Lagrangian and Hamiltonian Dynamics

Final project proposal

Benjamin Howe

I. Description

The pendulum on a rotating rim. A simple pendulum of length b and mass m moves on a mass-less rim of radius a rotating with constant angular velocity ω. Find the equation of motion for the mass.

II. Diagram

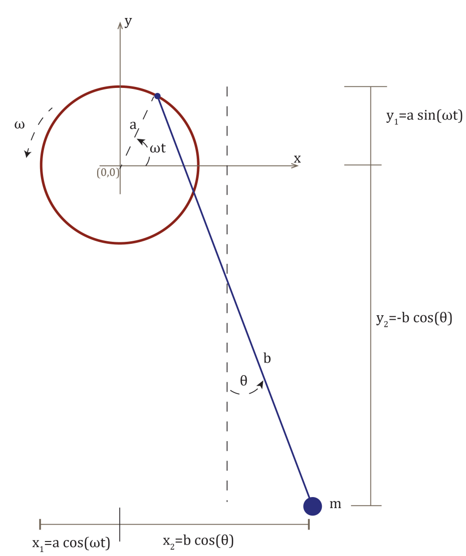

Figure 1: Diagram of setup

III. Pre-Analysis

1. The rim is mass-less, so all energy the terms are only for the pendulum mass

2. The pendulum, however, has both linear and angular momentum
$$T = 1/2mv^2 + 1/2I\omega^2 \tag{1}$$
3. A change of coordinates from Cartesian to polar is probably the most efficient way to proceed.
4. The plan is to get a differential equation, or two, in just θ and $\dot{\theta}$.
5. Using python code we can run a 4th order Runge-Kutta numerical analysis and plot the pendulum's position $x(t)$ and $y(t)$.
6. I'm going to need some initial values for this

IV. Equations

$$\begin{aligned} x &= a\cos(\omega t) + b\sin\theta & \dot{x} &= -a\omega\sin(\omega t) + b\sin(\theta)\dot{\theta} \\ y &= a\sin(\omega t) - b\cos\theta & \dot{y} &= a\omega\cos(\omega t) + b\sin(\theta)\dot{\theta} \end{aligned} \tag{2}$$

$$\begin{aligned} \mathbf{T} &= \frac{1}{2}m\left(a^2\omega^2 + b^2\dot{\theta}^2 + 2ab\omega\dot{\theta}\left[\sin\theta\cos\omega t - \sin\omega t\cos\theta\right]\right) \\ &= \frac{1}{2}m\left(m^2\omega^2 + b^2\dot{\theta}^2 + 2ab\omega\dot{\theta}\sin(\theta - \omega t)\right) \end{aligned} \tag{3}$$

$$\mathbf{U} = mg\left(a\sin\omega t - b\cos\theta\right) \tag{4}$$

$$\begin{aligned} \mathcal{L} &= \mathbf{T} - \mathbf{U} \\ &= \frac{1}{2}m\left(m^2\omega^2 + b^2\dot{\theta}^2 + 2ab\omega\dot{\theta}\sin(\theta - \omega t)\right) - mg\left(a\sin\omega t - b\cos\theta\right) \end{aligned} \tag{5}$$

V. Code outline

References

[1] Rovelli, Carlo, et al. *Reality Is Not What It Seems: the Journey to Quantum Gravity.* Riverhead Books, 2018.

[2] "Physics and Astronomy - Western University." Western University Physics And Astronomy, www.physics.uwo.ca/.

[3] "Chapter 7: Hamilton's Principle." Classical Dynamics of Particles and Systems, by Stephen T. Thornton and Jerry B. Marion, Brooks/Cole, 2004.