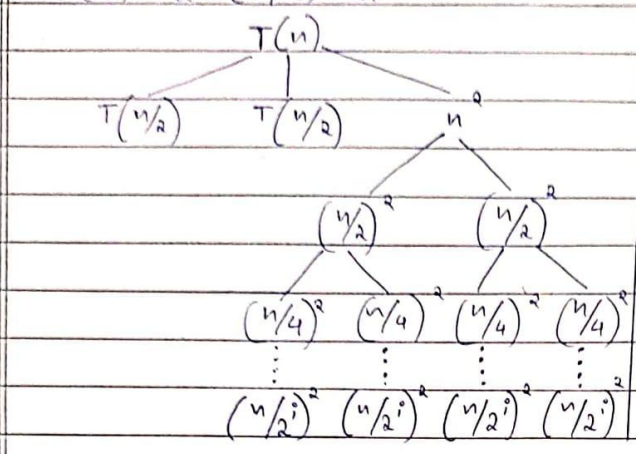
ASSIGNMENT-3

1. Use the recursion tree method to solve the recurrence:-

$T(n) = 2T(n/2) + n^2$

→



Level	no. of nodes	Running time
0	$2^0 = 1$	n^2
1	$2^1 = 2$	$2^1(n/2)^2$
2	$2^2 = 4$	$2^2(n/2^2)^2 = 4(n/4)^2$
i^0	2^i	$2^i(n/2^i)^2$
		$\sum_{i=0}^{\log_2 n} 2^i(n/2^i)^2$

∴ Num of $T(1) = 2^{\log_2 n} = n^{\log_2 2}$

Let $2^i = a$:-

$$\sum_{i=0}^{\log_2 n} a(n/a)^2 = \sum_{i=0}^{\log_2 n} a(n^2/a^2) = \sum_{i=0}^{\log_2 n} n^2/a = \sum_{i=0}^{\log_2 n} n^2/2^i$$

$$\Rightarrow \sum_{n=0}^{\log_2 n} n^2/2^i = \sum_{i=0}^{\log_2 n} n^2(1/2^i) = \sum_{i=0}^{\log_2 n} n^2(1/2)^i$$

∴ According to the equation:- $\sum_{i=m}^{n} ar^i = a \cdot \frac{r^m - r^{n+1}}{1-r}$; $r \neq 1$

$$\therefore \sum_{i=0}^{\log_2 n} n^2(1/2)^i = n^2 \cdot \frac{(1/2)^0 - (1/2)^{\log_2 n + 1}}{1 - 1/2} = n^2 \cdot \frac{1 - (1/2)^{\log_2 n + 1}}{1/2}$$

$$= 2n^2\left[1 - (1/2)^{\log_2 n} \cdot (1/2)^1\right]$$

$$= 2n^2\left[1 - (1/n)\cdot(1/2)\right]$$

$$= 2n^2\left[1 - 1/2n\right]$$

$$= 2n^2 - 2n^2/2n$$

$$= 2n^2 - n \Rightarrow O(n^2)$$

2. Use Master theorem to solve (if master theorem can not be applied, write the reason):-

a. $T(n) = 9T(n/3) + n$

→ Here, $a = 9, b = 3, k = 1, p = 0$

$f(n) = \theta(n^1 \log^0 n)$

Case 1:- $\log_b a > k \Rightarrow \log_3 9 > 1 \Rightarrow 2 > 1 \Rightarrow$ TRUE

$\therefore \theta(n^{\log_b a}) = \theta(n^{\log_3 9}) = \theta(n^2)$

b. $T(n) = 9T(n/3) + 1000n^2$

→ Here, $a = 9, b = 3, k = 2, p = 0$

$f(n) = \theta(n^2 \log^0 n)$

Case 1:- $\log_b a > k \Rightarrow \log_3 9 > 2 \Rightarrow \log 2 > 2 \Rightarrow$ False

Case 2:- $\log_b a = k \Rightarrow \log_3 9 = 2 \Rightarrow 2 = 2 \Rightarrow$ True

i) $p > -1 \Rightarrow 0 > -1 \Rightarrow$ True

$\therefore \theta(n^k \log^{p+1} n) = \theta(n^2 \log^{0+1} n) = \theta(n^2 \log n)$

c. $T(n) = 9T(n/3) + 1000n^3$

→ Here, $a = 9, b = 3, k = 3, p = 0$

$f(n) = \theta(n^3 \log^0 n)$

Case 1: $\log_b a > k \Rightarrow \log_3 9 > 3 \Rightarrow \log 2 > 3 \Rightarrow$ False.

Case 2: $\log_b a = k \Rightarrow \log_3 9 = 3 \Rightarrow 2 = 3 \Rightarrow$ False

Case 3: $\log_b a < k \Rightarrow \log_3 9 < 3 \Rightarrow 2 < 3 \Rightarrow$ True

i) $p \geq 0 \Rightarrow 0 \geq 0 \Rightarrow$ True

$\therefore \theta(n^k \log^p n) = \theta(n^3 \log^0 n) = \theta(n^3)$

2. d. $T(n) = 9T(n/3) + n^2 \log n$

→ Here, $a = 9$, $b = 3$, $k = 2$, $p = 1$

$f(n) = \theta(n^2 \log^1 n)$

Case 1:- $\log_b a > k \Rightarrow \log_3 9 > 2 \Rightarrow 2 > 2 \Rightarrow$ False

Case 2:- $\log_b a = k \Rightarrow \log_3 9 = 2 \Rightarrow 2 = 2 \Rightarrow$ True

i) $p > -1 \Rightarrow 1 > -1 \Rightarrow$ True

$\therefore \theta(n^k \log^{p+1} n) = \theta(n^2 \log^{1+1} n) = \theta(n^2 \log^2 n)$

e) $T(n) = 0.5\, T(n/2) + n$

→ Here, $a = 0.5$, $b = 2$, $k = 1$, $p = 0$

$f(n) = \theta(n^1 \log^0 n)$

Case 1:- $\log_b a > k \Rightarrow \log_2 0.5$ where $a = 0.5$

$\therefore a$ does not satisfy the condition $a \geq 1$.

Hence, we cannot apply master's theorem here.

f. $T(n) = 2T(n/2) - n$

→ Here, $a = 2$, $b = 2$, $k = 1$, $p = 0$

But this recurrence form given does not satisfy master's theorem condition where the equation is $T(n) = aT(n/b) + f(n)$

Hence, we cannot apply master's theorem here.

g. $T(n) = nT(n/2) + n \log n$

→ Here, $a = n$, $b = 2$, $k = 1$, $p = 1$

$\therefore a = n$ and it should be a constant where $a \geq 1$. as per the condition.

Hence, we cannot apply master's theorem here.

h. $T(n) = T(n-2) + n^2$

→ Here, $T(n)$ should be in dividing form and according to the recurrence form given, it is in subtraction form. Hence, we cannot apply master's theorem here.

i. $T(n) = T(7n/10) + n^2$

→ Here, $a = 1$, $b = 10/7$, $k = 1$, $p = 0$

$f(n) = \theta(n^1 \log^0 n)$

Case 1:- $\log_b a > k \Rightarrow \log_{10/7} 1 > 1 \Rightarrow 0 > 1 \Rightarrow$ False.

Case 2:- $\log_b a = k \Rightarrow \log_{10/7} 1 = 1 \Rightarrow 0 = 1 \Rightarrow$ False.

Case 3:- $\log_b a < k \Rightarrow \log_{10/7} 1 < 1 \Rightarrow 0 < 1 \Rightarrow$ True

i) $p \geq 0 \Rightarrow 0 \geq 0 \Rightarrow$ True.

$\therefore \theta(n^k \log^p n) = \theta(n^1 \log^0 n) = \theta(n)$

j. $T(n) = 4T(n/2) + n^2 \log n$

→ Here, $a = 4$, $b = 2$, $k = 2$, $p = 1$

$f(n) = \theta(n^2 \log n)$

Case 1:- $\log_b a > k \Rightarrow \log_2 4 > 2 \Rightarrow 2 > 2 \Rightarrow$ False.

Case 2:- $\log_b a = k \Rightarrow \log_2 4 = 2 \Rightarrow 2 = 2 \Rightarrow$ True

i) $p > -1 \Rightarrow 1 > -1 \Rightarrow$ True.

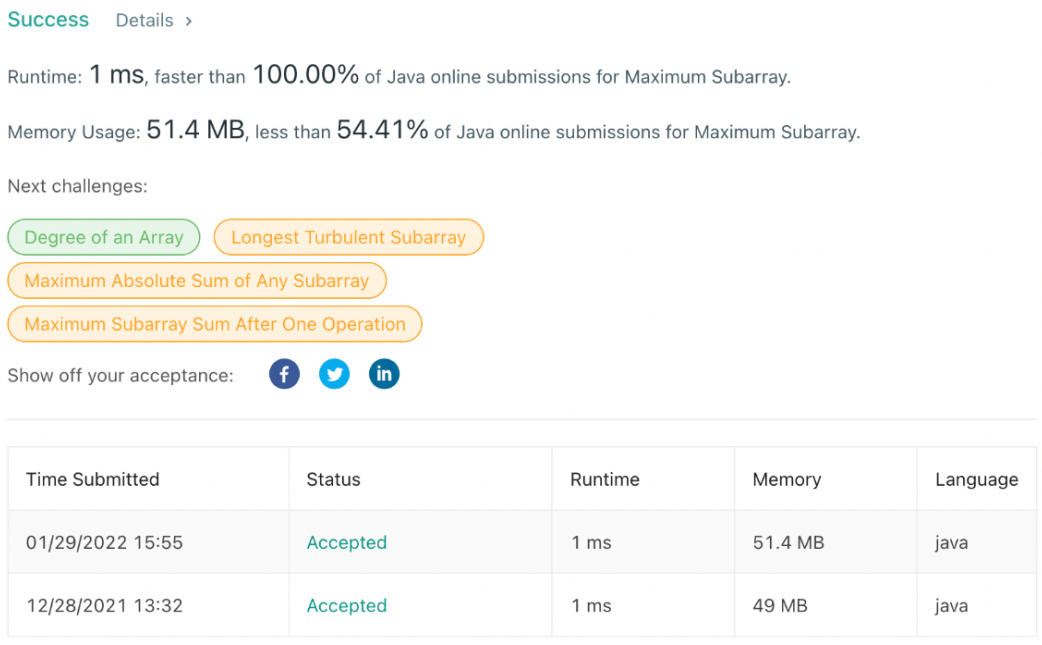
$\therefore \theta(n^k \log^{p+1} n) = \theta(n^2 \log^{1+1} n) = \theta(n^2 \log^2 n)$

3. Solve the leetcode question no 53 (Max Subarray)
Implement a solution that submission can be accepted.
Provide a screenshot of your submission.
(Check discussion for solution if you cannot figure it out yourself,
a linear solution can be found in the file bentley-max-subarray.pdf in camino)

Solution:

```
class Solution {
    public int maxSubArray(int[] nums) {
        int sum = nums[0];
        int result = nums[0];
        for (int i=1; i<nums.length; i++) {
            sum = Math.max(sum + nums[i], nums[i]);
            result = Math.max(result, sum);
        }
        return result;
    }
}
```

Screenshot:



Time Submitted	Status	Runtime	Memory	Language
01/29/2022 15:55	Accepted	1 ms	51.4 MB	java
12/28/2021 13:32	Accepted	1 ms	49 MB	java

4. Solve the leetcode question no. 240 (Search a 2D Matrix II)
Implement a solution that can be accepted.
Provide a screenshot of your submission.
(You can check "discussion" if you cannot figure out an efficient solution).

Solution:

```
class Solution {
    public boolean searchMatrix(int[][] matrix, int target) {
        if(matrix == null || matrix.length < 1 || matrix[0].length < 1) {
            return false;
        }

        int col = matrix[0].length - 1;
        int row = 0;

        while(col >=0 && row<=matrix.length-1) {
            if(target == matrix[row][col]) {
                return true;
            } else if(target < matrix[row][col]) {
                col--;
            } else if(target > matrix[row][col]) {
                row++;
            }
         }
        return false;
    }
}
```

Screenshot: