Section 12.3 Velocity and Acceleration

MATH211 Calculus III

Instructor: Ben Huang

MESSIAH UNIVERSITY

DEPARTMENT OF COMPUTING, MATHEMATICS AND PHYSICS

Learning objectives

Learning objectives: After studying this section, students will be able to

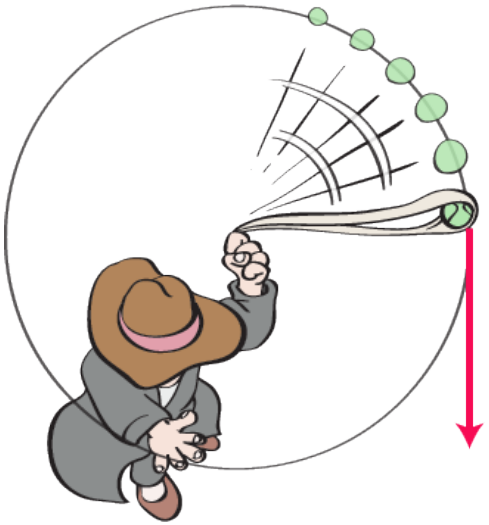
- find the **displacement**, **velocity** and **acceleration** of an object in motion with given conditions.
- use vector-valued functions to analyze **circular motion** and **projectile motion**.

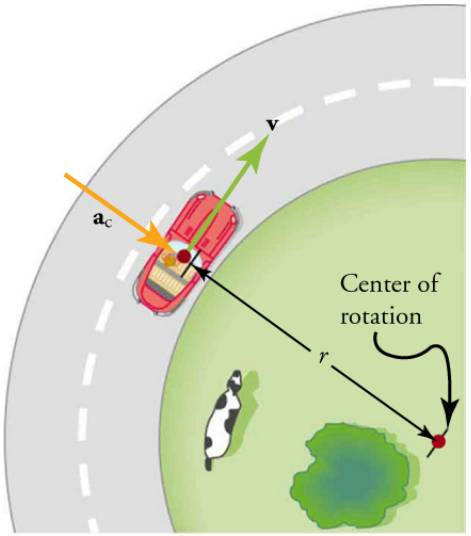
Review of mechanics

	Differentiation	Integration
Displacement $\mathbf{p}$ (Position)	$\mathbf{p}' = \mathbf{v}$	
Velocity $\mathbf{v}$	$\mathbf{v}' = \mathbf{a}$	$\int_{t_0}^{t_1} \mathbf{v}(t)\, \mathrm{d}t = \mathbf{p}(t_1) - \mathbf{p}(t_0)$
Acceleration $\mathbf{a}$	$\mathbf{a}' = \mathbf{jerk}$	$\int_{t_0}^{t_1} \mathbf{a}(t)\, \mathrm{d}t = \mathbf{v}(t_1) - \mathbf{v}(t_0)$

Newton's second law: $\mathbf{F} = m\mathbf{a}$.

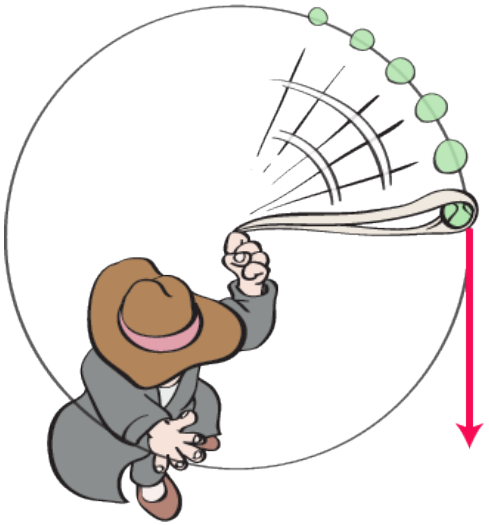
Circular motion

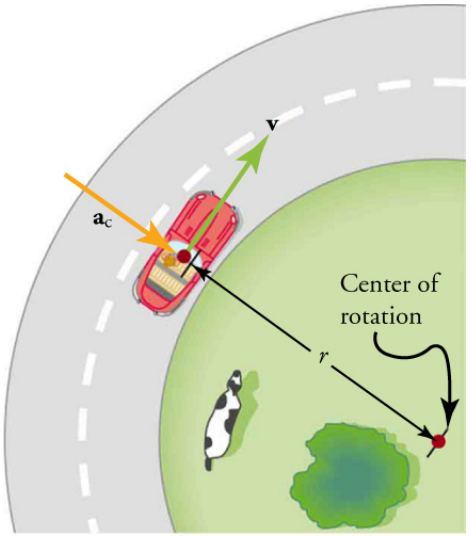




Car around corner

Circular motion





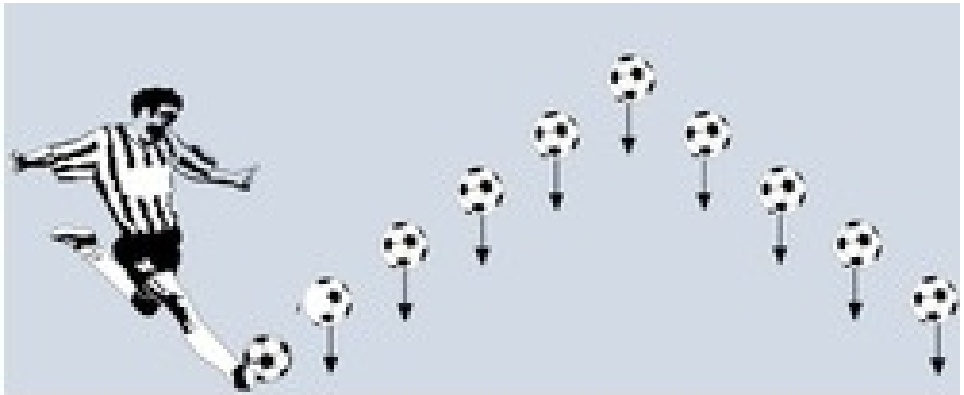
Car around corner

Circular Motion with Constant Speed:

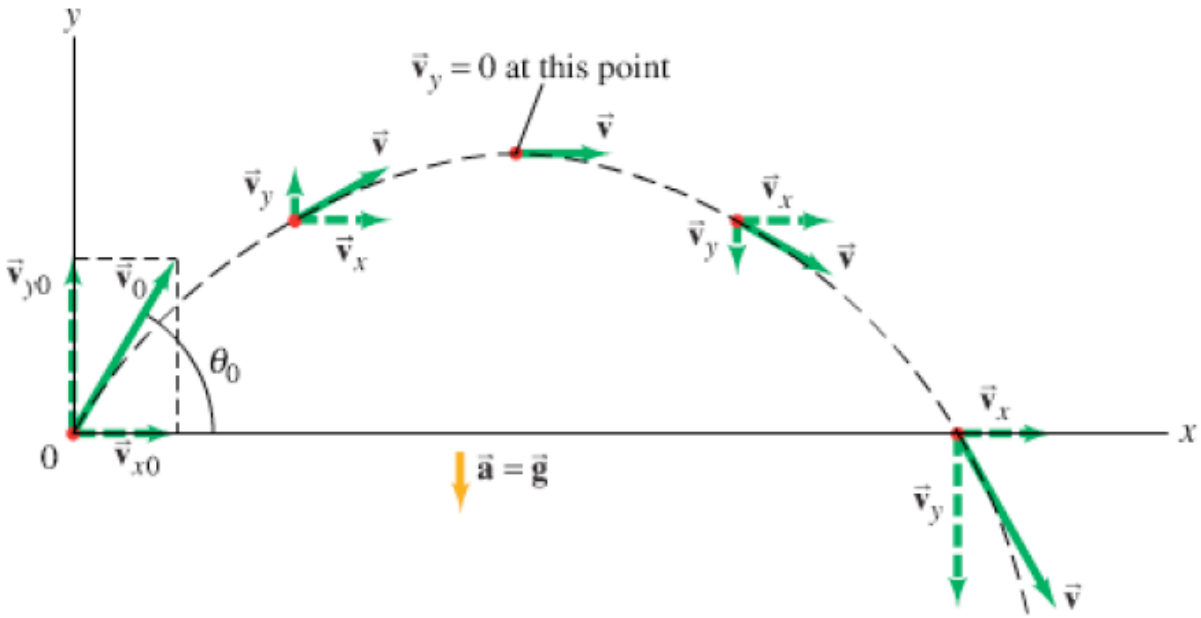
$$\mathbf{r}(t) = b\cos(\omega t)\mathbf{i} + b\sin(\omega t)\mathbf{j}$$

Projectile motion

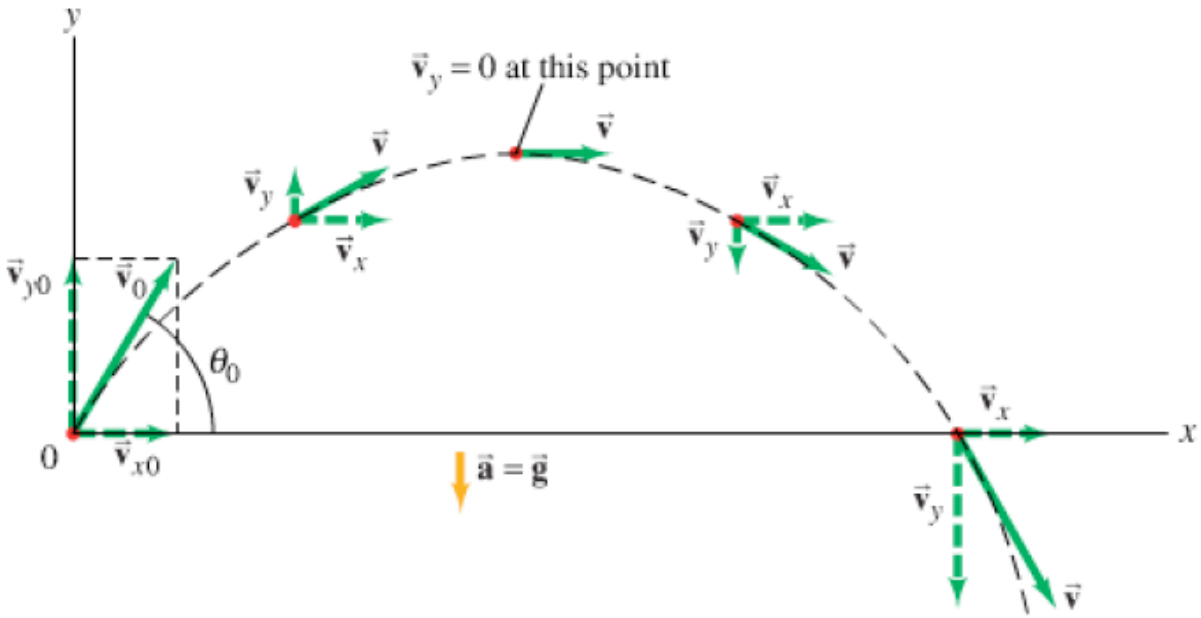
Projectile motion



Projectile motion



Projectile motion



$$\mathbf{a}(t) = \mathbf{g} = -g\mathbf{j}$$

$$\mathbf{v}(t) = \int_0^t \mathbf{a}(s)\, ds + \mathbf{v}_0 = -gt\mathbf{j} + \mathbf{v}_0$$

$$\mathbf{r}(t) = \int_0^t \mathbf{v}(s)\, ds + \mathbf{r}_0 = -\frac{1}{2}gt^2\mathbf{j} + \mathbf{v}_0 t + \mathbf{r}_0$$