Numerical Methods
Lesson 6

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Goals for this Lesson

3 The Monte Carlo method

- 3.1 Uniformly distributed random numbers
- 3.2 The transformation method
- 3.3 The acceptance–rejection method
- 3.4 Applications of the Monte Carlo method

Generation of uniform random numbers

- Several methods (routines) in the literature
- Simple method from the book:

$$n_{i+1} = (an_i) \bmod m.$$

- Where n_0 is the "seed" or first number which must be chosen.
- Mod means modulo or remainder after division.
- a and m parameters from the book:
  $a = 40692$, $m = 2147483399$
- Finally, transform the numbers to the [0,1] interval by simple division: $r_i = n_i / m$

Method1: Analytical Transformation

- The second step in the Monte Carlo method is to transform the numbers : $r_i \rightarrow x_i$
  to have the desired p.d.f.: f(x)
- The necessary condition to find this transformation is :

$$\int_{-\infty}^{x(r)} f(x')dx' = \int_{-\infty}^{r} g(r')dr'$$
$$= r.$$

  where g(r) is the uniform distribution for the values r_i

Example: the exponential distribution

- f(x) = 1/ξ exp(- x / ξ) :

$$\int_0^{x(r)} \frac{1}{\xi} e^{-x'/\xi} dx' = r.$$

gives the transfor:

$$x(r) = -\xi \log r$$

Method 2: Acceptance-rejection (von Neumann)

- In general, the analytical method in the previous slide is not possible because the p.d.f. $f(x)$ may not be integrable.
- The "acceptance-rejection" method is a numerical method to generate a set of numbers which follow the distribution $f(x)$
- This method requires **two steps of uniformly distributed random numbers**

Acceptance-rejection method procedure

- **Step 1**: generate a set of uniform random numbers in the range [0,1] as described before. Then transform this numbers to the desired range $[x_{min}, x_{max}]$:

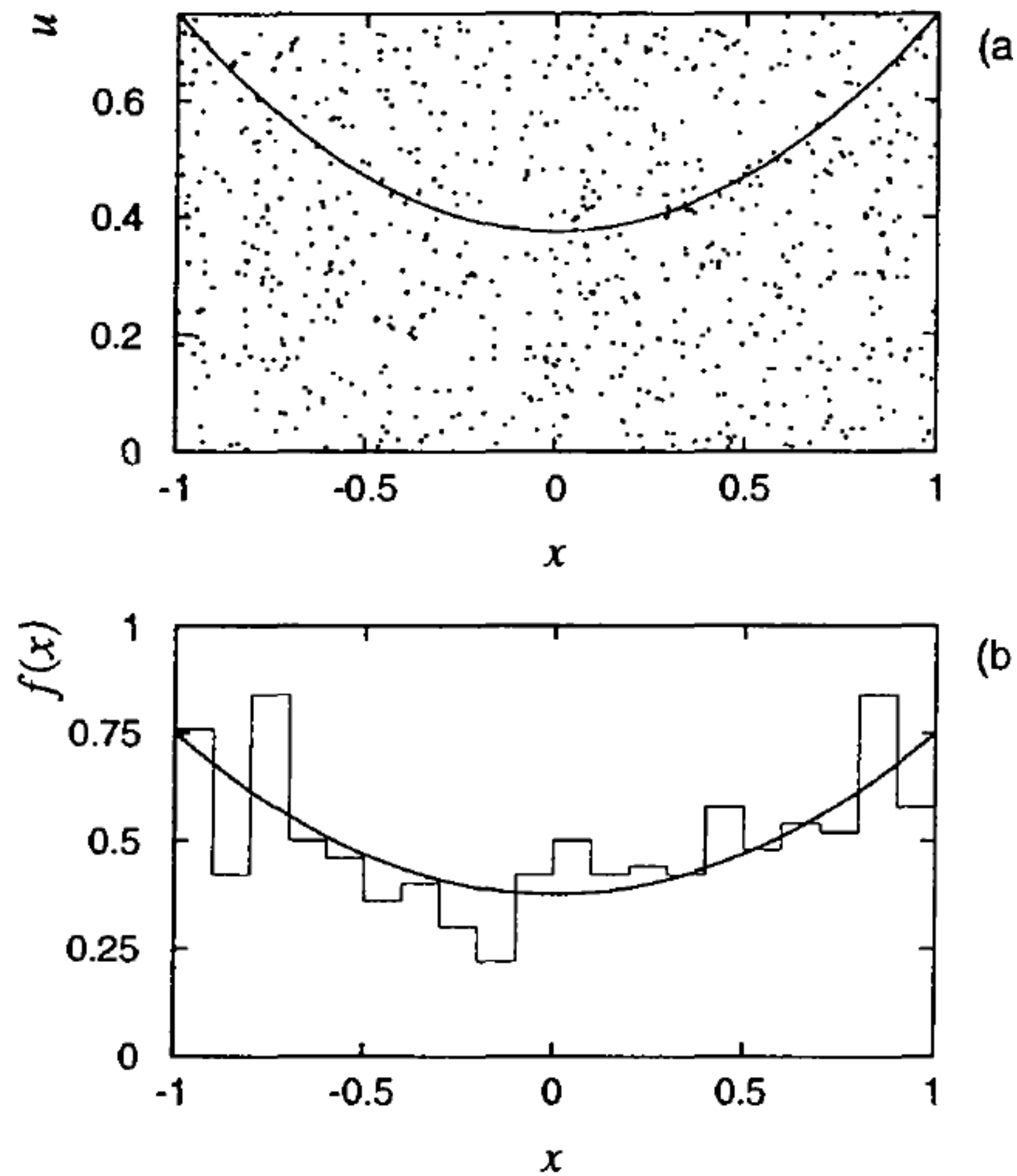
  $$x_i = x_{min} + r_i * (x_{max} - x_{min})$$

- **Step 2**: for each value x_i, generate a second uniformly distributed random number in the range: $[0, f_{max}]$. This second random number can be called u_i. Now we have a pair (xi,ui).
- **Step 3**: x_i is selected if $u_i < f(x_i)$

The selected set of x_i numbers from this procedure will follow the distribution $f(x)$.

Example:

$$f(x) = \frac{3}{8}(1 + x^2), \quad -1 \le x \le 1.$$



- Top graph shows the uniform random numbers generated: u_i and x_i
- The bottom graph shows the counts of selected x_i values.
- The final distribution follows the desired p.d.f. $f(x)$

Exercise

- Update the code
  https://github.com/benitezj/MetodosNumericosCourse

```
cd ~/ MetodosNumericosCourse
git pull
```

- Execute the root script:
  `Lesson6/generate_random_numbers.C`
  It will apply the accept-reject method to generate the distribution from the book.
- **Exercise**:
  Change the example to generate a Guassian distribution, use mean of 0 and sigma of 0.1, with 10000 events in the range (-1,1).

  Publish your results.