# Section 12.4 Tangent Vectors and Normal Vectors

## MATH211 Calculus III

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# Knowledge Checks



What is the **unit tangent vector** and the **principal unit normal vector**?

$$\textbf{Unit Tangent Vector, } \mathbf{T} = \frac{\mathbf{r}'}{\|\mathbf{r}'\|},$$

$$\textbf{Principal Unit Normal Vector, } \mathbf{N} = \frac{\mathbf{T}'}{\|\mathbf{T}'\|},$$

(Bonus) **Binormal Vector, $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.**

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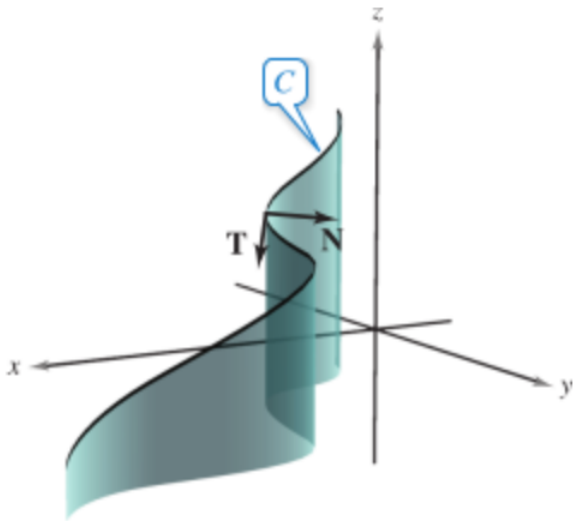


$$\mathbf{T} \cdot \mathbf{N} = 0$$

(Not very intuitive though. Make sure you read the textbook and understand why it is true.)

# Knowledge Checks



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(Again, it's not very intuitive that $\mathbf{a}$ can be decomposed in this way in 3D space. Make sure you read the textbook and understand the proof.)
Key consequences:

$$a_T\mathbf{T} = \text{proj}_{\mathbf{v}}\,\mathbf{a}$$

$$a_N\mathbf{N} = \mathbf{a} - \text{proj}_{\mathbf{v}}\,\mathbf{a}$$

$$\mathbf{N} = \frac{\mathbf{a} - \text{proj}_{\mathbf{v}}\,\mathbf{a}}{a_N} = \frac{\mathbf{a} - \text{proj}_{\mathbf{v}}\,\mathbf{a}}{\sqrt{\|\mathbf{a}\|^2 - a_T^2}} = \frac{\mathbf{a} - \text{proj}_{\mathbf{v}}\,\mathbf{a}}{\|\mathbf{a} - \text{proj}_{\mathbf{v}}\,\mathbf{a}\|}$$