HW-9

1) Consider a dataset with 2 points in 1D :-

$(x_1 = 0, y_1 = -1)$ $(x_2 = \sqrt{2}, y_2 = 1)$ $\phi(x) = [1, \sqrt{2}x, x^2]^T$

a) Find corresponding points in 3D.

$\phi(x_1) = [1, \sqrt{2} \times 0, (0)^2]^T = [1, 0, 0]^T$

$\phi(x_2) = [1, \sqrt{2} \times \sqrt{2}, (\sqrt{2})^2]^T = [1, 2, 2]^T$

b) Value of the margin i.e. distance between the 2 points

$= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2} = \sqrt{(1-1)^2 + (0-2)^2 + (0-2)^2}$

$= \sqrt{0 + 4 + 4} = \sqrt{8} = 2\sqrt{2}$

$\therefore$ for margin $= \frac{2\sqrt{2}}{2} = \sqrt{2}$

c) Given :- margin $= \frac{1}{\|w\|} = \sqrt{2}$ .... (from b)

$$w = \sum_{i=1}^{n} \alpha_i y_i \phi(x_i) = \alpha_1 y_1 \phi(x_1) + \alpha_2 y_2 \phi(x_2)$$

$$= \alpha_1 (-1) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \alpha_2 (1) \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -\alpha_1 + \alpha_2 \\ 2\alpha_2 \\ 2\alpha_2 \end{bmatrix}$$

we know, $\sum_{i=1}^{n} \alpha_i y_i = 0$

$\therefore \alpha_1 y_1 + \alpha_2 y_2 = 0$

$-\alpha_1 + \alpha_2 = 0$

Substitute it in w, $w = \begin{bmatrix} 0 \\ 2\alpha_2 \\ 2\alpha_2 \end{bmatrix}$

$\therefore \|w\| = \frac{1}{\sqrt{2}}$ i.e. $\sqrt{0 + 4\alpha_2^2 + 4\alpha_2^2} = \frac{1}{\sqrt{2}}$

i.e. $\frac{1}{\sqrt{2}} = 2\sqrt{2}\,\alpha_2$ $\therefore \alpha_2 = \frac{1}{4}$

$\therefore w = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}$

d) Find w_0 using value of w and previous equation.

$$y_1 \left(w^T \phi(x_1) + w_0 \right) = 1$$ ($\because$ points on decision boundary for inequalities becomes equalities)

i.e. $$-1 \left[\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + w_0 \right] = 1$$

$\therefore$ $-w_0 = 1$ i.e. $w_0 = -1$ //

e) $$g(x) = w_0 + w^T \phi(x)$$

$$= w_0 + \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{2}\,x \\ x^2 \end{bmatrix}$$

$$= w_0 + \frac{\sqrt{2}}{2} x + \frac{x^2}{2}$$

$$= -1 + \frac{x}{\sqrt{2}} + \frac{x^2}{2}$$ ($\because w_0 = -1$ from d) //

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