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Source: *The American Journal of Psychology*, Apr., 1948, Vol. 61, No. 2 (Apr., 1948), pp. 183-193

Published by: University of Illinois Press

Stable URL: http://www.jstor.com/stable/1416964

AN EXPERIMENTAL STUDY OF THE AUCTION-VALUE OF AN UNCERTAIN OUTCOME

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Students of elementary probability theory will recall that a prize with a known value V, for which the probability of winning is known to be p, is said to have a mathematical expectation equal to the product of its value and its probability, *i.e.* Vp. The mathematical expectation has a certain usefulness in defining 'rational' behavior. It can be shown theoretically, and it is often demonstrated by experience, that a long series of plays will result in systematic losses, if the player characteristically pays in excess of the mathematical expectation for the privilege of playing. Equally, systematic undervaluation of the privilege of playing will result in systematic gains for the player fortunate enough to find such a game to play. It goes without saying that, irrespective of the rationality of playing games which involve wagering, the payment of amounts in excess of the mathematical expectation for the privilege of participation is peculiarly irrational.

In addition to whatever the player may possess of the theory of mathematical expectations, his play is undoubtedly affected by other considerations. Thus, many players hold unsupported convictions about the dependence of chance events, *i.e.* a long run in black at roulette is believed by many to signify that the probability for a red on the next play has been thereby increased. Goodfellow[1] and Fernberger[2] have shown that the Zenith radio public, as well as university students, believe that roulette wheels will give repetition and simple alternation much less frequently than, in fact and in theory, such sequences will appear. In addition to such beliefs about probability, affecting the price paid for the privilege of playing games of chance, are such considerations as the peculiar value which often attaches to a given winning because it completes a larger whole, *i.e.* the special value attaching to the last fifty points neces-

* Accepted for publication November 14, 1947.

[1] L. D. Goodfellow, A psychological interpretation of the results of the Zenith radio experiments in telepathy, *J. Exper. Psychol.*, 23, 1938, 601-632.

[2] S. W. Fernberger, 'Extra-sensory Perceptions' or Instructions? *ibid.*, 22, 1938, 602-607.

sary to win a game in competition with others, the tendency towards contagious bidding characteristic of games in which several people compete against each other, and the limitation upon the amount offered which sometimes is observed when funds run low.

The foregoing considerations may be systematized by examining them relative to three parameters of the game situation. These parameters are the probability of a win, the prize value and the price paid. Thus, there are certain of the considerations which imply that failure of the price to correspond with the mathematical expectation is due to peculiarities of the player's notion of the meaning of a given *probability*. Players for whom a probability of 0.01 means that they have a fair chance of winning are behaving as if a probability of 0.01 was a probability of 0.10 or more. It is understandable if such a person pays in excess of the mathematical expectation for a chance to win the prize.

A second set of the considerations previously mentioned implies a difference between the objective value of the *prize* and its psychological value. The player who needs 50 points to complete his game may value that particular set of 50 points more than is justified by the purely local circumstances. It is understandable that he will pay in excess of the mathematical expectation to win the prize.

Finally, we may note that failure of the price paid to match the mathematical expectation may be due to the fact that the *price* may have a psychological value other than its objective value. Players with plenty of funds may, on that account, regard a small price as smaller than it is by objective standards, and under that circumstance will characteristically offer too much for a given opportunity.

The foregoing possibilities raise questions of both practical and theoretical importance in an area more general and more important than the relatively specific and trivial area of games of chance;[3] namely, the area of the concept. This fact is equally clear in the instance of each of the three possibilities just discussed. In the matter of the meaning which

[3] This fact is not surprising in view of the frequency with which games of chance have been studied for general rather than specific reasons, not only by psychologists (*e.g.*, E. M. Riddle, Aggressive behavior in a small social group, *Arch. Psychol.*, 1925, #78), but also by purely social scientists (*e.g.* J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, 1944, 1-641). Von Neumann and Morgenstern have examined games of chance from the point of view of set-theory as a basis for identifying optimal strategies to be pursued in economic ventures. It is interesting to note that these writers appear to hold the understanding of economic phenomena without recourse to psychological theory as a worthwhile ideal (a familiar theme for those acquainted with the efforts in psychology to understand psychological phenomena without recourse to physiological theory).

attaches to a given probability of a win, for example, we may inquire as to whether one can identify a *psychological* scale of probabilities to which mathematical probabilities correspond.[4] The present research will present evidence on this point. It will also present evidence on the accessory question as to the extent to which knowledge and experience with the mathematical theory of probability effect a reconciliation with what might be called the *psychological* probabilities.

Procedure. The data of the present experiment were obtained from a game invented for the purpose. The game was played with 42 cards. Each card offered an opportunity to win a certain number of points at certain odds. The number of points was one of six *i.e.* 5, 50, 100, 250, 500, or 1000. The odds offered were one of seven, corresponding to probabilities of 0.01, 0.05, 0.25, 0.50, 0.75, 0.95 and 0.99. With six prize quantities and seven probabilities, it is clear that there were 42 combinations of prize and probability. Each combination of prize and probability appeared once in the series of 42 cards. Each of the combinations was presented in a story-setting which, it was hoped, would enlist accessory interest in the proceedings, and thereby diminish the monotony of the game.

In principle the game could be played by any number of players; the present data, however, were obtained from pairs and from sets of four players. In either case the procedure was the same:

(1) Each player was given play money totalling 4000 points. This sum was known as the endowment. The amount of the endowment in the present experiment was fixed at approximately 67% of the amount resulting when the 42 mathematical expectations were summed.

(2) The cards were placed in random order by use of Tippett's Tables. This order was the same for all games.

(3) A set of cards was in the possession of each player; the players faced each other around a table.

(4) The first card was turned up and was read aloud by the experimenter. The opportunity offered by the card was then auctioned off to the highest bidder.

(5) The successful bidder was then permitted to roll a set of dice in an effort to make a point, the probability for which corresponded to the probability stated on the card.

(6) If the point was made, the successful bidder received the prize, less his bid. If the point was not made, the successful bidder paid for his bid from his endowment.

(7) This procedure was continued throughout the 42 cards.

(8) The player with the most play money at the end of the game received his or her choice of candy, cigarettes, or cigars.

(9) The data of the experiment consisted in the set of 42 winning bids.

Subjects. Data were collected on 20 games. Five games were played by pairs

[4] Teachers of elementary courses in statistics will recognize this question (perhaps with mixed feelings) as a paraphrase of a question about the precise meaning which attaches to the statistical significance associated with a given probability, often asked by their students, and upon which the text books are sometimes as silent as the students are vocal.

of men and five by pairs of women—all the players being undergraduate students. Five additional games were played by groups of four undergraduates; and finally, five by pairs of faculty members. The faculty members were, for the most part, of professorial rank in the departments of mathematics, statistics, and psychology. It should be pointed out that the latter group included men with substantial acquaintance with probability theory, at the levels of both theory and practice. It may be remarked that in many cases they were observed making active use of this theory.

Results. The results of the experiment will be presented in such a way as to bear upon the following matters:

(1) The existence in the game situation of a scale of *psychological probability* and its functional relationship to the scale of mathematical probability.

(2) The lack of existence in the game situation of scales of *psychological* prize value and price as distinct from the numerically defined prize values and prices.

(3) The presence of an indifference point in the psychological probability scale, when it is plotted relative to the scale of mathematical probability.

(4) The effect of the size of the prize upon the indifference point.

(5) The extent to which psychological probability is independent of formal experience with mathematical probability.

(6) The effect of the number of players upon the indifference point.

(7) The effect of the number of players upon the prices paid at extreme probability values.

Table I gives the mathematical expectation, the mean winning bid, and the ratio of mean winning bid to expectation, from the data of the entire set of 20 games. The rows of the table are governed by the probabilities and the columns by the prizes. Using the set of upper left hand entries for purposes of illustration: with a probability of 0.01 and a prize of 5 points, the mathematical expectation, denoted by E, is 0.05, the average winning bid was 0.51 points and the ratio of 0.51 to 0.05 is 10.02.

Various trends are apparent in the table. *(1)* For all values of prize from 5 to 1000, the mean winning bid exceeds the mathematical expectation for small values of the probability and is less than the mathematical expectation for large values of the probability. To see this, note that for $p = 0.01$ or 0.05 the ratios uniformly exceed 1.00; for $p \gtreqless 0.25$, the ratios without exception are less than 1.00.

(2) The probability for which the mean successful bid can be pre-

sumed to equal the mathematical expectation is uniformly between 0.05 and 0.25, *i.e.* the indifference point of the probability scale is somewhat less than 0.25.

(3) The ratio shows little evidence of systematic variation with the value of the prize (across the rows of the table). If the table is examined carefully by noting the variation of the ratios across the rows, it may be concluded that there is *some* tendency for the ratios to be large at extreme values of the prize. The lack of consistency in this respect forbids, however, any clear-cut conclusion, and we prefer to leave the tendency for later investigation.

TABLE I

MATHEMATICAL EXPECTATIONS (E), MEAN SUCCESSFUL BIDS (V), AND RATIO OF EXPECTATION TO BID (R), FOR EACH PLAY

Probability		Prize 5	50	100	250	500	1000
	E	.05	.50	1.00	2.50	5.00	10.00
.01	V	.51	4.44	4.86	12.75	19.59	59.96
	R	10.2	8.88	4.86	5.10	3.92	6.00
	E	.25	2.50	5.00	12.50	25.00	50.00
.05	V	.98	2.66	5.52	27.27	27.85	85.40
	R	3.92	1.06	1.10	2.16	1.11	1.71
	E	1.25	12.50	25.00	62.50	125.00	250.00
.25	V	1.06	10.21	14.74	35.53	114.95	231.25
	R	.85	.82	.59	.57	.92	.93
	E	2.50	25.00	50.00	125.00	250.00	500.00
.50	V	1.93	21.84	41.56	110.47	242.80	488.50
	R	.77	.87	.83	.88	.97	.97
	E	3.75	37.50	75.00	187.50	375.00	750.00
.75	V	3.73	29.98	71.88	168.30	304.70	716.40
	R	.995	.80	.96	.90	.81	.96
	E	4.75	47.50	95.00	237.50	475.00	950.00
.95	V	3.41	37.71	73.48	161.00	397.80	790.75
	R	.72	.80	.77	.68	.84	.83
	E	4.95	49.50	99.00	247.50	495.00	990.00
.99	V	3.67	41.72	84.25	226.35	384.20	913.18
	R	.74	.84	.85	.92	.78	.92

Since we have systematic variation from the mathematical expectation in the columns of Table I, with little evidence of the same in rows of the same table, we can exclude the size of the prize as a factor in the variation. No matter what the prize, the same tendency is evident, *i.e.* prizes with small probabilities are paid for too generously and prizes with large probabilities are taken as bargains. That this is not due to any characteristic of *price* paid follows from the design of the experiment. In the experiment the various opportunities were given in random order. Any theory which supposed that the peculiar variability of Table I was due to the price being misconceived uniformly when p was low or high would have to show that the p-values of a given kind appeared uniformly with

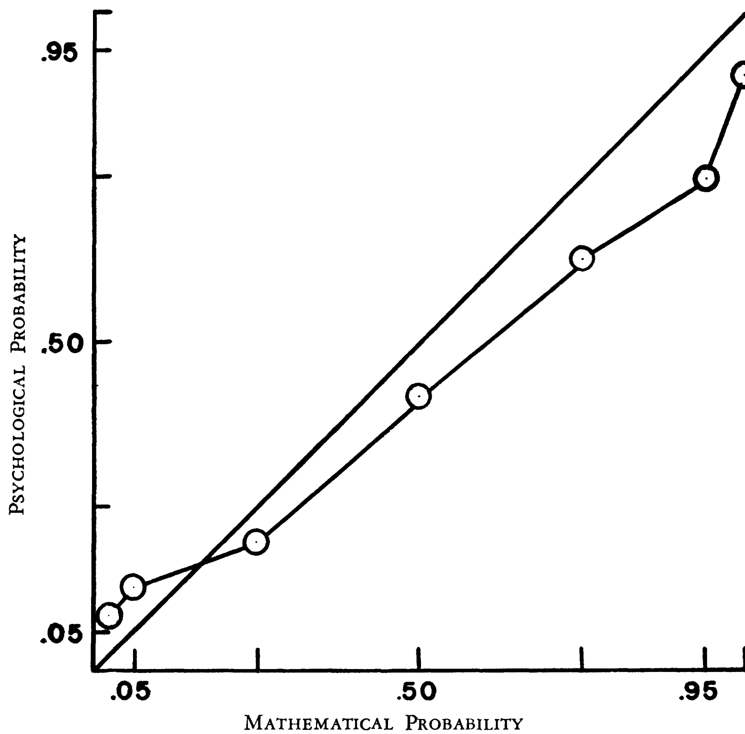

Fig. 1. Functional Relationship Between Psychological and Mathematical Probability

conditions which produced over- or undervaluation of the price. One such condition might be the size of the largest sum in the possession of the players. This quantity would have to be uniformly large with $p \leq 0.05$ and uniformly small with $p \gtreqless 0.25$ to account for the data of Table I. The random ordering of the opportunities eliminates such an hypothesis from serious consideration. Hence, we are left with the hypothesis that when $p \lesseqgtr 0.05$, players uniformly conceive the *probability* as somewhat higher, while with $p \geqq 0.25$ players uniformly conceive the *probability* as somewhat lower. How much higher and how much lower can be estimated by taking the ratio of the price paid to the prize, assuming we are correct in our conclusion that no great effect exists in either the price itself or the prize. The ratio of the price paid to the prize gives an estimate of the psychological probability. For example, if a prize of $50 is won by a price of $4.50 when $p = 0.01$, we may say that such a

TABLE II

RATIOS OF MEAN SUCCESSFUL BID TO MATHEMATICAL EXPECTATION FOR SOPHISTICATED (S) AND UNSOPHISTICATED MALE PAIRS (UM) AND UNSOPHISTICATED FEMALE PAIRS (UF)

(Each mean based on five games)

Probability	*S*	Prize					
		5	50	100	250	500	1000
.01	S	7.4	11.1	1.6	4.8	1.03	11.0
	UM	1.2	1.1	2.6	4.7	3.2	2.5
	UF	11.0	11.7	7.2	4.8	4.5	4.2
.05	S	3.7	.44	.33	4.1	.80	2.8
	UM	1.2	1.2	.45	1.1	1.09	.48
	UF	5.8	1.5	1.9	2.4	1.2	2.3
.25	S	.76	.68	.46	.43	1.1	.85
	UM	1.0	.46	.44	.85	.81	.87
	UF	.80	.75	.54	.38	.83	.71
.50	S	.69	.72	.66	.84	.98	.90
	UM	.80	1.2	1.02	1.03	1.2	1.1
	UF	.68	.50	.62	.66	.71	.70
.75	S	1.1	.89	.96	.93	.86	.87
	UM	.80	.91	1.09	1.07	1.01	1.05
	UF	.67	.47	.81	.67	.51	.76
.95	S	.41	1.00	.80	.82	.91	.90
	UM	1.02	.95	.93	.75	1.01	.99
	UF	.83	.40	.56	.51	.48	.54
.99	S	.87	.91	.92	.94	.86	.98
	UM	.85	.96	.97	.99	.97	.98
	UF	.53	.64	.65	.85	.43	.78

person is behaving as if a p of 0.01 were a p of 0.09. The p of 0.09 we call a psychological probability. The psychological probability is that probability which must be used in order to bring the price paid into rational relationship with the prize.

Having defined a psychological probability we may inquire as to its functional relationship with mathematical probability. Fig. 1 shows the functional relationship of psychological probability and mathematical probability as it appeared in the mean results of our game, *i.e.* from the data of Table I. This figure plots average psychological probability (the mean has been taken over the six prize values) for each of the seven mathematical probabilities. The figure discloses clearly that psychological probability exceeds mathematical probability at low values of p and is exceeded by it at high values of p.

Both Fig.1 and Table I show an indifference point in the scale of p at a point somewhat below 0.25. Table I indicates (within the limits of precision inherent in the experiment) that the indifference point is *not* a function of the size of the prize offered, since it appears uniformly in all columns within the interval $p = 0.05$ and $p = 0.25$.

TABLE III

RATIOS OF MEAN SUCCESSFUL BID TO MATHEMATICAL EXPECTATION FOR GROUPS OF TWO AND FOUR PLAYERS

(Each mean based on five games)

Probability	S	Prize 5	50	100	250	500	1000
.01	two	1.2	1.1	2.6	4.7	3.2	2.5
	four	21.4	11.6	8.0	6.1	6.9	6.2
.05	two	1.2	1.2	.45	1.1	1.2	.48
	four	5.0	1.1	1.7	1.1	1.3	1.2
.25	two	1.0	.46	.44	.85	.81	.87
	four	.83	1.4	.93	.61	.98	1.3
.50	two	.80	1.2	1.02	1.03	1.2	1.1
	four	.95	1.05	1.02	1.00	.95	1.2
.75	two	.80	.91	1.09	1.07	1.01	1.05
	four	1.4	.92	.97	.92	.88	1.14
.95	two	1.02	.95	.93	.75	1.01	.99
	four	.61	.84	.80	.65	.91	.90
.99	two	.85	.96	.97	.98	.97	.98
	four	.73	.87	.86	.92	.84	.96

Table II gives ratios from the data of the three types of *S*s used in pairs, *i.e.* the sophisticated, the unsophisticated men and the unsophisticated women, at each combination of probability and prize. The data show clearly that the sophisticated *S*s exhibit the same phenomena as do the unsophisticated, *i.e.* the indifference point is below 0.25, where the probability is in excess of 0.25 the prize is undervalued, and where the probability is less than 0.25 the prize is overvalued. Clearly knowledge of the theory of probability, while it may reduce them, does not eliminate the effects.

Table III permits a comparison between data obtained from unsophisticated *S*s playing in groups of two and four. This table indicates that the effect of increasing the number of players in the game appears to consist in increasing the amount of over- and undervaluation of the prize at extreme values since in all cases at $p = 0.01$ the ratio is *larger* for groups of four and in all cases at $p = 0.99$ the ratio is *smaller* for groups of four.

Discussion. One of the obvious results of the experiment, requiring additional consideration as well as additional investigation, is the presence of an indifference point in the scale of the probabilities. This point is in many respects analogous to the classical indifference points studied by Hollingsworth, Woodrow, and others. It is not, however, a reflection of a time error or a time-order error since there is nothing in the design of the experiment which permits the appearance of errors of this kind. A more useful analogy perhaps may be drawn between the present fact and the fact of the adaptation level recently used by Helson in the study of frames of reference.[5] This analogy recommends itself on several grounds. In the first place, Helson has shown the adaptation-level concept to be applicable not only to the problem of predicting the apparent brightnesses of figures seen against backgrounds of brightnesses of various compositions, but also to the problem of predicting the outcome of judging intensities in other sense modalities. In the second place, he has shown it to be a function essentially of the logarithms of the intensities of the elements in the perceptual field. In the present experiment, the indifference point is not at 0.50, the center of the range of the series and the arithmetic mean of the probabilities used, but is rather between 0.05 and 0.25. The geometric mean of the series is 0.24. This fact suggests that an additional experiment should be performed in which the range of p should be constricted, the game played a second time and observation directed to the new indifference point. Such an experiment is now in process.

A third ground on which Helson's theory of the adaptation-level recommends itself is related to the function of the initial endowment in the experimental situation. In the present investigation, the players were

[5] Harry Helson, Adaptation-level as frame of reference for prediction of psychophysical data, this JOURNAL, 60, 1947, 1-29.

endowed at the outset with an amount equal to 67% of the sum of the 42 mathematical expectations. Other endowments could have been chosen, and one may speculate on the outcome of possible choices. Increase in the endowment, for example, should result in increase in the average amount paid per play, since it is well known, particularly in these days, that prices go up when money is plentiful. Such a consequence would result in an increase in the indifference point. On the other hand, if the endowment were reduced, the average price should go down, resulting in a decrease in the indifference point. In short, the indifference point in this situation under certain circumstances may be a function of the original endowment. General increase of the endowment would undoubtedly change the atmosphere of the game. Psychologically it might be thought of as changing the background of the game. The theory of the adaptation level states that background contributes heavily to the location of the indifference point. With the indifference point known at two values of background (endowment), the constants can be determined by means of which a quantitative statement could be made as to the indifference point at a third background. In other words, the applicability of the theory of the adaptation level is capable of an experimental test. Such an experiment is in process.

The fact that the slope of the curve of psychological probability as a function of mathematical probability appears to vary systematically at the extremes, depending upon the number of players, is also a fact requiring further consideration. Such a fact suggests that at low probabilities increasing the number of players also increases the willingness of the players to play recklessly, while at high probabilities an increase in the number of players also increases the conservatism of the players. That the outcomes are not due to mere increase in variability due to increase in the number of players is shown by the fact that both effects occur. If increase in the number of players increased the likelihood of a reckless player appearing in the game, the overvaluation of p at values less than 0.05 would be explained; however, such an explanation would also require an overvaluation of all values of p, unless some peculiar interaction were at work. Such is not observed. On the contrary large p values are systematically undervalued.

Summary and Conclusions

(1) The phenomenon of the indifference point is demonstrated in the field of the concept.

(2) The indifference point in the scale of probabilities, under the conditions of the game studied in this investigation, is in the neighborhood of 0.20.

(3) The indifference point appears in the range of probabilities in the neighborhood of the geometric mean of the probabilities used.

(4) Probabilities of less than 0.25 are subject to systematic overestimation. Probabilities of more than 0.25 are subject to systematic underestimation.

(5) The foregoing effects are characteristic of the behavior of mathematicians, statisticians, and psychologists of many years acquaintanceship with the theory of probability as well as of the behavior of college students of various degrees of naïveté in this respect.

(6) Increase in the number of players of the game used in the experiment does not appear to affect the indifference point in the scale of probability but increases both the amount of overestimation of low probabilities and the amount of underestimation of high probabilities.

(7) The foregoing facts are interpreted with the help of a recently published theory of the adaptation level formulated from work on the perception of brightnesses.