The Need for Discontinuous Probability Weighting Functions: How Cumulative Prospect Theory is torn between the Allais Paradox and the St. Petersburg Paradox*

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Abstract

Cumulative Prospect Theory (CPT) must embrace probability weighting functions with a discontinuity at probability zero to pass the two most prominent litmus tests for descriptive decision theories under risk: the Allais paradox and the St. Petersburg paradox. We prove in a nonparametric framework that, with continuous preference functions, CPT cannot explain both paradoxes simultaneously. Thus, Kahneman and Tversky's (1979) originally proposed *discontinuous* probability weighting function has – when applied in a rank-dependent framework, of course – much more predictive power compared to all other popular, but continuous weighting functions, including e.g. Tversky and Kahneman's (1992) proposal. Neo-additive weighting functions constitute another parsimonious, yet promising class of discontinuous weighting functions. In other words, if we rashly restricted CPT to continuous preference functions we might erroneously jump to the conclusion that risk preferences are not stable over similar tasks or even reject CPT.

Keywords: Allais paradox, St. Petersburg paradox, Cumulative Prospect Theory, probability weighting, bounded rationality.

JEL Classification: C91, D81

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1 Introduction

Descriptive theories of decision making under risk are typically required to pass one or more litmus tests. Such tests help to determine whether a theory of decision making is able to predict human behavior to a desired extent. The most prominent tests in decision theory include Bernoulli's (1738, 1954) St. Petersburg paradox and the Allais paradox (Allais, 1953). Both paradoxes have paved the way for new decision theories and helped define the prevailing standard of a certain era. Specifically, the St. Petersburg paradox criticized Expected Value Theory (EVT) and fostered the dominance of Expected Utility Theory (EUT) thereafter. Similarly, the Allais paradox revealed inconsistencies of EUT with actually observed choice behavior and, thus, initiated the development of descriptive decision theories such as Kahneman and Tversky's (1979) prominent Prospect Theory. The advanced version, Cumulative Prospect Theory (CPT) by Tversky and Kahneman (1992) which is based on Rank Dependent Utility Theory (RDU; Quiggin, 1982), is largely considered to be the most powerful model to describe individual decision making under risk and uncertainty.

Virtually all theories of decision making under risk are motivated by *either* the St. Petersburg paradox *or* the Allais paradox. We propose the *joint* consideration of both paradoxes as the new minimum standard to test descriptive decision theories.

In the present paper, we show that CPT is torn between both paradoxes in the following way. If value and weighting functions are continuous, CPT cannot explain both the choice behavior in the Allais paradox and the finite willingness to pay to participate in the St. Petersburg lottery at the same time. For example, consider the originally proposed parametrization in Tversky and Kahneman (1992) where the probability weighting function is given by $w(p) = p^{\gamma}/\left(p^{\gamma} + (1-p)^{\gamma}\right)^{1/\gamma}$ with $\gamma > 0$ and the value function over gains is given by $v(x) = x^{\alpha}$ with $\alpha > 0$. Finite willingness to pay for the St. Petersburg lottery requires the parameter restriction $\alpha < \gamma$ while predicting Allais' common ratio effect requires the opposite inequality $\alpha \geq \gamma$. The more interesting novelty of the present paper stems from the fact that we generalize this result to *all* continuous and strictly increasing value functions v and *all* continuous and strictly increasing probability weighting functions w with $w(0) = 0$ and $w(1) = 1$. Put differently, this joint test dismisses large classes of popular probability weighting functions and considerably reduces the set of potentially promising weighting functions.

If the probability weighting function is discontinuous at probability zero, however, a solution to both paradoxes is possible. The prime example of such a probability weighting function was proposed by Kahneman and Tversky (1979) in their Figure 4. It depicts a hypothesized, yet discontinuous weighting function which "is relatively shallow in the open interval and changes abruptly near the end-points where $w(0) = 0$ and $w(1) = 1$" and which "is not well-behaved near the end-points" (Kahneman and Tversky, 1979, p. 282f.). We find it stunning how much more predictive power Kahneman and Tversky's (1979) originally proposed discontinuous probability weighting function has – when applied in a rank-dependent framework, of course – compared to Tversky and Kahneman's (1992) weighting function or other continuous weighting functions prominently advocated afterwards (e.g. Goldstein and Einhorn, 1987; Prelec, 1998; Diecidue et al., 2009).

Importantly, without a proper elicitation of discontinuities we might erroneously jump to the conclusion that risk preferences are not stable over similar tasks or even reject CPT if we rashly restricted CPT to continuous preference functions. The present literature has not yet used infinitesimally small probabilities to elicit weighting functions directly. Instead, previous studies rely on (continuous) interpolation or parametric fitting of only *finitely* many elicited points of the probability weighting function and, thus, cannot unveil discontinuities unambiguously. The St. Petersburg lottery, however, is a prime candidate for lotteries involving infinitesimally small probabilities and, thus, enables the detection of a potential discontinuity at probability zero. We exploit this advantage of the St. Petersburg lottery in a nonparametric CPT framework with potentially discontinuous probability weighting functions. An axiomatic foundation of CPT consistent with our assumptions can be found in Chateauneuf and Wakker (1999, see Theorem 2.3).[1]

Our theoretical findings are consistent with and, in fact, strongly support the empirical estimates in Barseghyan et al. (2013). Based on more than 4,000 households' insurance deductible choices, they fit a quadratic polynomial on the probability interval $[0, 0.16]$ and estimate the intercept at 0.061, indicating a discontinuity at probability zero. Thus, our theoretical results virtually echo their estimate which they find "is striking in its resemblance to the probability weighting function originally posited by Kahneman and Tversky (1979). In particular, it is consistent with a probability weighting function that [...] trends toward a positive intercept as [the probability] approaches zero [...]. By contrast, the probability weighting functions later suggested by Tversky and Kahneman (1992), Lattimore, Baker and Witte (1992), and Prelec (1998) – which are commonly used in the literature [...] – will not fit our data well, because they trend toward a zero intercept [...]" (Barseghyan et al., 2013, p. 2515).

To provide some intuition for our results, recall that the St. Petersburg paradox describes the fact that hardly anyone would be willing to pay an infinite amount of money for the lottery with infinite expected value which promises an amount of $\$2^k$ with probability 2^{-k} for $k = 1, 2, 3 \dots$. This fact was used as evidence against EVT and was a key motivation to include risk aversion in normative decision theory – such as EUT – to restore a minimum level of descriptive power. Based on the ratio test,

[1] Theorem 25d in Wakker (1994) states additional axioms to guarantee continuity of the probability weighting function.

we then show that, under CPT, a necessary condition for finite willingness to pay is

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \leq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \tag{STP}$$

For continuous (and strictly increasing) probability weighting functions w, this necessary condition is equivalent to the following simpler necessary condition which is useful when analyzing the common ratio effect:

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \leq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)} \tag{STP^*}$$

Now consider the Allais paradox. The Allais paradox exists in different versions and uncovers a violation of EUT's independence axiom. We focus on the common ratio version which involves choices between equal mean lotteries such as $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$z/2, \pi; \$0, 1 - \pi)$ where π is a probability. Kahneman and Tversky (1979) use payoff $z = 6,000$ and $\pi \in \{0.002, 0.9\}$ in Problems 7 and 8. Empirically, subjects choose the safer lottery L_2 for high probabilities π and the riskier lottery L_1 for low probabilities π. EUT's independence axiom, however, does not allow for this change in preference over L_1 and L_2 for varying probabilities π. In our analysis, we make explicit use of Allais' (1953) notion that the common ratio effect emerges in particular for large payoffs z in lotteries L_1 and L_2. He used payoffs in the millions. We then derive the following necessary condition for the common ratio effect:

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \geq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \tag{CRE^*}$$

When comparing conditions (CRE^*) and (STP^*), both inequalities turn into a single equality and we also rule out this equality as a potentially remaining case. Put differently, with continuous (and strictly increasing) v and w there does not exist a simultaneous solution to both paradoxes – independent of the exact parametrizations. To the best of our knowledge, we are the first to prove this general result.

Furthermore, in the CPT framework, any simultaneous solution to both paradoxes must drive a wedge between $\lim_{\pi \to 0^+} \frac{w(0.5\pi)-w(0.25\pi)}{w(\pi)-w(0.5\pi)}$ and $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)}$ and thus involves discontinuous probability weighting functions. Neo-additive weighting functions, as formalized by Wakker (2010) via $w(0) = 0$, $w(1) = 1$, and $w(p) = a + bp$ for $p \in (0,1)$ with $a, b > 0$, $a + b \leq 1$, presumably constitute the simplest class of such weighting functions with a discontinuity at zero.[2] For those functions it holds $\lim_{\pi \to 0^+} \frac{w_{neo}(0.5\pi)-w_{neo}(0.25\pi)}{w_{neo}(\pi)-w_{neo}(0.5\pi)} = 0.5$ and $\lim_{\pi \to 0^+} \frac{w_{neo}(0.5\pi)}{w_{neo}(\pi)} = 1$. Specifically, if we choose, for example, $a = 0.1$, $b = 0.8$, and $v(x) = x^{0.7}$, the CPT decision maker is willing to pay \$5.89 to be entitled to the St. Petersburg lottery and exhibits the typical choice pattern between the common ratio lotteries L_1 and L_2 with preference reversal probability $\pi^* = 0.42$.

To the best of our knowledge, we are the first to show that solving the St. Petersburg paradox rules out practically all CPT preferences that explain the common ratio version of the Allais paradox as long as preferences are given by the same continuous value and weighting function across both paradoxes. A discontinuity of the probability weighting function at probability zero is indispensable to accommodate both Allais' common ratio effect and the St. Petersburg paradox. Some authors analyze the restrictions that finite willingness to pay for the St. Petersburg lottery places on CPT

[2] Kilka and Weber (2001, p. 1717) use neo-additive weighting functions to approximate continuous weighting functions while Baillon et al. (2018) regard it as a full-fledged alternative to popular continuous weighting functions. Although a comparison of weighting functions is not in their focus, results in Baillon and Placido (2019) hint at a relatively better fit of neo-additive weighting functions compared to other popular weighting functions. Neo-additive weighting functions are popular for decision making under ambiguity (e.g. Abdellaoui et al., 2011; Baillon et al., 2017; Chateauneuf et al., 2007). To account for more complex choice behavior for moderate probabilities (Harless and Camerer, 1994; Wu and Gonzalez, 1996) neo-additive weighting functions might be too restrictive and should be amended by some non-linearities.

(e.g. Blavatskyy, 2005; Camerer, 2005; Rieger and Wang, 2006; De Giorgi and Hens, 2006; Cox and Sadiraj, 2008; Pfiffelmann, 2011), but the conflict with the common ratio effect, independent of parametrizations, has not been discovered before.

As a byproduct, we clarify that the slope of smooth probability weighting functions at probability zero is less important for the St. Petersburg paradox than often thought. Rather the trade-off between the limits of the w-ratio $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)}$ and v-ratio $\lim_{z \to \infty} \frac{v(0.5z)}{v(z)}$ is key. We further prove that a strictly concave value function v over gains and a positive, but finite slope $w'(0)$ cannot explain the common ratio effect.

For smooth weighting functions, Dierkes and Sejdiu (2019) show that the w-ratio limit is equivalent to a probabilistic counterpart of the Arrow-Pratt measure of relative risk aversion, $\lim_{\pi \to 0^+} \pi \frac{w''(\pi)}{w'(\pi)}$. The probabilistic and classical Arrow-Pratt measures of relative risk aversion allow for an intuitive interpretation in, for example, Tversky and Kahneman's (1992) original parametrization. The sum of probabilistic risk aversion at probability $\pi \to 0^+$ and the value function's risk aversion has to be strictly positive for finite willingness to pay in the St. Petersburg paradox, while the common ratio effect emerges only for negative overall risk aversion (risk proclivity).

Next to discontinuous probability weighting functions, another potential explanation for both paradoxes within the CPT framework might be varying preferences across both paradoxes[3]. It is well known that CPT preferences can be driven by, for example, affect (Rottenstreich and Hsee, 2001), feelings (Hsee and Rottenstreich, 2004), or perceived self-competence (Kilka and Weber, 2001). Similarly, Harrison and Rutström (2009) deliberately model decision makers with a latent process which switches between evaluation according to EUT or CPT. Whether differences in the Allais paradox and St. Petersburg paradox trigger such changes in preferences is an

[3]In the insurance context, for example, Barseghyan et al. (2011) conclude from field data that risk preferences are not stable across home deductible and auto deductible choices.

open question, though.

Using large payoffs, $z \to \infty$, in the Allais paradox above might appear extreme at first glance, but is supported by experimental evidence. If CPT's value function is parameterized by the power value function $v(x) = x^\alpha$, as is most often the case in empirical calibration studies, then the v-ratio $\frac{v(0.5z)}{v(z)}$ is independent of payoff z and using large payoffs is irrelevant. In other words, the power value function inhibits a solution to both paradoxes if the weighting function is continuous. With other value functions and continuous weighting functions, solutions might theoretically exist for moderate payoffs only - at odds with Camerer (1989), Conlisk (1989), Fan (2002), Huck and Müller (2012), and Agranov and Ortoleva (2017) who report less frequent Allais-type violations of EUT for small payoffs. A sensitivity analysis in Appendix B, however, explicitly rejects other typical value functions (exponential, logarithmic, and HARA) because of their unrealistic predictions.

Our results are not fabricated by the infinite expected payoff of the original St. Petersburg lottery. An analysis of truncated St. Petersburg lotteries does not change our conclusions (see Appendix C). Further, Rieger and Wang's (2006) arguments show that CPT can predict *infinite* willingness to pay (certainty equivalent) even in cases of risks with *finite* expected value.

Finally, the discrepancy between both the St. Petersburg and the Allais paradox is not an artifact of the preference reversal phenomenon whereby there can occur inconsistencies between choice and valuation tasks (see e.g. Lichtenstein and Slovic (1971) for an early reference).[4] We argue below that, if anything, the preference reversal phenomenon makes CPT's difficulties to predict both paradoxes even greater.

The remainder of this paper illustrates our conclusions by formal proofs and

[4]Schmidt et al.'s (2008) third generation Prospect Theory enhances CPT to allow, for example, for such preference reversals.

numerical examples.

2 The Allais - St. Petersburg Conflict

We make the following assumptions throughout our discussion.

Assumption 1 (Mathematical Notation) *Whenever we use limits, e.g.* $\lim_{x\to z} f(x)$*, we implicitly assume these limits exist in a weak sense, i.e. limes superior and limes inferior coincide and* $\lim_{x\to z} f(x) \in [-\infty, \infty]$*.*

Assumption 2 (Preference Calculus)

a) The decision maker's utility for a lottery $(x_1, p_1; x_2, p_2; \dots)$ *is given by Cumulative Prospect Theory. That is the decision maker has a value function* v *and a probability weighting function* w*. Assuming without loss of generality that payoffs are rank ordered such that* $0 \le x_1 \le x_2 \le \dots$*, the CPT value is given by* $v(x_1)[w(p_1+p_2+\dots)-w(p_2+p_3+\dots)]+v(x_2)[w(p_2+p_3+\dots)-w(p_3+p_4+\dots)]+\dots$*.*

b) The value function v *is continuous and strictly monotonically increasing with* $v(0) = 0$*.*

c) The probability weighting function w *is strictly monotonically increasing with* $w(0) = 0$ *and* $w(1) = 1$*.*

d) The reference point is the current wealth level. In particular, all lottery payoffs considered here are perceived as gains.[5]

[5]Without loss of generality, we follow the typical assumption that the reference point is fixed at zero in both paradoxes. In particular, theories with stochastic reference points (Kőszegi and Rabin, 2006) are not applicable.

2.1 The St. Petersburg Paradox under CPT

Bernoulli's (1738, 1954) St. Petersburg lottery L_{STP} promises an amount of \$$2^k$ with probability 2^{-k} for $k \in \mathbb{N}_{>0}$. Although the expected value of L_{STP} is infinite, real decision makers are only willing to pay a low price for lottery L_{STP}.[6] Under CPT, the decision maker assigns the following utility to the St. Petersburg lottery L_{STP}:

$$\begin{aligned} CPT(L_{STP}) &= \sum_{k=1}^{\infty} v\left(2^k\right) \cdot \left[w\left(\sum_{i=k}^{\infty} \frac{1}{2^i}\right) - w\left(\sum_{i=k+1}^{\infty} \frac{1}{2^i}\right)\right] \\ &= \sum_{k=1}^{\infty} v\left(2^k\right) \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right]. \end{aligned} \tag{1}$$

A CPT decision maker's willingness to pay for the lottery L_{STP} is given by the certainty equivalent $v^{-1}(CPT(L_{STP}))$. The following theorem states conditions for a finite certainty equivalent under CPT.

Theorem 1 (Emergence of the St. Petersburg paradox) *Let $v : \mathbb{R} \to \mathbb{R}$ be a strictly increasing value function and $w : [0,1] \to [0,1]$ be a strictly increasing probability weighting function with $w(0) = 0$ and $w(1) = 1$. Then, it holds for the St. Petersburg lottery L_{STP}:*

a) A CPT decision maker reports finite willingness to pay for L_{STP} if v is bounded from above.

b) Assume v is unbounded. Then, a CPT decision maker reports finite willingness to pay for L_{STP} if (sufficient condition)

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} < \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \tag{2}$$

[6] A comprehensive analysis of the St. Petersburg paradox is provided by Samuelson (1977).

c) A necessary condition for finite willingness to pay for L_{STP} is

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \leq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \tag{3}$$

Proof: Using a bounded value function $v^b(\cdot)$ is the simplest way to guarantee a finite CPT value. Assuming that v^b is monotonically increasing, strictly concave and bounded, i.e $\lim_{z\to\infty} v^b(z) = c$, it is straightforward to prove statement a) that, independent of the specification of the probability weighting function w, Equation (1) is always strictly smaller than c:

$$\sum_{k=1}^{\infty} v^b\left(2^k\right) \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right] < \sum_{k=1}^{\infty} c \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right] = c. \tag{4}$$

Hence, the maximum willingness to pay (certainty equivalent) for lottery L_{STP} is finite.

For unbounded value functions v, finite willingness to pay is equivalent to convergence of the infinite sum (1). The ratio test to assess the convergence of (1) in case of unbounded value functions implies finite willingness to pay if

$$\lim_{k\to\infty} \left| \frac{v\left(2^{k+1}\right) \cdot \left[w\left(2^{-k}\right) - w\left(2^{-k-1}\right)\right]}{v\left(2^k\right) \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right]} \right| < 1. \tag{5}$$

If we substitute π for 2^{1-k} (probability) and z for 2^{k+1} (payoff), we can restate the convergence criterion as

$$\lim_{z\to\infty} \frac{v(z)}{v(0.5z)} \cdot \lim_{\pi\to 0^+} \left| \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \right| < 1, \tag{6}$$

which corresponds to part b). Part c) follows because, according to the ratio test, a necessary condition for convergence is the weak version of the inequalities above.

Note that statement c) holds for bounded as well as unbounded value functions because for bounded and strictly increasing value functions we yield $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$. To see this, recall that $v(0) = 0$ so that strict monotonicity and boundedness lead to $\lim_{x\to\infty} v(x) = c$ for some upper bound $c > 0$ and, thus, $\frac{v(0.5z)}{v(z)} \underset{z\to\infty}{\longrightarrow} \frac{c}{c} = 1$. ■

The next theorem specializes to the case of continuous preference functions and already adumbrates that, for smooth weighting functions, a finite derivative of the probability weighting function at zero, $w'(0) < \infty$, does not guarantee a finite certainty equivalent as long as we allow for various forms for the value function. Rather, the trade-off between the limit of the w-ratio $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)}$ and the limit of the v-ratio $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)}$ is important.

Theorem 2 (Continuous weighting functions and the St. Petersburg paradox) *Let $v : \mathbb{R} \to \mathbb{R}$ be a continuous and strictly increasing value function and $w : [0, 1] \to [0, 1]$ be a continuous and strictly increasing probability weighting function with $w(0) = 0$ and $w(1) = 1$. Then, it holds for the St. Petersburg lottery L_{STP}:*

a) A CPT decision maker reports finite willingness to pay for L_{STP} if (sufficient condition)

$$\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} < \lim_{z\to\infty} \frac{v(0.5z)}{v(z)}. \tag{7}$$

b) A necessary condition for finite willingness to pay for L_{STP} is

$$\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} \leq \lim_{z\to\infty} \frac{v(0.5z)}{v(z)}. \tag{STP^*}$$

c) If in part b) the limits are equal and less than one, that is

$$\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} \in (0, 1), \tag{8}$$

then the decision maker's willingness to pay is arbitrarily large. Put differently, no reported finite willingness to pay for L_{STP} can be captured by these CPT preferences.

Proof: The case of bounded value functions is clear from Theorem 1. So, let us consider unbounded value functions. Lemma 1 in Appendix A proves that for continuous probability weighting functions, it holds:

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} = \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \ . \tag{9}$$

Then, statements a) and b) are clear from Theorem 1.

In the situation of statement c), Lemma 2 in Appendix A shows that for all $\epsilon > 0$ there exists $\pi_0 \in (0,1)$ such that

$$w(\pi) \geq \text{const} \cdot \left(\frac{\pi}{2}\right)^{\gamma+\epsilon} \tag{10}$$

for all $\pi \in (0, \pi_0]$. Similarly, Lemma 3 in Appendix A ensures that for all $\epsilon > 0$ there exist $x_0 > 0$ such that

$$v(x) \geq \text{const} \cdot x^{\alpha-\epsilon} \tag{11}$$

for all $x \geq x_0$.

Now let $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 0.5^\gamma \in (0,1)$ for some $\gamma > 0$. Observe that v is unbounded because otherwise, with our convention $v(0) = 0$, we would have had $\lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 1$. We use Lemma 3 and Lemma 2 which give lower boundaries for the value function v for larges payoffs z and for the probability weighting function w for small probabilities, respectively. For any $\epsilon_1, \epsilon_2 > 0$ the CPT value can be assessed as

$$CPT(L_{STP}) = \sum_{k=1}^{\infty} v\left(2^k\right) \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right] \tag{12}$$

$$= \sum_{k=1}^{\infty} v\left(2^k\right) w\left(2^{1-k}\right) \cdot \left[1 - \underbrace{\frac{w\left(2^{-k}\right)}{w\left(2^{1-k}\right)}}_{\approx 0.5^{\gamma} \text{ for sufficiently large } k \text{ and some } \gamma > 0}\right] \tag{13}$$

$$\geq \text{const} \cdot \sum_{k=k_0}^{\infty} \left(2^k\right)^{\gamma-\epsilon_1} \left(\frac{2^{1-k}}{2}\right)^{\gamma+\epsilon_2} \tag{14}$$

$$= \text{const} \cdot \sum_{k=k_0}^{\infty} \left(2^{-\epsilon_1-\epsilon_2}\right)^k \tag{15}$$

where k_0 is a sufficiently large index. Equation (15) equals infinity if and only if $\epsilon_1 + \epsilon_2 = 0$. Since we can choose $\epsilon_1 > 0$ and $\epsilon_2 > 0$ arbitrarily small, the sum in (15), and hence the willingness to pay, grows arbitrarily large. ■

Without probability weighting (i.e. $w(\pi) = \pi$), the v-ratio limit $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)}$ has to be strictly greater than 0.5 to ensure a finite subjective CPT value. The value function's concavity alone does not automatically imply a v-ratio limit greater than 0.5. For example, Bell's (1988) one-switch function $v(x) = \beta x - e^{-\alpha x} + 1$ with $\alpha, \beta > 0$ has a v-ratio limit of 0.5 (see Example 4). Hence, this concave value function yields infinite willingness to pay for the St. Petersburg lottery under EUT (Theorem 2, statement c).

Continuity of the probability weighting function in Theorem 2 is crucial:

Example 1 *Assume a neo-additive probability weighting function w. That is*

$$w(\pi) = \begin{cases} 0 & \textit{for } \pi = 0 \\ a + b \cdot \pi & \textit{for } \pi \in (0,1) \\ 1 & \textit{for } \pi = 1 \end{cases}, \tag{16}$$

where $a+b \leq 1$ *and* $a, b > 0$. *Note that although* $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \frac{a}{a} = 1$, *finite willingness to pay is well possible for* $\lim_{z \to \infty} \frac{v(0.5z)}{v(z)} < 1$ *(contrary to the case of continuous weighting functions) because the CPT value is given by*

$$CPT(L_{STP}) = \sum_{k=1}^{\infty} v\left(2^k\right) \cdot \left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right] \tag{17}$$

$$= \sum_{k=1}^{\infty} v\left(2^k\right) \cdot \left[a + b \cdot \left(2^{1-k}\right) - a - b \cdot \left(2^{-k}\right)\right] \tag{18}$$

$$= \sum_{k=1}^{\infty} v\left(2^k\right) \cdot b \cdot 2^{-k}. \tag{19}$$

If $\lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 0.5$ *then Lemma 3 in Appendix A applies and* (19) *is larger than* $b \sum_{k=k_0}^{\infty} 2^{(1-\epsilon)k-k} = b \sum_{k=k_0}^{\infty} \left(2^{-\epsilon}\right)^k$ *for any* $\epsilon > 0$ *and sufficiently large* k_0. *Since we can choose* $\epsilon > 0$ *arbitrarily small, the CPT value of the St. Petersburg lottery,* $CPT(L_{STP})$, *is unbounded if* $\lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 0.5$. *Furthermore,* v *cannot be bounded from above. Together with the ratio test applied to* (19), *it follows that a necessary and sufficient condition for finite willingness to pay for* L_{STP} *is*

$$\lim_{z \to \infty} \frac{v(0.5z)}{v(z)} > \frac{1}{2}. \tag{20}$$

In other words, neo-additive probability weighting functions have the same implications for the St. Petersburg paradox as EUT despite the w*-ratio limit being equal to one. An obvious value function that now produces finite willingness to pay is* $v(x) = x^{0.88}$ *because* $\frac{v(0.5z)}{v(z)} = 0.5^{0.88} = 0.543$.

It is clear that the limit $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)}$ is in the interval $[0, 1]$. Intuitively, the limit of this w-ratio is an index of w's concavity at probability $\pi = 0$. More precisely, for sufficiently smooth weighting functions, Dierkes and Sejdiu (2019) show that it relates to a probabilistic counterpart of relative risk aversion at $\pi = 0$ as defined here.

Definition 1 *Let w be a probability weighting function which is twice continuously differentiable on a subset $(0, \pi_0)$, $\pi_0 \in (0,1)$, and strictly increasing. Then we define a probabilistic counterpart of relative risk aversion at infinitesimally small probabilities:*

$$RRA_w^0 = \lim_{\pi \to 0^+} \pi \frac{w''(\pi)}{w'(\pi)}. \tag{21}$$

Note that $RRA_w^0 > 0$ indicates probabilistic risk aversion and $RRA_w^0 < 0$ probabilistic risk proclivity when processing infinitesimally small probabilities. Using our previous definition, Dierkes and Sejdiu (2019) show that for smooth w and all $\Delta \in (0,1)$ it holds

$$\lim_{\pi \to 0^+} \frac{w(\Delta\pi)}{w(\pi)} = \Delta^{1+RRA_w^0}\ . \tag{22}$$

That is, the limit $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)}$ is informative about the curvature of w at $\pi = 0$. In particular, a higher w-ratio limit indicates more concavity of w at $\pi = 0$.

The discussion of the power value function $v_{Power}(x) = x^\alpha$ for $\alpha \in (0,1)$ is now rather simple and a particularly worthwhile example because it is by far the most frequently used parametrization for CPT's value function. Recall that v_{Power} exhibits constant relative risk aversion equal to $1 - \alpha$. There is now an intuitive interpretation for finite willingness to pay for the St. Petersburg lottery L_{STP}. Corollary 1 below shows that willingness to pay for L_{STP} is finite if and only if the decision maker exhibits strictly positive total relative risk aversion. Here, total relative risk aversion is the sum of probabilistic relative risk aversion RRA_w^0 induced by the probability weighting function and relative risk aversion of the value function as defined by the Arrow-Pratt measure. Loosely put, to produce a lower certainty equivalent than the expected value (infinity in this case) the decision maker must exhibit strictly positive risk aversion. In the CPT framework, risk aversion is driven by both the value and the probability weighting function. Conversely, with risk neutrality or risk proclivity, gambling for an infinite expected payoff is desirable and decision makers are willing to pay any amount.

Corollary 1 *Provided w is twice continuously right-differentiable at zero and strictly increasing and the value function is given by $v_{Power}(x) = x^\alpha$ with $\alpha > 0$ then the CPT decision maker has finite willingness to pay for the St. Petersburg lottery L_{STP} if and only if*

$$RRA_w^0 + RRA_v > 0, \tag{23}$$

where $RRA_v = -x\frac{v''(x)}{v'(x)} = 1 - \alpha$ is the constant relative risk aversion of the power value function v.

Proof: Note that $\frac{v(0.5z)}{v(z)} \in (0,1)$ for all $\alpha > 0$. According to Theorem 2, parts a) and c), it suffices to show that $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} < 0.5^\alpha$ is equivalent to Equation (23). Using $\Delta = 0.5$ in Equation (22), we get

$$\begin{aligned} & \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} && < 0.5^\alpha && (24)\\ \Leftrightarrow\quad & 0.5^{RRA_w^0+1} && < 0.5^{-RRA_v+1} && (25)\\ \Leftrightarrow\quad & RRA_w^0 && > -RRA_v && (26) \end{aligned}$$

which is equivalent to Equation (23). ■

To illustrate the applications of our findings, we discuss some typical parametrizations of v and w from the literature.

Example 2 *The limits of the w-ratio $\frac{w(0.5\pi)}{w(\pi)}$ for tiny probabilities for commonly employed probability weighting functions w are given as follows:*

a) For $w_{TK92}(\pi) = \pi^\gamma / (\pi^\gamma + (1-\pi)^\gamma)^{1/\gamma}$, $\gamma \in (0,1)$ proposed by Tversky and Kahneman (1992), we have $\lim_{\pi \to 0^+} \frac{w_{TK92}(0.5\pi)}{w_{TK92}(\pi)} = 0.5^\gamma$.

b) For $w_{log-odds}(\pi) = \delta\pi^\gamma / (\delta\pi^\gamma + (1-\pi)^\gamma)$, $\gamma \in (0,1)$, $\delta > 0$ *proposed by Goldstein and Einhorn (1987)*[7]*, we have* $\lim_{\pi \to 0^+} \frac{w_{log-odds}(0.5\pi)}{w_{log-odds}(\pi)} = 0.5^\gamma$.

c) For $w_{Prelec}(\pi) = e^{-(-\log\pi)^\gamma}$, $\gamma \in (0,1)$ *proposed by Prelec (1998), we have*

$$\lim_{\pi \to 0^+} \frac{w_{Prelec}(0.5\pi)}{w_{Prelec}(\pi)} = \begin{cases} 1 & \text{if } \gamma \in (0,1), \\ 0.5 & \text{if } \gamma = 1, \\ 0 & \text{if } \gamma > 1. \end{cases}$$

d) Consider polynomial probability weighting functions $w_{Poly}(\pi) = \sum_{i=1}^N a_i \cdot \pi^i$ *with parameters* $a_i \in \mathbb{R}$ *for* $i = 1, \ldots, N$ *and* $a_N \neq 0$*. Let* j *be the smallest index* $i \leq N$ *such that* $a_j \neq 0$*, i.e.* $w_{Poly}(\pi) = \sum_{i=j}^N a_i \cdot \pi^i$*. Then the limit is* $\lim_{\pi \to 0^+} \frac{w_{Poly}(0.5\pi)}{w_{Poly}(\pi)} = 0.5^j$.

The third degree polynomial weighting function $w_{RW06}(\pi) = \frac{3-3b}{a^2-a+1}(\pi^3 - (a+1)\pi^2 + a\pi) + \pi$ *with* $a, b \in (0,1)$ *proposed by Rieger and Wang (2006) is a special case with limit* $\lim_{\pi \to 0^+} \frac{w_{RW06}(0.5\pi)}{w_{RW06}(\pi)} = 0.5$ *because* $a, b \in (0,1)$ *imply* $a_1 \neq 0$.

Before we get to the crux of our paper, it is now easy to restate some selected results from the literature on the willingness to pay for the St. Petersburg lottery L_{STP} (see, e.g., Blavatskyy (2005) and Rieger and Wang (2006)):

Example 3 *Suppose the CPT decision maker exhibits a power value function* $v_{Power}(x) = x^\alpha$ *with* $\alpha \in (0,1)$*. Hence,* $\frac{v(0.5z)}{v(z)} = 0.5^\alpha$ *is independent of* z*. From Theorem 2 and Example 2, we know that:*

(i) With parametrization of the probability weighting function w *as in Tversky and Kahneman (1992) or Goldstein and Einhorn (1987), i.e.* $w_{TK92}(\pi) =$

[7] It is also used by e.g Tversky and Fox (1995), and Lattimore et al. (1992).

$\pi^\gamma / (\pi^\gamma + (1-\pi)^\gamma)^{1/\gamma}$ or $w_{log-odds}(\pi) = \delta\pi^\gamma / (\delta\pi^\gamma + (1-\pi)^\gamma)$, respectively, the willingness to pay for the St. Petersburg lottery L_{STP} is finite if and only if $\alpha < \gamma$.

(ii) With the Prelec (1998) parametrization $w_{Prelec}(\pi) = e^{-(-\log \pi)^\gamma}$, finite willingness to pay for the St. Petersburg lottery L_{STP} implies $\gamma \geq 1$. In other words, finite willingness to pay is not possible with the inverse S-shaped probability weighting function w_{Prelec}.

(iii) The CPT decision maker states finite willingness to pay for L_{STP} for all polynomial probability weighting functions because $0.5^\alpha > 0.5^j$ for all coefficient indices $j = 1, 2, \ldots$ as in Example 2, part d).

Our last example in this subsection is interesting because it explicates that a probability weighting function's finite slope at zero is not sufficient for finite willingness to pay in the St. Petersburg paradox.

Example 4 *Suppose the CPT decision maker exhibits Bell's (1988) one-switch function $v(x) = \beta x - e^{-\alpha x} + 1$ which is unbounded, strictly increasing, and strictly concave for $\alpha > 0$ and $\beta > 0$. Then,*

$$\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = \lim_{z\to\infty} \frac{\beta 0.5z - e^{-\alpha 0.5z} + 1}{\beta z - e^{-\alpha z} + 1} \overset{l'Hospital}{=} \lim_{z\to\infty} \frac{0.5\beta + 0.5\alpha e^{-\alpha 0.5z}}{\beta + \alpha e^{-\alpha z}} = 0.5.$$

Consider polynomial probability weighting functions $w_{Poly}(\pi) = \sum_{i=1}^N a_i \cdot \pi^i$ with parameters $a_i \in \mathbb{R}$ for $i = 1, \ldots, N$ and $a_N \neq 0$. Theorem 2, statement c) and Example 2 show that willingness to pay for L_{STP} is finite if and only if $a_1 = 0$.

2.2 The common ratio effect under CPT

The Allais paradox is a traditional counterexample against EUT and comes along in different versions. We focus on the common ratio version which involves choices between equal mean lotteries[8]

$$L_1(\pi) = (\$z, 0.5\pi; \$0, 1-0.5\pi) \quad \text{and} \quad L_2(\pi) = (\$0.5z, \pi; \$0, 1-\pi) \tag{27}$$

where $z > 0$ denotes a payoff amount and π a probability. Empirically, subjects choose the safer lottery L_2 for high probabilities π and the riskier lottery L_1 for low probabilities π.[9] EUT's independence axiom, however, does not allow for this change in preference over L_1 and L_2 for varying probabilities π.

By introducing probability weighting, CPT is able to explain this choice behavior. In the CPT framework, a risk seeking choice is predicted when

$$CPT(L_1) > CPT(L_2)$$

$$\Leftrightarrow \qquad v(z) \cdot w(0.5\pi) + v(0) \cdot [1 - w(0.5\pi)] > v(0.5z) \cdot w(\pi) + v(0) \cdot [1 - w(\pi)] \tag{28}$$

$$\Leftrightarrow \qquad \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z) - v(0)}{v(z) - v(0)} > 0 \tag{29}$$

and a risk averse choice results vice versa. Using Assumption 2 whereby $v(0) = 0$, we define the common ratio effect with the help of Equation (29).

Definition 2 (Common ratio effect) *Let v be a value function and w be a probability weighting function. We say that, in this CPT framework with equal mean*

[8]More generally, the results of this section easily extend to lotteries $L_1(\pi) = (\$z, \Delta\pi; \$0, 1-\Delta\pi)$ and $L_2(\pi) = (\$\Delta z, \pi; \$0, 1-\pi)$ with $\Delta \in (0, 1)$. However, to analyze the conflict with the St. Petersburg paradox, we focus on lotteries with payoff and probability ratio of $\Delta = 0.5$ which is a common choice in experiments.

[9]Lottery $L_1(\pi)$ is called riskier than the lottery $L_2(\pi)$ because $L_1(\pi)$ is a mean-preserving spread of $L_2(\pi)$.

lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ *and* $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$ *with payoff* $z > 0$*, the common ratio effect is predicted if and only if there exists exactly one sign change in function*

$$f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}, \tag{30}$$

that is, there exists exactly one preference reversal probability π^* *such that* $f_{CRE}(\pi) > 0$ *for* $\pi \in (0, \pi^*)$ *and* $f_{CRE}(\pi) < 0$ *for* $\pi \in (\pi^*, 1)$.

This definition already foreshadows the conflict between the common ratio effect and the necessary conditions for finite willingness to pay for the St. Petersburg Lottery as stated in Theorem 2. This definition is also a little stricter than we need for our purposes. We can relax the assumption of a single preference reversal probability as long as there exists a probability π^* such that $f_{CRE}(\pi) > 0$ for $\pi \in (0, \pi^*)$. Put differently, we need to assume that, for sufficiently small probabilities π, the decision maker chooses the riskier lottery which we find is an intuitive criterion. Nevertheless, multiple preference reversal probabilities appear awkward as they would imply, at odds with lab results, rather erratic behavior.

The following proposition is a trivial consequence of the previous definition and explicitly states necessary conditions for the common ratio effect.

Proposition 1 (Emergence of the common ratio effect) *Let* v *be a value function and* w *be a probability weighting function. Consider the equal mean lotteries* $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ *and* $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$*. Then it holds:*

a) For fixed payoff z*, one necessary condition*[10] *for the prediction of the common*

[10] There are more necessary conditions, of course. This one, however, unveils the conflict with finite willingness to pay for the St. Petersburg lottery L_{STP}. Another obvious necessary condition is $\lim_{\pi \to 1} \frac{w(0.5\pi)}{w(\pi)} = w(0.5) \leq \frac{v(0.5z)}{v(z)}$.

ratio effect as defined in Definition 2 is

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \geq \frac{v(0.5z)}{v(z)}. \qquad (CRE)$$

b) Allais (1953) suggests that statement a) holds for all sufficiently large payoffs z, in particular large payoffs. This leads to the necessary condition:

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \geq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \qquad (CRE^*)$$

Clearly, if the v-ratio $\frac{v(0.5z)}{v(z)}$ is independent of the payoff z then the two necessary conditions CRE and CRE^* coincide. This is the case for the power value function $v_{Power}(x) = x^\alpha$, $\alpha \in (0, 1)$ which is the, by far, most often employed parametrization of the value function. Further, the necessary conditions CRE^* in Proposition 1 and STP^* in Theorem 2, respectively, leave at best a corner solution for many CPT calibrations. Put together, both restrictions require for continuous w

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \qquad (31)$$

However, statement c) in Theorem 2 rules out such cases where

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z \to \infty} \frac{v(0.5z)}{v(z)} \in (0, 1). \qquad (32)$$

In particular, using a power value function v_{Power} for which always $\lim_{z \to \infty} v(0.5z)/v(z) \in (0, 1)$, will always result in a strict conflict between the restrictions on the St. Petersburg and the Allais paradox if the probability weighting function is continuous.

The remaining case $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 1$ is ruled out by the following proposition.

Proposition 2 *Let w be a strictly increasing and continuous probability weighting function and v be a strictly increasing value function with $v(0) = 0$. If $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$ then the common ratio effect does not emerge for large payoffs z in lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1-0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1-\pi)$. Put differently, by increasing z, the preference reversal probability π^*, given by the intersection of function $f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}$ with the abscissa, can be moved arbitrarily close to zero if it exists at all.*

Proof: Since $0 \leq \frac{w(0.5\pi)}{w(\pi)} \leq 1$ for all probabilities π and $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$, the function $f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}$ is, for $\pi \to 0^+$, either negative or arbitrarily close to zero for sufficiently high payoffs z. That is, the intersection probability π^*, also denoted the preference reversal probability, moves for large z arbitrarily close to zero if it exists at all. ■

Bounded value functions always imply finite willingness to pay for the St. Petersburg lottery. However, the following corollary shows that they have difficulties predicting the Allais paradox because bounded value functions always have a v-ratio limit equal to one.

Corollary 2 *Let v be a bounded and strictly increasing value function with $v(0) = 0$. Then the common ratio effect does not emerge for large payoffs z in lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1-0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1-\pi)$ because, by increasing z, the preference reversal probability π^*, given by the intersection of function $f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}$ with the abscissa, can be moved arbitrarily close to zero if it exists at all.*

Proof: For bounded strictly increasing value functions v with $v(0) = 0$, it holds:

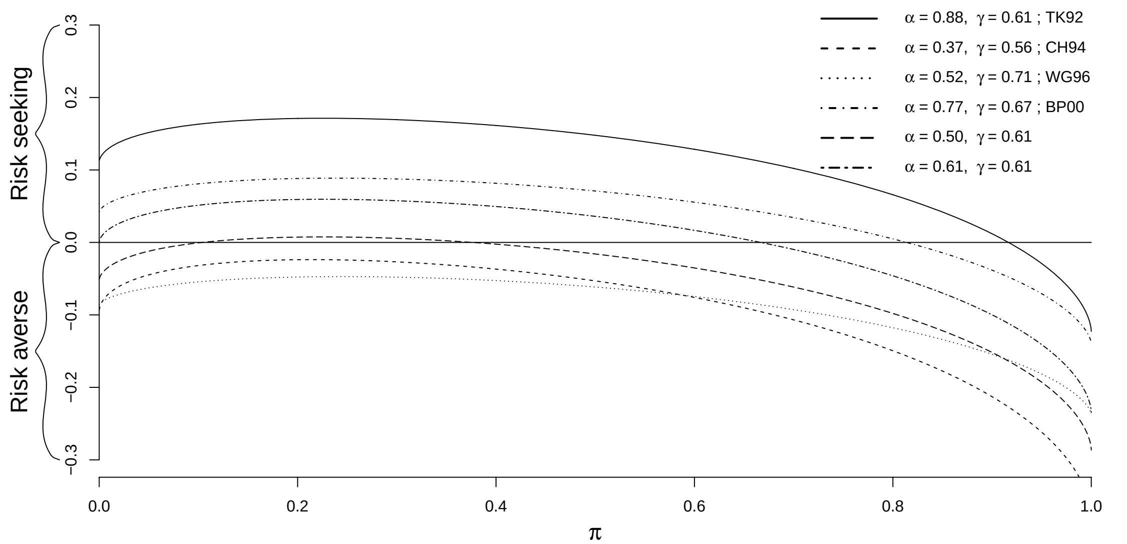
$\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$. Assume $\lim_{x\to\infty} v(x) = b$ for some upper bound $b > 0$. Hence, $\frac{v(0.5z)}{v(z)} \underset{z\to\infty}{\longrightarrow} \frac{b}{b} = 1$. Then, the statement follows from Proposition 2. ■

Before we summarize these findings in our main Theorem 4 and discuss further the, by now clear, tension between the two presumably most prominent paradoxes in decision theory, we illustrate implications of Definition 2 with Figure 1. This illustration is worthwhile for several reasons. First, some calibrations from the literature do not predict the common ratio effect while others do. Second, we show that several preference reversal points are theoretically possible. Third, it nicely hints at the role of Proposition 2 although we use the power value function. Fourth, it indicates similarities and differences between the weighting functions w_{TK92}, $w_{log-odds}$, w_{Prelec}, w_{Poly}, and w_{neo}.

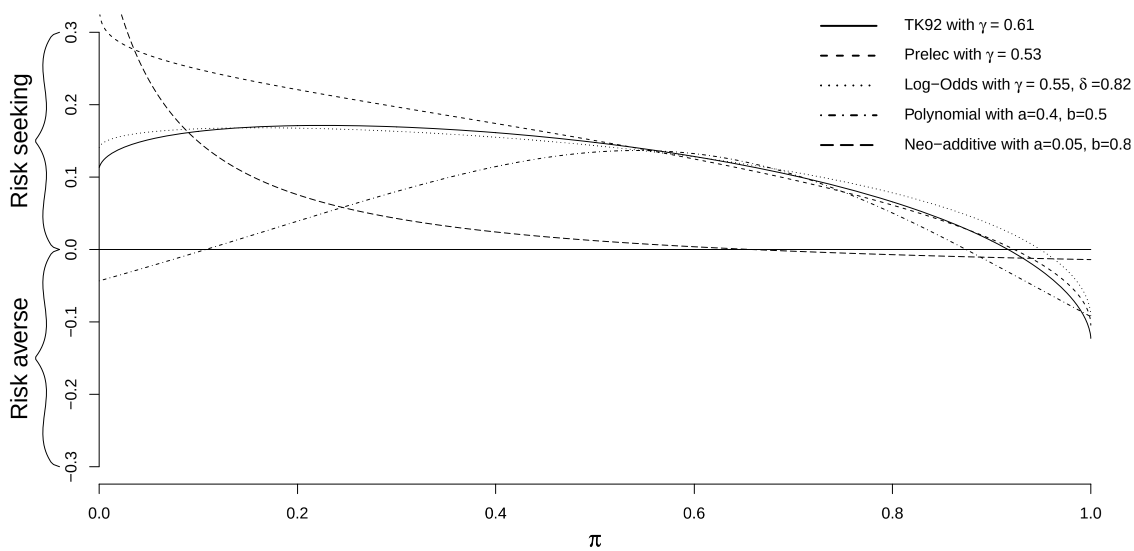
Figure 1 depicts function f_{CRE} as defined in Definition 2. We focus on Tversky and Kahneman's (1992) CPT parametrization in Panel A and vary the specification of the probability weighting function in Panel B. Since we assume the power value function $v_{Power}(x) = x^{\alpha}$, no assumption about the lottery payoffs z is needed. Specifically, Panel A depicts the following function:

$$f_{TK92}(\pi) = 0.5^{\gamma} \cdot \left[\frac{\pi^{\gamma} + (1-\pi)^{\gamma}}{(0.5\pi)^{\gamma} + (1-0.5\pi)^{\gamma}}\right]^{\frac{1}{\gamma}} - 0.5^{\alpha}. \tag{33}$$

Positive function values of (33) indicate risk seeking behavior and negative values risk averse behavior. The black solid line depicts an individual's risk attitude when assuming Tversky and Kahneman's (1992) suggested median parameters $\alpha = 0.88$ and $\gamma = 0.61$. In line with Definition 2, a risk seeking choice is predicted for $\pi < 0.91$ and a risk averse choice otherwise. Estimates $(\alpha, \gamma) = (0.77, 0.67)$, taken from Bleichrodt and Pinto (2000), make similar predictions. The decision maker behaves risk averse roughly for probabilities $\pi > 0.81$ and risk seeking otherwise. Interestingly, parameter



Panel A: Common ratio effect for different parameter sets (α, γ) of Tversky and Kahneman's (1992) CPT parametrization.



Panel B: Common ratio effect for various probability weighting functions.

Figure 1: Common ratio effect under CPT. This figure illustrates the common ratio effect under various parametrizations of CPT by depicting the function $f_{CRE}(\pi) = w(0.5\pi)/w(\pi) - v(0.5z)/v(z)$ as a function of probability π. The decision maker has the choice between the equal mean lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$. For positive function values f_{CRE}, she behaves risk seeking and prefers the riskier lottery L_1 over the safer lottery L_2. Conversely, for negative values f_{CRE}, she behaves risk averse and prefers L_2 over L_1. In Panel A, the individual's preferences are given by the value function $v(x) = x^{\alpha}$ and the weighting function w_{TK92} for different parameter sets (α, γ) including those estimated in Tversky and Kahneman (1992), Camerer and Ho (1994), Wu and Gonzalez (1996), and Bleichrodt and Pinto (2000) denoted in the legend by TK92, CH94, WG96, and BP00, respectively. In Panel B, the value function is fixed as $v(x) = x^{0.88}$ (Tversky and Kahneman, 1992) and the weighting function takes the forms w_{TK92}, $w_{log-odds}$, w_{Prelec}, w_{RW06}, and w_{neo}. Due to the form of the value function, no assumption about z is needed. Function values of f_{CRE} are depicted for weighting functions with parameter estimates of Tversky and Kahneman (1992) for TK92, estimates of Bleichrodt and Pinto (2000) for Prelec and Log-odds, parameter values motivated by Rieger and Wang (2006) for a cubic weighting function, and the neo-additive weighting function with intercept $a = 0.05$ and slope $b = 0.8$, respectively.

estimations by Camerer and Ho (1994) and Wu and Gonzalez (1996) who estimate the parameter sets $(\alpha, \gamma) = (0.37, 0.56)$ and $(\alpha, \gamma) = (0.50, 0.71)$, respectively, uniformly display risk aversion for all probabilities π and, hence, do not explain the common ratio effect.[11] However, as argued in Section 2.1, these parameter combinations are in line with finite willingness to pay for the St. Petersburg lottery because $\alpha < \gamma$.

In addition, we plot two illustrative pairs of preference parameters. Cases where $\alpha = \gamma < 1$ are interesting in the Tversky and Kahneman (1992) parametrization because they robustly predict a single preference reversal point with risk prone behavior for low probabilities π (positive f_{CRE}) and risk averse behavior for larger probabilities π (negative f_{CRE}), but the limiting case in Equation (33) yields $\lim_{\pi\to 0} f_{CRE}(\pi) = 0$. Panel A illustrates the case $\alpha = \gamma = 0.61$.

Interestingly, the pair $\alpha = 0.5$ and $\gamma = 0.61$ has two preference reversal points, yielding risk prone behavior for intermediate probabilities between 0.1 and 0.38 and risk averse behavior otherwise. Such cases are bad empirical predictors because real decision makers exhibit a single preference reversal point with lower probabilities typically leading to more risk prone choices.

Panel B of Figure 1 illustrates that this problem also arises for weighting functions with finite slope at zero such as the cubic weighting function $w_{RW06}(\pi) = \frac{3-3b}{a^2-a+1}\left(\pi^3 - (a+1)\pi^2 + a\pi\right) + \pi$ of Rieger and Wang (2006), see also Proposition 3 and Example 5 below. In the context of our paper, the failure of the polynomial weighting function w_{RW06} to explain the common ratio effect is particularly interesting since it was primarily designed to explain the St. Petersburg paradox (Rieger and

[11] Neilson and Stowe (2002) note that the parameter estimates of Camerer and Ho (1994) and Wu and Gonzalez (1996) are unable to predict gambling on unlikely gains and the choice behavior of Allais' original common consequence example. In contrast to Neilson and Stowe (2002), we do not call for new parametrizations of CPT. Our Allais - St. Petersburg test evaluates CPT in its most general form and does not rely on specific parametrizations of the value and probability weighting function.

Wang, 2006).

Moreover, Panel B fixes the value function as $v(x) = x^{0.88}$ and shows that the image of f_{CRE} is for the approximate range $0.56 \leq \pi \leq 1$ very similar for all four continuous probability weighting functions w_{TK92}, $w_{log-odds}$, w_{Prelec}, and w_{RW06} with standard parameter values but significantly different for lower probabilities $0 \leq \pi \leq 0.56$. While w_{TK92} and $w_{log-odds}$ seem to treat all probabilities very similar, w_{Prelec} and w_{RW06} go into the opposite direction when probabilities get smaller. This discrepancy nicely foreshadows the conflict between the conditions of the Allais and St. Petersburg paradox since both weighting functions are motivated by one paradox, respectively. Prelec (1998) motivates his probability weighting function $w_{Prelec}(\pi) = e^{-(-\log \pi)^{\gamma}}$ with Allais' common ratio effect. Similarly, the discontinuous neo-additive weighting function results in a strictly decreasing function f_{CRE} and predicts the common ratio effect. Figure 1 also shows that f_{CRE} does not need be a monotone function in π. Monotonicity is guaranteed, however, if we use a subproportional probability weighting function such as w_{Prelec} (Prelec, 1998) or neo-additive ones.

The next proposition shows that a combination of a probability weighting function with finite slope at zero and a strictly concave value function cannot explain the common ratio effect.

Proposition 3 *Let the value function v be strictly concave and let the probability weighting function w be right-differentiable around zero and the slope of w at zero be finite, i.e. $w'(0^+) = c$ with $0 \leq c < \infty$. Then, $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \leq 0.5$ and the CPT decision maker always prefers the safer lottery $L_2(\pi) = (\$0.5z, \pi; \$0, 1-\pi)$ over $L_1(\pi) = (\$z, 0.5\pi; \$0, 1-0.5\pi)$ for any payoff $z > 0$ when the probabilities π and 0.5π tend to zero. In particular, the common ratio effect does not emerge.*

Proof: If w is monotonically increasing with $w(0) = 0$, $w(1) = 1$ and $w'(0^+) = c$,

$0 < c < \infty$, then applying the rule of l'Hopital directly shows that

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \overset{l'Hopital}{=} \lim_{\pi \to 0^+} \frac{w'(0.5\pi) \cdot 0.5}{w'(\pi)} = \frac{w'(0) \cdot 0.5}{w'(0)} = 0.5 \ . \tag{34}$$

For $w'(0) = 0$, the general monotonicity assumption of w ensures that there exists a $\pi_0 \in (0, 1]$ such that w is convex for $\pi \in [0, \pi_0]$, i.e. $w(0.5\pi) \leq 0.5w(\pi)$.

Since v is strictly concave it follows that $0.5 < \frac{v(0.5z)}{v(z)} < 1$ for any $z > 0$ and

$$\lim_{\pi \to 0^+} f_{CRE}(\pi) = \underbrace{\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)}}_{\leq 0.5} - \underbrace{\frac{v(0.5z)}{v(z)}}_{>0.5} < 0 \tag{35}$$

which is equivalent to $L_2 \succ L_1$. This, however, contradicts the empirical evidence of the common ratio effect and, hence, Definition 2. ■

In light of Example 1, the next proposition shows that neo-additive weighting functions are a prime candidate to solve both paradoxes.

Proposition 4 *Let w be the neo-additive probability weighting function with intercept a and slope b with $a, b > 0$ and $a + b \leq 1$. Then the common ratio effect emerges between Lotteries L_1 and L_2 with payoffs determined by z if and only if*

$$\frac{a + 0.5b}{a + b} < \frac{v(0.5z)}{v(z)} < 1. \tag{36}$$

Proof: Observe that the function $f_{CRE}(\pi) = \frac{w_{neo}(0.5\pi)}{w_{neo}(\pi)} - \frac{v(0.5z)}{v(z)}$ is strictly decreasing in π:

$$\frac{\partial f_{CRE}}{\partial \pi} = -\frac{0.5 \cdot a \cdot b}{(a + b \cdot \pi)^2} < 0 \quad \forall \ \pi \in (0, 1) \text{ if } a, b > 0. \tag{37}$$

Since $\lim_{\pi \to 0^+} f_{CRE}(\pi) = 1 - \frac{v(0.5z)}{v(z)}$ and $\lim_{\pi \to 1} f_{CRE}(\pi) = \frac{a+0.5b}{a+b} - \frac{v(0.5z)}{v(z)}$, the common ratio effect then emerges if and only if the conditions in the proposition are fulfilled.

■

The class of neo-additive weighting functions is the simplest class of weighting functions that allows a solution to both paradoxes. In Subsection 2.3, we will show how to extend the class of weighting functions so as to accommodate more complex choice behavior for intermediate probabilities (Harless and Camerer, 1994; Wu and Gonzalez, 1996). We conclude this subsection with two examples.

Example 5 *Let the value function v be strictly concave. With polynomial probability weighting functions $w_{Poly}(\pi) = \sum_{i=1}^{N} a_i \cdot \pi^i$ with parameters $a_i \in \mathbb{R}$ for $i = 1, \ldots, N$ and $a_N \neq 0$, the choice behavior in the common ratio effect according to Definition 2 cannot be predicted. To see this, note that it holds $0 \leq w'_{Poly}(0) < \infty$. Then, Proposition 3 applies.*

Example 6 *Consider $w_{Prelec}(\pi) = e^{-(-\log \pi)^\gamma}$ proposed by Prelec (1998). It is subproportional and $\lim\limits_{\pi \to 0^+} \frac{w_{Prelec}(0.5\pi)}{w_{Prelec}(\pi)} = 1$ for $\gamma \in (0, 1)$, see Example 2. Assume the value function is given by $v_{Log}(x) = \log(1 + x)$. Then, $\lim\limits_{\pi \to 0^+} \frac{w_{Prelec}(0.5\pi)}{w_{Prelec}(\pi)} = \lim\limits_{z \to \infty} \frac{v_{Log}(0.5z)}{v_{Log}(z)} = 1$ is a corner case for the necessary conditions (CRE^*) and (STP^*). However, the empirically observed common ratio effect cannot be predicted because of Proposition 2. Intuitively, by using ever larger payoffs z in the common ratio lotteries L_1 and L_2, as is supported by experimental evidence in Allais (1953), we can move the preference reversal probability π^* arbitrarily close to zero.*

Interestingly, Example 8 in Appendix B shows that the combination of v_{Log} and w_{Prelec} produces finite willingness to pay for the St. Petersburg lottery. Theoretically then, smaller payoffs z might offer a solution to both paradoxes with v_{Log} and w_{Prelec}. However, a sensitivity analysis in Appendix B unveils for various combinations of v and continuous w that only unreasonably small z would do the trick.

2.3 Summary: Allais - St. Petersburg conflict in CPT

We summarize our findings in the following theorems which distinguish between discontinuous and continuous probability weighting functions. Specifically, in the latter case of continuous weighting functions, no solution exists in particular for large payoffs. In the former case of discontinuous weighting functions, the class of neo-additive weighting functions opens the door for a broader class of weighting functions, including e.g. Kahneman and Tversky's (1979) weighting function depicted in their Figure 4. Such weighting functions can additionally accommodate more complex choice behavior as found, for example, by Harless and Camerer (1994) than neo-additive functions which are linear for probabilities in $(0, 1)$.

In the following, we shall include explicit statements about the power value function because of its predominant use in the literature although they constitute a trivial corollary of more general results. Cases where for fixed payoff z and $z/2$ in lotteries L_1 and L_2, respectively, it holds $\frac{v(0.5z)}{v(z)} < \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z \to \infty} \frac{v(0.5z)}{v(z)} = 1$ are not covered here. They might theoretically allow for finite willingness to pay for the St. Petersburg lottery L_{STP} and the emergence of the common ratio effect for small or maybe even moderately large payoffs z. A sensitivity analysis in Appendix B, however, rules out any realistic cases for specific parametrizations.

Theorem 3 (Simultaneous solution to both paradoxes) *Let w_{neo} be defined by*

$$w(\pi) = \begin{cases} 0 & \textit{for } \pi = 0 \\ a + b \cdot \pi & \textit{for } \pi \in (0, 1) \\ 1 & \textit{for } \pi = 1 \end{cases}, \tag{38}$$

with $a, b > 0$ and $a + b \leq 1$. Let further v be a continuous and strictly increasing value function. The common ratio lotteries are given by $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and

$L_2(\pi) = (\$0.5z, \pi; \$0, 1-\pi)$ *with* $z > 0$. *Then it holds:*

a) Let the probability weighting function be given by w_{neo}. *Assume the decision maker strictly prefers the risky lottery* L_1 *over* L_2 *for probabilities near zero and the safe lottery* L_2 *over* L_1 *for probabilities near one for all sufficiently high payoffs* z. *This is equivalent to* $\frac{a+0.5b}{a+b} < \frac{v(0.5z)}{v(z)} < 1$ *for all payoffs* $z \geq z_0$ *for some* $z_0 > 0$. *Furthermore, in those cases, the decision maker states finite willingness to pay for playing the St. Petersburg lottery* L_{STP}.

In other words, $\frac{a+0.5b}{a+b} < \frac{v(0.5z)}{v(z)} < 1$ *for all payoffs* $z \geq z_0$ *for some* $z_0 > 0$ *is equivalent to the simultaneous solution of both the St. Petersburg paradox and the common ratio version of the Allais paradox for all sufficiently large payoffs in the common ratio lotteries.*

b) Assume the probability weighting function is given by $w(\pi) = w_{neo} \circ w_{cont.}(\pi) = w_{neo}(w_{cont.}(\pi))$, *where* $w_{cont.}$ *is a continuous and strictly increasing probability weighting function. If the decision maker states finite willingness to pay for playing the St. Petersburg lottery* L_{STP} *and strictly prefers the risky lottery* L_1 *over* L_2 *for probabilities near zero and the safe lottery* L_2 *over* L_1 *for probabilities near one for all sufficiently high payoffs* $z \geq z_0$, $z_0 > 0$ *then (necessary conditions) for all* $z \geq z_0$ *it holds*

$$\frac{a + bw_{cont.}(0.5)}{a+b} < \frac{v(0.5z)}{v(z)}, \tag{39}$$

$$1 > \frac{v(0.5z)}{v(z)}, \tag{40}$$

$$\lim_{\pi \to 0^+} \frac{w_{cont.}(0.5\pi)}{w_{cont.}(\pi)} \leq \lim_{z \to \infty} \frac{v(0.5z)}{v(z)}. \tag{41}$$

c) Assume that, in the situation of part b), all inequalities hold strictly and the *function* $f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}$ *has, for every fixed* z*, a single intersection point* $\pi^* \in (0,1)$ *with the abscissa, that is, there is a single preference reversal probability* π^* *between the common ratio lotteries* L_1 *and* L_2*. Then, both Allais' common ratio effect for all fixed payoffs* $z(\geq z_0)$ *and a finite willingness to pay for the St. Petersburg lottery emerge.*

Proof: We start by proving a). The common ratio effect for all payoffs $z \geq z_0$ for some $z_0 > 0$ is, by Proposition 4, equivalent to $1 > v(0.5z)/v(z) > \frac{a+0.5b}{a+b}$ for all $z \geq z_0$. In those cases, it holds that $1 \geq \lim\limits_{z\to\infty} \frac{v(z)}{v(0.5z)} \geq \frac{a+0.5b}{a+b} > 0.5$ because $\frac{a+0.5b}{a+b} > 0.5$ for $a > 0$. From Example 1, then, $\lim\limits_{z\to\infty} \frac{v(z)}{v(0.5z)} > 0.5$ is equivalent to the finite willingness to pay for playing L_{STP}.

For statement b), similar arguments as for Proposition 4 show that the common ratio effect implies $\frac{a+bw_{cont.}(0.5)}{a+b} < v(0.5z)/v(z) < 1$ which proves Equations (39) and (40). Theorem 1 gives a necessary condition for convergence of the CPT value for the St. Petersburg lottery as follows:

$$\lim_{z\to\infty} \frac{v\left(z\right)}{v\left(0.5z\right)} \cdot \lim_{\pi\to 0^+} \left| \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \right| \leq 1. \tag{42}$$

After substituting $w(\pi) = w_{neo}(w_{cont.}(\pi))$ we get

$$\lim_{z\to\infty} \frac{v\left(z\right)}{v\left(0.5z\right)} \cdot \lim_{\pi\to 0^+} \left| \frac{w_{cont.}(0.5\pi) - w_{cont.}(0.25\pi)}{w_{cont.}(\pi) - w_{cont.}(0.5\pi)} \right| \leq 1 \tag{43}$$

which, given the continuity of $w_{cont.}$ and Lemma 1 in Appendix A, leads to

$$\lim_{\pi\to 0^+} \frac{w_{cont.}(0.5\pi)}{w_{cont.}(\pi)} \leq \lim_{z\to\infty} \frac{v(0.5z)}{v(z)}. \tag{44}$$

This last equation is the same as Equation (41).

Statement c) is clear because the first two Inequalities (39) and (40) guarantee the common ratio effect with a single preference reversal point and the previous arguments with the ratio test show that the strict version of the last Inequality (41) is sufficient for the solution of the St. Petersburg paradox (see also Theorem 1). ■

Note that although Prelec's (1998) probability weighting function itself is a prime candidate to predict the common ratio effect it is likely less successful to also predict the St. Petersburg paradox when combined with the neo-additive probability weighting function because of condition (41).

We provide two simple examples for a solution to both paradoxes.

Example 7 *a) Let the value function be given by $v(x) = x^\alpha$ with $\alpha = 0.7$ and the neo-additive probability weighting function be given by $a = 0.1$ and $b = 0.8$. Then, the certainty equivalent or, equivalently, the willingness to pay for the St. Petersburg lottery equals*

$$CE(L_{STP}) := v^{-1}\left(\sum_{k=1}^{\infty} v\left(2^k\right)\cdot\left[w\left(2^{1-k}\right) - w\left(2^{-k}\right)\right]\right) \tag{45}$$

$$= \left[\sum_{k=1}^{\infty} 2^{\alpha k}\cdot b\cdot 2^{-k}\right]^{\frac{1}{\alpha}} \tag{46}$$

$$= \left[b\cdot\sum_{k=1}^{\infty}\left(0.5^{1-\alpha}\right)^k\right]^{\frac{1}{\alpha}} \tag{47}$$

$$\overset{\alpha\leq 1}{=} \left[\frac{b}{2^{1-\alpha}-1}\right]^{\frac{1}{\alpha}} = 5.892. \tag{48}$$

It it also clear from Theorem 3, part a), that the common ratio effect is predicted because $\frac{a+0.5b}{a+b} = \frac{5}{9} < \frac{v(0.5z)}{v(z)} = 0.5^\alpha = 0.616 < 1$. The preference reversal *probability is given by*

$$\frac{a + b \cdot 0.5\pi^*}{a + b \cdot \pi^*} \stackrel{!}{=} 0.5^\alpha \tag{49}$$

$$\Leftrightarrow \qquad \pi^* = \frac{a\,(1 - 0.5^\alpha)}{b\,(0.5^\alpha - 0.5)} = 0.416 \tag{50}$$

and is well in line with ranges proposed by Kahneman and Tversky (1979) and Starmer and Sugden (1989).

b) *Let the value function be given by $v(x) = x^\alpha$, $\alpha > 0$, and the probability weighting function be given by $w(\pi) = w_{neo} \circ w_{cont.}(\pi) = w_{neo}(w_{cont.}(\pi))$ where the neo-additive weighting function w_{neo} is given by intercept $a > 0$ and slope $b > 0$, $a + b < 1$, and the continuous weighting function $w_{cont.}(\pi) = \pi^\gamma$, $\gamma > 0$.*

Then, Inequality (40) is fulfilled because $\frac{v(0.5z)}{v(z)} = 0.5^\alpha < 1\ \forall \alpha > 0$; Inequality (41) holds strictly if $\alpha < \gamma$. Function $f_{CRE}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \frac{v(0.5z)}{v(z)}$ is strictly decreasing in π since its derivative is negative for all $\pi \in (0,1)$:

$$\frac{\partial f_{CRE}}{\partial \pi} = -\frac{ab\gamma\,(1 - 0.5^\gamma)\,\pi^{\gamma-1}}{(a + b\pi^\gamma)^2} < 0 \tag{51}$$

and Inequality (39) then ensures a single intersection with the abscissa.

Specifically, for $\alpha = 0.88$, $a = 0.1$, $b = 0.8$, and $\gamma = 2$, the weighting function $w = w_{neo} \circ w_{cont.}$ resembles Kahneman and Tversky's (1979) discontinuous weighting function and Equation (39) is fulfilled because $\frac{0.1+0.8\cdot 0.5^2}{0.1+0.8} = \frac{1}{3} < 0.5^{0.88} = 0.543$. The preference reversal probability equals

$$\pi^* = \left[\frac{a\,(1 - 0.5^\alpha)}{b\,(0.5^\alpha - 0.5^\gamma)}\right]^{\frac{1}{\gamma}} = 0.441. \tag{52}$$

The certainty equivalent of the St. Petersburg lottery is

$$CE(L_{STP}) = \left[\sum_{k=1}^{\infty} 2^{\alpha k} \cdot b \cdot 2^{-\gamma k} \cdot (2^{\gamma} - 1)\right]^{\frac{1}{\alpha}} \tag{53}$$

$$= \left[b(2^{\gamma} - 1) \cdot \sum_{k=1}^{\infty} \left(0.5^{\gamma-\alpha}\right)^{k}\right]^{\frac{1}{\alpha}} \tag{54}$$

$$= \left[\frac{b(2^{\gamma} - 1)}{2^{\gamma-\alpha} - 1}\right]^{1/\alpha} = 2.255. \tag{55}$$

However, when preferences are given by continuous functions, no simultaneous solution to both paradoxes exists:

Theorem 4 (Continuity and the conflict between both paradoxes) *Assume decision makers behave according to CPT with continuous and strictly increasing value function v and continuous and strictly increasing probability weighting function w. The common ratio lotteries are given by $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$ with $z > 0$.*

- a) *Assume the common ratio effect shows up for all payoffs z, in particular for large payoffs z as argued by Allais (1953). Then, there does not exist a simultaneous solution to both the St. Petersburg paradox and the common ratio effect.*

- b) *Assume $v(x) = x^{\alpha}$ with $\alpha > 0$. Then, there does not exist a simultaneous solution to both the St. Petersburg paradox and the common ratio effect.*

Proof: We start by proving statement a). Any solution to both paradoxes requires that $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)}$ because of Theorem 2, part b), and Proposition 1. Now, two cases can occur:

1) If $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} \in (0, 1)$ then the decision maker always states infinite willingness to pay for the St. Petersburg lottery L_{STP} (see Theorem 2, statement

c)). This is at odds with the empirical observation whereby decision makers report finite willingness to pay.

2) If $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$ then, by increasing the payoff z in lotteries L_1 and L_2, we can move the preference reversal probability π^* arbitrarily close to zero which rules out a solution to the common ratio effect. This is proven in Proposition 2.

The case $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 0$ can be ruled out. Observe that a risk averse choice in the common ratio effect for large probabilities ($\pi \to 1$) implies the necessary condition $\lim_{\pi\to 1} \frac{w(0.5\pi)}{w(\pi)} = w(0.5) \leq \lim_{z\to\infty} \frac{v(0.5z)}{v(z)}$. Since w is strictly increasing and $w(0) = 0$, this necessary condition can never be fulfilled when $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 0$.

Statement b) is a trivial corollary of case 1) because the v-ratio $\frac{v(0.5z)}{v(z)}$ equals $0.5^\alpha \in (0, 1)$ independent of payoff z. ∎

Admittedly, allowing payoffs z and $0.5z$ in lotteries L_1 and L_2, respectively, to grow infinitely large in Condition (CRE^*) is extreme. In special cases, such as the power value function, this is not necessary because then the v-ratio $\frac{v(0.5z)}{v(z)}$ is independent of z. Here, the conflict between both paradoxes appears directly. For other parametrizations of the value function, calibration exercises in Appendix B unveil that already moderately large payoffs z run counter real world decision makers' behavior.

Note also that the conflict between finite willingness to pay in the St. Petersburg paradox and choice behavior as in the common ratio effect is not a simple artifact of the preference reversal phenomenon, reported in e.g. Lichtenstein and Slovic (1971). The preference reversal phenomenon describes the puzzling fact that while subjects choose lottery A over lottery B they simultaneously state higher certainty equivalents for lottery B than for lottery A. Accounting for preference reversal effects, however, makes the conflict between both paradoxes even more severe. To see this, note that many studies of the preference reversal phenomenon involve mean-preserving spread lotteries, similar to our L_1 and L_2. Empirically, when subjects choose between L_1 and L_2, they have a tendency to exhibit a more risk averse choice behavior going for L_2 than their willingness to pay suggests. Suppose, for given payoff z, choices indicate indifference between L_1 and L_2 for some probability π^*. Then, using the same π^* and switching from a choice to a pricing task, i.e asking for certainty equivalents, would typically indicate a preference for the riskier lottery L_1 according the preference reversal effect. Tversky et al. (1990) attribute this to more extreme overweighting of small probabilities in pricing tasks. In the Tversky and Kahneman (1992) framework, for example, this would be equivalent to an even lower probability weighting parameter $\gamma_{\text{Certainty Equivalent}}$ than revealed by previous choices, that is $\gamma_{\text{Certainty Equivalent}} < \gamma_{\text{Choice}}$. But quite to the contrary, the St. Petersburg paradox, which also involves a pricing task by stating a certainty equivalent, requires a *higher* curvature parameter γ for finite willingness to pay.

Finally, truncating the St. Petersburg lottery does not help much for typical parametrizations. Appendix C analyzes various specifications with truncation levels up to a payment of $\$2^{40}$, that is, roughly one trillion dollar. The essence of our previous conclusions remains unchanged.

3 Conclusion

It is striking that so many textbooks on decision theory start by outlining the St. Petersburg paradox and the Allais paradox when motivating EUT and CPT, respectively. However, a joint consideration of both paradoxes without parametric assumptions on preference functions has, to the best of our knowledge, never been done before. Since CPT is widely accepted as the gold standard of descriptive theories of decision making under risk and uncertainty, we study a potential discrepancy between these two paradoxes within the framework of CPT. Our results can be extended to other theories of decision making under risk with similar additively separable utility across states.

The main result of our paper is that CPT with continuous preference functions is not able to simultaneously explain the two most prominent paradoxes in decision making under risk – the St. Petersburg paradox and the Allais paradox. All attempts to solve to the Allais - St. Petersburg conflict by changing the parametrizations of the CPT preference calculus within the class of continuous functions are in vain. Rather, future research must embrace discontinuous weighting functions, such as neo-additive weighting functions (Wakker, 2010) and their obvious nonlinear extensions such as the one depicted in Kahneman and Tversky's (1979) Figure 4.

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Online Appendix to: The Need for Discontinuous Probability Weighting Functions: How Cumulative Prospect Theory is torn between the Allais Paradox and the St. Petersburg Paradox*

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A Lemmata

Lemma 1 *Suppose both limits below exist. Then, for a continuous and strictly increasing probability weighting function w, it holds*

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} = \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \ . \tag{56}$$

Proof: Strict monotonicity of w ensures that $\frac{w(0.5\pi)}{w(\pi)} \in [0,1]$. Therefore, $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \in [0,1]$. The same is true for $\frac{w(0.5\pi)-w(0.25\pi)}{w(\pi)-w(0.5\pi)}$ as the following arguments show.

Monotonicity of w implies that $\frac{w(0.5\pi)-w(0.25\pi)}{w(\pi)-w(0.5\pi)} \geq 0$. Assume, for proof by contradiction, that

$$\lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} = \lambda > 1. \tag{57}$$

Then, we can find $\pi_0 \in (0,1)$ such that

$$\frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} > 1 \tag{58}$$

for all sufficiently small probabilities $\pi \in (0, \pi_0]$ and for those π we have

$$w(0.5\pi) - w(0.25\pi) > w(\pi) - w(0.5\pi). \tag{59}$$

Note that the last inequality also applies to probabilities $\hat{\pi} = 0.5\pi$ which leads to

$$w(0.5^2\pi) - w(0.5^3\pi) > w(0.5\pi) - w(0.5^2\pi) > w(\pi) - w(0.5\pi) \tag{60}$$

and by iteration

$$w(0.5^n\pi) - w(0.5^{n+1}\pi) > w(\pi) - w(0.5\pi) \ \text{ for all } n = 1, 2, \ldots. \tag{61}$$

Then $w(\pi)$ can be written as a telescoping series for all probabilities $\pi \in (0, \pi_0)$ and we yield the following inequality:

$$w(0.5\pi) = \sum_{n=1}^{\infty} \left(w\left(0.5^n \pi\right) - w(0.5^{n+1}\pi) \right) \geq \sum_{n=1}^{\infty} \left(w(\pi)\right) - w(0.5\pi)) = \infty \tag{62}$$

which is a contradiction. Hence, $\frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \in [0, 1]$.

We now prove Equation (56) by distinguishing cases. As a first case, suppose that $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} < 1$. Then

$$\begin{aligned} \lim_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} &= \lim_{\pi \to 0^+} \frac{1 - w(0.25\pi)/w(0.5\pi)}{1 - w(0.5\pi)/w(\pi)} \cdot \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \\ &= \lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} \end{aligned}$$

and both limits are equal.

In particular, if $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} = 1$ then it cannot be that $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} < 1$. Hence, Equation (56) holds if $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} = 1$.

As the last case, suppose $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = 1$. Assume, for proof by contradiction, that $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} < 1$. By similar arguments as above, we can find $\lambda \in (0, 1)$ such that

$$\frac{w(0.5\pi) - w(0.25\pi)}{w(\pi) - w(0.5\pi)} \leq \lambda < 1 \tag{63}$$

for all sufficiently small probabilities π less than some $\pi_0 \in (0, 1)$. By iteration and using a telescoping series, we get

$$w(0.5\pi) = \sum_{n=1}^{\infty} \left(w(0.5^n \pi) - w(0.5^{n+1}\pi) \right) \leq \sum_{n=1}^{\infty} \lambda^n \left(w(\pi)\right) - w(0.5\pi)) = \frac{\lambda}{1 - \lambda} \left(w(\pi)\right) - w(0.5\pi)) . \tag{64}$$

This last inequality is equivalent to

$$\frac{w(0.5\pi)}{w(\pi)} \leq \lambda \tag{65}$$

and, given that $\lambda < 1$, this is a contradiction. Hence, in any case, Equation (56) holds provided both limits exist. ∎

Lemma 2 *Let* $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = 0.5^\gamma \in (0,1)$*. Then, for all* $\epsilon > 0$ *there exists* $\pi_0 \in (0,1)$ *such that*

$$w(\pi) \geq const \cdot \left(\frac{\pi}{2}\right)^{\gamma+\epsilon} \tag{66}$$

for all $\pi \in (0, \pi_0]$*.*

Proof: Assume $\lim_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = 0.5^\gamma \in (0,1)$. Then for any $\epsilon > 0$ it holds $0.5^\gamma \geq 0.5^{\gamma+\epsilon}$. Hence, it exists a $\pi_0 \in (0,1)$ such that for all $\pi \in (0, \pi_0]$

$$w\left(0.5\pi\right) \geq 0.5^{\gamma+\epsilon} w(\pi)\ . \tag{67}$$

Equation (67) holds in particular for π_0 and iterating n times yields

$$w\left(0.5^n\pi_0\right) \geq 0.5^{n(\gamma+\epsilon)} w(\pi_0) = \left(0.5^n\pi_0\right)^{\gamma+\epsilon} \frac{w(\pi_0)}{\pi_0^{\gamma+\epsilon}} \geq \left(0.5^n\pi_0\right)^{\gamma+\epsilon} \frac{w(\pi_0)}{\pi_0^{\gamma}}\ . \tag{68}$$

For any $\pi \in (0, \pi_0]$ we choose $n \in \{1, 2, \ldots\}$ such that

$$0.5^n\pi_0 < \pi \leq 0.5^{n-1}\pi_0. \tag{69}$$

Then, it holds

$$
\begin{aligned}
w(\pi) &> w\left(0.5^n \pi_0\right) && (70)\\
&\geq \left(0.5^n \pi_0\right)^{\gamma+\epsilon} \frac{w(\pi_0)}{\pi_0^\gamma} && (71)\\
&= 0.5^{\gamma+\epsilon}\left(0.5^{n-1}\pi_0\right)^{\gamma+\epsilon} \frac{w(\pi_0)}{\pi_0^\gamma} && (72)\\
&\geq \left(\frac{\pi}{2}\right)^{\gamma+\epsilon} \frac{w(\pi_0)}{\pi_0^\gamma} && (73)\\
&= \text{const} \cdot \left(\frac{\pi}{2}\right)^{\gamma+\epsilon} && (74)
\end{aligned}
$$

■

Lemma 3 *Let* $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 0.5^\alpha \in (0,1)$*. Then, for all* $\epsilon > 0$ *there exist* $x_0 > 0$ *such that*

$$v(x) \geq const \cdot x^{\alpha-\epsilon} \tag{75}$$

for all $x \geq x_0$.

Proof: Assume $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 0.5^\alpha \in (0,1)$. Then for any $\epsilon > 0$ it holds $0.5^\alpha \leq 0.5^{\alpha-\epsilon}$. Hence, it exists a $x_0 > 0.5$ such that for all $x \geq x_0$

$$v(0.5x) \leq 0.5^{\alpha-\epsilon} v(x). \tag{76}$$

Equation (76) holds in particular for x_0 and iterating n times yields

$$v(2^n x_0) \geq 2^{n(\alpha-\epsilon)} v(x_0). \tag{77}$$

For any $x \geq x_0$, we choose $n \in \{0, 1, 2, \ldots\}$ such that

$$2^n x_0 \leq x < 2^{n+1} x_0. \tag{78}$$

Then, it holds

$$
\begin{aligned}
v(x) \geq & v\left(2^n x_0\right) && (79)\\
\geq & 2^{n(\alpha-\epsilon)} v(x_0) && (80)\\
= & \left(2^{n+1} x_0\right)^{\alpha-\epsilon} \frac{v(x_0)}{(2x_0)^{\alpha-\epsilon}} && (81)\\
\geq & x^{\alpha-\epsilon} \frac{v(x_0)}{(2x_0)^{\alpha}} && (82)\\
= & \text{const} \cdot x^{\alpha-\epsilon} && (83)
\end{aligned}
$$

■

B Sensitivity analysis for the common ratio effect with small payoffs

From our analyses in the main part of the paper it is clear that potentially interesting cases require small or only moderately large payoffs z in lotteries L_1 and L_2 and value functions with $\lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$. This restriction excludes Tversky and Kahneman's (1992) suggested power value function $v(x) = x^\alpha$ and Bell's (1988) one-switch function. Linear and quadratic utility functions are clearly unable to explain both paradoxes. Table 1 lists the remaining typical forms of promising value functions. For these ones, we perform a sensitivity analysis to gauge the set of payoff amounts z for which the common ratio effect can be predicted.

Table 1: Functional forms for the value function v in gains.

Type	Function $v(x)$	Parameter restriction	Bounded utility	$\frac{v(0.5z)}{v(z)}$	$\lim_{x\to\infty}\frac{v(0.5z)}{v(z)}$
Exponential	$1-e^{-\alpha x}$	$\alpha > 0$	yes	$\frac{1-e^{-\alpha 0.5z}}{1-e^{-\alpha z}}$	1
Logarithmic	$\log(1+\alpha x)$	$\alpha > 0$	no	$\frac{\log(1+\alpha 0.5z)}{\log(1+\alpha z)}$	1
HARA	$\frac{1-\alpha}{\alpha}\left[\left(\frac{x}{1-\alpha}+\beta\right)^{\alpha}-\beta^{\alpha}\right]$	$\alpha < 0,\ \beta > 0$	yes	$\frac{[0.5z/(1-\alpha)+\beta]^{\alpha}-\beta^{\alpha}}{[z/(1-\alpha)+\beta]^{\alpha}-\beta^{\alpha}}$	1

B.1 Exponential value function

A promising alternative parametrization is the use of the exponential value function $v(x) = 1 - e^{-\alpha x}$ with $\alpha > 0$. This value function is bounded and, thus, automatically ensures finite willingness to pay for the St. Petersburg lottery (Theorem 2, a)). It holds that $\lim_{z\to\infty}\frac{v(0.5z)}{v(z)} = 1$. To simultaneously predict the common ratio effect, a probability weighting function with $\lim_{\pi\to 0^+}\frac{w(0.5\pi)}{w(\pi)} = 1$ is presumably most promising. A prime candidate would be the commonly used weighting function $w_{Prelec}(\pi) = e^{-(-\log\pi)^{\gamma}}$ with $\gamma \in (0,1)$ as proposed by Prelec (1998).

Recall that, by Definition 2 of the common ratio effect, the decision maker behaves risk seeking for all probabilities $\pi \in (0,\pi^*)$ and risk averse for all $\pi \in (\pi^*, 1]$. Technically, we solve the following equation for any pair $(\pi^*, \alpha z)$:

$$CPT(L_1) \stackrel{!}{=} CPT(L_2)$$

$$\Leftrightarrow \quad \frac{w(0.5\pi^*)}{w(\pi^*)} = \frac{v(0.5z)}{v(z)} \tag{84}$$

$$\Leftrightarrow \quad \frac{e^{-(-\log(0.5\pi^*))^{\gamma}}}{e^{-(-\log\pi^*)^{\gamma}}} = \frac{1-e^{-0.5\alpha z}}{1-e^{-\alpha z}} \tag{85}$$

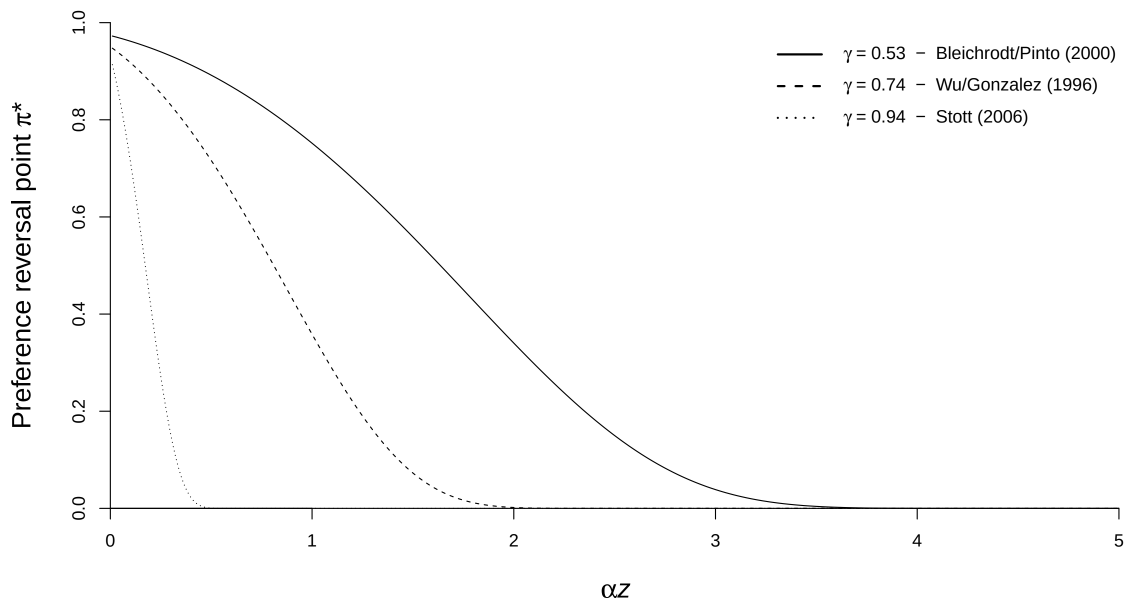


Figure 2: Preference reversal points for the exponential value and Prelec weighting function combination. This figure depicts preference reversal points π^* under CPT by solving $w(0.5\pi^*)/w(\pi^*) - v(0.5z)/v(z) = 0$ for the pairs $(\pi^*, \alpha z)$. The decision maker has the choice between the equal mean lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$. The individual's preferences are given by the bounded value function $v(x) = 1 - e^{-\alpha x}$, $\alpha > 0$ and Prelec's (1998) probability weighting function $w(\pi) = e^{-(-\log \pi)^\gamma}$, $\gamma \in (0, 1)$. The three lines indicate individual's preference reversal points for $\gamma = \{0.53, 0.74, 0.94\}$. The use of these three parameter estimates is motivated by Bleichrodt and Pinto (2000), Wu and Gonzalez (1996), and Stott (2006), respectively.

Equation (85) determines the decision maker's indifference point between the lotteries L_1 and L_2 in (27) for given constant absolute risk aversion coefficient α and payoff z.

Figure 2 depicts the preference reversal point π^* as a function of αz for the three weighting function parameter estimates $\gamma = \{0.53, 0.74, 0.94\}$ reported in Bleichrodt and Pinto (2000), Wu and Gonzalez (1996), and Stott (2006), respectively. For all three specifications, π^* quickly converges to zero when αz increases. Problems 7 and 8 in Kahneman and Tversky (1979) conservatively suggest that, empirically, $0.002 < \pi^* < 0.9$. Thus, even the lowest estimate $\gamma = 0.53$ of Bleichrodt and Pinto (2000) requires $\alpha z < 3.617$ to ensure the lower boundary of the empirically observed reversal point $\pi^* > 0.002$. The typical payoff $z = \$6,000$ leads to $\alpha < 3.617/6,000 \approx 0.000603$ which is an unreasonably low constant absolute risk aversion. To the contrary, De Giorgi and Hens (2006) suggest for the exponential value function the parameter value $\alpha \approx 0.2$. The typical $z = \$6,000$ implies $\alpha z = 1,200$ which results in a v-ratio that is just infinitesimally smaller than one. Hence, only when γ tends to zero – quite at odds with all calibration studies – the w-ratio equals one and the common ratio effect emerges theoretically. An alternative interpretation is that CPT counterfactually predicts a preference for risky lottery $L_1(\pi = 0.002)$ only for $z < \$18.08$. This is at odds with experiments that show *less* frequent Allais type behavior for low payoffs (Camerer, 1989; Conlisk, 1989; Fan, 2002; Huck and Müller, 2012; Agranov and Ortoleva, 2017). We conclude that the combination of the exponential value function and Prelec's (1998) subproportional weighting function is no viable solution to both paradoxes.

Recall that for probability weighting functions in, e.g., Tversky and Kahneman (1992) and Goldstein and Einhorn (1987), it holds $\lim\limits_{\pi \to 0^+} \frac{w(0.5\pi)}{w(\pi)} = 0.5^\gamma$. Because $\lim_{\gamma \to 0} 0.5^\gamma = 1$, a sufficiently low γ and sufficiently low payoff z, can theoretically explain both paradoxes in similar cases with low αz combinations. Table 2 provides a sensitivity analysis. It depicts for exponential, logarithmic and HARA value functions and various (α, z) combinations the maximum curvature parameter γ of the probability weighting function that still predicts the common ratio effect. Evidently, with the exponential value functions, payoffs of the order of $z = 6.000$ are sufficient to wipe out the common ratio effect unless unrealistic preference parameters are assumed. For example, with $z = 6.000$ and an extremely low $\alpha = 0.001$, the curvature parameter γ cannot exceed 0.07. Larger α values are even more problematic.

Table 2: Numerical upper boundaries for the curvature parameter γ of the Tversky and Kahneman (1992) or Goldstein and Einhorn (1987) weighting function for different values of α and z such that the common ratio effect is predicted.

			Payoff z				
Type v	Upper boundary for γ		\$100	\$1,000	\$6,000	\$20,000	\$100,000
Exponential	$\frac{\log\left(\frac{1-e^{-\alpha 0.5z}}{1-e^{-\alpha z}}\right)}{\log(0.5)}$	$\alpha = 0.001$	0.96	0.68	0.07	0	0
		$\alpha = 0.01$	0.68	0.01	0	0	0
		$\alpha = 0.1$	0.01	0	0	0	0
Logarithmic	$\frac{\log\left(\frac{\log(1+\alpha 0.5z)}{\log(1+\alpha z)}\right)}{\log(0.5)}$	$\alpha = 0.5$	0.27	0.17	0.13	0.11	0.10
		$\alpha = 1$	0.23	0.15	0.12	0.10	0.09
		$\alpha = 2$	0.2	0.14	0.11	0.10	0.08
HARA ($\beta = 1$)	$\frac{\log\left(\frac{[0.5z/(1-\alpha)+\beta]^{\alpha}-\beta^{\alpha}}{[z/(1-\alpha)+\beta]^{\alpha}-\beta^{\alpha}}\right)}{\log(0.5)}$	$\alpha = -0.25$	0.14	0.06	0.04	0.03	0.02
		$\alpha = -0.5$	0.08	0.02	0.01	0.01	0
		$\alpha = -1$	0.03	0	0	0	0

B.2 Logarithmic value function

Next, we consider a logarithmic value function $v(x) = \log(1+\alpha x)$ with $\alpha > 0$. This value function exhibits sufficiently high risk aversion such that the CPT decision maker reports finite willingness to pay for the St. Petersburg lottery for typical probability weighting functions w_{TK92}, $w_{log-odds}$, or w_{Prelec}. While the former two cases are a simple application of Theorem 2, finite willingness to pay in the latter case (w_{Prelec}) deserves special attention in Example 8 below because $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$.

Example 8 *Assume the value function is given by $v(x) = \log(1+\alpha x)$ with $\alpha > 0$ and the probability weighting function is given by $w_{Prelec}(\pi) = e^{-(-\log\pi)^{\gamma}}$ proposed by Prelec (1998). Then $\lim_{\pi\to 0^+} \frac{w(0.5\pi)}{w(\pi)} = \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = 1$ and the CPT decision maker states finite willingness to pay for L_{STP}.*

Proof: Observe that

$$w_{Prelec}\left(2^{1-k}\right) = e^{-((k-1)\log(2))^{\gamma}} < e^{-(k-1)^{\gamma}\log(2)} = 2^{-(k-1)^{\gamma}}. \tag{86}$$

For any given $\alpha > 0$, it exists a $\bar{k} \in \mathbb{N}_{\geq 0}$ such that for all $k \in \mathbb{N}_{\geq \bar{k}}$

$$\log\left(1+\alpha 2^{k}\right) < \log\left(\alpha 2^{k+1}\right) = \log(2\alpha) + k\log(2). \tag{87}$$

Then, the CPT value in Equation (1) is finite. As a first step we have

$$\begin{aligned}
CPT(L_{STP}) &= \sum_{k=1}^{\infty} \log\left(1+\alpha 2^{k}\right) \cdot \left[w_{Prelec}\left(2^{1-k}\right) - w_{Prelec}\left(2^{-k}\right)\right] && (88)\\
&< \sum_{k=1}^{\infty} \log\left(1+\alpha 2^{k}\right) \cdot w_{Prelec}\left(2^{1-k}\right) && (89)\\
&= \underbrace{\sum_{k=1}^{\bar{k}-1} \log\left(1+\alpha 2^{k}\right) \cdot w_{Prelec}\left(2^{1-k}\right)}_{=c<\infty} + \sum_{k=\bar{k}}^{\infty} \log\left(1+\alpha 2^{k}\right) \cdot w_{Prelec}\left(2^{1-k}\right) && (90)\\
&< c + \sum_{k=\bar{k}}^{\infty} \left[\log(2\alpha) + k\log(2)\right] \cdot 2^{-(k-1)^{\gamma}} && (91)\\
&= c + \sum_{k=\bar{k}-1}^{\infty} \left[\log(2\alpha) + (k+1)\log(2)\right] \cdot 2^{-k^{\gamma}} && (92)\\
&= c + \log(4\alpha) \sum_{k=\bar{k}-1}^{\infty} 2^{-k^{\gamma}} + \log(2) \sum_{k=\bar{k}-1}^{\infty} k 2^{-k^{\gamma}}. && (93)
\end{aligned}$$

The first term in (93) is a constant and the series $\sum_{k=\bar{k}-1}^{\infty} 2^{-k^{\gamma}}$ is strictly smaller than the series $\sum_{k=\bar{k}-1}^{\infty} k 2^{-k^{\gamma}}$. The integral test, using the substitution $x = k^{\gamma} \cdot \log(2)$ with

$\frac{dk}{dx} = \frac{1}{\gamma}\left(\frac{1}{\log(2)}\right)^{\frac{1}{\gamma}} x^{\frac{1}{\gamma}-1}$, shows convergence because

$$\begin{aligned}\int_0^\infty k2^{-k^\gamma} dk &= \int_0^\infty ke^{-k^\gamma \cdot \log(2)} dk &(94)\\ &= \frac{1}{\gamma}\log(2)^{-\frac{2}{\gamma}} \int_0^\infty x^{\frac{2}{\gamma}-1} e^{-x} dx &(95)\\ &= \frac{1}{\gamma}\log(2)^{-\frac{2}{\gamma}} \cdot \Gamma\left(\frac{2}{\gamma}\right) < \infty, &(96)\end{aligned}$$

where Γ is the well known Gamma function. ■

Further, observe that w_{Prelec} is subproportional and $\gamma \in (0,1)$ is a necessary condition for the common ratio effect. The following function, corresponding to function (30) depicted in Figure 1,

$$f_{CRE*}^{w_{Prelec},v}(\pi) = \frac{w(0.5\pi)}{w(\pi)} - \lim_{z\to\infty} \frac{v(0.5z)}{v(z)} = \frac{w(0.5\pi)}{w(\pi)} - 1, \tag{97}$$

is, thus, strictly monotonically decreasing in π and $\lim_{\pi\to 0^+} f_{CRE*}^{w_{Prelec},v}(\pi) = 0$. In other words, there is no preference reversal point π^* and, thus, no common ratio effect.

A sensitivity analysis is worthwhile because there might be a realistically large set of moderate payoffs z for which condition (CRE) is satisfied. Solving Equation (84) with w_{Prelec}, $v(x) = \log(1+x)$, and $z = 6,000$ gives preference reversal points of virtually zero. The value is roughly $\pi^* \approx 7.32 \times 10^{-11}$ for $\gamma = 0.53$ reported in Bleichrodt and Pinto (2000). For larger γ, as reported in e.g. Wu and Gonzalez (1996) and Stott (2006), the preference reversal point is even closer to zero.

Figure 3 depicts the preference reversal probability π^* dependent on the payoff z in lotteries L_1 and L_2. It unveils that π^* quickly converges to zero for moderately large payoffs. Convergence is faster for higher curvature parameters γ. For $z = 400$, the Bleichrodt and Pinto (2000) estimate $\gamma = 0.53$ is not distinguishable from zero. In fact, this moderate case implies $\pi^* \approx 4.5 \times 10^{-5}$.

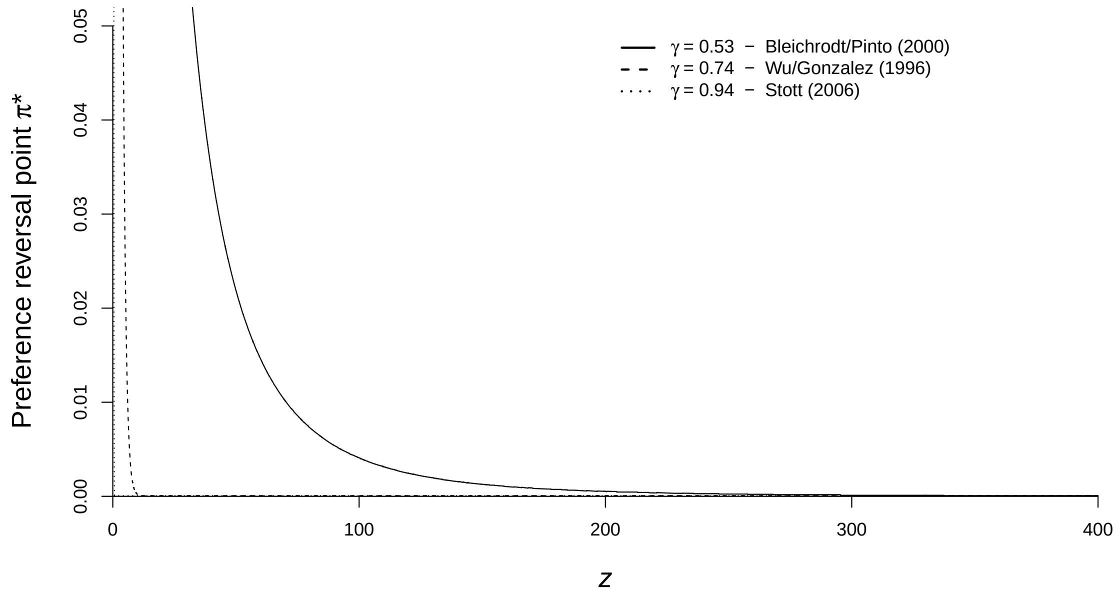


Figure 3: Preference reversal points for the logarithmic value and Prelec weighting function combination. This figure depicts preference reversal points π^* under CPT by solving $w(0.5\pi^*)/w(\pi^*) - v(0.5z)/v(z) = 0$ for the pairs (π^*, z). The decision maker chooses between the equal mean lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$. The individual's preferences are given by the bounded value function $v(x) = \log(1 + x)$ and Prelec's (1998) probability weighting function $w(\pi) = e^{-(-\log \pi)^\gamma}$, $\gamma \in (0, 1)$. The three lines indicate individual's preference reversal points for $\gamma = \{0.53, 0.74, 0.94\}$. The use of these three parameter estimates is motivated by Bleichrodt and Pinto (2000), Wu and Gonzalez (1996), and Stott (2006), respectively.

Table 2 kills any hope for w_{TK92} and $w_{log-odds}$. For example, using the typical amount $z = 6,000$, condition (CRE) implies $\gamma < 0.12$ which is unrealistically low. Further, Camerer and Ho (1994), Rieger and Wang (2006), and Ingersoll (2008) show that for $\gamma \leq 0.28$, w_{TK92} is not monotonically increasing.

B.3 HARA value function

Our last candidate value function is the HARA value function $v_{HARA}(x) = \frac{1-\alpha}{\alpha}\left(\left(\frac{x}{1-\alpha} + \beta\right)^\alpha - \beta^\alpha\right)$ with $\alpha < 0$ and $\beta > 0$. As the function is bounded from above it automatically ensures finite willingness to pay in the St. Petersburg paradox.

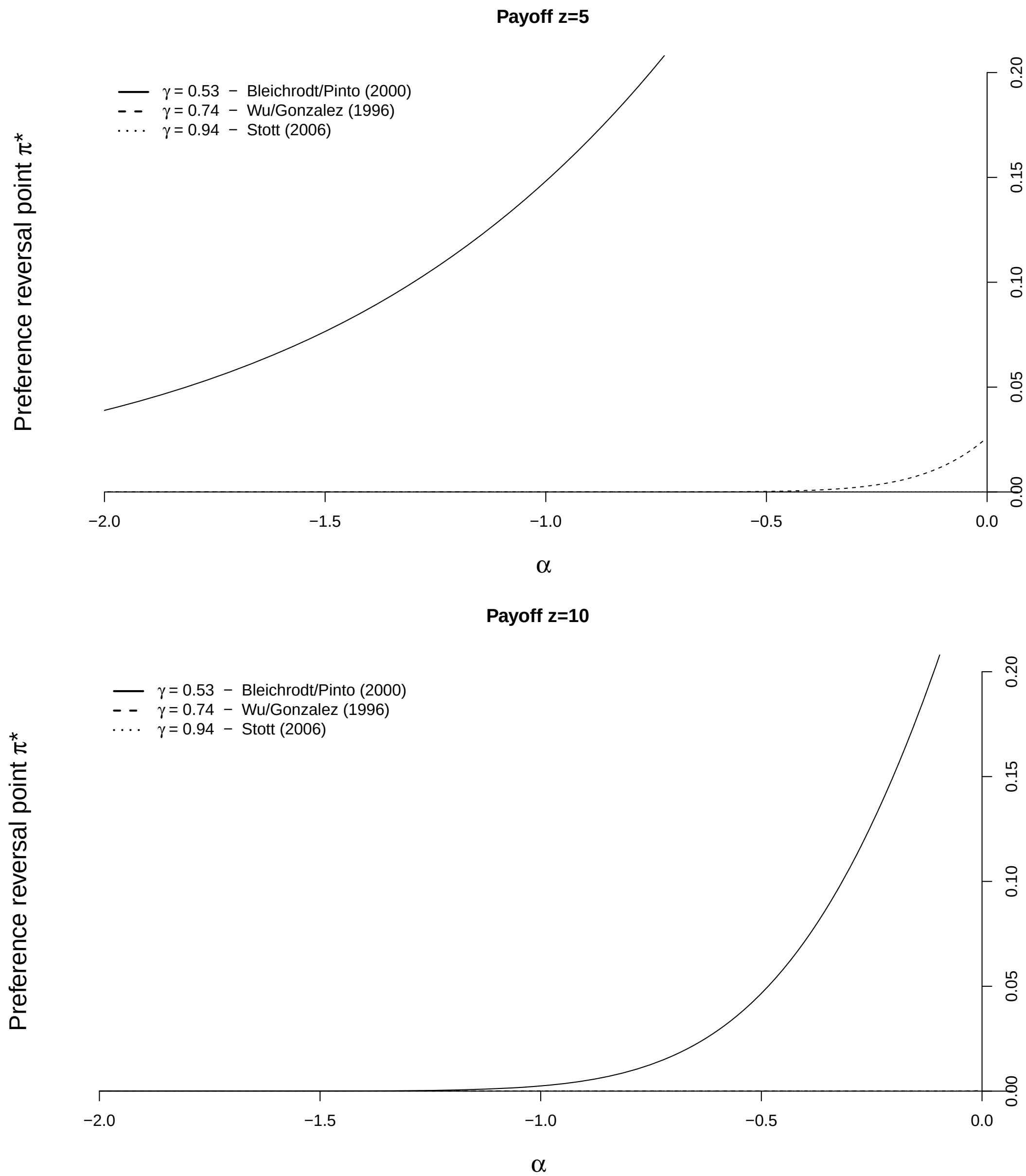


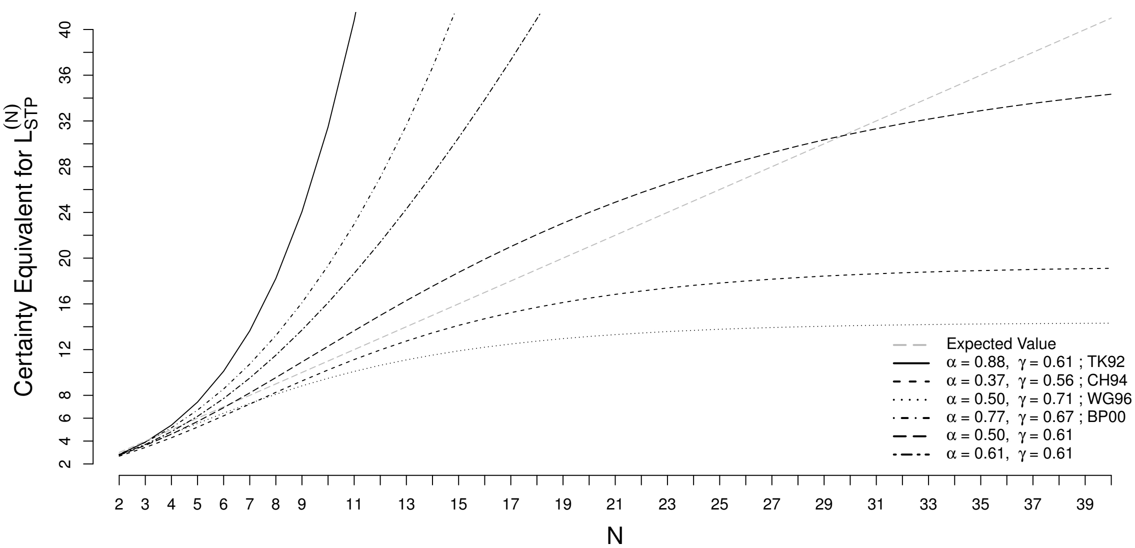
Figure 4: Preference reversal points for the HARA value and Prelec weighting function combination. This figure depicts preference reversal points π^* under CPT on the ordinate and risk aversion parameter α on the abscissa by solving $w(0.5\pi^*)/w(\pi^*) - v(0.5z)/v(z) = 0$ for the pairs (π^*, α). The individual's preferences are given by the bounded value function $v(x) = \frac{1-\alpha}{\alpha}\left(\left(\frac{x}{1-\alpha} + \beta\right)^{\alpha} - \beta^{\alpha}\right)$ with $\alpha < 0$ and normalized $\beta = 1$ and Prelec's (1998) probability weighting function $w(\pi) = e^{-(-\log \pi)^{\gamma}}$, $\gamma \in (0,1)$. The three lines indicate individual's preference reversal points for $\gamma = \{0.53, 0.74, 0.94\}$. The use of these three parameter estimates is motivated by Bleichrodt and Pinto (2000), Wu and Gonzalez (1996), and Stott (2006), respectively. The decision maker chooses between the equal mean lotteries $L_1(\pi) = (\$z, 0.5\pi; \$0, 1 - 0.5\pi)$ and $L_2(\pi) = (\$0.5z, \pi; \$0, 1 - \pi)$. The upper panel depicts the risk aversion parameter α if we fix $z = 5$ and the lower panel shows α for $z = 10$.

However, Table 2 unveils even more unrealistic calibrations for the curvature parameter γ for the Tversky and Kahneman (1992) or Goldstein and Einhorn (1987) probability weighting functions. For example, if we use the typical payoff $z = 6,000$ and value function parameters $\alpha = -1$ and $\beta = 1$ the largest plausible parameter would be $\gamma = 0.00048$ which is neither supported by empirical calibrations nor supported by – in the Tversky and Kahneman (1992) case – our stochastic dominance assumption of a strictly increasing w (see e.g. Camerer and Ho (1994), Rieger and Wang (2006), or Ingersoll (2008)).

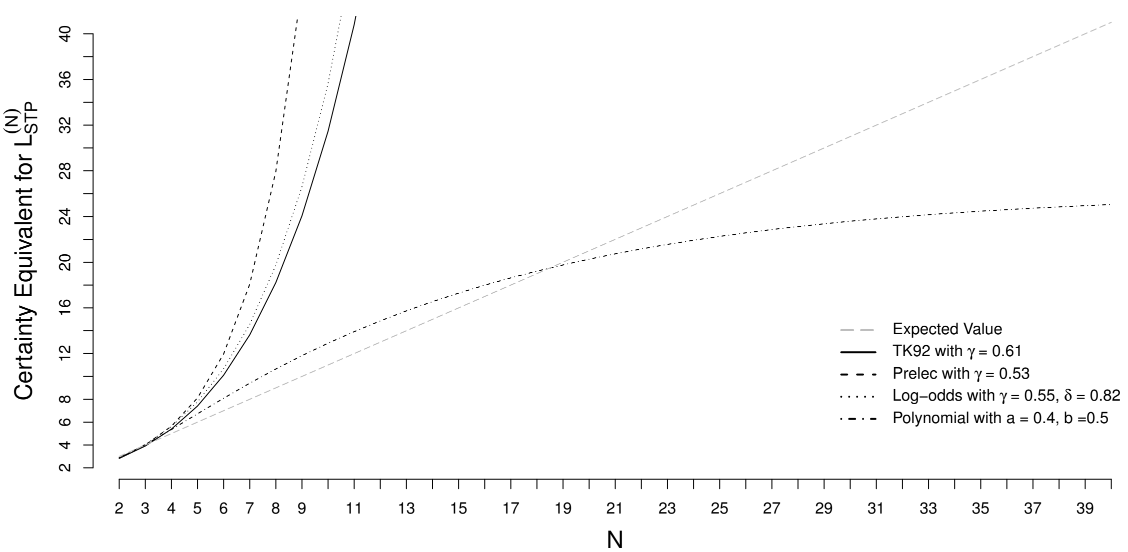
Figure 4 provides the results of a similar analysis with Prelec's (1998) probability weighting function w_{Prelec}. It depicts preference reversal points π^* on the ordinate and risk aversion parameter α on the abscissa by solving $\frac{w(0.5\pi^*)}{w(\pi^*)} - \frac{v(0.5z)}{v(z)} = 0$ for the pairs (π^*, α). Since the convergence of the preference reversal probability π^* to zero is so fast we depict the special case $z = 5$ and $z = 10$ in the upper and lower panel, respectively. As before, we use $\gamma = \{0.53, 0.74, 0.94\}$. For $\gamma = 0.94$ convergence is too quick to visualize it in one of the graphs. These effects are considerably stronger for larger values z. We conclude that using the HARA value function does not yield practical solutions to both paradoxes.

C Truncated St. Petersburg lotteries

As our testing ground, we propose the St. Petersburg paradox and the Allais paradox because of their outstanding prominence and importance for the development of new theories of decision making under risk throughout the history of risky decision making. Some readers might feel tempted to change this playing field. Especially the infinite expected payoff of the St. Petersburg lottery sometimes spurs criticism. We consider this to be scientific foul play. True, willingness to pay for the original



Panel A: $CE\left(L_{STP}^{(N)}\right)$ for different parameter sets (α, γ) of Tversky and Kahneman's (1992) parametrization.



Panel B: $CE\left(L_{STP}^{(N)}\right)$ for various probability weighting functions.

Figure 5: Certainty equivalents for the truncated St. Petersburg gamble under CPT. This figure illustrates the willingness to pay $CE\left(L_{STP}^{(N)}\right) = v^{-1}\left(CPT\left(L_{STP}^{(N)}\right)\right)$ for the truncated St. Petersburg lottery where $N = 2, \ldots, 40$ determines the maximum payoff \$$2^N$. The truncated gamble promises a payoff of \$$2^k$ with probability 0.5^k for $k = 1, \ldots, N-1$ and a payoff of \$$2^N$ with probability 0.5^{N-1}. In Panel A, the individual's preferences are given by the value function $v(x) = x^\alpha$ and the probability weighting function $w_{TK92}(\pi) = \pi^\gamma / (\pi^\gamma + (1-\pi)^\gamma)^{1/\gamma}$ for different parameter sets (α, γ) including those estimated in Tversky and Kahneman (1992), Camerer and Ho (1994), Wu and Gonzalez (1996), and Bleichrodt and Pinto (2000) denoted in the legend by TK92, CH94, WG96, and BP00, respectively. In Panel B, the value function is consistently $v(x) = x^{0.88}$ (Tversky and Kahneman, 1992) and the weighting function takes the forms w_{TK92}, $w_{log-odds}(\pi) = \delta\pi^\gamma / (\delta\pi^\gamma + (1-\pi)^\gamma)$, $w_{Prelec}(\pi) = e^{-(-\log \pi)^\gamma}$, and $w_{RW06}(\pi) = \frac{3-3b}{a^2-a+1}\left(\pi^3 - (a+1)\pi^2 + a\pi\right) + \pi$. Certainty equivalents CE_{STP} are depicted for weighting functions with parameter estimates of Tversky and Kahneman (1992) for w_{TK92}, estimates of Bleichrodt and Pinto (2000) for w_{Prelec} and $w_{log-odds}$, and parameter values motivated by Rieger and Wang (2006) for w_{RW06}, respectively. The gray dashed line represents the expected value of the lottery.

St. Petersburg lottery L_{STP} is difficult to elicit with monetary incentives. Nevertheless, stated willingness to pay in hypothetical scenarios is reliable, though noisy and real incentives would not change our conclusions (Holt and Laury, 2002, 2005).[1]

A typical concern is that subjects do not trust promises to payout in the St. Petersburg lottery above a certain threshold (Tversky and Bar-Hillel, 1983). Resorting to truncated versions of the St. Petersburg lottery, however, is not much more than grasping at straws, as we shall see. Let $L_{STP}^{(N)}$ denote the truncated St. Petersburg lottery which yields a payoff $\$2^k$ with probability 0.5^k for $k = 1, \ldots, N-1$ and a payoff of $\$2^N$ with probability 0.5^{N-1}. The expected value of this lottery equals $N+1$ and corresponds to $N-1$ possible rounds of coin flipping.[2]

By all indications, subjects behave risk averse in the original as well as the truncated St. Petersburg lottery $L_{STP}^{(N)}$ (Bernoulli, 1738, 1954; Bottom et al., 1989; Rivero et al., 1990; Baron, 2008; Hayden and Platt, 2009; Neugebauer, 2010; Cox et al., 2011; Seidl, 2013; Erev et al., 2017; Cox et al., 2019). Specifically, Hayden and Platt (2009) show in an experimental study with real monetary payments that individuals' willingness to pay is hardly affected by truncating the lottery. They find that bids on truncated St. Petersburg lotteries are typically smaller than twice the smallest payoff, that is $4 for $L_{STP}^{(N)}$. Cox et al. (2011) show that the majority of their subjects behave risk averse in the finite St. Petersburg gamble especially for $N \geq 6$. For $N = 9$, 83% of their participants rejected the gamble for a price of $8.75, thus indicating risk averse behavior. The proportion of subjects rejecting the gamble for a price slightly lower than the expected value increased monotonically with N. Moreover, Erev et al. (2017) show that risk aversion in the St. Petersburg paradox is also robust to feedback (with

[1] Note that infinite willingness to pay can also emerge from finite expected payoff gambles (Rieger and Wang, 2006).

[2] For example, for $N = \{10, 20, 30, 40\}$ the maximum payoffs are $\$2^{10}$, $\$2^{20}$, $\$2^{30}$, and $\$2^{40}$ which roughly equal 1 thousand, 1 million, 1 billion, and 1 trillion dollars, respectively.

real monetary incentives).

In the CPT framework, Figure 5 depicts the willingness to pay $CE\left(L_{STP}^{(N)}\right) = v^{-1}\left(CPT\left(L_{STP}^{(N)}\right)\right)$ as a function of N for the truncated St. Petersburg gamble, where $N = 2, \ldots, 40$ determines the maximum payoff $\$2^N$, for the most common parametrizations. Panel A shows the certainty equivalents $CE\left(L_{STP}^{(N)}\right)$ for Tversky and Kahneman's (1992) parametrization of the value and probability weighting function, that is $v_{Power}(x) = x^\alpha$ and $w_{TK92}(\pi) = \pi^\gamma / \left(\pi^\gamma + (1-\pi)^\gamma\right)^{1/\gamma}$, for different parameter sets (α, γ) as estimated in Tversky and Kahneman (1992), Camerer and Ho (1994), Wu and Gonzalez (1996), and Bleichrodt and Pinto (2000). Parameter combinations with $\alpha \geq \gamma$ predict a certainty equivalent that increases exponentially with N and illustrate once more infinite willingness to pay for $N \to \infty$ (see Theorem 2 or Example 3). The empirical evidence on truncated St. Petersburg lotteries clearly rejects such parameter combinations.[3] Surprisingly, the hypothetical parameter combination $(\alpha, \gamma) = (0.50, 0.61)$ implies risk proclivity for $N = 7, \ldots, 29$, that is higher certainty equivalents than expected payoff (gray dashed line) although this preference combination yields finite willingness to pay for $N \to \infty$. Predicted and actual willingness to pay can deviate by substantial amounts, though both are finite. One interpretation is that our formerly applied criterion of *finite* willingness to pay is rather conservative if benchmarked against actual willingness to pay.

We derive similar conclusion from Panel B which fixes the value function as $v(x) = x^{0.88}$ and uses probability weighting functions $w_{TK92}(\pi) = \pi^\gamma / \left(\pi^\gamma + (1-\pi)^\gamma\right)^{1/\gamma}$ with $\gamma = 0.61$, $w_{log-odds}(\pi) = \delta\pi^\gamma / \left(\delta\pi^\gamma + (1-\pi)^\gamma\right)$ with $\gamma = 0.55$ and $\delta = 0.82$, $w_{Prelec}(\pi) = e^{-(-\log\pi)^\gamma}$ with $\gamma = 0.53$, and $w_{RW06}(\pi) = \frac{3-3b}{a^2-a+1}\left(\pi^3 - (a+1)\pi^2 + a\pi\right) +$

[3] The empirical evidence mentioned above overwhelmingly supports risk averse behavior in truncated St. Petersburg lotteries and risk proclivity is merely a thought experiment (Tversky and Bar-Hillel, 1983) or an artificially induced observation where individuals were framed to a risk-seeking choice (Erev et al., 2008).

π with $a = 0.4$ and $b = 0.5$. The parameter values are motivated by Tversky and Kahneman (1992) for w_{TK92}, Bleichrodt and Pinto (2000) for w_{Prelec} and $w_{log-odds}$, and Rieger and Wang (2006) for the cubic weighting function w_{RW06}.

Just like with w_{TK92}, we see that the standard parametrization of the two weighting functions w_{Prelec} and $w_{log-odds}$ also predict unreasonably high willingness to pay for finite values of N. Interesting is the case of the polynomial weighting function w_{RW06} which implies a willingness to pay of \$26.18 for $N \to \infty$. We yield risk seeking behavior for $N = 3, \ldots, 18$ which correspond to maximum payoffs of $\$8; \$16; \ldots; \$262,144$. This prediction does not match the empirical fact mentioned above.

In summary, truncating the original St. Petersburg lottery does not change the essence of our previous conclusions.