Section 12.5 Arc Length and Curvature

MATH211 Calculus III

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Knowledge Checks

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$$s = \int_a^b \|\mathbf{r}'(t)\| \, \mathrm{d}t = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, \mathrm{d}t$$

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Note: This is just the non-constant version of the elementary fact

$$\text{distance} = \text{speed} \cdot \textit{time}$$

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1. $K(t) = \dfrac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

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2. If $C : y = y(x)$,

$$K(x) = \frac{|y''(x)|}{\{1 + [y'(x)]^2\}^{3/2}}$$

Knowledge Checks

Physics implications (Read the textbook for the proof):

$$\mathbf{a} = \|v\|'\mathbf{T} + K\|v\|^2\mathbf{N},$$

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