Section 13.8 Extrema of Functions of Two Variables

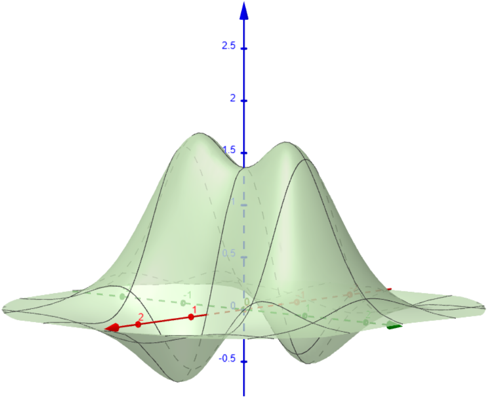
MATH211 Calculus III

Instructor: Ben Huang

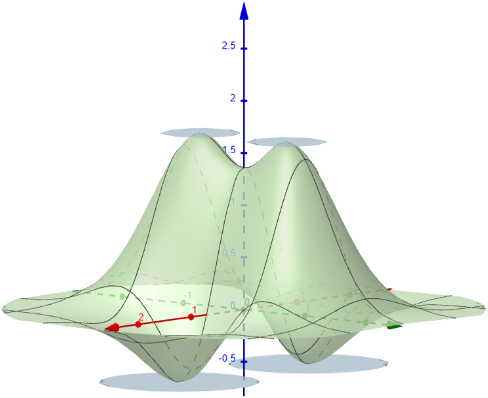
MESSIAH UNIVERSITY

DEPARTMENT OF
COMPUTING, MATHEMATICS
AND PHYSICS

Relative Extrema

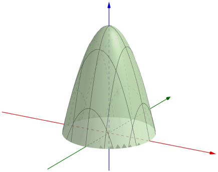


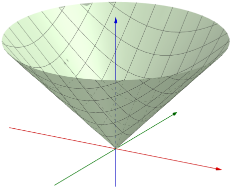
Relative Extrema

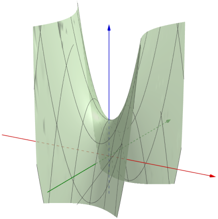


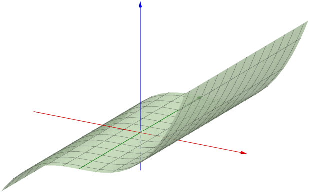
Critical points

Where are the critical points?









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Critical points

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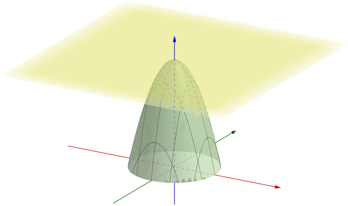
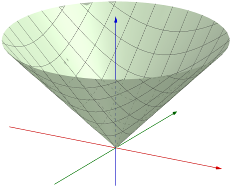
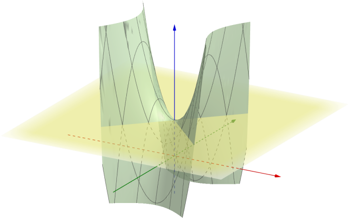
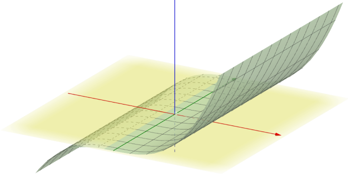
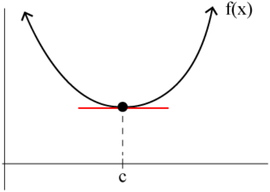






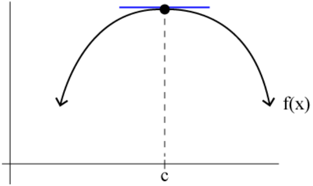


Second Derivative Test From Calculus I



$f'(c) = 0$ **and** $f''(c) > 0$

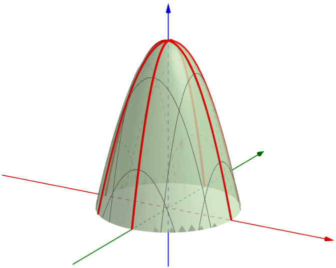
$f(c)$ is a local minimum



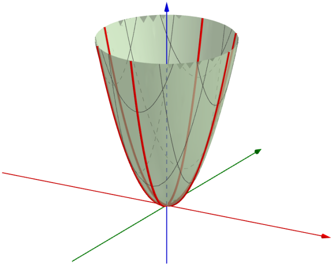
$f'(c) = 0$ **and** $f''(c) < 0$

$f(c)$ is a local maximum

Classification of Critical Points



$f_{xx} < 0$ & $f_{yy} < 0$
at the critical point
local maximum



$f_{xx} > 0$ & $f_{yy} > 0$
at the critical point
local minimum

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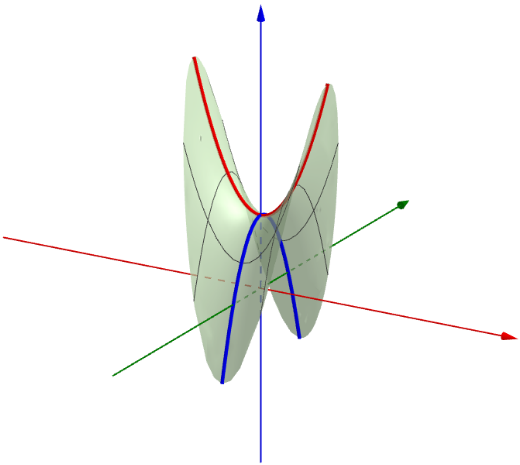
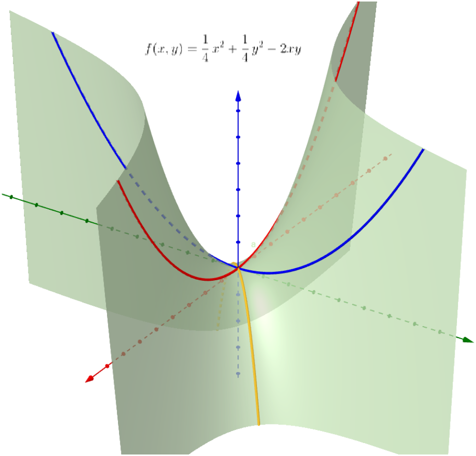
Classification of Critical Points



$f_{xx} > 0$ & $f_{yy} < 0$ at the critical point
saddle point

Classification of Critical Points



$f_{xx} > 0$ & $f_{yy} > 0$ at the critical point
saddle point

Failure of the Second Partials Test

When $d(x, y) = f_{xx} f_{yy} - f_{xy}^2 = 0$ at the critical point in question, the Second Partials Test is **inconclusive**.

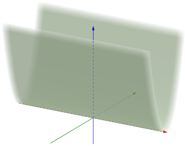


Figure 1: $z = y^2$

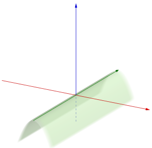


Figure 2: $z = -x^2$

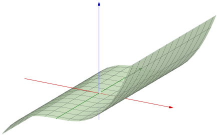


Figure 3: $z = \frac{1}{50}x^3$