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Rossby waves

Introduction

Rossby waves are inertial waves which occur in rotating fluids due to the conservation of vorticity. In the atmosphere, these take the form of large-scale meanders in the high level winds which circle the poles. In the ocean they propogate along the thermocline and are caused by wind stress anomalies at the surface.

Physical basis

Rossby waves occur in an inviscid, barotropic fluid of constant depth, such as the atmosphere.

- Inviscid - viscous (frictional) forces are considered to be zero in the free atmosphere.
- Barotropic - surfaces of constant pressure and constant density coincide ($\nabla\rho\times\nabla p = 0$). Pressure depends only on density.

In this case, the divergence of the horizontal velocity is zero, given by:

$$\nabla_h \cdot \tilde{\mathbf{u}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The vorticity of the fluid is given by the curl of the velocity:

$$\nabla \times \tilde{\mathbf{u_a}} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f = \eta \tag{2}$$

The absolute vorticity is given by the curl of the absolute velocity ($\tilde{\mathbf{u_a}}$), which is the air velocity relative to an inertial frame. For this we consider only the vertical component of the relative vorticity, $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, and the vorticity of the earth, f to give the absolute vorticity: $\eta = \zeta + f$.

For the barotropic, non-divergent (equation 1) fluid case, the absolute vorticity is conserved following the motion:

$$\frac{D}{Dt}(\zeta + f) = 0 \tag{3}$$

In the more general case, we consider a baroclinic atmosphere where density depends on both temperature and pressure, where the geostrophic wind varies with height. In this case, the Rossby wave is a potential vorticity-conserving motion which occurs due to gradients in potential vorticity. The potential vorticity is given by:

$$\mathcal{Q} = \alpha(2\mathbf{\Omega} + \nabla \times \mathbf{u}) \cdot \nabla\theta \tag{4}$$

Where $\mathbf{\Omega}$ is the angular velocity of the earth, α is the specific volume $(1/\rho)$, $\mathbf{u}$ is the three dimensional vector velocity, and θ is the potential temperature $(\theta = T(p_0/p)^{\kappa})$. In the absence of friction and heating, the potential vorticity (ertel PV, $\mathcal{Q}$) is conserved following the motion of the fluid.

Emergence of Rossby waves

For an idealized case, we consider a barotropic fluid with a constant density and depth, where variation in the coriolis parameter is given by:

$$f = f_0 + \beta y \tag{5}$$

Where:

f_0 is the coriolis parameter at ϕ_0, the latitude of the equator ($y = 0$).

$\beta \equiv (df/dy)_{\phi_0} = 2\Omega \cos\phi_0/a$ is the Rossby parameter which accounts for the the meridional variation of the Coriolis force caused by the spherical shape of the earth. Where Ω is the angular velocity of the earth, and a is the radius of the earth.

y is the distance from the equator.

Relevance

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