

Homework 1  
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1. Use mathematical induction to prove:

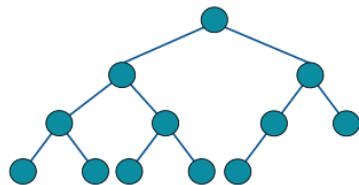
For any natural number  $n$ ,  $n^3 - n$  is divisible by 3

2. Write algorithm to calculate Factorial by following the requirements:
  - a. Write algorithm using loop. then prove the algorithm is correct
  - b. Write algorithm using recursion, then calculate the running time

3. draw the function curve to compare the following different running times ( $n$  is the input size)

$\lg n$
$\sqrt{n}$
$n$
$n \lg n$
$n^2$
$n^3$
$2^n$
$n!$

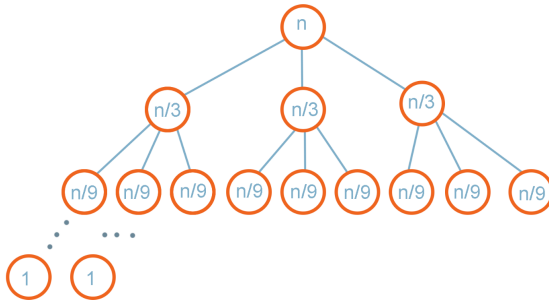
4. Given two sorted arrays, use any language you are familiar to implement algorithm to merge the two arrays so that the resulting array is sorted. what is time complexity of this merge?
5. A complete binary tree is a binary tree that every level, except possibly the last, is completely filled, and all nodes are as far left as possible.



if total number of nodes is  $N$ , what is the level of the tree?

6. Assuming  $n = 3^k$

if we build a tree that each node has a value. with root having value  $n$ , and each node has 3 children, each having value of  $1/3$  of the parent node value...and so forth, like this:



if the minimum node value is 1, then what is the level of the tree.