

Homework 1

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2022/01/07
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1. [20 points] Use mathematical induction to prove:

For any natural number n , $n^3 - n$ is divisible by 3

answer:

$n = 1$, $1 - 1 = 0$, $0/3 = 0$
($n = 2$, $8 - 2 = 6$)

assume $n = k$ works,

then $(k+1)^3 - k - 1 = k^3 + 3k^2 + 3k + 1 - k - 1 = k^3 - 5k = (k^3 - k) + 6k$

is divisible by 3

2. [40 points] Write algorithm of calculating Factorial by following the requirements:

- Write the algorithm using loop (as oppose to recursion)
- Explain what "loop invariant" is and prove the algorithm is correct
- Calculate the running time of this algorithm (you can assume any constant)
- Write algorithm using recursion

answer:

a.

```
Factorial(n) {  
    fac = 1  
    k = 1  
    while (k <= n) {  
        fac = fac * k  
        k=k+1  
    }  
}
```

b. correctness:

$n = 1$, it return 1 the algorithm is correct

loop invariant: each iteration, fac is factorial of $(k-1)$ times k , which is $k!$

assuming $n = k$ is correct.

each iteration, it calculate $k!$ times the new value $(k+1)$, so it calculate a new factorial of 1 bigger.

loop will terminate after reach input value n, by this time, fac is factorial of n

```
c. Factorial(n) {  
    if n == 1  
        return(1)  
    return n*Factorial(n-1)  
}
```

d. running time: $T(n) = T(n-1) + 1$
 $= T(n-2) + 1 + 1 = \Theta(n)$

3. [40 points] Solve the "Maximum Ascending Subarray Sum" problem (no, 1800) on leetcode.com, then follow the requirements:

a. Make a successful submission. Attach the screen shot image of your submission.

Note. your submission image should be something like this
(that is, pass all test cases, instead of an accept page of a particular test case)

The screenshot shows a 'Success' message with details for a submission. It indicates a runtime of 52 ms, faster than 57.69% of Ruby online submissions, and a memory usage of 209.9 MB, less than 23.08% of Ruby online submissions. Below this, there are three 'Next challenges' listed in orange buttons: 'Container With Most Water', 'Minimum Operations to Reduce X to Zero', and 'Minimum Length of String After Deleting Similar Ends'. There are also social media sharing icons for Facebook, Twitter, and LinkedIn. At the bottom, a table shows the submission details.

Time Submitted	Status	Runtime	Memory	Language
04/01/2021 15:59	Accepted	52 ms	209.9 MB	ruby

- b. Calculate the running time of your algorithm as a function of your input size.
(Do not just give the result, show how you calculate the result)