Problems.

1. (**Taylor's Theorem with the Integral Form Remainder**) If a function f is differentiable through order $n+1$ in an interval I containing c , then, for each x in I,

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x)$$

where

$$R_n(x) = \frac{1}{n!}\int_c^x (x-t)^n f^{(n+1)}(t)\,\mathrm{d}t$$

Prove the theorem.

2. Use the Taylor's Theorem to prove that, given any $a > 0$ and $h > 0$,

$$1 + ah < \mathrm{e}^{ah}$$

3. Suppose f'' is continuous and $|f''| \leq M$ on the interval I. Let $h = x - c$, where $x > c$. Prove that

$$|R_1(x)| \leq \frac{Mh^2}{2}$$