THE UTILITY OF WEALTH[1, 2]

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1.1. Friedman and Savage[3] have explained the existence of insurance and lotteries by the following joint hypothesis:

(1) Each individual (or consumer unit) acts as if he (*a*) ascribed (real) numbers (called utility) to every level of wealth[4] and

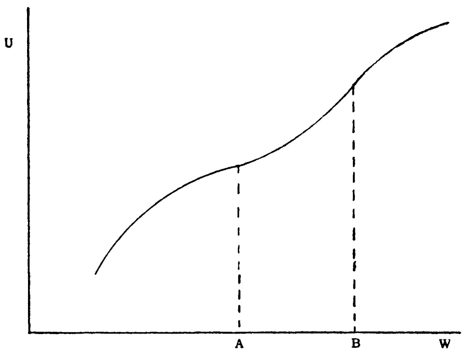

FIG. 1

(*b*) acts in the face of known odds so as to maximize expected utility.

(2) The utility function is as illustrated in Figure 1. We may assume it to be a continuous curve with at least first and second derivatives.[5] Let U be utility and W be wealth. Below some point A, $(\partial^2 U)/(\partial W^2) < 0$; between A and B, $(\partial^2 U)/(\partial W^2) > 0$; above B, $(\partial^2 U)/(\partial W^2) < 0$.

To tell geometrically whether or not an individual would prefer W_0 with certainty or a "fair"[6] chance of rising to W_1 or falling to W_2, draw a line from the point $(W_1, U(W_1))$ to the point $(W_2, U(W_2))$. If this line passes above the point $(W_0, U(W_0))$, then the expected utility of the fair bet is greater than $U(W_0)$; the bet is preferred to having W_0 with certainty. The opposite is true if the line $\overline{(W_1, U(W_1)), (W_2, U(W_2))}$ passes below the point $(W_0, U(W_0))$. In Figure 2, W_0 is preferred to a fair chance of rising to W_1 or

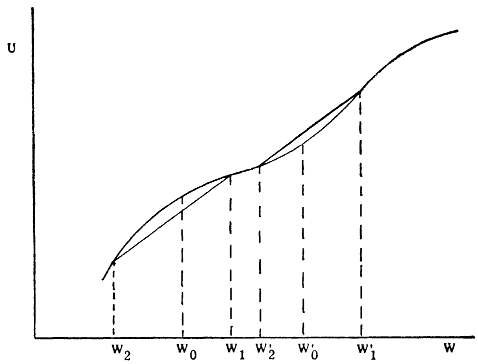

FIG. 2

1 This paper will be reprinted as Cowles Commission Paper, New Series, No. 57.

2 I have benefited by conversations with M. Friedman, C. Hildreth, E. Malinvaud, L. J. Savage, and others. While the present paper takes issue with the article of Friedman and Savage, quoted in n. 3, I take it as axiomatic that the Friedman-Savage article has been one of the major contributions to the theory of behavior in the face of risk. The present paper leads only to a small modification of the Friedman-Savage analysis. This modification, however, materially increases the extent to which commonly observed behavior is implied by the analysis.

3 M. Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, LVI (August, 1948), 279–304.

4 I wish to avoid delicate questions of whether the relevant utility function is the "utility of money" or the "utility of income." I shall assume that income is discounted by some interest rate, and I shall speak of the "utility of wealth."

5 The existence of derivatives is not essential to the hypothesis. What is essential is that the curve be convex below A and above B; concave between A and B. The discussion would be essentially unaffected if these more general assumptions were made.

6 A fair bet is defined as one with expected gain or loss of wealth equal to zero. In particular if α is the probability of W_1 and $(1 - \alpha)$ is that of W_2, then $\alpha W_1 + (1 - \alpha)W_2 = W_0$.

falling to W_2. The chance of rising to W_1' or falling to W_2' is preferred to having W_0' with certainty. The first example may be thought of as an insurance situation. A person with wealth W_1 would prefer to be sure of W_0 than to take a chance of falling to W_2. The second example may be thought of as a lottery situation. The person with wealth W_0' pays $(W_0' - W_2)$ for a lottery ticket in the hope of winning $(W_1' - W_2')$. Even if the insurance and the lottery were slightly "unfair,"[7] the insurance would have been taken and the lottery ticket bought.

Thus the Friedman-Savage hypothesis explains both the buying of insurance and the buying of lottery tickets.

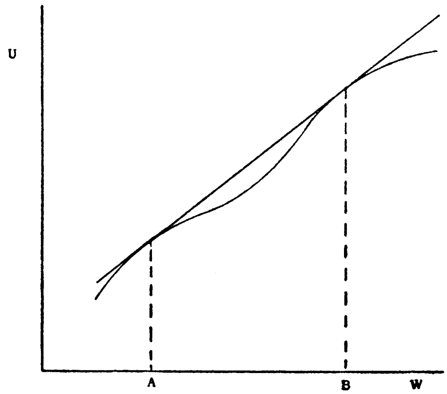

Fig. 3

1.2. In this section I shall argue that the Friedman-Savage (F-S) hypothesis contradicts common observation in important respects. In the following section I shall present a hypothesis which explains what the F-S hypothesis explains, avoids the contradictions with common observation to which the F-S hypothesis is subject, and explains still other phenomena concerning behavior under uncertainty.

In Figure 3 a line l has been drawn tangent to the curve at two points.[8] A person with wealth less than C is presumably "poor"; a person with wealth greater than D is presumably well to do. Friedman and Savage go so far as to suggest that these may represent different social classes. The amount $(D - C)$ is the size of the optimal lottery prize (i.e., the size of prize which it is most profitable for lottery managers to offer). Those poorer than C will never take a fair bet. Those richer than D will never take a fair bet. Those with wealth between C and D will take some fair bets.

We shall now look more closely at the hypothesized behavior of persons with various levels of wealth. We shall see that for some points on the W axis the F-S hypothesis implies behavior which not only is not observed but would generally be considered peculiar if it were. At other points on the curve the hypothesis implies less peculiar, but still questionable, behavior. At only one region of the curve does the F-S hypothesis imply behavior which is commonly observed. This in itself may suggest how the analysis should be modified.

Consider two men with wealth equal to $C + \frac{1}{2}(D - C)$ (i.e., two men who are midway between C and D). There is nothing which these men would prefer, in the way of a fair bet, rather than one in which the loser would fall to C and the winner would rise to D. The amount bet would be $(D - C)/2$—half the size of the optimal lottery prize. At the flip of a coin the loser would become poor; the winner, rich. Not only would such a fair bet be acceptable to them but none would please them more.

We do not observe persons of middle income taking large symmetric bets. We expect people to be repelled by such bets. If such a bet were made, it would certainly be considered unusual and probably irrational.

Consider a person with wealth slightly less than D. This person is "almost rich." The bet which this person would like most, according to the F-S hypothesis, is one which if won would raise him to D, if lost would lower him to C. He would be willing to take a small chance of a large loss for a large chance of a small gain. He would not insure against a loss of wealth to C. On the

[7] I.e., even if $\alpha W_1 + (1 - \alpha)W_2 > W_0$, $\alpha' W_1' + (1 - \alpha')W_2' < W_0'$. For limits on the amount of unfairness which an individual would accept see Friedman and Savage, *op. cit.*, p. 291.

[8] *Ibid.*, p. 300, n. 35.

contrary he would be anxious to underwrite insurance. He would even be willing to extend insurance at an expected loss to himself!

Again such behavior is not observed. On the contrary we find that the insurance business is done by companies of such great wealth that they can diversify to the point of almost eliminating risk. In general, it seems to me that circumstances in which a moderately wealthy person is willing to risk a large fraction of his wealth at actuarially unfair odds will arise very rarely. Yet such a willingness is implied by a utility function like that of Figure 3 for a rather large range of wealth.

Another implication of the utility function of Figure 3 is worth noting briefly. A person with wealth less than C or more than D will never take any fair bet (and, a fortiori, never an unfair bet). This seems peculiar, since even poor people, apparently as much as others, buy sweepstakes tickets, play the horses, and participate in other forms of gambling. Rich people play roulette and the stock market. We might rationalize this behavior by ascribing it to the "fun of participation" or to inside information. But people gamble even when there can be no inside information; and, as to the joy of participation, if people like participation but do not like taking chances, why do they not always play with stage money? It is desirable (at least according to Occam's razor) to have an alternative utility analysis which can help to explain chance-taking among the rich and the poor as well as to avoid the less defensible implications of the F-S hypothesis.

Another level of wealth of interest corresponds to the first inflection point on the F-S curve. We shall find that the implications of the F-S hypothesis are quite plausible for this level of wealth. I shall not discuss these implicatons at this point, for the analysis is essentially the same as that of the modified hypothesis to be presented below.

2.1. I shall introduce this modified hypothesis by means of a set of questions and answers. I have asked these questions informally of many people and have typically received the answers indicated. But these "surveys" have been too unsystematic to serve as evidence; I present these questions and typical answers only as a heuristic introduction. After this hypothesis is introduced, I shall compare its ability and that of the F-S hypothesis to explain well-established phenomena. The hypothesis as a whole is presented on page 155.

Suppose a stranger offered to give you either 10 cents or else one chance in ten of getting $1 (and nine chances in ten of getting nothing). If the situation were quite impersonal and you knew the odds were as stated, which would you prefer?

(1) 10 cents with certainty or one chance in ten of getting $1?

Similarly which would you prefer (why not circle your choice?):

(2) $1 with certainty or one chance in ten of getting $10?

(3) $10 with certainty or one chance in ten of getting $100?

(4) $100 with certainty or one chance in ten of getting $1,000?

(5) $1,000 with certainty or one chance in ten of getting $10,000?

(6) $1,000,000 with certainty or one chance in ten of getting $10,000,000?

Suppose that you owed the stranger 10 cents, would you prefer to pay the

(7) 10 cents or take one chance in ten of owing $1?

Similarly would you prefer to owe

(8) $1 or take one chance in ten of owing $10?

(9) $10 or take one chance in ten of owing $100?

(10) $100 or take one chance in ten of owing $1,000?

(11) $1,000,000 or take one chance in ten of owing $10,000,000?

The typical answers (of my middle-income acquaintances) to these questions are as follows: most prefer to take a chance on $1 rather than get 10 cents for sure; take a chance on $10 rather than get $1 for sure. Preferences begin to differ on the choice between \$10 for sure or one chance in ten of \$100. Those who prefer the \$10 for sure in situation (3) also prefer \$100 for sure in situation (4); while some who would take a chance in situation (3) prefer the \$100 for sure in situation (4). By situation (6) everyone prefers the \$1,000,000 for sure rather than one chance in ten of \$10,000,000.

All this may be explained by assuming that the utility function for levels of wealth above present wealth is first concave and then convex (Fig. 4).

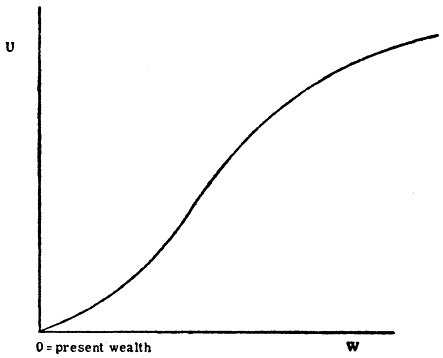

FIG. 4

Let us continue our heuristic introduction. People have generally indicated a preference for owing 10 cents for sure rather than one chance in ten of owing \$1; owing \$1 for sure rather than taking one chance in ten of owing \$10; \$10 for sure rather than one in ten of \$100. There comes a point, however, where the individual is willing to take a chance. In situation (11), for example, the individual generally will prefer one chance in ten of owing \$10,000,000 rather than owing \$1,000,000 for sure. All this may be explained by assuming that the utility function going from present wealth downward is first convex and then concave. Thus we have a curve as in Figure 5, with three inflection points. The middle inflection point is at present wealth. The function is concave immediately above present wealth; convex, immediately below.

How would choices in situations (1)–(11) differ if the chooser were rather rich? My guess is that he would take a chance on getting the \$10 rather than take \$1 for sure; take a chance on \$100 rather than take \$10 for sure; perhaps take a chance on \$1,000 rather than take \$100 for sure. But the point would come when he too would become cautious. For example, he would prefer \$1,000,000 rather than one chance in ten of \$10,000,000. In other words, he would act essentially the same, in situations (1)–(6), as someone with more moderate wealth, except that his third inflection point would be farther from the origin. Similarly we hypothesize that in situations (7)–(11) he would act as if his first inflection point also were farther from the origin.

Conversely, if the chooser were rather poor, I should expect him to act as if his first and third inflection points were closer to the origin.

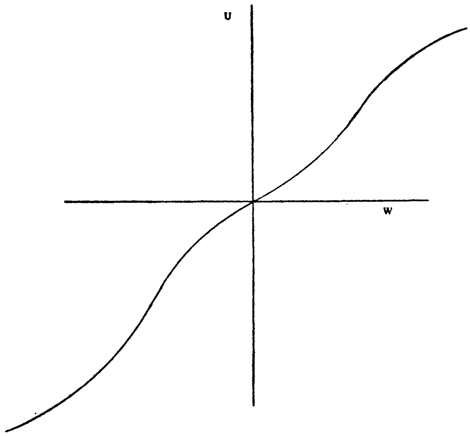

FIG. 5

Generally people avoid symmetric bets. This suggests that the curve falls faster to the left of the origin than it rises to the right of the origin. (I.e., $U(X) > |U(-X)|$, $X > 0$.)

To avoid the famous St. Petersburg Paradox, or its generalization by Cramer, I assume that the utility function is bounded from above. For analogous reasons I assume it to be bounded from below.

So far I have assumed that the second inflection corresponds to present wealth. There are reasons for believing that this is not always the case. For example, suppose that our hypothetical stranger, rather than offering *to* give you $\$X$ or a chance of $\$Y$, had instead first given you the $\$X$ and then had offered you a fair bet which if lost would cost you $-\$X$ and if won would net you $\$(Y - X)$. These two situations are essentially the same, and it is plausible to expect the chooser to act in the same manner in both situations. But this will not always be the implication of our hypotheses if we insist that the second inflection point always corresponds to present wealth. We can resolve this dilemma by assuming that in the case of recent windfall gains or losses the second inflection point may, temporarily, deviate from present wealth. The level of wealth which corresponds to the second inflection point will be called "customary wealth." Unless I specify otherwise, I shall assume that there have been no recent windfall gains or losses, and that present wealth and "customary wealth" are equal. Where the two differ, I shall let "customary wealth" (i.e., the second inflection point) remain at the origin of the graph. Later I will present evidence to support my contentions concerning the second inflection point and justify the definition of "customary wealth."

To summarize my hypothesis: the utility function has three inflection points. The middle inflection point is defined to be at the "customary" level of wealth. Except in cases of recent windfall gains and losses, customary wealth equals present wealth. The first inflection point is below, the third inflection point is above, customary wealth. The distance between the inflection points is a nondecreasing function of wealth.[9] The curve is monotonically increasing but bounded; it is first concave, then convex, then concave, and finally convex. We may also assume that $|U(-X)| > U(X), X > 0$ (where $X = 0$ is customary wealth). A curve which is consistent with our specifications is given in Figure 5.

2.2. An examination of Figure 5 will show that the above hypothesis is consistent with the existence of both "fair" (or slightly "unfair") insurance and "fair" (or slightly "unfair") lotteries. The same individual will buy insurance and lottery tickets. He will take large chances of a small loss for a small chance for a large gain.

The hypothesis implies that his behavior will be essentially the same whether he is poor or rich—except the meaning of "large" and "small" will be different. In particular there are no levels of wealth where people prefer large symmetric bets to any other fair bet or desire to become one-man insurance companies, even at an expected loss.

Thus we see that the hypothesis is consistent with both insurance and lotteries, as was the F-S hypothesis. We also see that the hypothesis avoids the contradictions with common observations to which the F-S hypothesis was subject.

2.3. I shall now apply the modified hypothesis to other phenomena. I shall only consider situations wherein there are objective odds. This is because we are concerned with a hypothesis about the utility function and do not want to get involved in questions concerning subjective probability beliefs. It may be hoped, however, that a utility function which is successful in explaining behavior in the face of known odds (risk) will also prove useful in the explanation of behavior under uncertainty.

It is a common observation that, in card games, dice games, and the like, people play more conservatively when losing moderately, more liberally when winning moderately. Anyone who wishes evidence of this is referred to an experiment of Mosteller and Nogee.[10] Participants in the experiment were asked to write instructions as to how their money should be bet by others. The instructions consisted of indicating what bets

[9] It may also be a function of other things. There is reason to believe, for example, that the distance between inflection points is typically greater for bachelors than for married men.

[10] "An Experimental Measurement of Utility," *Journal of Political Economy*, LIX (1951), 389. The above evidence would be more conclusive if it represented a greater range of income levels.

should be accepted when offered and "further (written) instructions." The "further instructions" are revealing; for example, "A-II—Play till you drop to 75 cents then *stop!!*"; "B-V—If you get low, play only very good odds"; "C-I—If you are ahead, you may play the four 4's for as low as $3"; "C-III—If player finds that he is winning, he shall go ahead and bet at his discretion"; "C-IV—If his winnings exceed $2.50, he may play any and every hand as he so desires, but, if his amount should drop below 60 cents, he should use discretion in regard to the odds and hands that come up." No one gave instructions to play more liberally when losing than when winning. The tendency to play liberally when winning, conservatively when losing, can be explained in two different ways. These two explanations apply to somewhat different situations.

A bet which a person makes during a series of games ("plays" in the von Neumann sense) cannot be explained without reference to the gains and losses which have occurred before and the possibilities open afterward. What is important is the outcome for the whole series of games: the winnings or losings for "the evening" as a whole. Suppose "the evening" consists of a series of independent games (say matching pennies); suppose that the probability (frequency) distribution of wins and losses for a particular game is symmetric about zero. Suppose that at each particular game the player has a choice of betting liberally or conservatively (i.e., he can influence the dispersion of the wins and losses). If he bet with equal liberality at each game, regardless of previous wins or losses, then the frequency distribution of final wins and losses (for the evening as a whole) would be symmetric. The effect of playing conservatively when losing, liberally when winning, is to make the frequency distribution of final outcomes skewed to the right. Such skewness is implied as desirable (in a large neighborhood of customary income) by our utility function. In sum, our utility function implies the desirability of some positive skewness of the final outcome frequency distribution, which in turn implies the desirability of playing conservatively when losing moderately and playing liberally when winning moderately.

This implication holds true whatever be the level of customary wealth of the individual. In the F-S analysis a person with wealth equal to D in Figure 3 would play liberally when losing, conservatively when winning, so as to attain negative skewness of the frequency distribution. This, I should say, is another one of those peculiar implications which flow from the F-S analysis.

Now let us consider the effect of wins or losses on the liberality of betting when we do not have the strategic considerations which were central in the previous discussion. For example, suppose that the "evening" is over. The question arises as to whether or not the game should be continued into the morning (i.e., whether or not a new series of games should be initiated). There is also a question of whether or not the stakes should be higher or lower. We abstract from fatigue or loss of interest in the game.

How do the evening's wins or losses affect the individual's preferences on these questions? Since his gain or loss is a "windfall," the individual is moved from the middle inflection point (presumably by the amount of the gain or loss).

A person who broke even would, by hypothesis, have the same preferences as at the beginning of the evening.

A person who had won moderately would (by definition of "moderate") be between the second and third inflection point. The moderate winner would wish to continue the game and increase the stakes.

A person who had won very much would (by the definition of "very much") be to the right of the third inflection point. He would wish to play for lower stakes or not to play at all. In the vernacular, the heavy winner would have made his "killing" and would wish to "quit while winning."

The moderate loser, between the first and second inflection points, would wish to play for lower stakes or not to play at all.

A person who lost extremely heavily (to the left of the first inflection point) would wish to continue the game (somewhat in desperation).

We see above the use of the distinction between customary and present wealth. In the explanation use was made of both the assumption that (*a*) before windfall gains or losses the second inflection point (customary income) is at present income and (*b*) immediately after such gains or losses customary income and present income are not the same.

To have an exact hypothesis—the sort one finds in physics—we should have to specify two things: (*a*) the conditions under which customary wealth is not equal to present wealth (i.e., the conditions referred to as recent windfall gains or losses) and (*b*) the value of customary wealth (i.e., the position of the second inflection point) when customary wealth is not equal to present wealth. It would be very convenient if I had a rule which in every actual situation told whether or not there had been a recent windfall gain or loss. It would be convenient if I had a formula from which customary wealth could be calculated when this was not equal to present wealth. But I do not have such a rule and formula. For some clear-cut cases I am willing to assert that there are or are not recent windfall gains or losses: the man who just won or lost at cards; the man who has experienced no change in income for years. I leave it to the reader's intuition to recognize other clear-cut cases. I leave it to future research and reflection to classify the ambiguous, border-line cases. We are even more ignorant of the way customary follows present wealth or how long it takes to catch up.

I have assumed that asymmetric bets are undesirable. This assumption could be dropped or relaxed without much change in the rest of the hypothesis; but I believe this assumption is correct and should be kept. Symmetric bets are avoided when moderate or large amounts are at stake. Sometimes small symmetric bets are observed. How can these be explained? I offer three explanations, one or more of which may apply in any particular situation. First, we saw previously that a symmetric bet may be part of a strategy leading to a positively skewed probability distribution of final outcome for the evening as a whole. Second, for very small symmetric bets the loss in utility from the bet is negligible and is compensated for by the "fun of participation." Third (this reason supplements the second), there is an inflection point at $W = 0$; therefore, the utility function is almost linear in the neighborhood of $W = 0$, and, therefore, there is little loss of utility from small symmetric bets.

3.1 Above I used the concept of "fun of participation." If we admit this—as we must—as one of the determinants of behavior under uncertainty, then we must con-

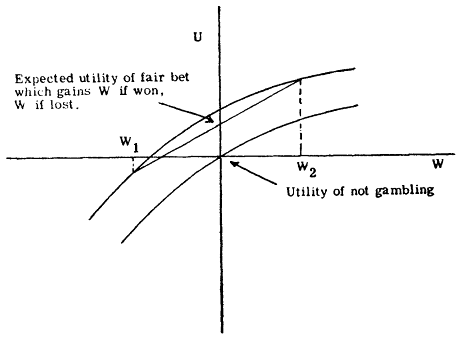

FIG. 6

tend with the following hypothesis: The utility function is everywhere convex; all (fair) chance-taking is due to the "fun of participation." This "classical" hypothesis is simpler than mine and is probably rather popular. If it explained observable behavior as well as my hypothesis, this classical hypothesis would be preferable to mine.

Before examining the hypothesis, we must formulate it more exactly. It seems to say that the utility of a gamble is the expected utility of the outcomes plus the utility of playing the game (the latter utility is independent of the outcome of the game). This can be presented graphically as in Figure 6. One implication of this hypothesis is that, for given (fair) odds, the smaller the amount bet, the higher the expected utility. In particular, when millionaires play poker together, they play for pennies; and no one will buy more than one lottery ticket. This contradicts observation.[11]

One might hypothesize that the utility of the game, to be added to the utility of the outcomes, is a function of the possible loss $(-W_2)$ or the difference between gain and loss $(W_1 - W_2)$. Neither of these hypotheses explains why people prefer small chances of large gains with large chances of small losses rather than vice versa. Nor do they explain why people play more conservatively when losing than when winning.

In short, the classical hypothesis may be consistent with the existence of chance-taking, but it does not explain the particular chances which are taken. To explain such choices, while maintaining simple hypotheses concerning "fun of participation," we must postulate a utility function as in Figure 5.

4.1. It may be objected that the arguments in this paper are based on flimsy evidence. It is true that many arguments are based on "a priori" evidence. Like most "a priori" evidence, these are presumptions of the writer which he presumes are also held by the reader. Such a priori evidence includes the implausibility of middle-income persons desiring large symmetric bets and the implausibility of the one (moderately rich) man insurance company. Perhaps the only evidence of mine which could, so to speak, "stand up in court" is the testimony of the Mosteller-Nogee experiment. But this does not fully suit our needs, since only a narrow range of wealth positions were sampled. I realize that I have not "demonstrated" "beyond a shadow of a doubt" the "truth" of the hypothesis introduced.[12] I have tried to present, motivate, and, to a certain extent, justify and make plausible a hypothesis which should be kept in mind when explaining phenomena or designing experiments concerning behavior under risk or uncertainty.

[11] The statement that millionaires "ought" to play for pennies is irrelevant. We seek a hypothesis to explain behavior, not a moral principle by which to judge behavior.

[12] Even now we are aware of one class of commonly observed phenomena which seems to be inconsistent with the hypothesis introduced in this paper, as well as the hypotheses which this one was intended to supersede. The existence of multiple lottery prizes with various sized prizes *may* contradict the theory presented. If we are forced to concede that the individual (lottery-ticket buyer) prefers, say, a fair multiple prize lottery to all other fair lotteries, then my hypothesis cannot explain this fact. Nor can any other hypothesis considered in this paper explain a preference for different sized lottery prizes. Nor can any hypothesis which assumes that people maximize expected utility. Even now we must seek hypotheses which explain what our present hypotheses explain, avoid the contradictions with observation to which they are subject, and perhaps explain still other phenomena.