HW - 3

1) Find the derivatives of the function

$f(x) = 5(x+47)^2$: $\frac{d}{dx}\left(5(x+47)^2\right) = 5\frac{d}{dx}(x+47)^2$

$= 5 \cdot 2(x+47) \cdot 1 = 10(x+47)$ //

2) Determine the minimum and maximum of the function $f(x) = 3x^3 + 15x^2$. Then sketch it.

→ Find any critical point of $f(x)$ is $f'(x) = 0$

$f'(x) = 0$

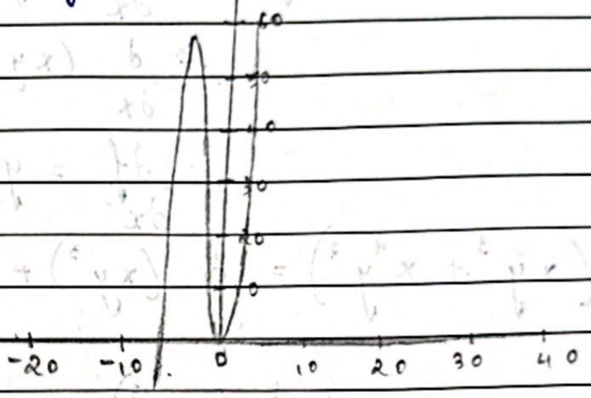
$\therefore 3 \cdot 3x^2 + 15 \cdot 2x = 0$

$\therefore 9x^2 + 30x = 0$

$\therefore 3x(3x+10) = 0$

$3x = 0$; $3x + 10 = 0$

$x = 0$; $x = \frac{-10}{3} = -3.33$



and $f''(x) = 18x + 30$

$\therefore$ The maximum and minimum value of the function

$f''(0) = 30 > 0$ and $f''\left(\frac{-10}{3}\right) = \cancel{18}^{6}\left(\frac{-10}{\cancel{3}}\right) + 30 = -60 + 30$

$= -30 < 0$

$\therefore f''(0) > 0$ and $f''\left(\frac{-10}{3}\right) < 0 \Rightarrow (-3.33) < 0 \rightarrow$ Maximum

$\therefore$ The maximum value is

$f\left(\frac{-10}{3}\right) = 3\left(\frac{-10}{3}\right)^3 + 15\left(\frac{-10}{3}\right)^2$

$= -\cancel{3}\left(\frac{1000}{\cancel{27}_9}\right) + \cancel{15}_5\left(\frac{100}{\cancel{9}_3}\right)$

$= \frac{-1000}{9} + \frac{500}{3} = \frac{-1000 + 1500}{9} = \frac{500}{9} \approx 55.56$

$\therefore$ Maximum value is $\frac{500}{9}$ // and $\frac{-10}{3} \Rightarrow$ Max: $(-3.33, 55.56)$

Minimum value of $f(x)$ is $f(0) = 3(0)^3 + 15(0)^2 = 0$

$f(0) = 0$ // $\Rightarrow$ Min: $(0, 0)$

$\therefore$ ~~The minimum and maximum value is (0, 55.56)~~ //

Find the partial derivatives $\frac{df}{dx}$ and $\frac{df}{dy}$ for the following functions:

3) $f(x,y) = 3x + 4y \Rightarrow \frac{d}{dx}(3x+4y) = \frac{d}{dx}(3x) + \frac{d}{dx}(4y)$

$\therefore \frac{d}{dx}(3x) = 3$ ($\because y$ is treated as a constant here)

$\therefore \frac{d}{dx}(4y) = 0$

$\frac{df}{dx}\cancel{(x+y)} = 3 + 0 = 3$

$\frac{d}{dy}(3x+4y) = \frac{d}{dy}(3x) + \frac{d}{dy}(4y)$

$\therefore \frac{d}{dy}(3x) = 0 \quad \frac{d}{dy}(4y) = 4$

$\therefore \frac{df}{dy} = 4$

4) $f(x,y) = xy^3 + x^2y^2 \Rightarrow \frac{d}{dx}(xy^3 + x^2y^2) = \frac{d}{dx}(xy^3) + \frac{d}{dx}(x^2y^2)$

$\therefore \frac{d}{dx}(xy^3) = y^3 \quad \therefore \frac{d}{dx}(x^2y^2) = 2xy^2$

$\therefore \frac{df}{dx} = y^3 + 2y^2x$

$\frac{d}{dy}(xy^3 + x^2y^2) = \frac{d}{dy}(xy^3) + \frac{d}{dy}(x^2y^2)$

$\therefore \frac{d}{dy}(xy^3) = 3xy^2 \quad \therefore \frac{d}{dy}(x^2y^2) = 2x^2y$

$\therefore \frac{df}{dy} = 3xy^2 + 2x^2y$

5) $f(x,y) = x^3y + e^x \Rightarrow \frac{d}{dx}(x^3y + e^x) = \frac{d}{dx}(x^3y) + \frac{d}{dx}(e^x)$

$\therefore \frac{d}{dx}(x^3y) = 3yx^2 \quad \therefore \frac{d}{dx}(e^x) = e^x$

$\therefore \frac{df}{dx} = 3yx^2 + e^x$

$\frac{d}{dy}(x^3y + e^x) = \frac{d}{dy}(x^3y) + \frac{d}{dy}(e^x)$

$\therefore \frac{d}{dy}(x^3y) = x^3 \quad \therefore \frac{d}{dy}(e^x) = 0$

$\therefore \frac{df}{dy} = x^3 + 0 = x^3$

6) $f(x,y) = xe^{2x+3y} = \frac{d}{dx}(xe^{2x+3y}) = \frac{d(x)}{dx}e^{2x+3y} + \frac{d}{dx}(e^{2x+3y})x$

$\therefore \frac{d}{dx}(x) = 1 \qquad \therefore \frac{d}{dx}(e^{2x+3y}) = e^{2x+3y}\cdot 2$

$\therefore \frac{df}{dx} = 1\cdot e^{2x+3y} + e^{2x+3y}\cdot 2x$

$= e^{2x+3y} + 2e^{2x+3y}x$

$\frac{df}{dy}(xe^{2x+3y}) = x\frac{d}{dy}(e^{2x+3y}) = e^{2x+3y}\frac{d}{dy}(2x+3y)$

$\therefore \frac{d}{dy}(2x+3y) = 3$

$\therefore \frac{df}{dx} = xe^{2x+3y}\cdot 3$

7) Given the function $J(w)$:-

$J(w_0, w_1) = \frac{1}{2m}\sum_{i=1}^{m}(w_0 + w_1 x^{(i)} - y_i)^2$

$\therefore \frac{dJ(w)}{dw_0} = \frac{1}{m}\sum_{i=1}^{m}(w_0 + w_1 x^{(i)} - y_i)$

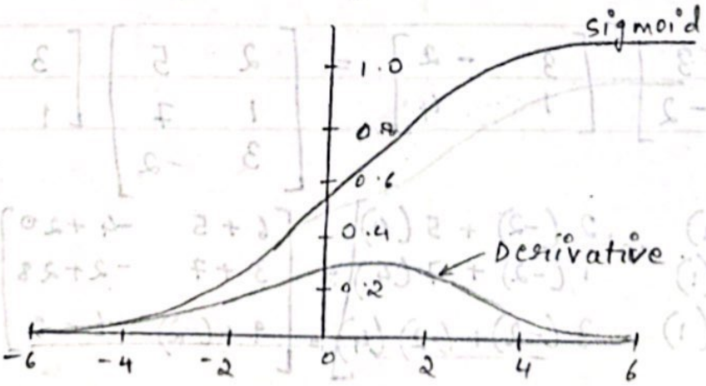
$\therefore \frac{dJ(w)}{dw_1} = \frac{1}{m}\sum_{i=1}^{m}(w_0 + w_1 x^{(i)} - y_i)\cdot x^{(i)}$

8) Find the derivative of the function $s(x) = \frac{1}{1+e^{-x}}$

(By $h = \frac{f}{g} \Rightarrow h' = \frac{f'g - fg'}{g^2}$)

$\therefore s'(x) = \frac{0-(-e^{-x})}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1}{1+e^{-x}}\right)\left(1-\frac{1}{1+e^{-x}}\right)$

$= s(x)(1-s(x))$



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