HW-10

1) a) Show that the weight correction Δw_{kj} for the n^{th} iteration is given by.

$$\Delta w_{kj}(n) = \eta \cdot \delta_k(n) \cdot y_j(n)$$

→ given: $\delta_k(n) = [d_k(n) - y_k(n)] \cdot [y_k(n)(1 - y_k(n))] \rightarrow ①$

$$V_k(n) = \sum_{j=0}^{m} w_{kj}(n) \cdot y_j(n)$$

$$y_k(n) = \phi(V_k(n))$$

$$\therefore \mathcal{E}(n) = \frac{1}{2} \sum_k e_k^2(n) = \frac{1}{2} \sum_k [d_k(n) - y_k(n)]^2$$

$$\frac{\partial \mathcal{E}(n)}{\partial w_{kj}(n)} = \frac{\partial \mathcal{E}(n)}{\partial e_k(n)} * \frac{\partial e_k(n)}{\partial y_k(n)} * \frac{\partial V_k(n)}{\partial w_{kj}(n)} \rightarrow ②$$

$$= e_k(n) * (-1) * \phi'(V_k(n)) * y_j(n)$$

$\because \phi$ is a sigmoid function

$$\phi'(V_k(n)) = \phi(V_k(n)) * (1 - \phi(V_k(n)))$$

$$= y_k(n) * (1 - y_k(n)) \rightarrow ③$$

$$\therefore e_k(n) = [d_k(n) - y_k(n)] \rightarrow ④$$

Substitute ③ & ④ in ②,

$$\Delta w_{kj}(n) = (-1)[d_k(n) - y_k(n)] * [y_k(n)(1 - y_k(n)] * y_j(n)$$

Substitute ①,

$$\therefore \Delta w_{kj}(n) = \eta \cdot \delta_k(n) \cdot y_j(n)$$

b) Show that the weight correction Δw_{ji} for the n^{th} iteration is given by

$$\Delta w_{ji}(n) = \eta \cdot \delta_j(n) \cdot y_i(n)$$

→ given: $\delta_j(n) = \sum_k \delta_k(n) * w_{kj}(n) \cdot y_j(n) \cdot (1 - y_j(n)) \rightarrow ①$

$$V_j(n) = \sum_{i=0}^{m} w_{ji}(n) * y_i(n)$$

$$y_j(n) = \phi(V_j(n)) = \phi\left[\sum_{i=0}^{m} w_{ji}(n) * y_i(n)\right]$$

$$\mathcal{E}(n) = \frac{1}{2} \sum_k e_k^2(n) = \frac{1}{2} \sum_k \left[d_k(n) - \phi(V_k(n))\right]$$

$$\frac{\partial \mathcal{E}(n)}{\partial w_{ij}(n)} = \sum_k e_k \cdot \frac{\partial e_k(n)}{\partial V_k(n)} * \frac{\partial V_k(n)}{\partial y_j(n)} * \frac{\partial y_j(n)}{\partial w_{ji}(n)}$$

$$= \sum_k e_k * (-1) * \phi'(V_k(n)) * w_{jk}(n) * \phi'(V_j(n)) * y_i(n)$$

$$= -\sum_k \delta_k(n) * w_{kj}(n) * \phi(V_j(n)) * y_i(n) \longrightarrow ②$$

$\because \phi$ is a sigmoid function.

$$\phi'(V_j(n)) = \phi(V_j(n)) * \left[1 - \phi(V_j(n))\right]$$
$$= y_j(n)\left[1 - y_j(n)\right] \longrightarrow ③$$

Substitute ③ in ②,

$$\Delta w_{ji}(n) = \sum_k \delta_k'(n) \cdot w_{kj}(n) \cdot y_j(n) \cdot \left[1 - y_j(n)\right] \cdot y_i(n)$$

Using ①,

$$\Delta w_{ji}(n) = \eta\, \delta_j(n)\, y_i(n)$$