MATH211

MESSIAH
UNIVERSITY

DEPARTMENT OF
COMPUTING, MATHEMATICS
AND PHYSICS

Written Assignment, Page 1

Problem. Consider the function defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}.$$

(a) Find $f_x(x,y)$ and $f_y(x,y)$ for $(x,y) \neq (0,0)$.

(b) Use the definition of partial derivatives to find $f_x(0,0)$ and $f_y(0,0)$.

(Recall that $f_x(0,0) = \lim\limits_{\Delta x \to 0} \dfrac{f(\Delta x, 0) - f(0,0)}{\Delta x}$, $f_y(0,0) = \lim\limits_{\Delta y \to 0} \dfrac{f(0, \Delta y) - f(0,0)}{\Delta y}$.)

(c) Use the definition of partial derivatives to find $f_{xy}(0,0)$ and $f_{yx}(0,0)$.

(d) According to Theorem 13.3 *Equality of Mixed Partial Derivatives* and the result of part (c), **by logic**, what can be inferred about the continuity of f_{xy} and f_{yx} at $(0,0)$? Type your reasoning. (Remark: Handwritten response receives NO credit for this part.)