TENTATIVELY: COMPARING PROSPECT THEORY AGAINST ERGODIC THEORIES OF DECISION MAKING

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ABSTRACT

Recent developments in decision sciences have seen the emergence of theories predicated on the concept of ergodicity. Ergodic considerations have predicted substantially different behavior under different dynamical settings [1, 2, 3]. A recent experiment found that risk preferences change when dynamics change, with the subjects approximating the time optimal strategy [4]. The study found that the ergodic model outperformed Prospect Theory and other behavioral economic models. However, it has been claimed that the Prospect Theory model tested was inadequate because it assumed each choice between gambles to be independent and did not incorporate probability weighting [5]. In this paper we investigate these claims by testing whether an implementation of Prospect Theory that evaluates each gamble in a consecutive manner and deploy probability weighting outperforms the version that was tested and if so re-run the model selection analysis to include the new implementation. We show that the new implementation of Prospect Theory does outperform the version that was tested. Re-running the full model selection, we further show that there is only anecdotal evidence in favor of the ergodic model.

Keywords: Ergodicity, Expected Utility Theory, Prospect Theory, Behavioral Economics, Hierachical Bayesian Modelling, Bayesian Model Comparison

1. INTRODUCTION

The prevailing behavioral economic models are all based on classic utility theory, which originates from the behavioral null model: individuals were assumed to optimize changes in the expectation values of their wealth (expected value). Recent developments in decision sciences, though, challenges this, and Peters et al. [1] argues that this a priori is a bad null model, as expected value in a general setting does not reflect what happens over time. They propose a different null model, which seek to eliminate the need for classic utility theory. These investigations are based on the branch of mathematics called *Ergodic Theory*, which investigates the question of whether the average over an observable's possible states, is the same as its average over time [6].[1] An observable is considered ergodic if it satisfy Birkhoff's equation

$$\underbrace{\lim_{T\to\infty}\frac{1}{T}\int_0^T f(\omega(t))dt}_{\text{Time average of } f} = \underbrace{\int_\Omega f(\omega)P(\omega)d\omega}_{\text{Expected value of } f}, \qquad \text{(eq. 1)}$$

where f is determined by the system's state ω. On the left-hand side, the state in turn depends on time t, where on the right hand side, we consider the ensemble and assign each state a timeless weight $P(\omega)$.

In decision science we investigate the behavior emerging from a gamble: a stochastic variable, δW, which represent the possible change in wealth, W, given a set of pairs of possible wealth changes and corresponding probabilities $\{(x, p)\}$.

$$W_{i+1} = W_i + \delta W_i = \begin{cases} W_i \circledast x_u, & p_u \\ W_i \circledast x_l, & 1 - p_u, \end{cases} \qquad \text{(eq. 2)}$$

where $\circledast$ is a wildcard operator. Decision making is predominantly studied in an additive setting, where the choice outcomes exert additive effects on the wealth, $\circledast = +$. In these gambles the change in wealth is ergodic and a linear utility function is optimal for maximizing the growth in wealth over time [2]. However, we wish to investigate the behavior under different dynamics and thus allow for a multiplicative setting as well, $\circledast = \times$. In contrast to the additive setting, the change in wealth in these gambles is non-ergodic and a logarithmic utility

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[1] Ergodic theory is a very technical branch of mathematics, but for the the purpose of this paper, this very broad definition is sufficient.

function is optimal for maximizing the growth in wealth over time [2].

Meder et al. [4] present an experiment in which they experimentally manipulated the ergodic properties of a simple gamble environment to evaluate what effects this has on the utility functions that best account for choice behavior. A detailed description of the experiment is given in [4] - here we will only present the outline. In the experiment two environments were set up: an additive, in which all gambles resulted in additive effects on the wealth and a multiplicative, where the changes in wealth corresponded to some proportion of the wealth. For each subject ($n = 18$) the experiment stretched over two days: day$^+$ and day$^\times$, which only differed in the dynamics of wealth change. On each day the subjects were given 312 choices between two gambles, each with a fifty-fifty probability distribution. The choices were consequential, with a DKK1,000 ($\sim$ USD150) initial endowment and a payout dependant on the wealth accumulated through the experiment. Based on the data from the experiment, Meder et al. [4] present a model selection in which they estimate a hierarchical latent mixture model, to model latent mixtures of the Time Optimal model (eq. 6) presented in [1], as well as two of the prevailing behavioral economic models: *Prospect Theory* [7, 8] and *Isoelastic utility* [9, 10]. They show that most subjects had most of their probability mass located over the Time Optimal model and that it had very strong evidence for being the most frequent model.

It has, though, been argued that Prospect Theory and expected utility is applied incorrectly by Meder et al. [4] in two aspects:

1. The subjects are aware that only the outcome of the final choice is received and that this can never be negative. Therefore each choice should be considered only an intermediate choice, where the wealth trajectory until time i is considered in a consequential manner and all gambles are viewed as gains only gambles [11, 12]; and

2. despite the fact that the probabilities of all gambles are equal, $p = \frac{1}{2}$, probability weighting cannot be omitted [7, 8, 12].

In this paper we re-visit the model selection analysis from [4]. Our aim is to test if a new implementation of Prospect Theory outperforms the original implementation. And if so we re-run the full model selection to also include this new implementation. We refer to the two implementations of Prospect Theory as the original and the new implementation, or use the abbreviations PT and PG respectively.

2. METHODS

Model-space. Each of the four models we consider can be described by specifying three functions [13]: a utility function, a probability weighting function and a stochastic choice function. The objective is to compare the utility functions with regards to the choice data in both the additive and multiplicative setting.

Prospect Theory (PT) as implemented by Meder et al. [4], where the choice sequence can be seen as a markov chain in which the utility is described by a power function that depends only on the initial wealth at time i and the possible change at time i and thus have changing reference point:

$$\delta u = \begin{cases} (\delta x)^{\alpha_{gain}} & \text{if } \delta x > 0 \\ -\lambda|(\delta x)|^{\alpha_{loss}} & \text{if } \delta x \leq 0 \end{cases}, \qquad \text{(eq. 3)}$$

where α_{gain} and α_{loss} are risk preference parameters that can take values in the unit interval and λ is a loss aversion parameter that can take values in the interval$[1, \infty]$.

Prospect Theory (PG) is the new implementation of Prospect Theory, where each choice is seen only as an intermediate choice that is dependent on the wealth trajectory from time 0 to time i. Further the reference point is fixed at 0, which eliminates the need for considerations of loss aversion:

$$W_{i-1} + \delta u_i = (W_{i-1} \circledast x)^{\alpha}, \qquad \text{(eq. 4)}$$

where α, as in eq. 3, is a risk preference parameter that can take values in the unit interval, and $\circledast$ is the wildcard operator introduced in eq. 2.

Isoelastic utility where the utility function is given by:

$$\delta u = \delta x \cdot x^{-\eta}, \qquad \text{(eq. 5)}$$

where η is a risk aversion parameters that can take any value in $\mathbb{R}$. Here $\eta > 0$ represents risk aversion, $\eta = 0$ risk neutral and $\eta < 0$ risk seeking.

Time optimal utility where utility is determined by a conditional function, with linear utility in additive dynamics and logarithmic utility in multiplicative dynamics:

$$\delta u = \begin{cases} \delta x & \text{if } \textit{additive dynamics} \\ \delta \ln x & \text{if } \textit{multiplicative dynamics} \end{cases}, \qquad \text{(eq. 6)}$$

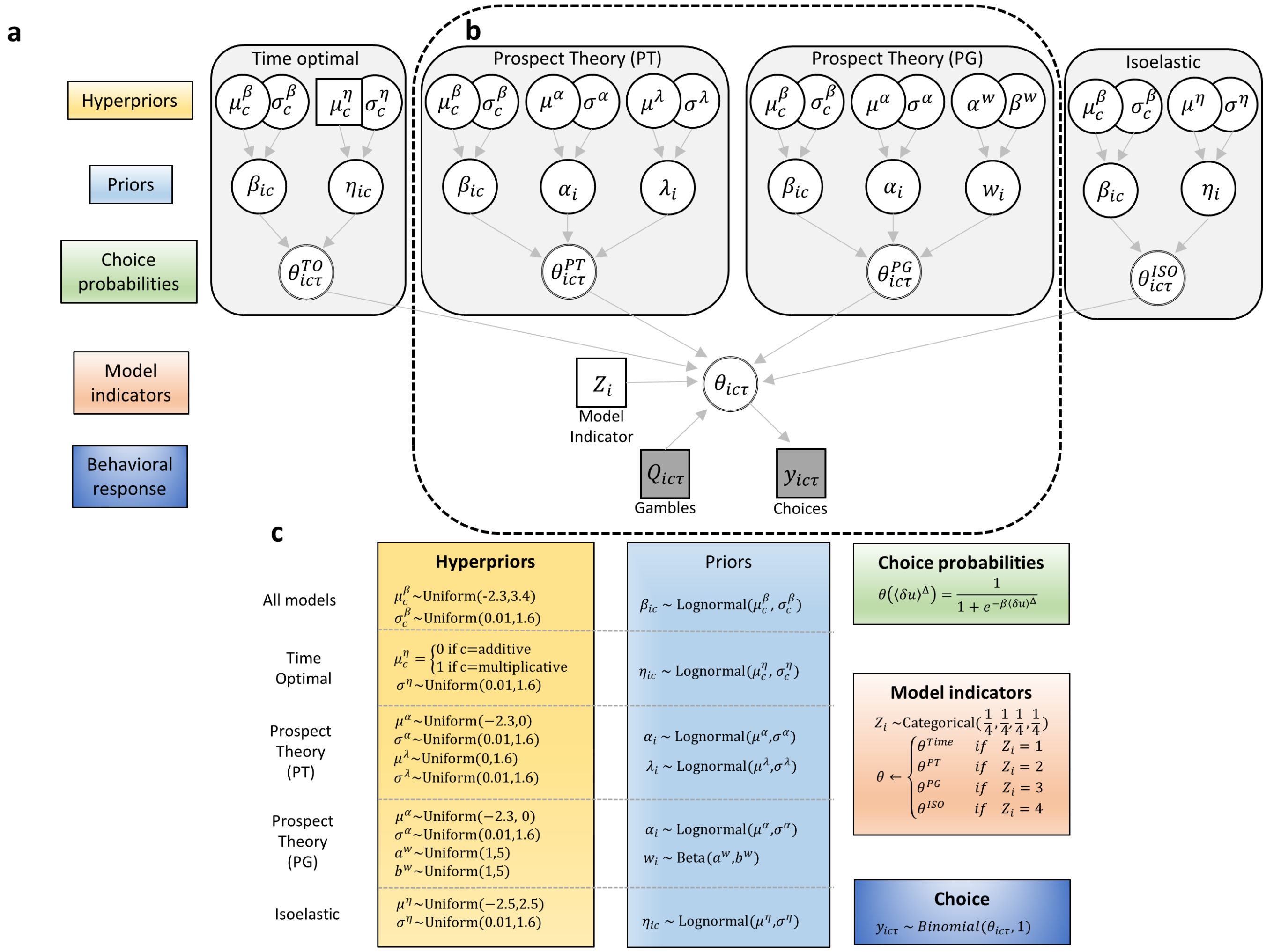

Fig. 1: **Bayesian hierarchical latent mixture model.** **a,** graphical representation of hierarchical Bayesian model for estimating latent mixtures for four different utility models. Circular nodes denote continuous variables, square nodes discrete variables; shaded nodes denote observed variables, unshaded nodes unobserved variables; single bordered nodes denote deterministic variables, double bordered nodes stochastic variables. **b,** part of the graphical representation that is included in the first part of the analysis. **c,** hyperprior and prior distributions, including choice functions and choice generating distributions.

Expected utility. For each gamble the expected utility is calculated as it is proposed by Kahnemann and Tversky [7]:

$$\langle \delta u^{Left} \rangle = w(p) \cdot \delta u_1^{Left} + w(q) \cdot \delta u_2^{Left} \qquad \text{(eq. 7)}$$

and equivalent for the right-hand gamble. Here w is a probability weighting function. For the three utility models presented by Meder et al. [4] (eq. 3, eq. 5, eq. 6) probability weighting is not deployed and thus $w(p) = w(q) = \frac{1}{2}$. In contrast, we allow $w(p)$ to take any value in the unit interval for the new implementation of Prospect Theory (eq. 4). In the literature, probability weights in general need not to sum to one, but can be both smaller and greater. This is, however, not the case for gambles with only positive outcomes, where the probability weights do sum to one [8]. And as probability weights are only deployed for PG, where all gambles are gains only we set $w(q) = 1 - w(p)$.

The difference in utility between the left-hand and right-hand gamble is denoted Δ and thus the difference in utility for each choice is

$$\langle \delta u \rangle^{\Delta} = \langle \delta u^{Left} \rangle - \langle \delta u^{Right} \rangle. \qquad \text{(eq. 8)}$$

Stochastic choice function. The stochastic choice function is identical for all models and is comprised of a logistic function:

$$\theta\left(\langle\delta u\rangle^{\Delta}\right)=\frac{1}{1+e^{-\beta_c\langle\delta u\rangle^{\Delta}}}, \qquad \text{(eq. 9)}$$

where β is a sensitivity parameter that is dependent on the dynamic and determines the sensitivity of the choice probability to differences in the expected change in utility between the two gambles. And θ evaluates to the probability of choosing the left-hand gamble. We emphasize that the sensitivity parameter is model specific and free to vary across subjects. Thus, there are two sensitivity parameters for each subject, for each utility model - sub-/superscripts are suppressed in Fig. 1 and eq. 9 for clarity.

Sampling procedure. Rather than computing point estimates, which ignore the uncertainty within the estimation of the parameters, Bayesian modelling affords computation of full probability distributions of the parameters. Due to the hierarchical structure, individual parameter estimates come from group level distributions [14]. This is done with Monte-Carlo Markov Chain sampling via JAGS (v4.03), called from MATLAB™ (v9.4.0.813654 R2018a, Mathworks®, mathworks.com) via the interface MATJAGS (v1.3, psiexp.ss.uci.edu/research/programs_data/jags). For all models we used: burn-in > 500, 10^3 samples per chain and four chains. Convergence was established via monitoring R-hat values between 0.99 to 1.01 [15]. The sampling procedures were efficient, as indicated by low autocorrelations of the sample chains, R-hat values, and visual inspections of the chain plots (See supplementary methods and results - A. Convergence).

Model selection. We estimate four utility models using a hierarchical latent mixture (HLM) model. Fig. 1a, shows a graphical representation of the HLM model (note that the initial analysis only considers the two implementations of Prospect Theory - Fig. 1b) and the distributional and structural equations are listed in Fig. 1c. Though the four utility models technically are only submodels of the HLM model, we refer to them as utility models for consistency. We follow Nilsson et al. [14] and set weakly informative hyperpriors. We assume that the group mean of β, which is common for all four models, lie in the interval 0.01 to ~ 30. Thus, we have log-normal group mean distributed: $\mu_c^{\beta} \sim \text{Uniform}(-2.3, 3.4)$ and log-normal standard deviation distributed: $\sigma_c^{\beta} \sim \text{Uniform}(0.01, 1.6)$. *Prospect Theory (PT):* has three extra parameters; a risk preference parameter for gains, α_{gain}, and for losses, α_{loss}, which are both restricted to lie in the unit interval. We assume that each come from uninformative log-normal distributions, with uninformative priors on group mean and standard deviation: $\mu^{\alpha} \sim \text{Uniform}(-2.3, 0)$, $\sigma^{\alpha} \sim \text{Uniform}(0.01, 1.6)$. And a loss aversion parameter, λ, which is assumed to lie in the interval 1 to 5 and thus we set uninformative priors on the log-normal group means and standard deviations: $\mu^{\lambda} \sim \text{Uniform}(0, 1.6)$, $\sigma^{\lambda} \sim \text{Uniform}(0.01, 1.6)$. *Prospect Theory (PG);* has, in the utility function, one extra parameter, α, which as in PT is a risk preference parameter. However in PG we deploy probability weighting, w, which take values in the unit interval. We assume the group distribution to be beta distributed with uninformative priors on the shape parameters: $\alpha^{w} \sim \text{Uniform}(1, 5)$, $\beta^{w} \sim \text{Uniform}(1, 5)$. We note that we always compute $w(p)$ and set $w(q) = 1 - w(p)$, and following Miyamoto [16] assign $w(p)$ to the highest outcome for both the left-hand and right-hand gamble. *Isoelastic utility;* Assuming uninformative uniform priors for the log-normal group mean of the risk aversion parameter $\mu^{\eta} \sim \text{Uniform}(-2.5, 2.5)$, $\sigma^{\eta} \sim \text{Uniform}(0.01, 1.6)$ for the log-normal standard deviation. *Time optimal;* which as evident from (eq. 6) is a restriction of the isoelastic utility with fixed mean conditioned on the dynamic: $\mu^{\eta} = 0$ for additive and $\mu^{\eta} = 1$ for multiplicative. Posterior distributions for all parameters for each utility model are presented in; Supplementary methods and results - B. Parameter recovery. *Latent mixtures;* are modelled via indicator variables, which allows comparison between qualitatively different utility models within the HLM model [17]. The model indicator variable, Z, is set with a uninformed uniform prior and is free to vary across subjects. The likelihood for each utility model can now be calculated using Bayes rule: $p(Z = j|D) \propto p(D|Z = j)p(Z = j)$ [17]. And since we use uniform priors on the utility models, we get that the probability of choosing model j is proportional to the posterior of that model. The posterior model probabilities (Fig. 2a & d), estimated model frequencies (Fig. 2b & e) and the protected exceedance probabilities (Fig. 2c & f) are estimated via the Variational Bayesian Analysis toolbox [18] (mbb-team.github.io/VBA-toolbox/).

3. RESULTS

Bayesian model selection supports new implementation of Prospect Theory. We initially tested if the new implementation of Prospect Theory, (eq. 4) dominates the original implementation, (eq. 3) - see Fig. 1b & c. The posterior model probabilities shows that all subjects has most of the probability mass located over the new implementation (Fig. 2a). This is supported from the estimated frequencies (Fig. 2b).

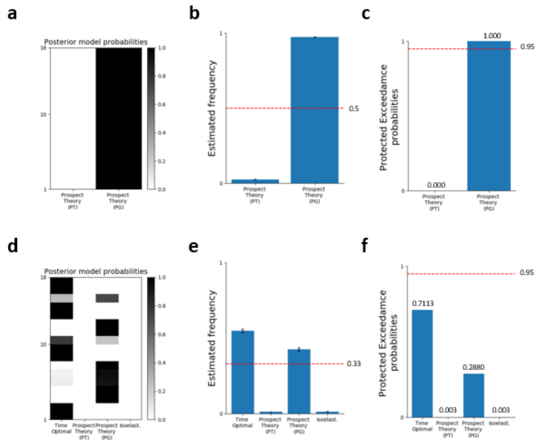

Fig. 2: **Model selection results, PT and PG only (a, b, c) and all four models (e, f, g). a & d,** posterior model probabilities for each utility model based on the model indicator variables representing each utility model. **b & e,** estimated model frequencies including error bars, with the red dashed line indicating prior probabilities. **c & f,** protected exceedance probability for each model being the most frequent, with red dashed line indicating 95 % confidence level.

Computing protected exceedance probabilities,[2] we find that the new implementation has an exceedance probability of 1.0 (Fig. 2c) corresponding to very strong evidence for it being most frequent ($\text{BF}_{\text{PG-PT}} = 36.9$).

Bayesian model selection afford no conclusions between Time Optimality and Prospect Theory. We next included the time optimal utility model (eq. 6) and isoelastic utility model (eq. 5) to obtain a HLM model with all four utility models - see Fig. 1a & c. The posterior model probabilities shows that subjects has most of the probability mass located over either the Time Optimal model or the new implementation of Prospect Theory (Fig. 2d), which is supported by the estimated frequencies (Fig. 2e). We find that the protected exceedance probability is 0.7113 for the Time Optimal and 0.2880 for PG, which is considered anecdotal evidence in favor of the Time Optimal model ($\text{BF}_{\text{TO-PG}} = 1.29$, $\text{BF}_{\text{TO-PT}} = 41.63$, $\text{BF}_{\text{TO-ISO}} = 41.04$)

4. DISCUSSION

Using Bayesian model comparison we here show that the new implementation of Prospect Theory (eq. 4) that has been suggested as critique to Meder et al. [4] dominates the implementation originally used (eq. 3). Including the Time Optimal utility model (eq. 6) and the isoelastic utility model (eq. 5) in the HLM model and thus do a four way Bayesian model comparison, we further show that there's only anecdotal evidence in favor of the Time Optimal utility model over the new implementation of Prospect Theory.

This paper relies fully on the experimental data from Meder et al. [4], and thus we adopt a number of the limitations they present. As noted in the paper the size of the cohort ($n = 18$) was constraint to concentrate power within each subject, and to enable the high-stake design, where each subject could win up to USD750. The advantage of this approach lies in the strength of the evidence and that it affords opportunity for stringent testing between utility models. However, in contrast to Meder et al. the effects from the cohort in our analysis was not consistent across all participants. We do, though, emphasize that the suggestion for this new implementation of Prospect Theory was made after having seen the data and data analysis done by Meder et al. Therefore conclusions about the validity of the utility model must be taken with caution.

These considerations motivates the need for further explorations of risk preferences in environments with changing dynamics. We here present the coded utility models and suggest that this is used to further explore the utility models by generating synthetic data in order to generate experiments that exploits the differences between the utility models.

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Suplementary methods and results

A. Convergence

R-Hat values were calculated for the indicator variable, as well as all model parameters for each subject (Supplementary Tab. 1 for the initial analysis and Supplementary Tab. 2 & 3 for the full model selection). All R-hat values lie within 0.99 and 1.1 indicating the chain has converged to the equilibrium distribution [15].

		Prospect Theory (PT)					Prospect Theory (PG)			
n	Z	α_{gain}	α_{loss}	λ	β_{add}	β_{mul}	α	w	β_{add}	β_{mul}
1	1.0000	1.0003	1.0007	1.0021	1.0001	1.0002	1.0009	1.0000	1.0003	1.0001
2	0.9999	1.0001	1.0006	1.0013	0.9999	1.0003	1.0027	1.0001	1.0022	1.0019
3	1.0000	1.0005	1.0005	1.0023	1.0000	0.9999	1.0026	1.0000	1.0013	1.0015
4	0.9999	1.0003	1.0006	1.0016	1.0000	1.0000	1.0022	1.0001	1.0011	1.0011
5	1.0001	1.0003	1.0006	1.0019	1.0000	1.0000	1.0017	1.0003	1.0010	1.0008
6	0.9999	1.0005	1.0004	1.0016	0.9999	1.0001	1.0015	1.0031	1.0009	1.0031
7	0.9999	1.0005	1.0003	1.0018	1.0000	1.0001	1.0009	1.0000	1.0003	1.0003
8	0.9999	1.0004	1.0005	1.0020	1.0000	1.0001	1.0004	1.0001	1.0000	1.0000
9	1.0000	1.0002	1.0003	1.0019	1.0001	1.0004	1.0011	1.0000	1.0005	1.0004
10	1.0001	1.0002	1.0002	1.0012	1.0001	1.0000	1.0014	1.0000	1.0008	1.0005
11	0.9999	1.0004	1.0003	1.0015	1.0000	1.0002	1.0032	1.0005	1.0018	1.0015
12	1.0000	1.0005	1.0004	1.0023	0.9999	1.0001	1.0009	1.0000	1.0003	1.0002
13	1.0000	1.0004	1.0006	1.0017	1.0000	1.0002	1.0019	1.0001	1.0006	1.0007
14	1.0000	1.0003	1.0007	1.0016	0.9999	1.0002	1.0009	0.9999	1.0002	1.0005
15	1.0001	1.0005	1.0005	1.0017	1.0002	1.0002	1.0014	1.0000	1.0009	1.0002
16	0.9999	1.0003	1.0004	1.0018	1.0000	1.0000	1.0005	1.0001	1.0005	1.0002
17	1.0000	1.0003	1.0005	1.0021	1.0000	1.0000	1.0007	1.0001	1.0001	1.0002
18	1.0001	1.0004	1.0007	1.0012	1.0000	1.0005	1.0011	1.0002	1.0003	1.0006

Supplementary Tab. 1: R-hat values for model selection: PT and PG.

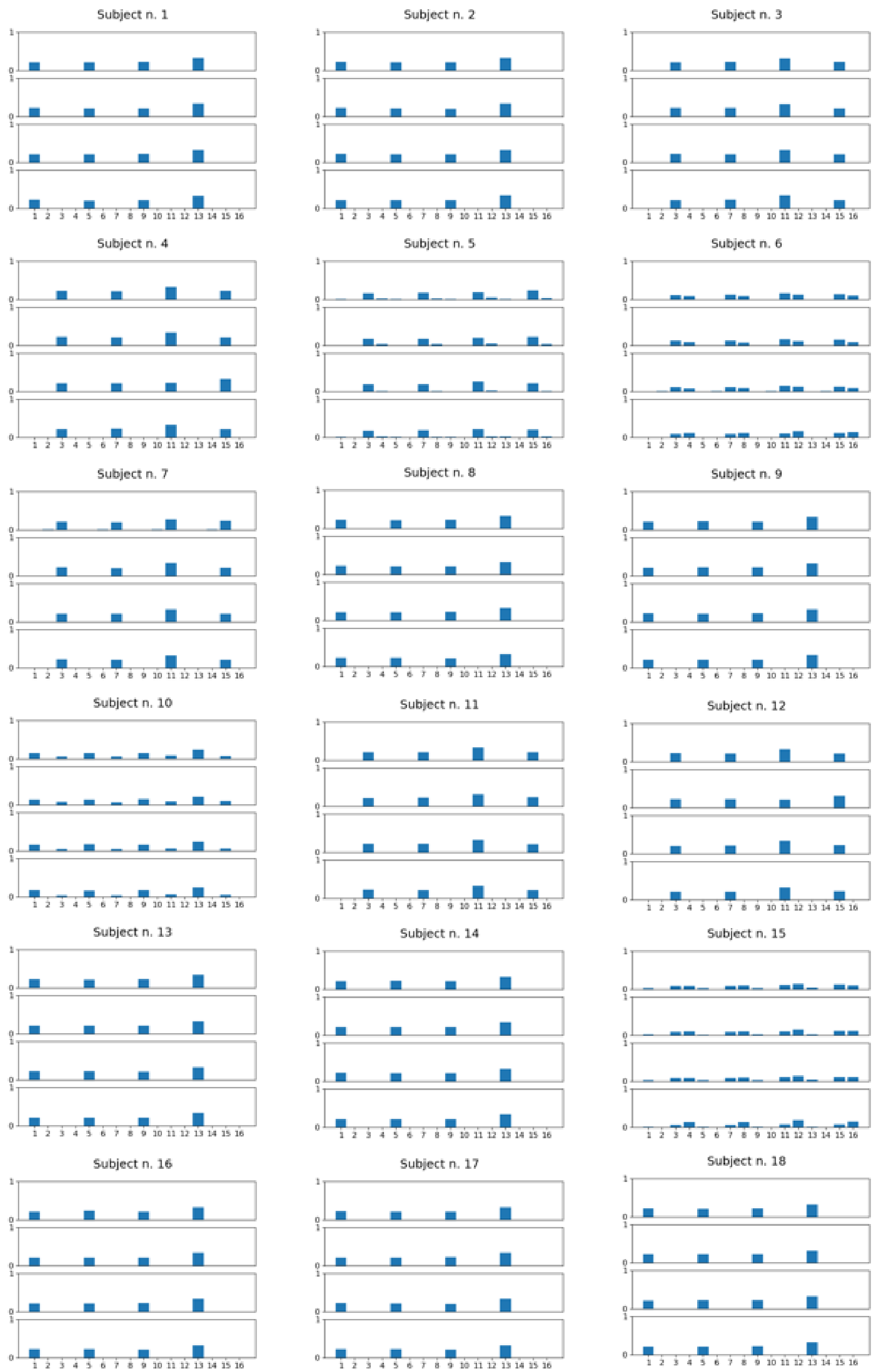
		Prospect Theory (PT)					Prospect Theory (PG)			
n	Z	α_{gain}	α_{loss}	λ	β_{add}	β_{mul}	α	w	β_{add}	β_{mul}
1	1.0000	1.0028	1.0006	1.0034	1.0004	1.0001	1.0034	1.0000	1.0003	1.0000
2	1.0001	1.0036	1.0004	1.0040	1.0011	1.0004	1.0031	1.0004	0.9999	1.0001
3	0.9999	1.0029	1.0007	1.0039	1.0009	1.0004	1.0068	1.0006	1.0067	1.0069
4	1.0013	1.0030	1.0006	1.0040	1.0000	1.0000	1.0056	1.0003	1.0029	1.0026
5	1.0003	1.0038	1.0003	1.0034	1.0008	1.0001	1.0046	1.0025	1.0041	1.0002
6	1.0006	1.0030	1.0004	1.0045	1.0006	1.0006	1.0079	1.0032	1.0014	1.0005
7	0.9999	1.0026	1.0006	1.0039	1.0009	1.0002	1.0061	1.0087	1.0034	1.0027
8	1.0000	1.0028	1.0002	1.0036	1.0005	1.0004	1.0040	1.0004	1.0000	1.0000
9	0.9999	1.0021	1.0006	1.0043	1.0012	1.0002	1.0040	1.0003	1.0000	1.0000
10	1.0004	1.0032	1.0003	1.0037	1.0001	1.0002	1.0032	1.0028	1.0002	0.9999
11	1.0001	1.0029	1.0003	1.0042	1.0006	1.0002	1.0120	1.0019	1.0126	1.0126
12	1.0005	1.0032	1.0003	1.0042	1.0008	1.0002	1.0062	1.0011	1.0052	1.0055
13	1.0000	1.0027	1.0003	1.0038	1.0003	1.0001	1.0038	1.0001	1.0002	1.0001
14	1.0000	1.0027	1.0004	1.0033	1.0006	1.0001	1.0034	1.0002	1.0001	1.0001
15	0.9999	1.0027	1.0005	1.0040	1.0007	1.0001	1.0054	1.0028	1.0006	1.0003
16	1.0001	1.0027	1.0007	1.0039	1.0012	1.0005	1.0038	1.0001	1.0000	1.0002
17	1.0000	1.0023	1.0007	1.0046	1.0008	1.0003	1.0045	1.0005	1.0002	1.0000
18	1.0000	1.0028	1.0005	1.0034	1.0003	1.0000	1.0041	1.0001	1.0002	0.9999

Supplementary Tab. 2: R-hat values for the four way model selection, part I.

	Isoelastic utility			Time optimal			
n	η	β_{add}	β_{mul}	η_{add}	η_{mul}	βadd	β_{mul}
1	1.0019	1.0011	1.0015	1.0009	1.0008	1.0006	1.0004
2	1.0028	1.0006	1.0001	1.0016	1.0005	1.0009	1.0009
3	1.0022	1.0008	1.0005	0.9999	1.0001	0.9999	0.9999
4	1.0025	1.0007	1.0011	0.9999	0.9999	1.0000	1.0000
5	1.0025	1.0012	1.0018	1.0001	1.0014	0.9999	1.0000
6	1.0018	1.0007	1.0008	1.0005	1.0003	0.9999	0.9999
7	1.0017	1.0013	1.0009	1.0002	0.9999	1.0000	1.0000
8	1.0026	1.0019	1.0015	0.9999	1.0015	1.0000	1.0012
9	1.0019	1.0003	1.0012	1.0011	1.0002	1.0002	1.0001
10	1.0015	1.0010	1.0010	1.0003	1.0014	1.0007	1.0005
11	1.0027	1.0009	1.0013	1.0002	1.0002	1.0001	1.0000
12	1.0025	1.0009	1.0026	1.0000	1.0000	1.0000	1.0000
13	1.0027	1.0008	1.0010	1.0008	1.0003	1.0008	1.0003
14	1.0020	1.0013	1.0014	1.0000	1.0000	1.0001	1.0001
15	1.0028	1.0013	1.0011	1.0002	1.0000	0.9999	0.9999
16	1.0020	1.0009	1.0005	1.0014	1.0007	1.0005	1.0010
17	1.0019	1.0011	1.0012	1.0007	0.9999	1.0006	1.0001
18	1.0025	1.0009	1.0001	1.0006	1.0001	1.0006	1.0000

Supplementary Tab. 3: **R-hat values for the four way model selection, part II.**

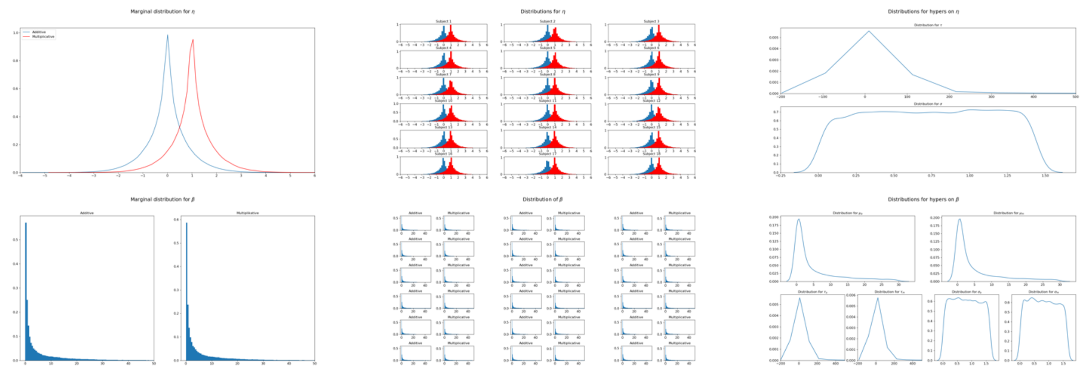
Visual inspection of chain plots. To better allow for convergence, we implemented parameter expansion of the indicator variable , such that four values of the indicator variable map to each utility specification: $\tilde{Z} \in [1, 5, 9, 13]$ map to the Time Optimal Utility model, $\tilde{Z} \in [2, 6, 10, 14]$ to PT, $\tilde{Z} \in [3, 7, 11, 15]$ to PG, and $\tilde{Z} \in [4, 8, 12, 16]$ to Isoelastic utility model. We further ran four independent chains, which act as an extra convergence check, as convergence will imply each chain to produce identical posterior distributions for the indicator variable. From Supplementary Fig. 1 we see that this, within an acceptable tolerance level, is the case and thus indicating convergence.



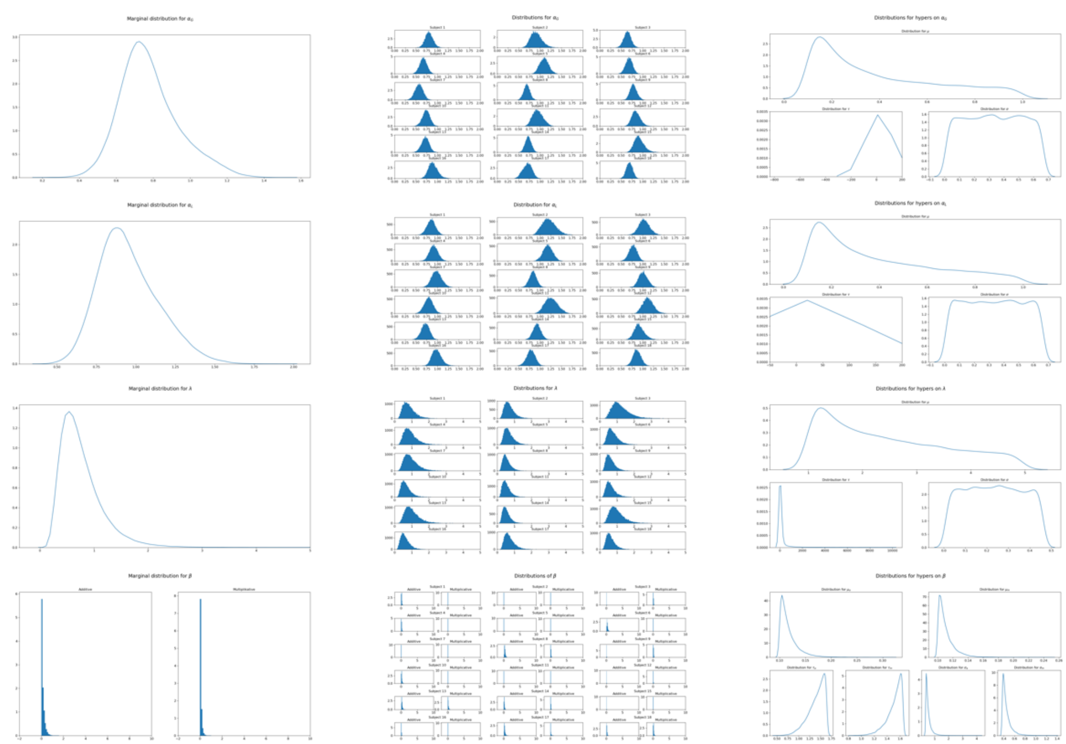
Supplementary Fig. 1: Visual convergence check. The expanded model indicator variable ($\tilde{Z}$) for each subject and each chain.

B. Parameter recovery

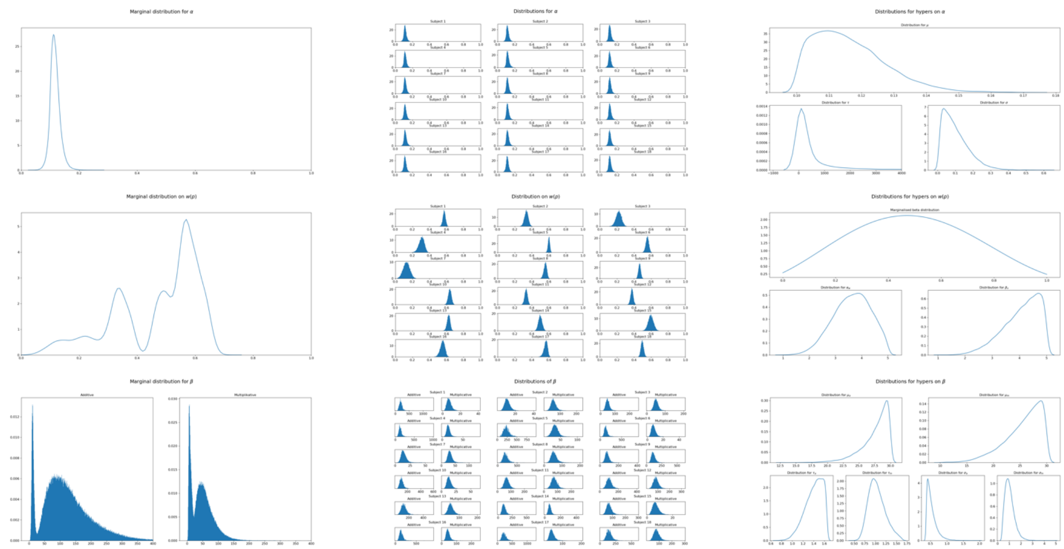
Here we plot the posterior distribution of the parameters for each of the four models. The parameter recovery is done setting the prior of the indicator variable to one for each utility model sequentially, which is equivalent to running the HLM model with only the specified utility model, such that the model parameters are optimized to the data. For each parameter we present the group means, μ, and standard deviation, σ/precision, τ (hypers), the posterior distribution for each subject, as well as the marginalised posterior distribution. We note that the group mean for the Time Optimal is deterministic and therefore not presented.



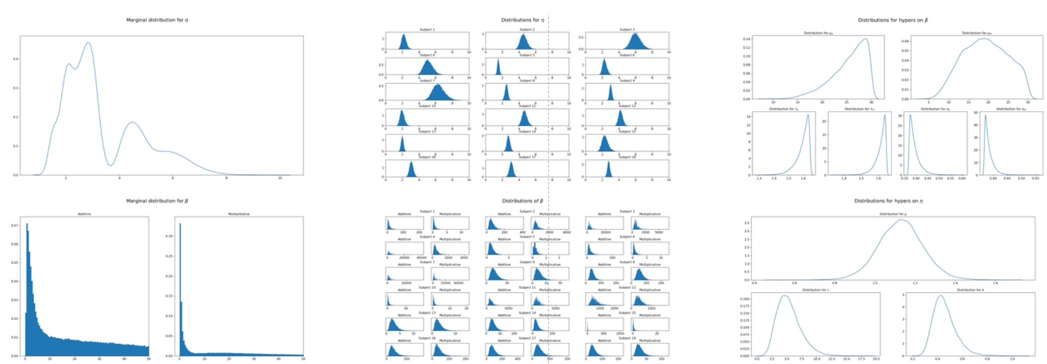
Supplementary Fig. 2: Parameter recovery for Time optimal utility model. Top row shows parameters relating to η and bottom row to β. for all rows, column 1 is the marginalised posterior distribution, column 2 the subject wise posterior distribution and column 3 the posterior distribution for the group-mean and group level precision/standard deviation.



Supplementary Fig. 3: Parameter recovery for original implementation of Prospect Theory. Top row shows parameters relating to α_{gain}, row two to α_{loss}, row three to λ and bottom row to β. For all rows, column 1 is the marginalised posterior distribution, column 2 the subject wise posterior distribution and column 3 the posterior distribution for the group-mean and group level precision/standard deviation.



Supplementary Fig. 4: Parameter recovery for new implementation of Prospect Theory. Top row shows parameters relating to α, row two to w and bottom row to β. For all rows, column 1 is the marginalised posterior distribution, column 2 the subject wise posterior distribution and column 3 the posterior distribution for the group-mean and group level precision/standard deviation.



Supplementary Fig. 5: Parameter recovery for Isoelastic utility model. Top row shows parameters relating to η and bottom row to β. For all rows, column 1 is the marginalised posterior distribution, column 2 the subject wise posterior distribution and column 3 the posterior distribution for the group-mean and group level precision/standard deviation.

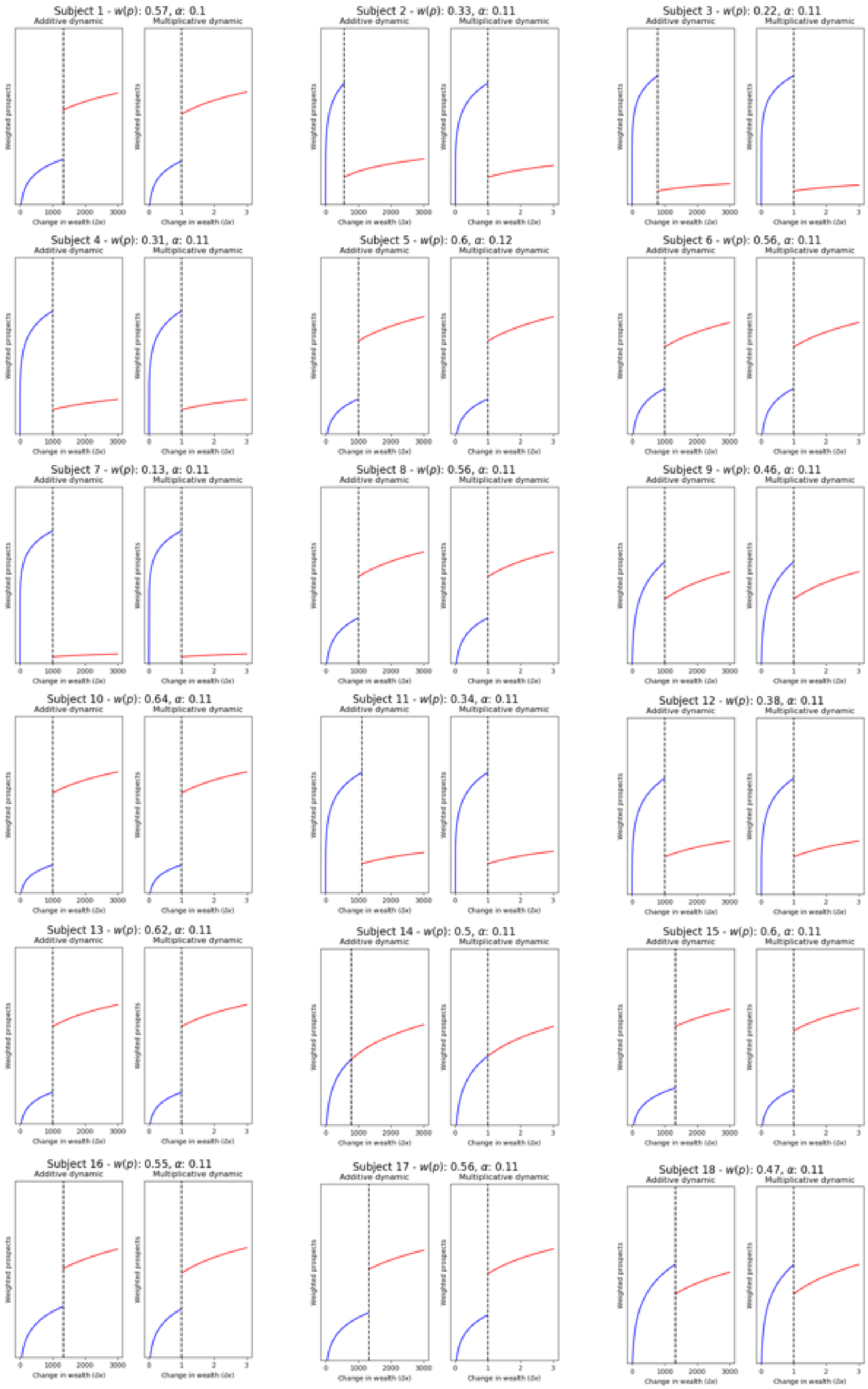
C - MAP utility function for Prospect Theory

We here present a mapping of the utilities (MAP) for the new implementation of Prospect Theory (Supplementary Fig. 6). The MAP is found subject wise and we use the mode values of the posterior distributions that are presented in Supplementary methods and results - B. Parameter recovery. The MAP function is a mixture of eq. 4 and eq. 7 and is given by:

$$\text{MAP}_{\text{add}} = \begin{cases} w(p) \cdot x^{\alpha} & \text{if } x \geq I \\ (1 - w(p)) \cdot x^{\alpha} & \text{if } x < I \ , \end{cases} \qquad \text{(eq. 10a)}$$

$$\text{MAP}_{\text{mul}} = \begin{cases} w(p) \cdot I \cdot x^{\alpha} & \text{if } x \geq 1 \\ (1 - w(p)) \cdot I \cdot x^{\alpha} & \text{if } x < 1 \ , \end{cases} \qquad \text{(eq. 10b)}$$

where I defines the initial wealth for the subject in the active session and x is the outcome of the gamble, which in eq. 10a causes additive changes and in eq. 10b multiplicative changes. Note that I is not restricted to be equal amongst subjects nor among dynamics within each subject.



Supplementary Fig. 6: Utility mapping for the new implementation of Prospect theory for each subject. Blue line represents the weighted prospects when $x < I$ for additive and $x < 1$ for multiplicative. And red prospects when $x \geq I$ for additive and $x \geq 1$ for multiplicative.