Section 11.7 Cylindrical and Spherical Coordinates

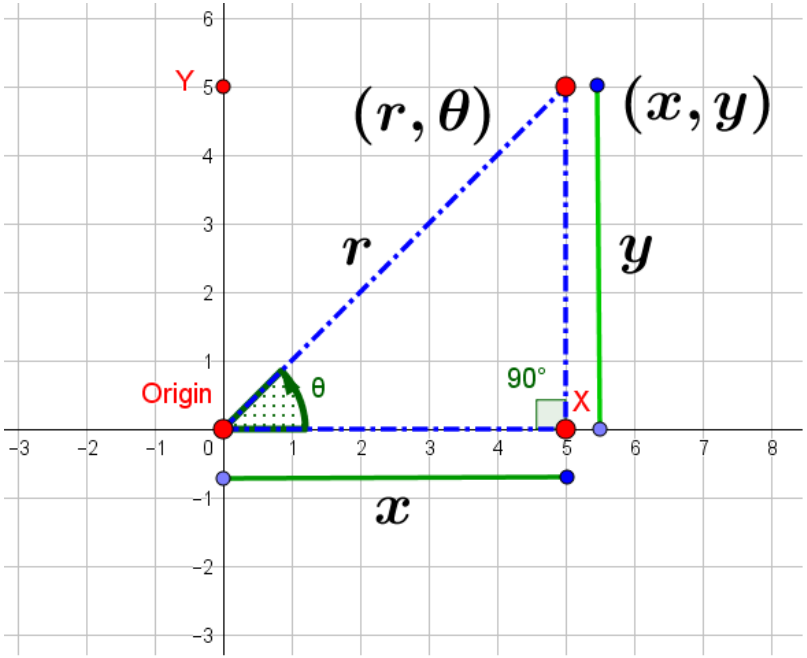
MATH211 Calculus III

Instructor: Ben Huang

MESSIAH UNIVERSITY

DEPARTMENT OF COMPUTING, MATHEMATICS AND PHYSICS

Knowledge Checks



How to express x and y in terms of r and θ?

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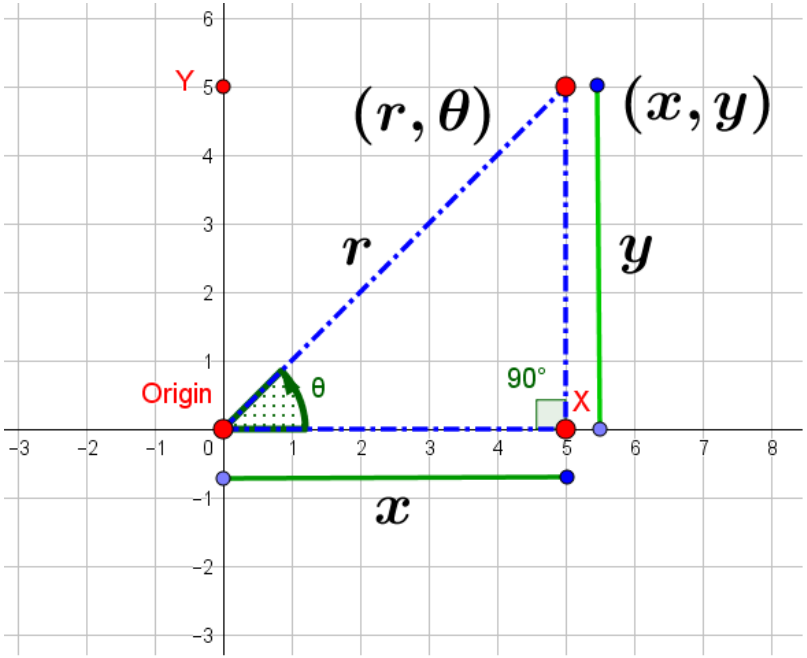

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$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

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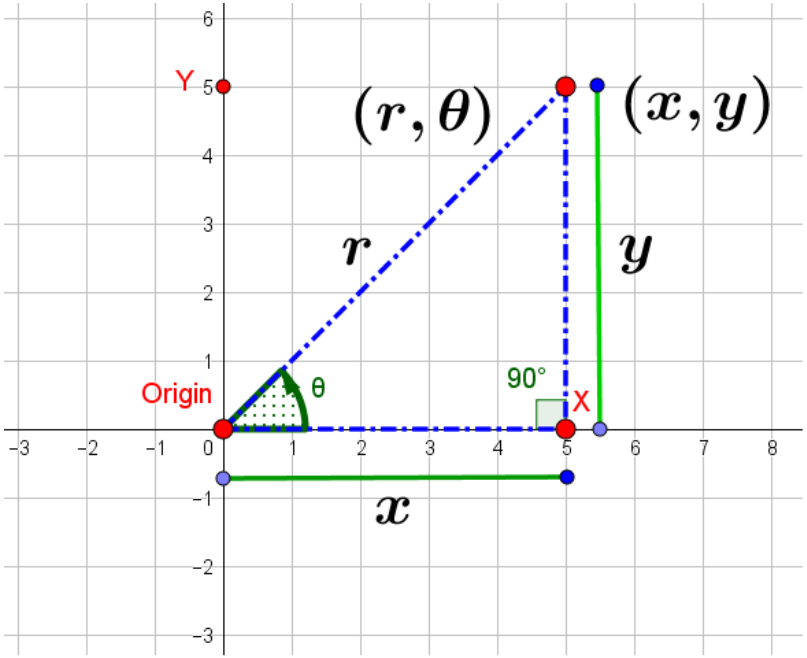

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Remark: (r, θ) is the polar coordinates of the point.

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What are the spherical coordiates in 3-D space?

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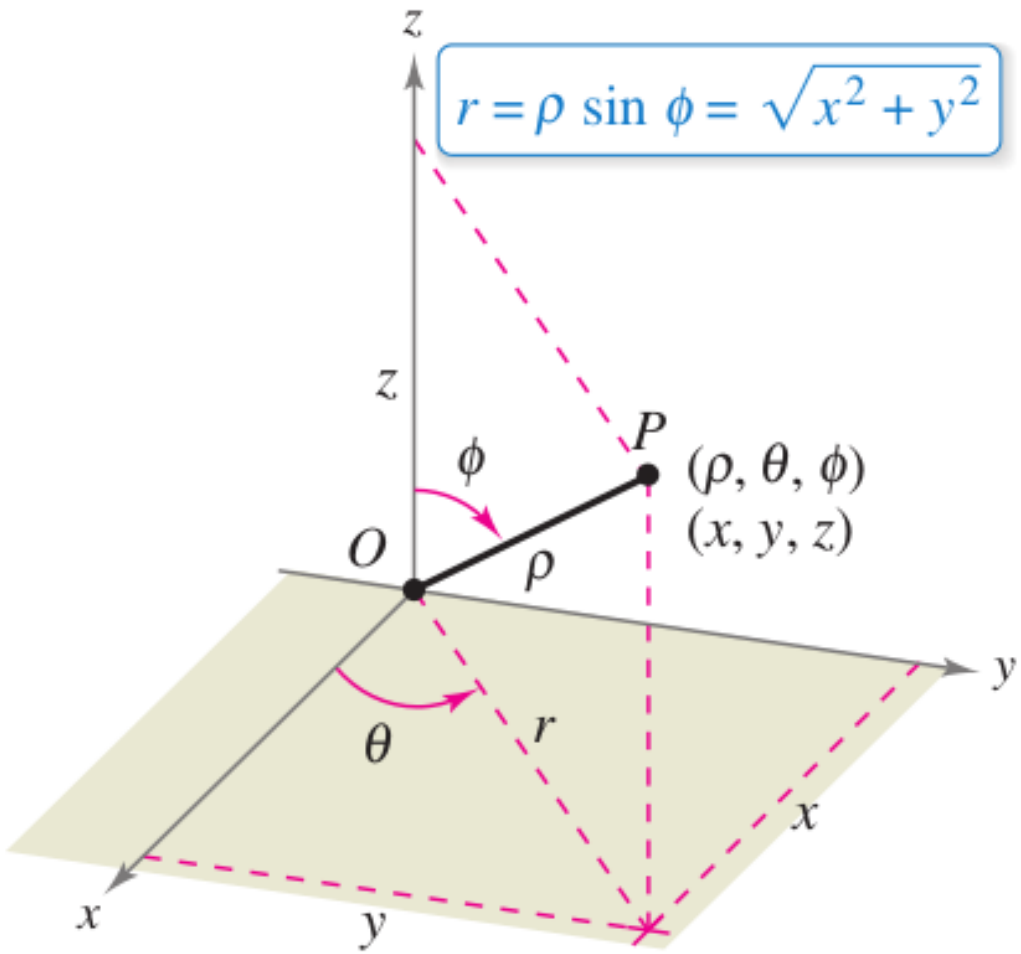

Spherical coordinates

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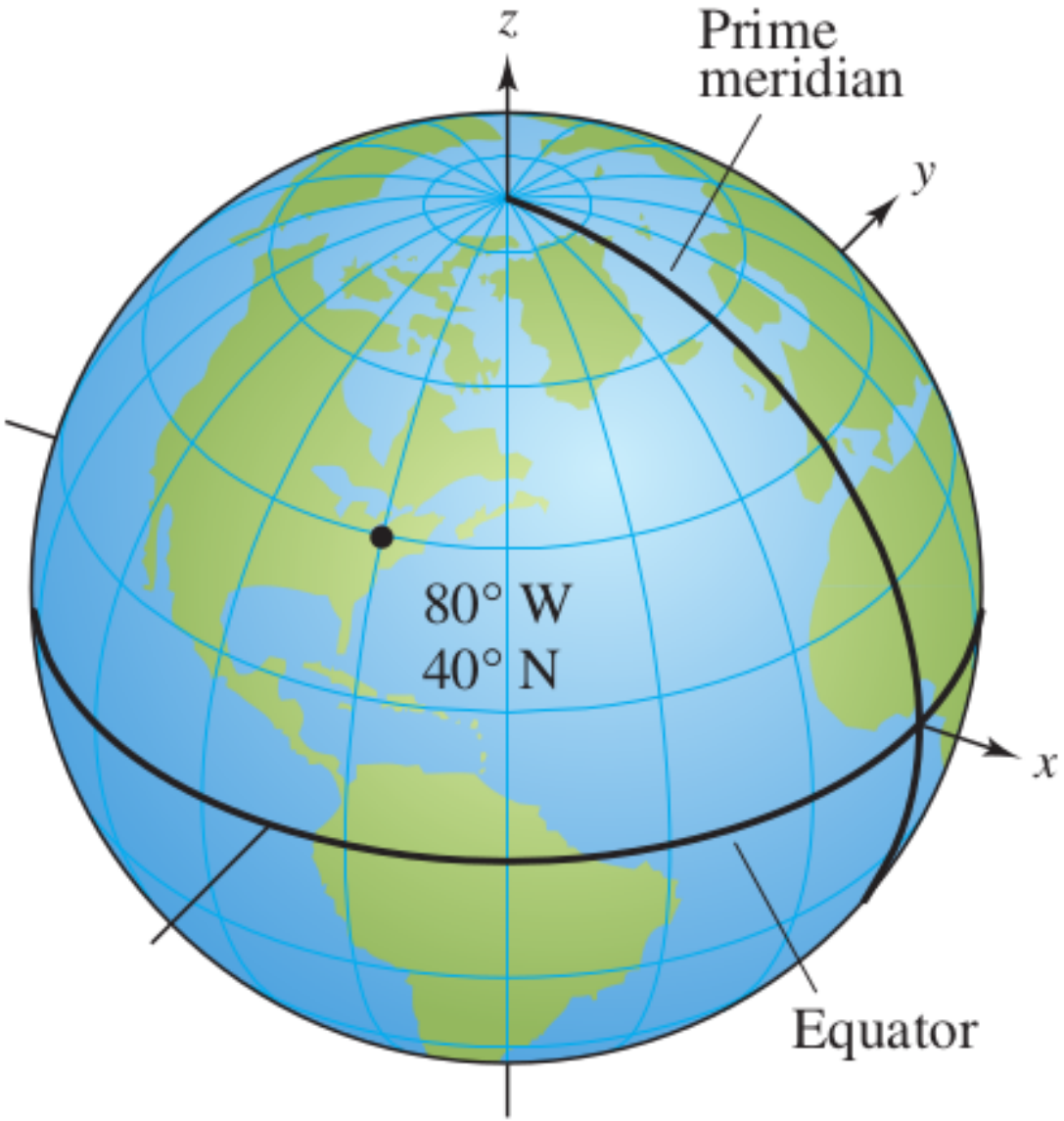

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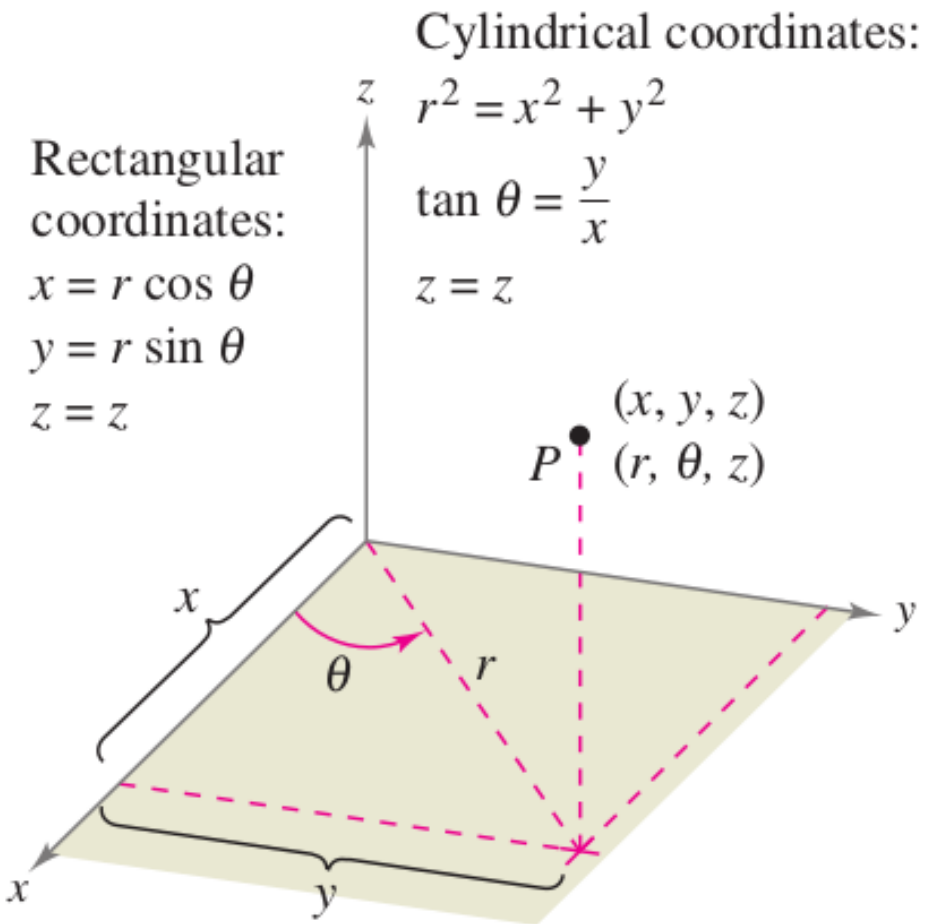

Spherical coordinates

Example

Sketch the graph of the spherical coordinates equation.

(a) $\phi = \frac{\pi}{4}$. (b) $\theta = \frac{\pi}{4}$. (c) $\rho = 1$.

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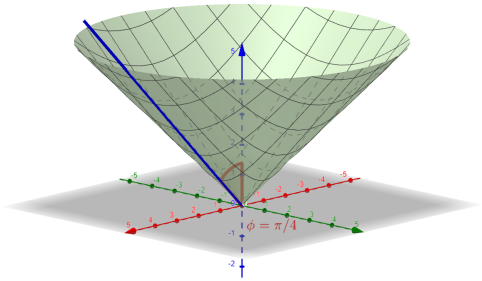

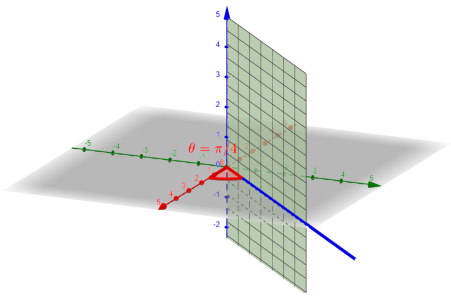

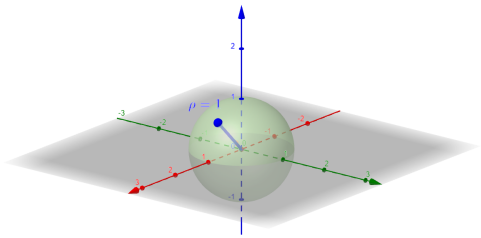

Spherical coordinates

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Convert the rectangular equation to an equation in spherical coordinates.

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Solution.

$$\begin{aligned}
(\rho\cos\theta\sin\phi)^2 + (\rho\sin\theta\sin\phi)^2 + (\rho\cos\phi)^2 &= 16 \\
(\cos^2\theta + \sin^2\theta)(\rho\sin\phi)^2 + (\rho\cos\phi)^2 &= 16 \\
\rho^2(\sin^2\phi + \cos^2\phi) &= 16 \\
\rho^2 &= 16 \\
\rho &= 4.
\end{aligned}$$

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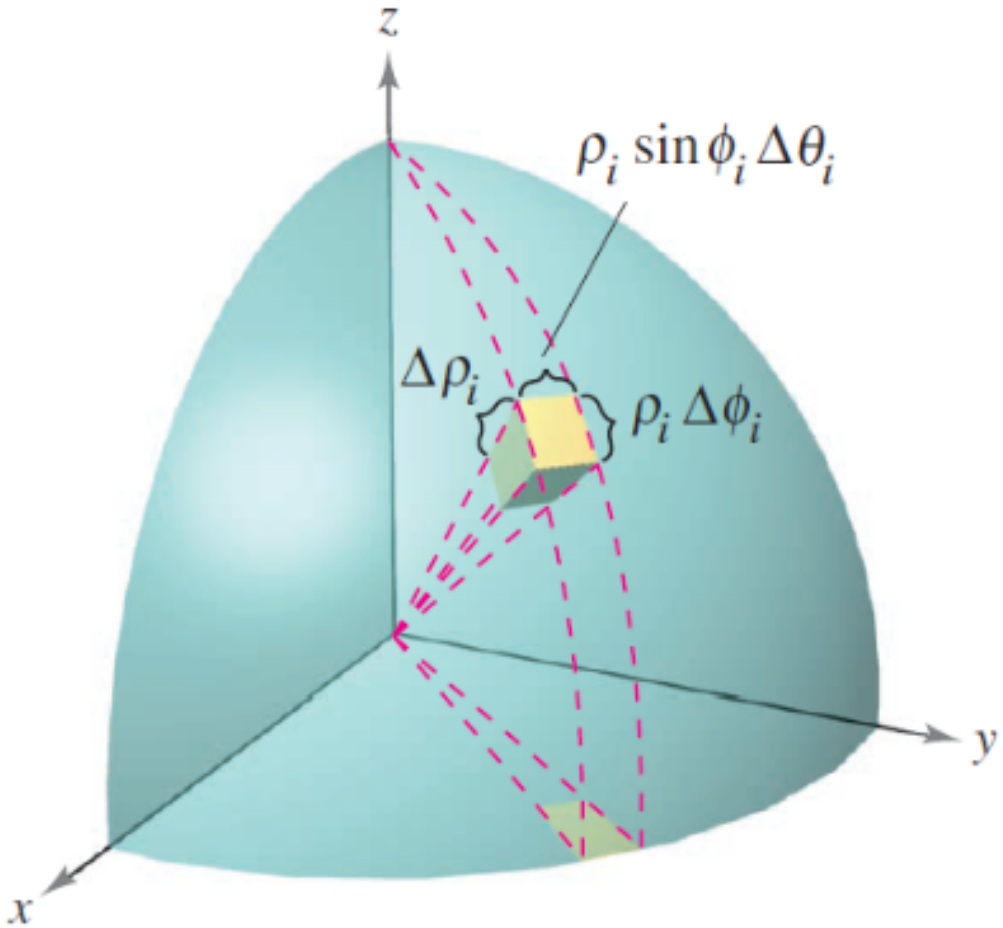
Triple Integrals in Spherical Coordinates



$$\Delta V_i \approx \rho_i^2 \sin\phi_i \Delta\rho_i \Delta\phi_i \Delta\theta_i$$

Spherical coordinates

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Consider the following points in spherical coordinates

$$A(\rho = 1,\ \theta = \pi/4,\ \phi = \pi/4), \qquad B(\rho = 1,\ \theta = 5\pi/4,\ \phi = 7\pi/4).$$

Plot these points in the coordinate space, and find the rectangular coordinates of them.

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Answer: Rectangular coordinates $A = (1/2,\ 1/2,\ \sqrt{2}/2) = B$.