INTRODUCTION

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EUCLIDEAN ALGORITHM

Basic observation:

1) $\gcd(a, 0) = a$
2) $\gcd(a, b) = \gcd(b, a \% b)$ for all $a, b \neq 0$

Ex.:
$$\gcd(49, 14) = \gcd(14, 49\%14 = 7)$$
$$= \gcd(14, 7) \checkmark$$
$$= \gcd(7, 14\%7 = 0)$$
$$= \gcd(7, 0)$$
$$= 7$$

$$\gcd(6, 9) = \gcd(9, 6\%9 = 6)$$
$$= \gcd(9, 6)$$
$$= \gcd(6, 9\%6 = 3)$$
$$= \gcd(3, 6\%3 = 0)$$
$$= 3$$

ANALYSIS OF FOR LOOPS

```
for (int i = 1; i <= n; ++i)
    sum = sum + i;
```

$$\sum_{i=1}^{n} (1) = n$$

How many additions + are performed by this loop? n

```
for (int i = a; i ≤ b; ++i)
    sum = sum + i;
```

$$\sum_{i=a}^{b} (1) = b - a + 1$$

Recall: $\sum_{i=1}^{n} (1) = \underbrace{1}_{i=1} + \underbrace{1}_{2} + \underbrace{1}_{3} + \cdots + \underbrace{1}_{n} = n$

$b - a + 1 = \sum_{i=a}^{b} (1) = 1 + 1 + 1 + \cdots + 1$

$i = a \quad a+1 \quad a+2 \quad \cdots \quad b$

$1 + \cdots + 1 + 1 + 1 + \cdots +$

$\underbrace{1 \quad 2 \quad \cdots \quad a-1 \quad a \quad a+1 \quad \cdots \quad b}$

$$b - (a-1)$$
$$b - a + 1$$

VARIATIONS:

```
for (int i = a; i < b; ++i)
    sum = sum + i;
```

same as $i \le b - 1$

$$\sum_{i=a}^{b-1} (1) = b - 1 - a + 1$$
$$= b - a$$

$b \quad \quad b$

```
for (int i = a; i ≤ b; ++i)
    sum = sum + i + 2;
```

$$\sum_{i=a}^{b}(2) = 2\sum_{i=a}^{b}(1)$$

$$= 2(b-a+1)$$

$$\sum_{i=a}^{b}(j) = j\sum_{i=a}^{b}(1)$$

AS LONG AS j is NOT related to i

$$\sum_{i=a}^{b}(x+y) = \sum_{i=a}^{b}(x) + \sum_{i=a}^{b}(y)$$

ANALYSIS OF INSERTION SORT

COUNTING "A[i] > A[i+1]"

$$C(n) = \sum_{j=1}^{n-1}\left(\sum_{i=0}^{j-1}(1)\right)$$

$$= \sum_{j=1}^{n-1}\left(j\cancel{-1} - 0 \cancel{+1}\right)$$

$$= \sum_{j=1}^{n-1} j = \underset{j=1}{1} + \underset{2}{2} + \underset{3}{3} + \cdots + \underset{n-1}{n-1} = 1+2+3+\cdots+n-1$$

$$= \frac{(n-1)(n-1+1)}{2}$$

$$= \frac{(n-1)n}{2} \sim \frac{n^2}{2}$$

ARITHMETIC SUM

$$\boxed{\sum_{i=1}^{m} i = \frac{m(m+1)}{2}, \quad m \geq 0}$$

$$\sum_{i=1}^{m} i = 1 + 2 + 3 + \cdots + m = ?\ S$$

$$m + m-1 + m-2 \quad + 1 = S$$

$$m+1 + m+1 + m+1 \cdots + m+1 = 2S$$

$$m(m+1) = 2S$$

$$\boxed{\frac{m(m+1)}{2} = S}$$