Assignment2
279/377
2022 winter

1. Use divide-and-conquer technique to calculate sum of an integer array.
   Use two different ways to define subproblems. For each way:

   a. write the pseudocode of the algorithm
   b. give the running time recurrence (recursive equation)
   c. calculate the running time in Θ notation

answer

a.

algorithm:

```
sum(A, p, r)
   if p == r
     return A[p]
   return A[p] + sum(A, p+1, r)
```

b.

recurrence:

$$T(n) = T(n-1) + 1$$

c.

running time

$$\begin{aligned} T(n) &= T(n-1) + 1 \\ &= T(n-2) + 1 + 1 \\ &= T(n-3) + 1 + 1 + 1 \\ &= \ldots \\ &= \Theta(n) \end{aligned}$$

2. Use divide-and-conquer technique to search a number in the sorted list of n numbers.

   a. Write the pseudocode of the algorithm.
   b. Write the recursive running time equation (recurrence)
   c. Guess the result of this recurrence

answer:

a. algorithm:

```
search(value, list, i, j)
  if i == j
    if list[i] == value
      return [true, i]
    else
      return false
    end
  end
  mid = (i+j)/2
  if value == list[mid]
    return [true, mid]
  else
    if value < list[mid]
      return search(value, list, i, mid-1)
    else
      return search(value, list, mid+1, j)
    end
  end
end
```

b. recursive running time

   $T(n) = T(n/2) + C$

c. $T(n) = \Theta(\log n)$

prove: $T(n) = O(\log n)$

need to prove $T(n) <= C1\log n$

$n = 2, T(2) = T(1) + C = C$
$C1\log n = C1$

let $C1 = C + 1$

induction: assume true for $k < n$

$T(n) = T(n/2) + C <= C1 \log(n/2) + C = C1 \log n - C1 \log 2 + C = C1\log n - (C1 - C)$

let $C1 = C+1$, $T(n) <= C1 \log n - 1 <= C1 \log n$

$T(n) = O(\log n)$ proved.

$T(n) = \Omega(\log n)$

similar to the above.