Exponential Smoothing

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Introduction

What is exponential smoothing?

Forecasting future observations using weighted averages of past observations, with the weights decaying exponentially as observations recede further into the past

- Presumably, more recent data points are more predictive than older points

Exponential Smoothing Models

1. Naive—all weight is given to the last observation
2. Average—each past observation is given equal weight
3. Exponential weighted average—Recent observations get higher weight, older observations less weight
4. Holt linear—Same as 3, but accounts for time series with trend
5. Holt-Winters—Same as 4, but also accounts for time series with seasonality
6. State space—

ES_1: Naive model

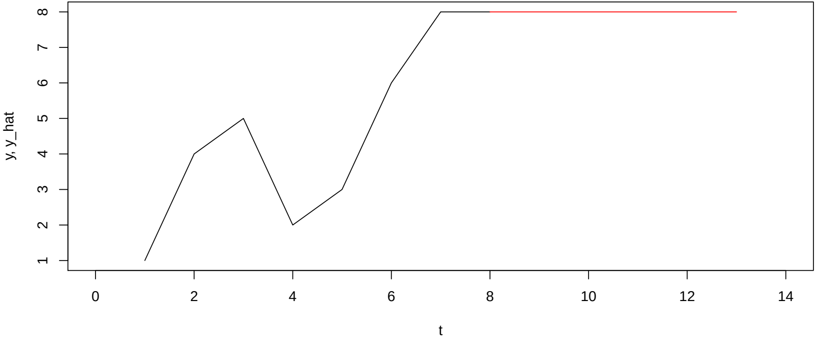
- The naive forecasting model can be thought of us exponential smoothing
- Where 100 percent of weight is given to the last observation:

```
forecast_naive <- function(y, h) {
    n <- length(y)
    y_hat <- rep(y[n], h)
    return( y_hat )
}
```

ES_1: Naive model: Example

```r
y <- c(1, 4, 5, 2, 3, 6, 8)
y_hat <- forecast_naive(y, h=7)

plot_forecast(y, y_hat)
```



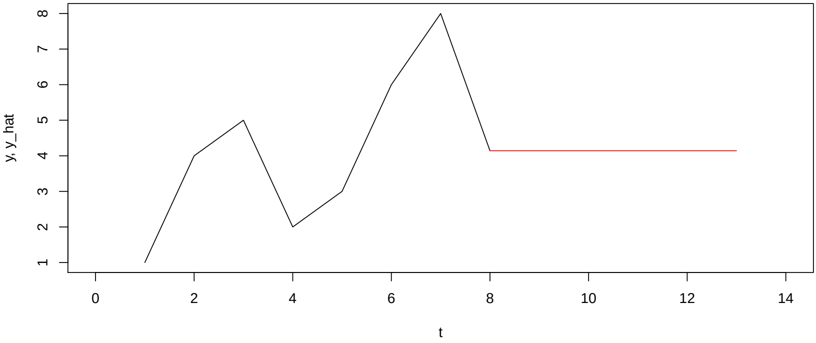
ES_2: Average model

- All future values are forecast as the average of the observed data
- Equivalent to to exponential smoothing where each observation is given equal weight

```
forecast_avg <- function(y, h) {
    y_hat <- rep(mean(y), h)
    return( y_hat )
}
```

ES_2: Average model: Example

```r
y_hat <- forecast_avg(y, h=7)
plot_forecast(y, y_hat)
```



ES_3: Simple exponential smoothing

- More sophisticated model would given recent observations more weight, and decreasing weight for past observations
- Parameter α controls smoothing; can be optimized

$$\hat{y}_T = \alpha y_T + (1 - \alpha)\hat{y}_{T-1}$$

- $\hat{y}_T \equiv$ predicted value of y at time t
- $\alpha \equiv$ user-chosen smoothing parameter, $0 \leq \alpha \leq$
  - 1 means no smoothing, 0 means tons of smoothing
- $\hat{y}_{T-1} \equiv$ predicted value of y at immediately previous period $t - 1$

ES_3: Simple exponential smoothing: Problem

Problem: What is $\hat{y}_{T-1}$ when $t = 1$?

$$\hat{y}_T = \alpha y_T + (1 - \alpha)\hat{y}_{T-1}$$

- Textbook: Component form, the level ℓ_0
- Set for mean or median of the time series

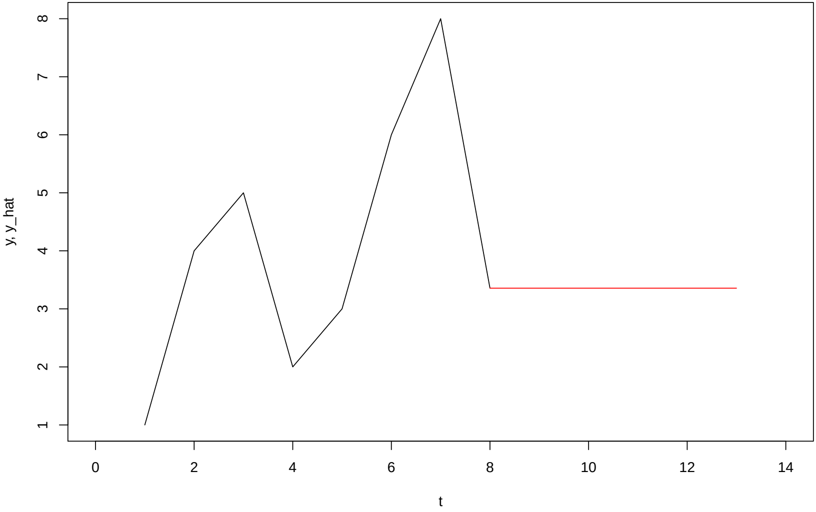
```r
forecast_simple <- function(y, h, alpha=0.1) {
    n <- length(y)
    l_0 <- mean(y)

    l <- c()
    for (i in 1:(n+1)) {
        if (i == 1) {
            l_i <- alpha * y[i] + (1 - alpha) * l_0
        } else {
            l_i <- alpha * y[i] + (1 - alpha) * l[i-1]
        }
        l <- append(l_i, l)
    }

    y_hat <- l[n+1]
    return( rep(y_hat, h) )
}
```

ES_3: Simple exponential smoothing: Example

```
y_hat <- forecast_simple(y, h=7, alpha=0.25)
plot_forecast(y, y_hat)
```



ES_3: Weighted average: Optimizing α^*

BEN: Not sure if we want to keep this??

Interlude

- The previous method is effective for time series without trend or seasonality
- But what if your time series has trend?

ES_4: Holt Linear Trend Model

Appropriate for time series that can be described with

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

where β_1 quantifies the trend

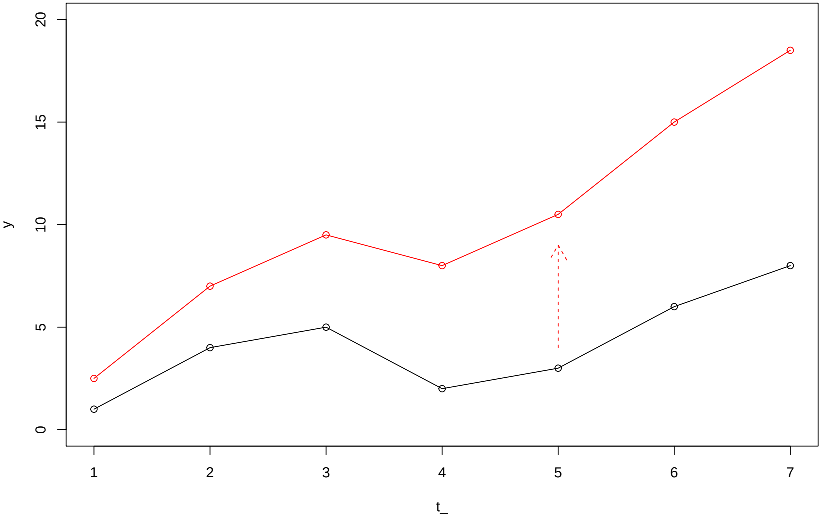
ES_4: Holt Linear Trend Model: Linear Time Series

- We can explicitly convert our previous time series to a trended one using this formula:
- Previous time series `y` = (1, 4, 5, 2, 3, 6, 8)

```
beta_0 <- 0
beta_1 <- 1.5
t_ <- 1:7
( y_1 <- beta_0 + y + beta_1*t_ )
```

```
## [1]  2.5  7.0  9.5  8.0 10.5 15.0 18.5
```

ES_4: Holt Linear Trend Model: Linear Time Series



Holt's linear trend + damped

See http://www.real-statistics.com/time-series-analysis/basic-time-series-forecasting/holt-linear-trend/ for excel formula implementation

Holt-Winters method + multiplicative + taxonomy

ETS modeling (Innovations state space models)

Conclusion

Blah blah