Section 11.3 The Dot Product of Two Vectors

MATH211 Calculus III

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MESSIAH UNIVERSITY

DEPARTMENT OF
COMPUTING, MATHEMATICS
AND PHYSICS

Knowledge Checks

The **dot product** (in $\mathbb{R}^3$) of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

Knowledge Checks

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$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3.$$

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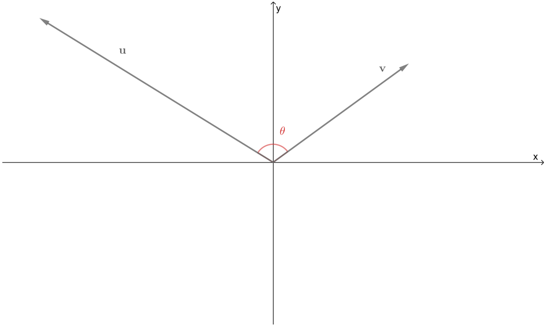

How does $\mathbf{u} \cdot \mathbf{v}$ relate to this figure?

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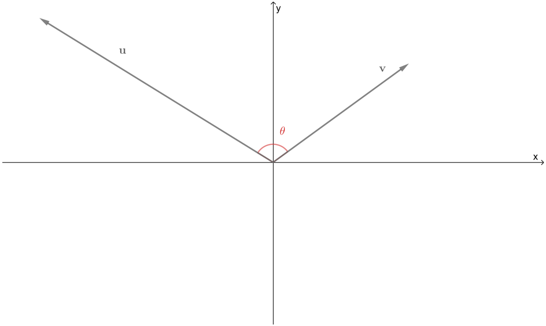
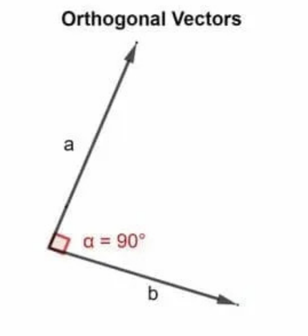


How does $\mathbf{u} \cdot \mathbf{v}$ relate to this figure?

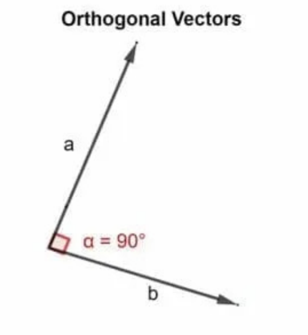
$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

Knowledge Checks



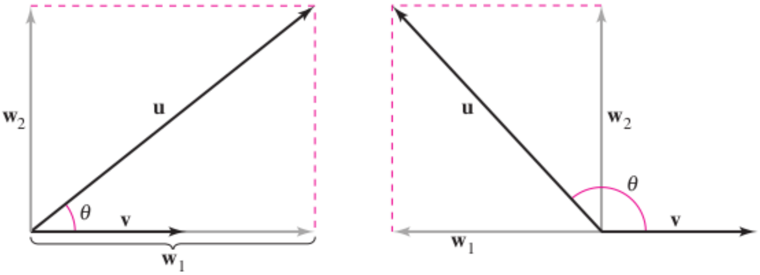
What is the dot product between a pair of orthogonal vectors?

Knowledge Checks



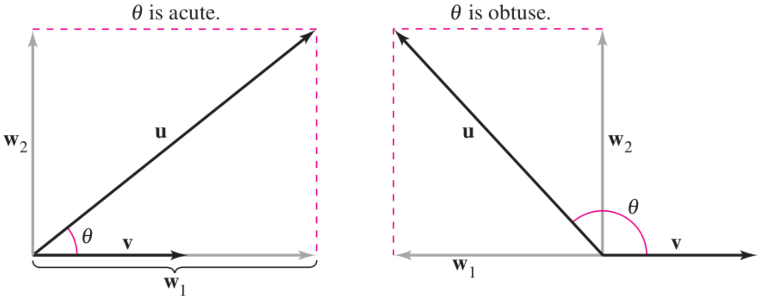
What is the dot product between a pair of orthogonal vectors? 0

Projections and vector components



What is $\mathbf{w}_1$ and $\mathbf{w}_2$ called?

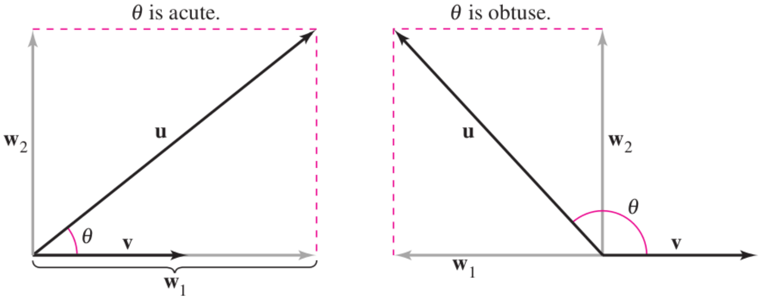
Projections and vector components



$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \text{projection of } \mathbf{u} \text{ onto } \mathbf{v} = \text{vector component of } \mathbf{u} \text{ along } \mathbf{v}$

$\mathbf{w}_2 = \text{vector component of } \mathbf{u} \text{ orthogonal to } \mathbf{v}$

Projections and vector components



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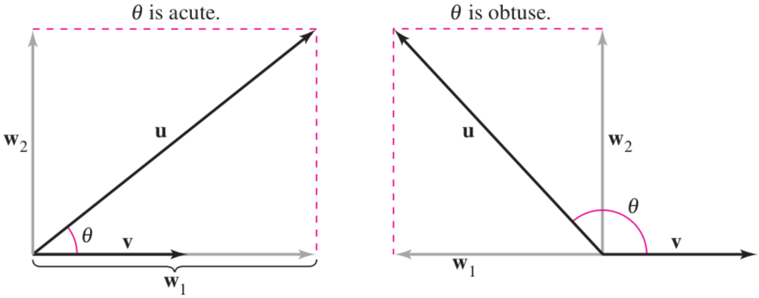
How to find $\mathbf{w}_1$ and $\mathbf{w}_2$?

Projections and vector components



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How to find $\mathbf{w}_1$ and $\mathbf{w}_2$?

$$\mathbf{w}_1 = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \mathbf{u} - \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\right)\mathbf{v}$$