Orthogonal Diagonalizability

Linear Algebra

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Theorem

Orthogonally diagonalizable matrices are symmetric.

Proof. Suppose $A = ODO^T$, where D is diagonal, and O is orthogonal. Then

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Theorem

Symmetric matrices are orthogonally diagonalizable.

Proof. The proof is highly nontrivial. Due to the lack of time, it's left to you to investigate.

However, the process of orthogonally diagonalizing a symmetric matrix is manageable:

Step 1: Find the eigenvalues of the matrix.

Step 2: Find a generic basis for each eigenspace.

Step 3: Apply the Gram-Schmidt process to the basis of each eigenspace.

Step 4: Put all the basis vectors side by side to form the orthogonal matrix.