

Section 11.7

B.H.

## Section 11.7 Cylindrical and Spherical Coordinates

MATH211 Calculus III

Instructor: Ben Huang

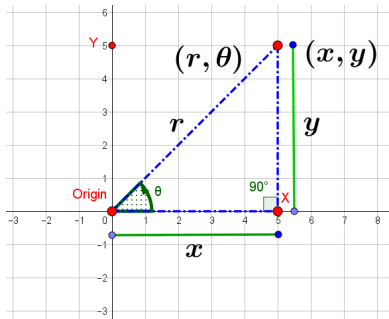


DEPARTMENT OF  
COMPUTING, MATHEMATICS  
AND PHYSICS

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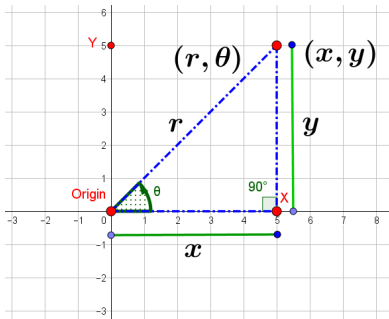


How to express  $x$  and  $y$  in terms of  $r$  and  $\theta$ ?

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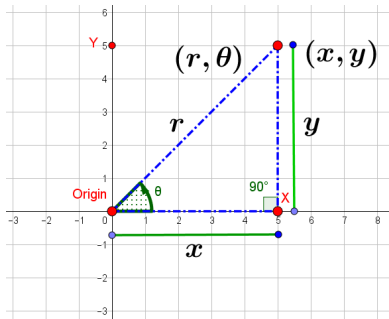
How to express  $x$  and  $y$  in terms of  $r$  and  $\theta$ ?

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

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How to express  $x$  and  $y$  in terms of  $r$  and  $\theta$ ?

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

Remark:  $(r, \theta)$  is the polar coordinates of the point.

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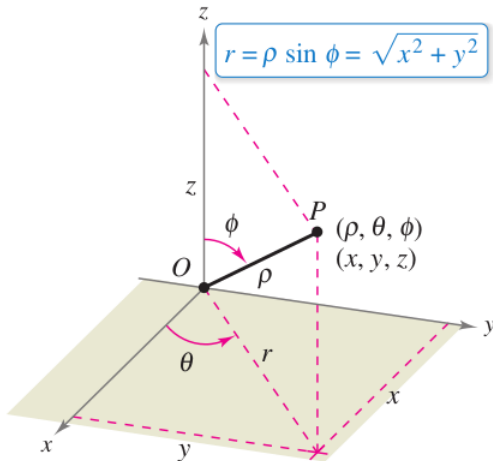
What are the spherical coordinates in 3-D space?

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What are the spherical coordinates in 3-D space?

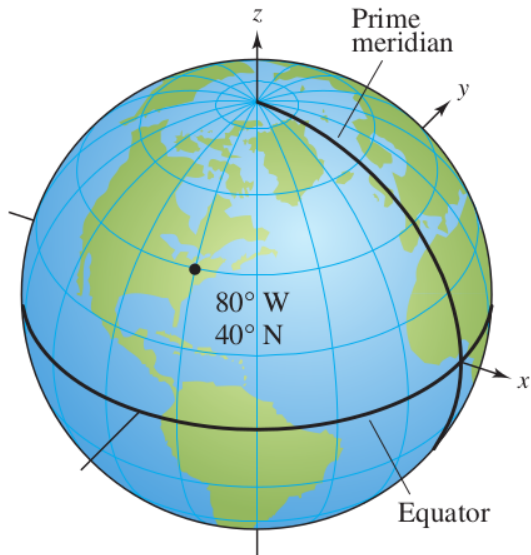


Spherical coordinates

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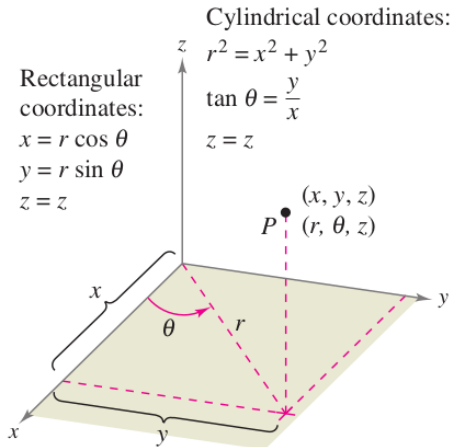


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What are the cylindrical coordinates in 3-D space?



# Spherical coordinates

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### Example

Sketch the graph of the spherical coordinates equation.

(a)  $\phi = \frac{\pi}{4}$ .

(b)  $\theta = \frac{\pi}{4}$ .

(c)  $\rho = 1$ .

# Spherical coordinates

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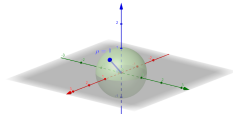
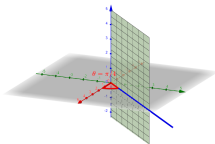
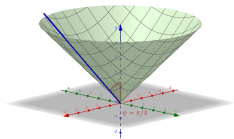
### Example

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### Example

Convert the rectangular equation to an equation in spherical coordinates.

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**Solution.**

$$(\rho \cos \theta \sin \phi)^2 + (\rho \sin \theta \sin \phi)^2 + (\rho \cos \phi)^2 = 16$$

$$(\cos^2 \theta + \sin^2 \theta)(\rho \sin \phi)^2 + (\rho \cos \phi)^2 = 16$$

$$\rho^2(\sin^2 \phi + \cos^2 \phi) = 16$$

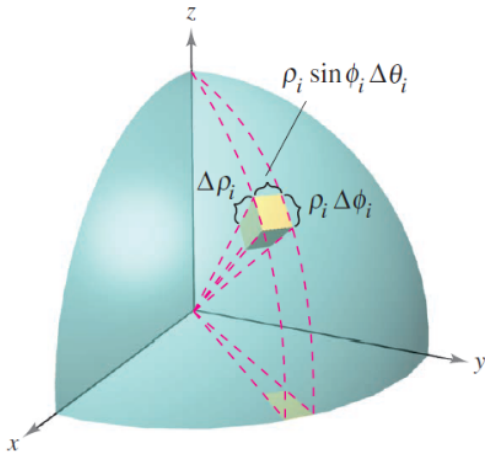
$$\rho^2 = 16$$

$$\rho = 4.$$

# Triple Integrals in Spherical Coordinates

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$$\Delta V_i \approx \rho_i^2 \sin \phi_i \Delta \rho_i \Delta \phi_i \Delta \theta_i$$

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### Example

Consider the following points in spherical coordinates

$$A(\rho = 1, \theta = \pi/4, \phi = \pi/4), \quad B(\rho = 1, \theta = 5\pi/4, \phi = 7\pi/4).$$

Plot these points in the coordinate space, and find the rectangular coordinates of them.

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### Example

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Plot these points in the coordinate space, and find the rectangular coordinates of them.

**Answer:** Rectangular coordinates  $A = (1/2, 1/2, \sqrt{2}/2) = B$ .

