Problem 1

Find the solution (x^*, y^*) to the following problem.

$$\text{optimize } xy$$

$$\text{subject to } x + y = 10$$

Problem 2

The SVM optimization can be defined by the primal form:

$$\min_w \frac{1}{2}\|\boldsymbol{w}\|^2$$

$$\text{subject to } y_i(\boldsymbol{w}^T\boldsymbol{x}_i + b) \geq 1, \qquad i = 1, \dots, N$$

Or by its the dual form:

$$\max_\alpha J(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i\alpha_j y_i y_j \left(\boldsymbol{x}_i^T\boldsymbol{x}_j\right)$$

$$\text{subject to } \alpha_i \geq 0, i = 1, \dots N \text{ and } \sum_{i=1}^{N} \alpha_i y_i = 0$$

What is the Lagrangian function $L(\boldsymbol{w}, b, \alpha)$ evaluated at $\boldsymbol{w}$ that minimizes that function?

Note this is the objective function $J(\alpha)$.

Hints:

1. Write the primal problem in standard form
2. Form the Lagrangian function $L(\boldsymbol{w}, b, \alpha)$
3. Find $\boldsymbol{w}$ and b that minimize $L(\boldsymbol{w}, b, \alpha)$
4. Plug the results back into $L(\boldsymbol{w}, b, \alpha)$