6 RANDOM VARIABLES

In this section, we will consider random quantities that are usually called random variables.

Introduction

In application of probability, we are often interested in a number associated with the outcome of a random experiment. Such a quantity whose value is determined by the outcome of a random experiment is called a **random variable**. It can also be defined as any quantity or attribute whose value varies from one unit of the population to another.

A **discrete** random variable is function whose range is finite and/or countable, Ie it can only assume values in a finite or count ably infiniteset of values. A **continuous** random variable is one that can take any value in an interval of real numbers. (There are *unaccountably* many real numbers in an interval of positive length.)

6.1 Discrete Random Variables and Probability Mass Function

A random variable X is said to be discrete if it can take on only a finite or countable number of possible values x. Consider the experiment of flipping a fair coin three times. The number of tails that appear is noted as a discrete random variable. *X*= "number of tails that appear in 3 flips of a fair coin". There are 8 possible outcomes of the experiment: namely the sample space consists of

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$
$$X = \{\ 0 \quad 1, \quad 1, \quad 2, \quad 1, \quad 2, \quad 2, \quad 3\}$$

are the corresponding values taken by the random variable *X*.

Now, what are the possible values that *X* takes on and what are the probabilities of *X* taking a particular value?

From the above we see that the possible values of *X* are the 4 values

$$X = \{0, 1, 2, 3\}$$

Ie the sample space is a disjoint union of the 4 events $\{ X = j \}$ for j=0,1,2,3

Specifically in our example :

$\{X = 0\} = \{HHH\}$ $\qquad$ $\{X = 1\} = \{HHT,\ HTH,\ THH\}$

$\{X = 2\} = \{TTH,\ HTT,\ THT\}$ $\qquad$ $\{X = 3\} = \{TTT\}$

Since for a fair coin we assume that each element of the sample space is equally likely (with probability $\frac{1}{8}$, we find that the probabilities for the various values of *X,* called the *probability distribution*of*X* or the *probability mass function (pmf)*. can be summarized in the following table listing the possible values beside the probability of that value

X	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Note: The probability that *X* takes on the value *x*, ie $p(X = x)$, is defined as the sum of the probabilities of all points in *S* that are assigned the value *x*.

We can say that this pmf places mass $\frac{3}{8}$ on the value $X = 2$.

The "masses" (or probabilities) for a pmf should be between 0 and 1.

The total mass (i.e. total probability) must add up to 1.

Definition: The **probability mass function** of a discrete variable is a, table, formula or graph that specifies the proportion (or probabilities) associated with each possible value the random variable can take. The mass function $P(X = x)$ (or just p(x) has the following properties:

$$0 \le \mathrm{p(x)} \le 1 \text{ and } \sum_{\text{all x}} \mathrm{p(x)} = 1$$

More generally, let X have the following properties

i) It is a discrete variable that can only assume values $x_1, x_2, \ldots. x_n$

ii) The probabilities associated with these values are $P(X = x_1) = p_1$, $P(X = x_2) = p_2 \ldots\ldots$ $P(X = x_n) = p_n$

Then X is a discrete random variable if $0 \le p_i \le 1$ and $\sum_{i=1}^{n} p_i = 1$

Remark: We denote random variables with capital letters while realized or particular values are denoted by lower case letters.

Example 1

Two tetrahedral dice are rolled together once and the sum of the scores facing down was noted. Find the pmf of the random variable 'the sum of the scores facing down.'

Solution

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$\mathrm{X} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Therefore the pmf is given by the table below

x	2	3	4	5	6	7	8
P(X=x)	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

This can also be written as a function

$$P(X = x) = \begin{cases} \frac{x-1}{16} \text{ for } x = 2, 3, 4, 5 \\ \frac{9-x}{16} \text{ for } x = 6, 7, 8 \end{cases}$$

Example 2

The pmf of a discrete random variable W is given by the table below

w	-3	-2	-1	0	1
P(W=w)	0.1	0.25	0.3	0.15	d

Find the value of the constant d, $P(-3 \le \mathrm{w} < 0)$, $P(\mathrm{w} > \text{-}1)$ and $P(-1 < \mathrm{w} < 1)$

Solution

$$\sum_{\text{all w}} \mathrm{p(W = w)} = 1 \quad \Rightarrow \quad 0.1 + 0.25 + 0.3 + 0.15 + d = 1 \quad \Rightarrow \quad d = 0.2$$

$$P(-3 \le \mathrm{w} < 0) = P(W = -3) + P(W = -2) + P(W = -1) = 0.65$$

$$P(\mathrm{w} > \text{-}1) = P(\mathrm{w} = 0) + P(\mathrm{w} = 1) = 0.15 + 0.2 = 0.35$$

$$P(-1 < \mathrm{w} < 1) = P(W = 0) = 0.15$$

Example 3

A discrete random variable Y has a pmf given by the table below

y	0	1	2	3	4
P(Y=y	c	2c	5c	10c	17

) | c

Find the value of the constant c hence computes $P(1 \leq \text{Y} < 3)$

Solution

$\sum_{\text{ally}} \text{p(Y = y)} = 1 \quad \Rightarrow \quad c(1+2+5+10+17) = 1 \quad \Rightarrow \quad c = \frac{1}{35}$

$P(1 \leq \text{Y} < 3) = P(Y=1) + P(Y=2) = \frac{2}{35} + \frac{5}{35} = \frac{1}{5}$

Exercise

1. A die is loaded such that the probability of a face showing up is proportional to the face number. Determine the probability of each sample point.
2. Roll a fair die and let X be the square of the score that show up. Write down the probability distribution of X hence compute $P(X < 15)$ and $P(3 \leq \text{X} < 30)$
3. Let X be the random variable the number of fours observed when two dice are rolled together once. Show that X is a discrete random variable.
4. The pmf of a discrete random variable X is given by $P(X = x) = kx \quad \text{for } x = 1,2,3,4,5,6$ Find the value of the constant k, $P(X < 4)$ and $P(3 \leq \text{X} < 6)$
5. A fair coin is flip until a head appears. Let N represent the number of tosses required to realize a head. Find the pmf of N c , $P(\text{N} < 2)$ and $P(\text{N} \geq 2)$
6. A discrete random variable Y has a pmf given by $P(Y = y) = c\left(\frac{3}{4}\right)^x \text{ for } y = 0,1,2,.....$ Find the value of the constant c , $P(\text{X} < 3)$ and $P(\text{X} \geq 3)$
7. Verify that $\text{f(x)} = \dfrac{2x}{k(k+1)} \text{ for } y = 0,1,2,.....k$ can serve as a pmf of a random variable X.
8. For each of the following determine c so that the function can serve as a pmf of a random variable X.
   a. $\text{f(x)} = c \quad \text{for } x = 1,2,3,4,5$
   b. $\text{f(x)} = cx \quad \text{for } x = 1,2,3,4,5$
   c. $\text{f(x)} = cx^2 \text{ for } x = 0,1,2,.....k$
   d. $\text{f(x)} = \frac{c}{2} \quad \text{for } x = -1,\ 0,\ 1,2$
   e. $\text{f(x)} = \frac{(x-2)}{c} \quad \text{for } x = 1,2,3,4,5$
   f. $\text{f(x)} = \frac{(x^2-x+1)}{c} \quad \text{for } x = 1,2,3,4,5$
   g. $\text{f(x)} = c(x^2+1) \text{ for } x = 0,1,2,3$
   h. g) $\text{f(x)} = cx({}_3C_x) \text{ for } x = 1,2,3$
   i. $\text{f(x)} = c\left(\frac{1}{6}\right)^x \text{ for } x = 0,1,2,3.....$
   j.
   $\text{f(x)} = c2^{-x} \text{ for } x = \text{ for } x = 0,1,2,....$
9. A coin is loaded so that heads is three times as likely as the tails.
   a. For 3 independent tosses of the coin find the pmf of the total number of heads realized and the probability of realizing at most 2 heads.
   b. A game is played such that you earn 2 points for a head and loss 5 points for a tail. Write down the probability distribution of the total scores after 4 independent tosses of the coin
10. For an on-line electronics retailer, X = “the number of Zony digital cameras returned per day” follows the distribution given by

x	0	1	2	3	4	5
P(X=x	0.0	0.1	t	0.2	0.2	0.1

) | 5 5

Find the value of t and $P(X > 3)$

11. Out of 5 components, 3 are domestic and 2 are imported. 3 components are selected at random (without replacement). Obtain the PMF of X ="number of domestic components picked" (make a table).

6.2 Continuous Random Variables and Probability Density Function

A **continuous** random variable can assume any value in an interval on the real line or in a collection of intervals. The sample space is uncountable. For instance, suppose an experiment involves observing the arrival of cars at a certain period of time along a highway on a particular day. Let T denote the time that lapses before the 1st arrival, the T is a continuous random variable that assumes values in the interval $[0,\infty)$

Definition: A random variable *X* is *continuous* if there exists a nonnegative function fso that, for every interval *B*, $P(X \in B) = \int_B f(x)\,dx.$ The function $f = f(x)$ is called the *probability density function*of *X*.

Definition: Let X be a continuous random variable that assumes values in the interval $(-\infty,\infty)$, The f(x) is said to be a probability density function (pdf) of X if it satisfies the following conditions

i) $f(x) \geq 0$ for all x , ii) $p(a \leq x \leq b) = \int_a^b f(x)\,dx = 1$ and iii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

The support of a continuous random variable is the smallest interval containing all values of x where f(x) >= 0.

Remark A crucial property is that, for any real number x, we have $P(X = x) = 0$ (implying there is no difference between $P(X \leq x)$ and $P(X < x)$); that is it is not possible to talk about the probability of the random variable assuming a particular value. Instead, we talk about the probability of the random variable assuming a value within a given interval.The probability of the random variable assuming a value within some given interval from $x = a$ to $x = b$ is defined to be the area under the graph of the probability density function between $x = a$ and $x = b$.

Example

Let X be a continuous random variable. Show that the function

$$f(x) = \begin{cases} \frac{1}{2}x, & 0 \leq x \leq 2 \\ 0, & elsewhere \end{cases}$$

is a pdf of X hence compute $P(0 \leq X < 1)$ and $P(-1 < X < 1)$

Solution

$f(x) \geq 0$ for all x in the interval $0 \leq x \leq 2$ and $\int_0^2 \frac{1}{2}x\,dx = \left[\frac{x^2}{4}\right]_0^2 = 1$. Therefore f(x)is indeed a pdf of X.

Now $P(0 \leq X < 1) = \int_0^1 \frac{1}{2}x\,dx = \left[\frac{x^2}{4}\right]_0^1 = \frac{1}{4}$ and

$$P(-1 < X < 1) = P(-1 < X < 0) + P(0 < X < 1) = 0 + \tfrac{1}{4} = \tfrac{1}{4}$$

Exercise

1) Suppose that the random variable X has p.d.f. given by $\mathrm{f(x)} = \begin{cases} cx, & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$ Find the value of the constant c hence determine m so that $P(\mathrm{X} \le \mathrm{m}) = \frac{1}{2}$

2) Let X be a continuous random variable with pdf $\mathrm{f(x)} = \begin{cases} \frac{x}{5} + k, & 0 \le x \le 3 \\ 0, & elsewhere \end{cases}$ Find the value of the constant k hence compute $P(1 < \mathrm{X} < 3)$

3) 2. A continuous random variable Y has the pdf given by $\mathrm{f(y)} = \begin{cases} k(1+y), & 4 \le x \le 7 \\ 0, & elsewhere \end{cases}$ Find the value of the constant k hence compute $P(Y < 5)$ and $P(5 < \mathrm{Y} < 6)$

4) 3. A continuous random variable X has the pdf given by $\mathrm{f(x)} = \begin{cases} k(1+x)^2, & -2 \le x \le 0 \\ 4k & 0 < x \le \frac{4}{3} \\ 0, & elsewhere \end{cases}$ Find the value of the constant k hence compute $P(X > 1)$ and $P(-1 < \mathrm{X} < 1)$

5) 4. A continuous random variable X has the pdf given by $\mathrm{f(x)} = \begin{cases} kx, & 0 \le x \le 2 \\ k(4-x) & 2 < x \le 4 \\ 0, & elsewhere \end{cases}$ Find the value of the constant k hence compute $P(X > 3)$ and $P(1 < \mathrm{X} < 3)$

6.3 Distribution Function of a Random Variables

Definition: For any random variable X, we define the**cumulative distribution function (CDF)**, *F(x)* as $\mathrm{F(x)} = P(X \le x)$ for every x.

If X is a discrete random variable with pmf f(x), then $\mathrm{F(x)} = \sum_{t=-\infty}^{x} \mathrm{f(t)}$ \\ t is introduced to facilitate summation// However, if X is a continuous random variable with pdf f(x), then

$\mathrm{F(x)} = \int_{-\infty}^{x} \mathrm{f(t)dt}$ \\ Again here t is introduced to facilitate integration//

Properties of any cumulative distribution function

- $\lim_{x\to\infty} \mathrm{F(x)} = 1$ and $\lim_{x\to-\infty} \mathrm{F(x)} = 0$
- F(x) is a non-decreasing function.
- F (x) is a right continuous function of *x*. In other words $\lim_{t\to x} \mathrm{F(t)} = \mathrm{F(x)}$

Reminder If the c.d.f. of X is F(x) and the p.d.f. is f(x), then differentiate $F_{(x)}$ to get $f_{(x)}$, and integrate $f_{(x)}$ to get F(x);

Theorem: For any random variable X and real values a < b, $P(a \le X \le b) = \mathrm{F}(b) - \mathrm{F}(a)$

Example 1

Let X be a discrete random variable with pmf given by $f(x)=\begin{cases}\frac{1}{20}(1+x) & \text{for } x=1,2,3,4,5\\ 0, & elsewhere\end{cases}$. Determine the cdf of X hence compute $P(X>3)$

Solution

$$F(x)=\sum_{t=-\infty}^{x} f(t)=\tfrac{1}{20}\sum_{t=1}^{x}(x+1)=\tfrac{1}{20}(2+3+....+x)=\tfrac{1}{20}\left\{\tfrac{x}{2}\left[4+(x-1)\right]\right\}=\tfrac{x(x+3)}{40}$$

$$F(x)=\begin{cases}0 & \text{for } x<1\\ \frac{x(x+3)}{40} & \text{for } x=1,2,3,4,5\\ 1 & \text{for } x>5\end{cases}$$

Recall for an AP $S_n=\tfrac{n}{2}\left[2a+(n-1)d\right]$

$$P(X>3)=1-P(X\le 3)=1-\frac{3(6)}{40}=\frac{11}{20}$$

***Example** 2*

Suppose is a continuous random variable whose pdf f(x) is given by $f(x)=\begin{cases}\frac{1}{2}x, & 0\le x\le 2\\ 0, & elsewhere\end{cases}$. Obtain the cdf of X hence compute $P(X>\tfrac{2}{3})$

Solution

$$F(x)=\int_{-\infty}^{x} f(t)dt=\int_{0}^{x}\tfrac{1}{2}t\,dt=\left[\frac{t^2}{4}\right]_0^x=\frac{x^2}{4}$$

thus

$$F(x)=\begin{cases}0, & x<0\\ \frac{x^2}{4}, & 0\le x\le 2\\ 1, & x>2\end{cases}$$

$$P(X>\tfrac{2}{3})=1-P(X\le\tfrac{2}{3})=1-\tfrac{1}{4}\left(\tfrac{2}{3}\right)^2=\tfrac{8}{9}$$

Exercise

1. The pdf of a continuous random variable X is given by $f(x)=\begin{cases}C/\sqrt{x}, & 0\le x\le 4\\ 0, & elsewhere\end{cases}$ Find the value of the constant C, the cdf of X and $P(X\ge 1)$
2. The pdf of a random variable X is given by $g(x)=\begin{cases}kx(1-x), & 0\le x\le 1\\ 0, & elsewhere\end{cases}$ Find the value of the constant k, the cdf of X and the value of m such that $G(x)=\tfrac{1}{2}$
3. Find the cdf of a random variable Y whose pdf is given by;

a) $f(x)=\begin{cases}1/3, & 0\le x\le 1\\ 1/3, & 2\le x\le 4\\ 0, & elsewhere\end{cases}$

b) $f(x)=\begin{cases}x/2, & 0\le x\le 1\\ 1/2, & 1\le x\le 2\\ (3-x)/2, & 2\le x\le 3\\ 0, & elsewhere\end{cases}$

4. If the cdf of a random variable Y is given by $F(x)=1-\frac{9}{y^2}$ for $Y\ge 3$ and $F(x)=0$ for $Y<3$, find $P(X\le 5)$, $P(X>8)$ and the pdf of X.

6.3 Expectation and Variance of a Random Variable

6.2.2 Expected Values

One of the most important things we'd like to know about a random variable is: what value does it take on average? What is the average price of a computer? What is the average value of a number that rolls on a die? The value is found as the average of all possible values, weighted by how often they occur (i.e. probability)

Definition: Let X be a discrete r.v. with probability function p(x). Then the **expected value** of X, denoted $E(X)$ or μ, is given by $\mathrm{E(x)} = \mu = \sum_{x=-\infty}^{\infty} xp(X = x).$

Similarly for a continuous random variable X with pdf f(x), $\mathrm{E(x)} = \mu = \int_{-\infty}^{\infty} x\mathrm{f}(x)dx.$

Theorem: Let Xbe a discrete r.v. with probability function $p(X=x)$ and let $g(x)$ be a real-valued function of X. ie, then the expected value of $g(x)$ is given by

$$\mathrm{E}[\mathrm{g(x)}] = \sum_{x=-\infty}^{\infty} g(x)p(X = x).$$

Similarly for a continuous random variable X with pdf f(x), $\mathrm{E}[\mathrm{g(x)}] = \int_{-\infty}^{\infty} \mathrm{g(x)} \times \mathrm{f}(x)dx.$

Theorem: Let X be a discrete r.v. with probability function p(x). Then

(i) $E(c) = c$, where c is any real constant;

(ii) $\mathrm{E}[\mathrm{ax+b}] == a\mu + b$ where a and b are constants

(iii) $\mathrm{E}[\mathrm{kg(x)}] = k\mathrm{E}[\mathrm{g(x)}]$ where $g(x)$ is a real-valued function of X

(iv) $\mathrm{E}[\mathrm{ag_1(x)} \pm b\mathrm{g_2(x)}] = a\mathrm{E}[\mathrm{g_1(x)}] \pm b\mathrm{E}[\mathrm{g_2(x)}]$ and in general $\mathrm{E}\left[\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{c_i g_i(x)}\right] = \sum_{\mathrm{i=1}}^{n} c_i \mathrm{E}[\mathrm{g_i(x)}]$

where $\mathrm{g_{i's}(x)}$ are real-valued functions of X.

This property of expectation is called *linearity property*

Remark: This theorem will also hold for a continuous random variable but we need to replace all the summation signs with integral signs.

Proof

(i) $\mathrm{E}[\mathrm{c}] = \sum_{all\ x} cP(X = x) = c\sum_{all\ x} P(X = x) = c(1) = c$

(ii) $\mathrm{E}[\mathrm{ax+b}] = \sum_{all\ x} (\mathrm{ax+b})P(x) = \sum_{all\ x} \mathrm{ax}P(x) + \sum_{all\ x} \mathrm{b}P(x) = a\sum_{all\ x} xP(x) + b\sum_{all\ x} P(x) = a\mu + b$

(iii) $\mathrm{E}[\mathrm{kg(x)}] = \sum_{all\ x} kg(x)P(X = x) = k\sum_{all\ x} g(x)P(X = x) = k\mathrm{E}[\mathrm{g(x)}]$

(iv) $\mathrm{E}[\mathrm{ag_1(x)} \pm b\mathrm{g_2(x)}] = \mathrm{E}[\mathrm{ag_1(x)}] \pm E[b\mathrm{g_2(x)}] = a\mathrm{E}[\mathrm{g_1(x)}] \pm b\mathrm{E}[\mathrm{g_2(x)}]$ from part iii

6.2.3 Variance and Standard Deviation

Definition: Let X be a r.v with mean $E(X) = \mu$, the **variance** of X, denoted σ^2 or Var(X) ,is given by $Var(X) = \sigma^2 = E(X - \mu)^2$. The units for variance are square units. The quantity that has the correct units is **standard deviation**, denoted σ. It'sactually the positive square root of Var(X).

$$\sigma = \sqrt{Var(X)} = \sqrt{E(X - \mu)^2}.$$

Theorem: $Var(X) = E(X - \mu)^2 = E(X)^2 - \mu^2$

Proof:
$Var(X)=E(X-\mu)^2=E\left(X^2-2X\mu+\mu^2\right)=E(X)^2-2\mu E(X)+\mu^2=E(X)^2-\mu^2$ Since $E(X)=\mu$

Theorem: $Var(aX+b)=a^2\,\mathrm{var}(X)$
Proof:
Recall that $E[aX+b]=a\mu+b$ therefore
$Var(aX+b)=E[(aX+b)-(a\mu+b)]^2=E[a(X-\mu)]^2==E\left[a^2(X-\mu)^2\right]=a^2E\left[(X-\mu)^2\right]=a^2\,\mathrm{var}(X)$

Remark

(i) The expected value of X always lies between the smallest and largest values of X.
(ii) In computations, bear in mind that variance cannot be negative!

Example 1

Given a probability distribution of X as below, find the mean and standard deviation of X.

x	0	1	2	3
P(X=x)	1/8	1/4	3/8	1/4

Solution

x	0	1	2	3	total
$p(X=x)$	$1/8$	$1/4$	$3/8$	$1/4$	1
$xp(X=x)$	0	$1/4$	$3/4$	$3/4$	$7/4$
$x^2p(X=x)$	0	$1/4$	$3/2$	$9/4$	4

$E(X)=\mu=\sum_{x=0}^{3}xp(X=x)=1.75$ and

standard deviation

$\sigma=\sqrt{E(X^2)-\mu^2}\;=\sqrt{4-1.75^2}\approx 0.968246$

Example 2

The probability distribution of a r.v X is as shown below, find the mean and standard deviation of; a) X b) $Y=12X+6$.

x	0	1	2
P(X=x)	1/6	1/2	1/3

Solution

x	0	1	2	total
$p(X=x)$	$1/6$	$1/2$	$1/3$	1
$xp(X=x)$	0	$1/2$	$2/3$	$7/6$
$x^2p(X=x)$	0	$1/2$	$4/3$	$11/6$

$E(X)=\mu=\sum_{x=0}^{2}xp(X=x)=7/6$ and

$E(X^2)=\sum_{x=0}^{2}x^2p(X=x)=11/6$

Standard deviation $\sigma=\sqrt{E(X^2)-\mu^2}\;=\sqrt{11/6-(7/6)^2}=\sqrt{17/6}\approx 1.6833$

Now $E(Y)=12E(X)+6=12(7/6)+6=20$

$Var(Y)=Var(12X+6)=12^2\times Var(X)=144\times\sqrt{17/6}\approx 242.38812$

***Example** 3*

A continuous random variable X has a pdf given by $\mathrm{f(x)} = \begin{cases} \frac{1}{2}x, & 0 \le x \le 2 \\ 0, & elsewhere \end{cases}$, find the mean and standard of X

Solution

$\mathrm{E(x)} = \int_{-\infty}^{\infty} x\mathrm{f}(x)dx = \int_0^2 \tfrac{1}{2}x^2 \mathrm{dx} = \left[\frac{x^3}{6}\right]_0^2 = \frac{4}{3}$ and $\mathrm{E(x^2)} = \int_{-\infty}^{\infty} x^2\mathrm{f}(x)dx = \int_0^2 \tfrac{1}{2}x^3 \mathrm{dx} = \left[\frac{x^4}{8}\right]_0^2 = 2$

Standard deviation $\sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{2 - (4/3)^2} = \frac{\sqrt{2}}{3}$

Exercise

1.
2. Suppose X has a probability mass function given by the table below

x	2	3	4	5	6
P(X=x)	0.01	0.25	0.4	0.3	0.04

   Find the mean and variance of; X

3. Suppose X has a probability mass function given by the table below

x	11	12	13	14	15
P(X=x)	0.4	0.2	0.2	0.1	0.1

   Find the mean and variance of; X

4.

5. Let X be a random variable with $P(X = 1) = 0.2$, $P(X = 2) = 0.3$, and $P(X = 3) = 0.5$. What is the expected value and standard deviation of; a) X b) $Y = 5X - 10$?
6. A random variable W has the probability distribution shown below,

w	0	1	2	3
P(W=w)	2d	0.3	d	0.1

Find the values of the constant d hence determine the mean and variance of W. Also find the mean and variance of $Y = 10X + 25$

7. A random variable X has the probability distribution shown below,

x	1	2	3	4	5
P(X=x)	7c	5c	4c	3c	c

Find the values of the constant c hence determine the mean and variance of X.

8. The random variable Z has the probability distribution shown below,

z	2	3	5	7	11
P(Z=z)	⅙	⅓	¼	X	y

If $E(Z) = 4\frac{2}{3}$, find the values of x and y hence determine the variance of Z

9. A discrete random variable M has the probability distribution $f(m) = \begin{cases} \frac{m}{36}, & m = 1,2,3,...,8 \\ 0, & elsewhere \end{cases}$, find the mean and variance of M
10. For a discrete random variable Y the probability distribution is $f(y) = \begin{cases} \frac{5-y}{10}, & y = 1,2,3,4 \\ 0, & elsewhere \end{cases}$, calculate $E(Y)$ and $\mathrm{var}(Y)$
11. Suppose X has a pmf given by $f(x) = \begin{cases} kx & \text{for } x = 1,2,3,4 \\ 0, & elsewhere \end{cases}$, find the value of the constant k hence obtain the mean and variance of X
12. A team of 3 is to be chosen from 4 girl and 6 boys. If X is the number of girls in the team, find the probability distribution of X hence determine the mean and variance of X
13. A fair six sided die has; '1' on one face, '2' on two of it's faces and '3' on the remaining three faces. The die is rolled twice. If T is the total score write down the probability distribution of T hence determine;
    a) the probability that T is more than 4 b) the mean and variance of T
14. The pdf of a continuous r.v R is given by $f(r) = \begin{cases} kr & \text{for } 0 \le r \le 4 \\ 0, & elsewhere \end{cases}$, (a) Determine *c*. hence Compute $P(10 \le r \le 2)$, E(X) and Var(X).
15. A continuous r.v M has the pdf given by $f(m) = \begin{cases} k(1 - \frac{m}{10}) & \text{for } M \le 10 \\ 0, & elsewhere \end{cases}$, find the value of the constant k, the mean and the variance of X
16. A continuous r.v X has the pdf given by $f(x) = \begin{cases} k(1 - x) & \text{for } 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$, findt the value of the constant k. Also find the mean and the variance of X

17. The lifetime of new bus engines, T years, has continuous pdf $\mathrm{f(t)} = \begin{cases} d/t^2 & if\ x \geq 1 \\ 0, & if\ x < 1 \end{cases}$ find the value of the constant d hence determine the mean and standard deviation of T
18. An archer shoots an arrow at a target. The distance of the arrow from the centre of the target is a random variable X whose p.d.f. is given by $\mathrm{f(x)} = \begin{cases} k(3 + 2\mathrm{x} - \mathrm{x}^2) & if\ x \leq 3 \\ 0, & if\ x > 3 \end{cases}$ find the value of the constant k. Also find the mean and standard deviation of X
19. A continuous r.v X has the pdf given by $\mathrm{f(x)} = \begin{cases} k(1+x), & -1 \leq x < 0 \\ 2k(1-x), & 0 \leq x \leq 1 \\ 0, & elsewhere \end{cases}$, find the value of the constant k. Also find the mean and the variance of X
20. A continuous r.v X has the pdf given by $\mathrm{f(x)} = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0, & elsewhere \end{cases}$, find the mean and standard deviation of; a) X b) $\mathrm{Y} = e^{\frac{3}{4}x}$