xi + 2\,R\alpha\,\delta\,\kappa - 2\,R\alpha\,\kappa\,\xi + \delta^2 + 2\,\delta\,\xi + \xi^2}$

4. $\lambda_4 = 1/2\,\lambda\,\kappa\,I - 1/2\,R\alpha\,\kappa - \delta/2 - \xi/2 + 1/2$
$\sqrt{I^2\kappa^2\lambda^2 - 2\,IR\alpha\,\kappa^2\lambda + R^2\alpha^2\kappa^2 - 2\,I\delta\,\kappa\,\lambda + 2\,\lambda\,\kappa\,I\xi + 2\,R\alpha\,\delta\,\kappa - 2\,R\alpha\,\kappa\,\xi + \delta^2 + 2\,\delta\,\xi + \xi^2}$

No matter the values given to the variables, there will always be at least one eigenvalue that will be positive, which makes equilibrium unstable.

The eigenvalues of equilibrium 2 were

1. $\lambda_1 = \kappa\,S\beta - S\kappa\,\lambda$
2. $\lambda_2 = 0$
3. $\lambda_3 = 0$
4. $\lambda_4 = S^2\eta^2\kappa^2 + 2\,S\eta\,\kappa\,\xi + \delta\,\xi + \xi^2$

When numerically calculating the eigenvalues of equilibrium 2, the eigenvalues were complex with negative real parts:

- $\lambda_1 = -2.79453109958139 \cdot 10^{-17} + i1.27433158109030 \cdot 10^{-10}$
- $\lambda_2 = -2.79453109958139 \cdot 10^{-17} - i1.27433158109030 \cdot 10^{-10}$
- $\lambda_3 = -1.09980239520000$
- $\lambda_4 = -0.00120000000000$

Since all the eigenvalues were not real, negative values, I was unable to conclude that the ISHR model will converge to a stable equilibrium.

5 Conclusions and Future Work

In this paper, the goal was to model the complexity of rumors within a population based on SIR epidemic models. In today's world with the power of social media, the growth of rumors can be extremely tricky to inhibit. As the spread of rumors can be seen as as "infection of the mind" or "thought contagion", their spread is analogous to the spread of an infectious disease. The ISR model did an acceptable job to show how quickly a rumor can propagate. Steady-state analysis was successfully conducted on the system of differential equations to show that the system has a stable equilibrium. The model isn't without its flaws, and a new model was considered to add more realistic factors to rumor spreading model to make it closer to real life. The more complex ISHR model was unsuccessful in demonstrating the propagation of rumors.

I would be curious to look at and model a data set of rumors from Twitter from a particular incident, such as the Boston Marathon Bombings. During that time, tens of thousands of tweets were sent and retweeted containing false information, such as how many bombs were set off, who the attackers were, ect. Then analyze what tweets had a high believability rate λ and be able to see if a predicted model would closely match the real model from the data set.

6 Acknowledgments

Special thanks to Dr. Patrick Shipman for his guidance and encouragement to continue and strive to make this project better. Also to Joshua de Jong for his ever-present willingness to help me through my coding errors. I also wanted to thank L. Zhao and J.R Peiqueira for their ISHR and ISR models, respectively.

References

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MATLAB Code

```
function [x,t] = SIR_Adapt(Kappa_input, Lambda_input, Alpha_input, Susceptibles, Spreaders, Stiflers, Plot_Option)
close all;
% Ben Hoobler
% The Mathematics of Gossip


%% The Mathematics of Gossip

% In this project, the goal is to create a numerical model to investigate
% the spread of rumors, lies, and misinformation that take place throughout
% the world today. The model will be an adapted SIR model used similar to
% the model of the transmission and vaccination of the Ebola virus.
% Different parameters will be tested, such as susceptibility of
% "infection" depending on the source of the information and the
% probability that an infected host can overcome the rumor and believe it
% to be false.


%% Recommended Initials for t = 0

% Susceptibles = .99
% Spreaders = .01
% Hibernators = 0
% Stiflers = 0


%%%%%%%%%%%
%Quick Copy
%%%%%%%%%%%
%SIR_Adapt(.3, .8, .3, .99, .01, 0,1);

%% Variables
global Alpha;
Alpha = Alpha_input;
global Kappa;
Kappa = Kappa_input;
global Lambda;
Lambda = Lambda_input;

%% Solver Input
Initials = [Susceptibles, Spreaders, Stiflers]; %for t = 0
TimeScale = [0:.1:100];

%% Solver
[t,x] = ode45( @system, TimeScale, Initials);

%% Plot
```

```
if(Plot_Option)
figure
plot(t,x(:,1),'g',t,x(:,2),'b', t,x(:,3),'k');
title('SIR Model of Rumor Spreading')
legend('Ignorant', 'Spreader', 'Stifler');
xlabel('time [hours]')
ylabel('Density of Classes in SIR Model')
end

end

function output = system(t,x)
% This function is to store all the equations

 %% Factors of Spreading
% Kappa is the degree of the network. If the origin came from a Twitter
% account with a very large following, Kappa would be much larger than an
% interation between friends on Facebook.

% Lamda is the probability that the person disseminates the rumor and
% changes into a Spreader. Lamda is the spreading rate of a rumor

% Alpha is the stifling rate. If a Spreader encounters another Spreader,
% Hibernator, or Stifler, the initiating Spreader becomes a Stifler at
% probability Alpha.


% Standard ISR Model
%     Kappa = .6;            % Degree of Network Reach
%
%     Lamda = .6;            % Rate of Ign-to-Spr
%
%     Alpha = .3;            % Stifling rate

%% Variables
global Alpha
global Kappa
global Lambda

    %% x Dump
    I = x(1);
    S = x(2);
    R = x(3);

    %% ODE System
    dI = -Lambda*Kappa*I*S;
    dS =  Lambda*Kappa*I*S - Alpha*Kappa*S*(S+R);
    dR =  Alpha*Kappa*S*(S+R);
```

```
    %% Format output
    output = [dI; dS; dR];

end



%%Rumor Script
clear all; close all;
co = [0         0.4470    0.7410;
      0.6350    0.0780    0.1840;
      0.9290    0.6940    0.1250;
      0.4660    0.6740    0.1880;
      0.4940    0.1840    0.5560;
      0.8500    0.3250    0.0980;
      0.3010    0.7450    0.9330];
set(groot,'defaultAxesColorOrder',co)
%% For Loops
Storage = zeros(1001,3,5);
Storage2 = zeros(1001,3,5);
Lambda = 0:.2:.8;

for i = 1:length(Lambda)
    [Storage(:,:,i), Time] = SIR_Adapt(.6, Lambda(i), .3, .99, .01, 0, 0);
end

for i = 1:length(Lambda)
    [Storage2(:,:,i), Time] = SIR_Adapt(Lambda(i), .6, .3, .99, .01, 0, 0);
end

%% Variable Lambda Spreaders
Fig1 = figure;
for i = 2:length(Lambda)
plot(Time,Storage(:,2,i));
hold on;
end
%savefig(Fig1,'Variable_Lambda_Spreaders')


%% Variable Lambda Ignorants
Fig2 = figure;
for i = 2:length(Lambda)
plot(Time,Storage(:,1,i));
hold on;
end
savefig(Fig2,'Variable_Lambda_Ignorants')

%% Variable Lambda Stiflers
Fig3 = figure;
for i = 2:length(Lambda)
```

```
plot(Time,Storage(:,3,i));
hold on;
end
savefig(Fig3,'Variable_Lambda_Stiflers')

%% Variable Kappa on Stiflers
Fig4 = figure;
for i = 2:length(Lambda)
plot(Time,Storage(:,3,i));
hold on;
end
savefig(Fig4,'Variable_Kappa_Stiflers')


% %SIR_Adapt(Kappa_input, Lambda_input, Alpha_input, Susceptibles, Spreaders, Stiflers,Plot_Option)
x = 0:.1:100;

%% Normal ISR
normal = SIR_Adapt(.6, .6, .3, .99, .01, 0, 0); % L = .6,   A =.3

figure(1),
plot(x, normal)
title('ISR Model of Rumor Spreading')
xlabel('Time')
ylabel('Density of Classes')
legend('Ignorant', 'Spreader', 'Stifler');

%% Alpha = Lambda
about_same = SIR_Adapt(.6, .5, .49, .99, .01, 0, 0); % L = .5,   A =.49

figure(2),
plot(x, about_same);
title('\lambda = .5, \alpha = .49, \kappa =.6')
xlabel('Time')
ylabel('Density of Classes')
legend('Ignorant', 'Spreader', 'Stifler');

%% Lambda >>> Alpha
lambda_over_alpha = SIR_Adapt(.6, .95, .05, .99, .01, 0, 0); % L = .95,   A =.05

figure(3),
plot(x, lambda_over_alpha);
title('\lambda = .95, \alpha = .05, \kappa =.6')
xlabel('Time')
ylabel('Density of Classes')
legend('Ignorant', 'Spreader', 'Stifler');

%% Alpha >>> Lambda
```

```
alpha_over_lambda = SIR_Adapt(.6, .05, .95, .99, .01, 0, 0); % L = .05,    A =.95

figure(4),
plot(x, alpha_over_lambda);
title('\lambda = .05, \alpha = .95, \kappa =.6')
xlabel('Time')
ylabel('Density of Classes')
legend('Ignorant', 'Spreader', 'Stifler');

%% Normal ISR with almost all stiflers at end
lambda8 = SIR_Adapt(.6, .8, .3, .99, .01, 0, 0); % L = .05,    A =.95

figure(5),
plot(x, lambda8);
title('\lambda = .8, \alpha = .3, \kappa =.6')
xlabel('Time')
ylabel('Density of Classes')
legend('Ignorant', 'Spreader', 'Stifler');
```

```
function [x,t] = rumors_SIHR(Susceptible, Spreaders, Hibernators, Stiflers)
close all;
% Ben Hoobler
% The Mathematics of Rumor Propogation
% Math 435
% SIHR Model

%%%%%%%%%%%
%Quick Copy
%%%%%%%%%%%
% rumors_SIHR(0.99, .01, 0, 0);


%% Solver Input
Initials = [Susceptible, Spreaders, Hibernators, Stiflers]; %for t = 0
TimeScale = [0:.1:200];

%% Solver
[t,x] = ode45( @system, TimeScale, Initials);

%% Plot
figure
plot(t,x);
title('SIHR Model of Rumor Spreading')
legend('Ignorant', 'Spreader', 'Hibernator', 'Stifler');
xlabel('Time')
ylabel('Density of Classes in SIHR Model')

end

function output = system(t,x)
% This function is to store all the equations

 %% Factors of Spreading
% Kappa is the degree of the network. If the origin came from a Twitter
% account with a very large following, Kappa would be much larger than an
% interation between friends on Facebook.

% Lambda is the probability that the person disseminates the rumor and
% changes into a Spreader. Lambda is the spreading rate of a rumor

% Beta is the probability an Ignorant will not believe the rumor. Beta is
% the rate a refusal when an Ignorant encounters a Spreader.

% Delta is the probability a Spreader spontaneously loses interest or
% forgets about the rumor. If a Spreader forgets about the rumor, they
% become a Hibernator.

% Xi is spontaneous remembering rate for Hibernators to become a Spreader.

```

```
% Eta is the probability a Hibernator becomes a Spreader when encountering
% a Spreader. This is the wakened remembering rate.

% Alpha is the stifling rate. If a Spreader encounters another Spreader,
% Hibernator, or Stifler, the initiating Spreader becomes a Stifler at
% probability Alpha.


% Coefficients
    Kappa = .6;            % Degree of Network Reach

    Lambda = .8;           % Rate of Ign-to-Spr

    Beta  = .1;            % Rate of rumor refusal

    Alpha = .1;            % Stifling rate

    Delta = .6;            % Rate of forgetting to be Hib

    Xi    = .8;            % Rate of spontaneous remembering

    Eta   = .8;            % Wakened remembered rate


    %% x Dump
    I = x(1);
    S = x(2);
    H = x(3);
    R = x(4);

    %% ODE System
    dI = (-Lambda + Beta)*Kappa*I*S;
    dS = Lambda*Kappa*S*I - Alpha*Kappa*S*(H + R + S) -Delta*S + Xi*H + Eta*Kappa*S*H;
    dH = Delta*S - Xi*H - Eta*Kappa*H*S;
    dR = Beta*Kappa*I*S + Alpha*Kappa*S*(S+H+R);

    %I(t) + S(t) + H(t) + R(t) = 1.

    %% Format output
    output = [dI; dS; dH; dR];

end
```

```
> restart:
#Ben Hoobler
#Math 435
#Maple Code
#The Mathematics of Rumor Propagation

> with(Student[NumericalAnalysis]):
with(DynamicSystems):
with(VectorCalculus):
with(LinearAlgebra):
with(linalg):
with(Student[LinearAlgebra]):

> Ldot:= -lambda*kappa*L*S:
> Sdot:= lambda*kappa*L*S-alpha*kappa*S*(S+R):
> Rdot:= alpha*kappa*S*(S+R):
> solve({Ldot=0,Sdot=0,Rdot=0},{L,S,R});
Equil1 := {L = L, R = R, S = 0};
Equil2 := {L = 0, R = -S, S = S};
```

$$\{L=L, R=R, S=0\}, \{L=0, R=-S, S=S\}$$

$$Equil1 := \{L=L, R=R, S=0\}$$

$$Equil2 := \{L=0, R=-S, S=S\} \tag{1}$$

```
> J0 := Jacobian([Ldot,Sdot,Rdot], [L, S, R]);
```

$$J0 := \begin{bmatrix} -\kappa S \lambda & -\lambda \kappa L & 0 \\ \kappa S \lambda & \lambda \kappa L - \alpha \kappa (S+R) - \alpha \kappa S & -\alpha \kappa S \\ 0 & \alpha \kappa (S+R) + \alpha \kappa S & \alpha \kappa S \end{bmatrix} \tag{2}$$

```
> J0_EQUIL1 := subs(L = L, R = R, S = 0,J0);
```

$$J0\_EQUIL1 := \begin{bmatrix} 0 & -\lambda \kappa L & 0 \\ 0 & \lambda \kappa L - \alpha \kappa R & 0 \\ 0 & \alpha \kappa R & 0 \end{bmatrix} \tag{3}$$

```
> EIGEN_JEQ1:= eigenvalues(J0_EQUIL1);
```

$$EIGEN\_JEQ1 := 0, 0, \lambda \kappa L - \alpha \kappa R \tag{4}$$

```
> J0_EQUIL2 := subs(L = 0, R = -S, S = S,J0);
```

$$J0\_EQUIL2 := \begin{bmatrix} -\kappa S \lambda & 0 & 0 \\ \kappa S \lambda & -\alpha \kappa S & -\alpha \kappa S \\ 0 & \alpha \kappa S & \alpha \kappa S \end{bmatrix} \tag{5}$$

```
> EIGEN_JEQ2:= eigenvalues(J0_EQUIL2);
```

$$EIGEN\_JEQ2 := -\kappa S \lambda, 0, 0 \tag{6}$$

```
> Num_Eigen_JEQ1:= subs(lambda=.8,alpha=.3,kappa=.3, L=.99,S=.01,
R=0,J0_EQUIL1);
Num_Eigen_JEQ2:= subs(lambda=.8,alpha=.3,kappa=.3, L=.99,S=.01,
R=0,J0_EQUIL2);
```

$$Num\_Eigen\_JEQ1 := \begin{bmatrix} 0 & -0.2376 & 0 \\ 0 & 0.2376 & 0 \\ 0 & 0. & 0 \end{bmatrix}$$

$$Num\_Eigen\_JEQ2 := \begin{bmatrix} -0.0024 & 0 & 0 \\ 0.0024 & -0.0009 & -0.0009 \\ 0 & 0.0009 & 0.0009 \end{bmatrix} \tag{7}$$

```
> eigenvalues(Num_Eigen_JEQ1);
```

$$0., 0.237600000000000, 0. \tag{8}$$

```
> eigenvalues(Num_Eigen_JEQ2);
```

$$0., 0., -0.00240000000000000 \tag{9}$$

```
> (-Rdot - Ldot) - Sdot; #Spreaders are a linear combination of
ignorants                                    and stiflers
```

$$0 \tag{10}$$

```
>
```