Problem. In this problem, we will analyze the truncation error in Euler's method. You may want to review the relevant material in the textbook first (page 99 - 102).

Consider the IVP

$$y' = f(x, y) = \mathrm{e}^{-y^2}, \quad y(0) = y_0$$

1. Find an upper bound of $|f(x, y)|$.

2. Find an upper bound of $|f_x(x, y)|$ and $|f_y(x, y)|$ respectively.

3. Find an upper bound M of $|f_x(x, y) + f_y(x, y)f(x, y)|$.

4. Use Taylor's Theorem to find an R such that

$$|f(x_i, y(x_i)) - f(x_i, y_i)| \leq R|e_i|$$

5. Let $b = 1$, and $h = \dfrac{b-0}{n} = \dfrac{1}{n}$. Find a K such that

$$|y(b) - y_n| \leq Kh$$

6. Lastly, in order for the truncation error to be less than 0.01, how great does n need to be?