HW-7

Consider N i.i.d samples drawn from a Poisson distribution. The PMF is defined as follows:

$$Poisson(x/\lambda) = e^{-\lambda} \frac{\lambda^{x}}{x!} \text{ for } x \in \{0, 1, 2, \dots\}$$

where $\lambda > 0$ is the rate parameter. Find λ_{MLE}

→ Likelihood function:

$$l(\lambda; x_n) = \prod_{i=1}^{n} e^{-\lambda} \frac{1}{x_i!}$$

$$l(\lambda/x_n) = \log\left(e^{-n\lambda} \frac{\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}\right)$$

$$= \log\left(e^{-n\lambda}\right) + \log\left(\lambda^{\sum_i x_i}\right) - \log\left(\prod_{i=1}^{n} x_i!\right)$$

$$= -n\lambda + \sum_{i=1}^{n} x_i \cdot \log\lambda - \log\left(\prod_{i=1}^{n} x_i!\right)$$

Taking partial derivative w.r.t λ,

$$\frac{\partial l}{\partial(\lambda)} = -n + \frac{1}{\lambda} \cdot \sum_{i=1}^{n} x_i$$

$$\frac{1}{\lambda} \sum_{i=1}^{n} x_i - n = 0$$

$$\lambda = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\lambda_{MLE} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

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