

Problem. Suppose ϕ is a function with a continuous derivative on $0 \leq x \leq 1$ satisfying there $\phi'(x) - 2\phi(x) \leq 1$, and $\phi(0) = 1$. Prove that

$$\phi(x) \leq \frac{3}{2}e^{2x} - \frac{1}{2}.$$