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Violations of Branch Independence in Choices between Gambles

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Branch Independence is weaker than Savage's independence axiom; it holds that if two gambles have a common outcome for an event of known probability, the value of that common outcome should have no effect on the preference order induced by other probability-outcome branches. Systematic violations of branch independence were obtained in two experiments with choices between gambles composed of three equally likely, positive outcomes. Most people prefer ($2, $40, $44) over ($2, $10, $98); however, most people prefer ($10, $98, $136) over ($40, $44, $136). These results refute Expected Utility theories. They also refute the theory that people edit and cancel common components in choice. The pattern is opposite that predicted by the weighting function of cumulative prospect theory. Results are consistent with rank dependent, configural weight theory, with $w_L > w_M > w_H$, where w_L, w_M, and w_H are the weights of the lowest, medium, and highest outcomes, respectively. In this theory, violations of branch independence depend on relations among weights: results indicate that $w_L/w_M < w_M/w_H$.

Subjective Expected Utility (SEU) theory (Savage, 1954) rests on and requires an axiom called the "sure thing" principle. According to this principle, if two alternatives give the same outcome under one state of the world, then the value of that common outcome should not affect the preference order produced by other aspects of the gambles. The Allais Paradox (Allais, 1979) and the Ellsberg Paradox (Ellsberg, 1961) have been interpreted as evidence that humans do not always follow Savage's axiom (e.g., Slovic & Tversky, 1974). However, these paradoxes do not really provide pure tests of the axiom, being cluttered with other assumptions (Luce, 1992; Schoemaker, 1982; Stevenson, Busemeyer, & Naylor, 1991). Edwards' (1954) psychological generalization of SEU, using a weighting function of probability, implies a weaker form of Savage's Axiom, called branch independence.

Branch independence corresponds to the "weak" independence axiom that Cohen and Jaffray (1988) find more tenable than Savage's "sure thing" principle. According to branch independence, if two gambles have a common branch (the same outcome at the same probability produced by the same event), then the value of that outcome should have no effect on the ordering (Birnbaum, Coffey, Mellers, & Weiss, 1992).

Branch independence means that given a choice between alternatives, common branches (probability-outcomes) of the alternatives will have no effect on the decision. Consider gambles A and B which share a common branch $(z, p(z))$:

$$\mathrm{A} = (z, p(z); a_2, p(a_2); \ldots; a_i, p(a_i); \ldots; a_n, p(a_n))$$
$$\mathrm{B} = (z, p(z); b_2, p(b_2); \ldots; b_j, p(b_j); \ldots; b_m, p(b_m))$$

where $p(a_i)$ and $p(b_j)$ are the probabilities to receive outcomes a_i and b_j given choice A and B, respectively; and $\Sigma_{i=2}^{n} p(a_i) = \Sigma_{j=2}^{m} p(b_j) = 1 - p(z)$. Branch Independence requires that subjects prefer gamble A to B if and only if they prefer gamble A′ to B′ where the common outcome is now z' as follows:

$$\mathrm{A}' = (z', p(z); a_2, p(a_2); \ldots; a_i, p(a_i); \ldots; a_n, p(a_n))$$
$$\mathrm{B}' = (z', p(z); b_2, p(b_2); \ldots; b_j, p(b_j); \ldots; b_m, p(b_m)).$$

In other words, if two gambles share a common branch

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$(z, p(z))$ then changing the outcome of that branch $(z', p(z))$ should not affect the preference order induced by the other probability-outcome combinations.

Birnbaum *et al.* (1992, pp. 338–339) found that ($8, .5; z, .5) received higher judgments than ($16, .1; $5, .4; z, .5) for values of $z >$ \$20; however, the order of the mean judgments was reversed for values of $z \leqslant$ \$20, thus violating branch independence. This result could be explained by the rank-dependent, configural-weight model of Birnbaum *et al.* (1992). Weber, Anderson, and Birnbaum (1992) found similar results for ratings of the attractiveness of gambles. However, Wakker, Erev, and Weber (1994) found little evidence of violations of branch independence with direct choices between gambles; they found no evidence to favor rank-dependent utility theories over EU theory.

Theories in which the weights of stimulus components depend on their ranks have become more popular in recent years to explain violations of independence. Birnbaum, Parducci, and Gifford (1971) and Birnbaum (1974) proposed configural-weight averaging models to account for deviations from constant-weight averaging models in psychophysical and social information integration tasks. Birnbaum (1974, p. 559) noted, "The configural-weight averaging model assumes that the weight of a stimulus depends on its rank within the set to be judged." In the case of judgments of likeableness of a person based on their personality traits or of the morality of a person based on their deeds, judgments were theorized to depend mostly on the person's worst trait or deed, respectively (Birnbaum, 1972, 1973; Riskey & Birnbaum, 1974; Birnbaum, Wong, & Wong, 1976).

Birnbaum and Stegner (1979) proposed revisions to configural weight theory and showed that it could explain judgments of buying and selling prices of uncertain prospects (used cars), based on information from sources of varied credibility and bias (see also Birnbaum & Stegner, 1981; Birnbaum & Mellers, 1983). Kahneman and Tversky (1979) proposed a model of choice between risky prospects that used rank and sign-dependent weights. Quiggin (1982) introduced a rank-dependent theory in economics. Luce and Narens (1985) showed for binary gambles that a rank-dependent utility theory is the most general type of a configural weight theory that yields interval scales of utility, and that rank-dependent theory can explain many of the paradoxes that had been considered evidence against Expected Utility theory. Rank dependent weighting has been adocated or investigated in a number of other recent papers (Birnbaum *et al.*, 1992; Birnbaum & Sutton, 1992; Birnbaum & Sotoodeh, 1991; Champagne & Stevenson, 1994; Lopes, 1990; Luce, 1992; Luce & Fishburn, 1991; 1995; Miyamoto, 1989; Wakker, 1993, 1994; Weber, 1994; Schmeidler, 1989; Tversky & Kahneman, 1992; Yaari, 1987).

The present experiment uses gambles with three equally likely outcomes, denoted (x, y, z). These three amounts are written on slips that will be mixed and drawn at random from a container. If slip 1 is drawn, the prize is x; if slip 2 is drawn, the prize is y; if slip 3 is drawn, the prize is z. In this case, branch independence can be written:

$$(x, y, z) \text{ preferred to } (x', y', z) \quad \text{if and only if} \quad (x, y, z') \text{ preferred to } (x', y', z'). \tag{1}$$

In other words, replacing the common outcome z with z' should not affect the direction of preference between (x, y) and (x', y'). For this situation (with fixed probabilities), branch independence is equivalent to joint independence (Krantz, Luce, Suppes, & Tversky, 1971, p. 339).

THEORETICAL ANALYSIS

It is useful to analyze the present experiment with respect to generic, rank-dependent configural weight theory, and to discuss related theories with respect to that theory. According to this theory, the weights of equally likely outcomes depend entirely on their ranks. Since there are three ranks in our three-outcome gambles, there are three weights, for Lowest, Medium, and Highest ranking outcomes ($\mathrm{w_L}$, $\mathrm{w_M}$, and $\mathrm{w_H}$, respectively). In this case, weights can be normalized to sum to one by dividing by their total. Consider the preference relation, $\succ$, between two such gambles, with a common value of z that is the lowest outcome in either gamble. Outcomes are chosen such that $0 < z < x' < x < y < y' < z'$. Suppose

$$(z, x, y) \succ (z, x', y').$$

According to the rank-dependent model, this preference relation will hold if and only if

$$\mathrm{w_L}u(z) + \mathrm{w_M}u(x) + \mathrm{w_H}u(y) > \mathrm{w_L}u(z) + \mathrm{w_M}u(x') + \mathrm{w_H}u(y'), \tag{2}$$

where $\mathrm{w_L}$, $\mathrm{w_M}$, and $\mathrm{w_H}$ are the weights of the lowest, medium, and highest outcomes, respectively, and $u(x)$ is the utility function of money. Subtracting $\mathrm{w_L}u(z)$ from both sides leaves:

$$w_M u(x) + w_H u(y) > w_M u(x') + w_H u(y')$$

Therefore

$$w_M[u(x) - u(x')] > w_H[u(y') - u(y)]$$

hence,

$$\frac{w_M}{w_H} > \frac{u(y') - u(y)}{u(x) - u(x')} . \tag{3}$$

Now suppose that there is a violation of branch independence when the common outcome is changed from the lowest to the highest outcome in both gambles ($0 < x' < x < y < y' < z'$). In this case, the preference relation would be as follows:

$$(x, y, z') \prec (x', y', z').$$

This relation will hold if and only if

$$w_L u(x) + w_M u(y) + w_H u(z') < w_L u(x') + w_M u(y') + w_H u(z'). \tag{4}$$

This expression implies

$$w_L u(x) + w_M u(y) < w_L u(x') + w_M u(y')$$

therefore,

$$\frac{w_L}{w_M} < \frac{u(y') - u(y)}{u(x) - u(x')} . \tag{5}$$

This analysis shows that if the ratios of weights of adjacent ranks are equal, then there would be no violations in any experiment due to any change in common outcome from lowest to highest (z to z'). In order to observe this violation of branch independence, the ratios of successive weights must "straddle" the ratio of differences in utility as follows:

$$\frac{w_L}{w_M} < \frac{u(y') - u(y)}{u(x) - u(x')} < \frac{w_M}{w_H} . \tag{6}$$

By reversing the preference relations and inequalities in the above derivation, one can see that the opposite pattern of violations of branch independence will occur when the following holds:

$$\frac{w_L}{w_M} > \frac{u(y') - u(y)}{u(x) - u(x')} > \frac{w_M}{w_H} . \tag{7}$$

Table 1 presents an analysis of the stimuli that were employed in Design 1 of the present experiment. Note that a wide range pair (x', y') is compared to a series of smaller range pairs (x, y) that vary in their totals ($x + y$). Each entry in the table shows the ratio of differences in utility as in Expressions 6 and 7. If the ratio of weights exceeds the ratio of utility differences in the table, then the gamble containing the smaller range pair (x, y) will be preferred to the one containing the larger range pair (x', y') when the common outcome is z; if the ratio of weights is less than the given ratio, then the gamble containing the wider range pair will be preferred. Four examples of utility functions are listed in Table 1, to illustrate how violations of branch independence depend on both the pattern of weights and the utility function, as in Expressions 6 and 7.

If the weights of Lowest, Medium, and Highest ranks are equal or stand in any fixed ratio (e.g., 4:2:1, 9:3:1, 1:1:1, 1:2:4), there will be no violations of branch independence between cases of the common outcome being lowest or highest. In a finite experiment such as Table 1, the ratios of weights must "straddle" the ratios of differences in utility specified by the experiment to produce a violation of branch independence. (Note: Changing the common outcome to the middle value can also produce violations of branch independence; derivations follow the same approach as Eqs. (2–7). Extensions to situations in which outcomes are varied in probability and the number of outcomes is changed are described in Appendix B).

Expected Utility Theory

Expected Utility (EU) theory assumes equal weights for all ranks, so EU theory and Savage's SEU predict no violations of branch independence. The psychological version of SEU theory (Edwards, 1954), which uses a weighting function of probability, would also have the same implication, because three equally probable events are also equal in weight. (In this experiment, branch independence and event independence coincide).

Editing and Cancellation Theory

Suppose the subject were to edit comparisons between gambles by canceling any equal probability, equal outcome that is common to both gambles. Such a subject would be following Savage's axiom, whether motivated by a principle of rationality or by a desire to simplify the decision problem. The theory that subjects edit and cancel equal aspects in making choices has been proposed by Tversky (1969; 1972), elaborated by Kahneman and Tversky (1979), and reiterated by Tversky and Kahneman (1992). If subjects canceled com-

TABLE 1

Rank-Dependent Utility Theory Analysis of Violations of Branch Independence in This Experiment

Row	Contrast (x, y)	Contrast (x', y')	Utility function $u(x) = x$	$u(x) = x^{.88}$	$u(x) = x^{.5}$	$u(x) = \log x$
1	(50, 54)	(12, 96)	1.11	.99	.68	.40
2	(45, 49)	(11, 97)	1.41	1.25	.84	.48
3	(40, 44)	(10, 98)	1.80	1.58	1.03	.58
4	(35, 39)	(12, 96)	2.48	2.18	1.45	.84
5	(30, 34)	(11, 97)	3.32	2.89	1.86	1.04
6	(25, 29)	(10, 98)	4.60	3.95	2.46	1.33

Note. Entries in the last four columns show ratios of differences in utility: $[u(y') - u(y)]/[u(x) - u(x')]$. According to rank-dependent theory, branch independence will be violated between the cases in which z is smallest or largest when the ratios of successive weights "straddle" the ratios specified by the experiment and the utility function. For example, if $u(x) = x$, then (z, \$40, \$44) will be preferred over (z, \$10, \$98), for $z <$ \$10 and (\$10, \$98, z') will be preferred over (\$40, \$44, z') for $z' >$ \$98 if $w_L/w_M < 1.8 < w_M/w_H$.

mon outcomes, there would be no violations of branch independence, apart from random error. Because this strategy might apply to choice but not to independent judgment, it might be possible to observe violations of branch independence in a judgment task (as in Birnbaum *et al.*, 1992), but not in a choice task (as in Wakker *et al.*, 1994).

It is also possible that a cancellation strategy might be induced by the context within the experiment, if most of the trials involved comparisons that would permit such a cancellation. There is evidence that the model that represents subjects' judgments can actually change, depending on the distribution of stimuli presented to the subjects, so this concern should not be taken lightly (see Mellers, Ordóñez, & Birnbaum, 1992, Experiment 3). The present experiment was designed to include a large number of "filler" judgments in which all six outcomes were distinct, so that fewer than half of the choices involved comparisons in which a cancellation would be possible. Wakker *et al.* (1994) used an experimental design in which all of the choices included at least one common branch.

Rank-Dependent Utility Theory

Suppose $u(x) = x$. If the weights stand in the ratios of 3:2:1 for Low, Medium, and High ranks, then a violation of branch independence will occur in the third row of Table 1 (since $\frac{3}{2} < 1.8 < \frac{2}{1}$), and subjects will express the preferences

$$(\$2, \$40, \$44) \succ (\$2, \$10, \$98)$$

and

$$(\$40, \$44, \$136) \prec (\$10, \$98, \$136).$$

In this case, the smaller range pair (x, y) will be preferred to (x', y') in the first two rows of Table 1; there will be a violation of branch independence in the third row; and the last three rows would have consistent preferences for the gambles containing the wider range, (x', y') pair of higher expected value.

Figure 1 illustrates the predictions of these parameters by showing the equivalent cash value of each gamble as a function of the common outcome, z. The gamble with the higher ordinate value should be preferred, according to the theory. The crossing of the curves shows a violation of branch independence.

For the same weights, assuming a square root function for utility, the pattern is similar but now the reversal is predicted to occur in the fifth row of the table,

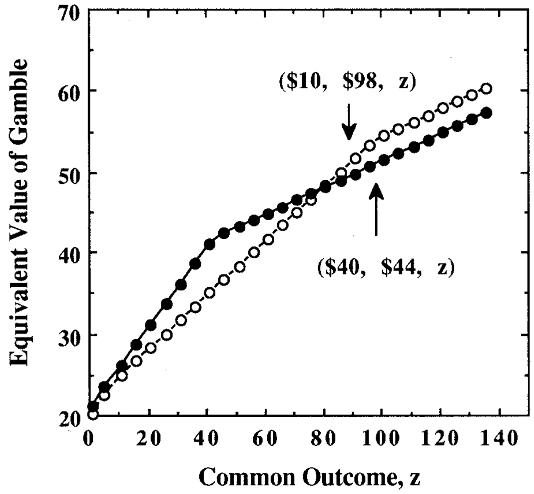

FIG. 1. Rank-dependent utility theory predictions, using weights of 3/6, 2/6, and 1/6 for lowest, middle, and highest outcomes, respectively. Crossing of the curves is a violation of branch independence.

since the ratios of weights straddle 1.86 ($\frac{3}{2} < 1.86 < \frac{2}{1}$). Changes in the utility function thus change the row(s) in which violations of branch independence would occur, holding the weights constant.

The above derivations (Expressions 2–7) show that rank-dependent utility theory implies no change in preference order when the common outcome maintains the same rank but is changed in value. The requirement of rank-dependent utility theory that there be no violations for gambles in which the common outcome maintains the same rank order is termed "comonotonic independence" by Wakker *et al.* (1994). In Fig. 1, the comonotonic independence prediction can be seen as the parallelism of the curves when the outcomes have the same rank (i.e., for $z < \$10$ and for $z' > \$98$).

Median Theory

Median Theory is a special case of rank-dependent utility theory, with weights of 0, 1, and 0 for Low, Medium, and High ranks, respectively. This model evaluates each gamble as equal to its median (which minimizes a loss function defined as the sum of absolute deviations of the outcomes about the gamble's value). According to this theory, there should be a violation of branch independence in every row of the experiment in Table 1. The subject should always choose the low range combination (x, y) when the common outcome is lowest (since the median is highest) and should always choose the high range combination when the common outcome (and the median) is highest. Median theory implies comonotonic independence, and it also implies that changes in the highest or lowest outcome of a three outcome gamble will not affect the gamble's utility. Therefore, this theory implies that there should be no effect of rows in Table 1 on preferences.

Cumulative Prospect Theory

Cumulative Prospect Theory, in this experiment, is a special case of rank-dependent utility theory in which middle values receive *lower* relative weight than higher or lower outcomes (Tversky & Kahneman, 1992, Eq. 6). Because of its weighting function, which has been confirmed by Wu and Gonzalez (in press), the cumulative prospect model predicts the opposite pattern of violations of branch independence from that in Fig. 1. As long as the middle outcome holds the least weight, Expression 7 follows, rather than Expression 6. For three equally likely positive outcomes, the parameters of Tversky and Kahneman (1992) yield weights of lowest, middle, and highest outcomes equal to .487, .177, and .336, respectively, yielding ratios of adjacent weights of $w_L/w_M = 2.75$ and $w_M/w_H = .53$, respectively,

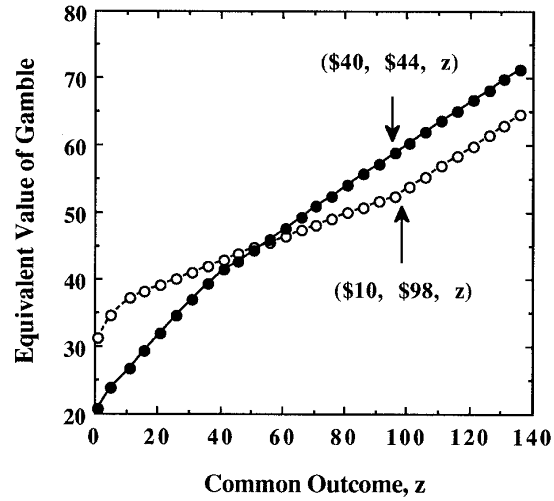

FIG. 2. Cumulative prospect theory predictions, plotted as in Fig. 1. Note that crossover pattern is opposite that in Fig. 1.

conforming to Expression 7. The cumulative prospect model also uses a power function for value of money, $u(x) = x^{.88}$ (see Table 1).

Therefore, the weighting function of cumulative prospect theory (apart from any editing assumptions that would eliminate systematic violations of branch independence) predicts that the subject should prefer the gamble containing the *wider* range pair (x', y') in all rows when the common outcome is lowest, but it implies that the subject should prefer the gamble with the smaller range pair (x, y) when the common outcome is highest in the first four rows. Thus, this theory predicts violations in the first four rows of Table 1 in the direction implied by Expression 7 and opposite that implied by Expression 6.

Because the model of Tversky and Kahneman (1992) places the least weight on the middle of three equally likely outcomes, it predicts preferences and violations of branch independence in the *opposite* direction from those predicted by the 3:2:1 pattern or median theory, which place relatively more weight on the middle outcome. The predictions of the cumulative prospect model are shown in Fig. 2, plotted for comparison with Fig. 1. Note that cumulative prospect theory also obeys the comonotonic independence assumption, yielding parallel curves for $z < \$10$ and $z' > \$98$.

Configural-Weight Theory

Birnbaum (1974) noted that the range model of Birnbaum, Parducci, and Gifford (1971) is a configural weighted averaging model in which the relative weight of a stimulus component depends on the rank of that stimulus component among the other components comprising the within-set context. Birnbaum and Stegner (1979) fit Birnbaum's (1974) range model as well as a revised rank-dependent, configural weight model that provided a better fit to buying, selling, and neutral's prices.

Birnbaum and Stegner's (1979, Eq. (10)) revised model is illustrated in Fig. 3 for three equally credible sources. According to this revised model, the absolute weight of a stimulus component is transferred according to rank in proportion to the absolute weight of the stimulus that loses the weight. Birnbaum and Stegner (1979) found that the proportion of weight transferred and the direction of transfer depends on the judge's point of view. For the buyer's point of view and the neutral's, weight is transferred from the highest value to the lowest; however, for the seller's point of view, weight is transferred from the lowest to the highest. Birnbaum and Sutton (1992) and Birnbaum *et al.* (1992) found similar changes of relative weight as a function of point of view, as did Birnbaum and Zimmermann (1995).

For the revised version of the model (Birnbaum & Stegner, 1979, Eq. (10)) for the buyer's point of view, the transfer of weights results in absolute weights of 1 + .385, 1, and 1 − .385, for the lowest, middle, and highest values, respectively. These absolute weights imply relative weights of .46, .33, and .21, which conform to Expression 6; they imply violations of branch independence in row two of present experiment, similar to those described in the section on rank-dependent utility theory for the 3:2:1 pattern of weights, illustrated in Fig. 1.

Configural-Weighting Derived from Asymmetric Loss Functions

Birnbaum *et al.* (1992) showed that for binary gambles, a rank-dependent model can be derived from the rationale that subjects act as if they minimize asymmetric loss functions (see also Weber, 1994). For three-outcome gambles, however, generalization of the loss function approach yields configural weights that depend on both the ranks and relative spacing of the outcomes. For a squared loss function, with asymmetric weights for over- vs underestimation, one can derive a configural weight model that is identical to a rank-dependent utility theory only on a limited subdomain. Over a global domain, the theory violates comonotonic independence. Comonotonic independence has been described as the key distinction between RDU and EU theories (Wakker *et al.*, 1994).

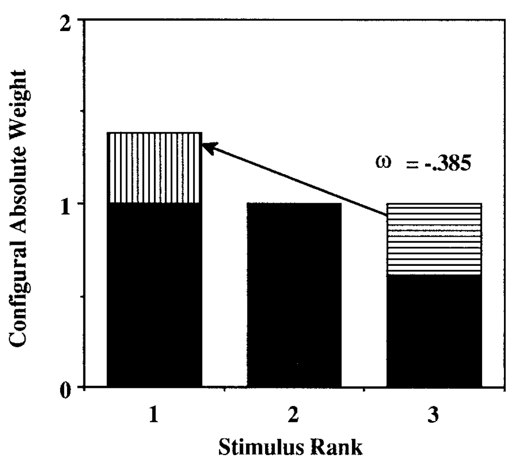

FIG. 3. Birnbaum and Stegner's revised model. The configural weight parameter, ω, represents the proportion of absolute weight transferred from stimulus losing weight to stimulus gaining weight.

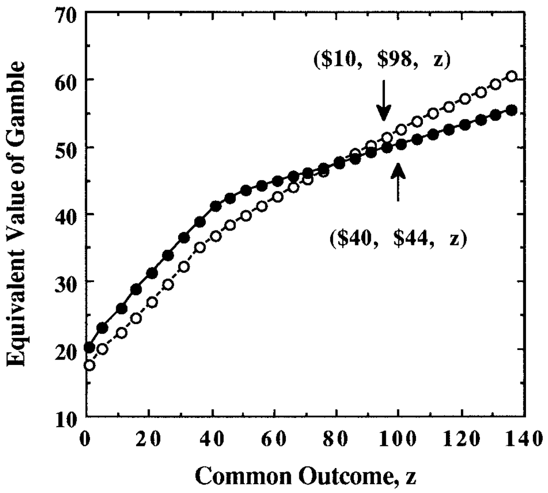

FIG. 4. Predictions of theory of minimizing asymmetric losses. Although similar to RDU in this case (Fig. 1), this theory can violate comonotonic independence.

Because this theory can violate comonotonic independence, the loss function theory is distinct from RDU theory in general, although it can be almost equivalent to RDU in restricted situations, such as the experiment of Table 1 under most parametric assumptions. An illustration of this theory is shown in Fig. 4. Gamble equivalent values, t, calculated to minimize an asymmetrically weighted loss function, L(t), are plotted in Fig. 4. The function minimized is as follows:

$$\mathrm{L}(t) = 2|u(x) - u(t)|^r \text{ if } x < \mathrm{t}$$
$$\mathrm{L}(t) = |u(x) - u(t)|^r \text{ if } x \geqslant \mathrm{t}.$$

Where $u(x) = x^b$, $b = .9$ and $r = 1.8$. These parameters were chosen to make Figs. 1 and 4 similar. Although similar, note that the curves in Fig. 4 are not parallel for comonotonic outcomes (unlike Fig. 1), which allows this theory to violate comonotonic independence.

Although the loss function approach may seem more complex than rank-dependent weighting, it can be simpler than a purely rank-dependent model, because only one parameter may be needed to represent the asymmetry of the loss function for any number of stimuli, rather than requiring a parameter for each rank position. Additional information on configural weighting derived from loss functions is presented in Appendix A, with an example illustrating violation of comonotonic independence.

METHOD

The subjects were given pairs of gambles, asked to choose between gambles, and to indicate their strength of preference using two different methods in two experiments. In Experiment 1, they judged the amount of money they would pay to receive their preferred gamble rather than the other gamble in each pair. In Experiment 2, they rated the strength of preference on a category rating scale.

Instructions

Subjects received printed instructions which were also read aloud to them. The instructions stated that each gamble consisted of three equally likely slips of paper with numbers written on them; these slips were to be mixed and one would be selected at random to determine the gamble's outcome.

One group of subjects were presented pairs of gambles and were instructed (in part), "Your task is to decide which of the two gambles you would prefer to play and to judge how much you would pay to play your preferred gamble rather than the other gamble." Subjects circled the gamble they would prefer to play and then judged the strength of their preference in dollars. For purposes of data analysis, a negative sign was associated with choice of the gamble on the left.

Another group of subjects received identical instructions and stimuli, except that their task was to judge the strength of preference on a 19-point rating scale, labeled as follows: −9 = Prefer the gamble on the left very very much more; −7 = Prefer the gamble on the left very much more; −5 = Prefer the gamble on the left much more; −3 = Prefer the gamble on the left more; −1 = Prefer the gamble on the left slightly more; 1 = Prefer the gamble on the right slightly more; 3 = Prefer the gamble on the right more; 5 = Prefer the gamble on the right much more; 7 = Prefer the gamble on the right very much more; 9 = Prefer the gamble on the right very very much more.

Stimuli

Each choice between gambles was presented using the format of the following example:

1._____($5, $45, $49) vs ($5, $11, $97)

The numbers within the parentheses represent equally likely outcomes of a gamble. The gamble on the left represents an equally likely opportunity of winning $5, $45, or $49. The gamble on the right represents equal chances to win either $5, $11, or $97. The values within each choice were presented in ascending order left to right.

The booklet included 94 pairs of gambles, displayed as in the example above. Each pair of gambles was numbered and preceded by a space in which the subjects were to write their judgments of strength of preference.

Design

The experiment consisted of two subdesigns. In the first subdesign, 36 pairs of gambles were of the form of (x, y, z) vs (x', y', z), with z common to both gambles. This subdesign was a 6 by 6 factorial in which the common outcome, z, could take on 6 levels ($2, $5, $33, $42, $108, or $136) and the 6 comparisons of (x, y) and (x', y') were: ($50, $54) vs ($12, $96), ($45, $49) vs ($11, $97), ($40, $44) vs ($10, $98), ($35, $39) vs ($12, $96), ($30, $34) vs ($11, $97), and ($25, $29) vs ($10, $98), as in Table 1. Note that the sum of x' and y' is constant (and the range is also large and nearly constant), but the sum of x and y was varied from 104 to 54 in steps of 10, with a constant small range of $4.

The second subdesign consisted of 48 pairs of gambles, in which all six values were distinct, (x, y, z) vs (x', y', z'), constructed from an 8 by 6 factorial design of the first and second gambles. The 8 levels of (x, y, z) choices were ($7, $8, $9), ($80, $8, $9), ($7, $8, $82), ($80, $8, $82), ($7, $81, $9), ($80, $81, $9), ($7, $81, $82) and ($80, $81, $82). (These 8 were composed of a 2 by 2 by 2 factorial design of x by y by z, where x = $7 or $80; y = $8 or $81; and z = $9 or $82). These 8 gambles were crossed with 6 (x', y', z') gambles [($13, $14, $15), ($13, $14, $48), ($13, $14, $92), ($84, $85, $15), ($84, $85, $48), or ($84, $85, $92)]. (These six were constructed from a 3 by 2 factorial design, in which z' = $15, $48, or $92 and ($x'$, y') = ($13, $14), or ($84, $85)). The second subdesign was included to ensure a majority of trials in which all six levels would be distinct, to reduce the possibility that the subjects might adopt a strategy to cancel common values, which seemed a possibility if all trials were from the first subdesign.

Procedure

The choices from both subdesigns were intermixed and printed in booklets in random order with the re-

TABLE 2

Percentage of Choices for (x, y, z) over (x', y', z) as a Function of Common Outcome (z)

Contrast		Common outcome ($)					
(x, y)	(x', y')	$z = 2$	$z = 5$	$z = 33$	$z = 42$	$z = 108$	$z = 136$
($50, $54)	($12, $96)	83	73	71	68	63	63
		77	69	58	69	65	71
($45, $49)	($11, $97)	70	78	65	68	53	52
		67	71	67	69	40	48
($40, $44)	($10, $98)	65	67	50	56	39	49
		75	71	33	50	40	33
($35, $39)	($12, $96)	61	53	48	39	37	37
		54	48	48	38	33	29
($30, $34)	($11, $97)	51	51	35	41	28	37
		38	42	33	33	15	29
($25, $29)	($10, $98)	37	42	36	27	20	16
		31	35	19	21	10	19

Note. Each entry is the percentage of choices of (x, y, z) over (x', y', z), as a function of z. Upper entry in each cell is for Experiment 1; lower entries are from Experiment 2. Branch independence implies that choice percentages should not change as a function of z.

strictions that successive trials did not repeat a row or column of either subdesign, and no two successive trials came from the first subdesign. (Thus, no two successive trials would permit a cancellation.) Each booklet contained two pages of instructions with example trials, six warm-up trials, followed by four unlabeled practice trials and 84 experimental trials.

The experimenter checked the first six warm up trials. Initial examples were very simple, such as the choice between ($10, $20) vs ($50, $100)—if the subject did not choose ($50, $100) in this instance, the experimenter would ask the subject to explain the choice, and direct the subject to reread the instructions as needed. The warm up examples increased in complexity to include choices like those of the actual experiment. After the warm ups were checked, subjects proceeded to 4 additional unlabeled practice trials (in which there were no common branches), followed by 84 experimental trials.

Subjects completed the experiment within one hour, working at their own paces.

Subjects

The subjects were 154 undergraduates enrolled in Introductory Psychology, who participated for extra credit. There were 106 who participated in Experiment 1, expressing their preferences in money, and 48 different participants in Experiment 2, who used the category rating scale.

RESULTS

Table 2 presents the percentage of subjects who preferred the (x, y, z) gamble over the (x', y', z) gamble. Within each cell, the upper and lower numbers show the results for Experiments 1 and 2, respectively. Recall that the (x, y) pairs have a small range and decrease in value down the rows; the (x', y') pairs have wide range and a constant total $(x' + y' = 108)$. The percentage of choices favoring (x, y, z) declines from the top row to the bottom, showing increasing preference for the wide range pair as the values of (x, y) decrease. This decrease is contrary to Median Theory.

Tests of Branch Independence

Columns of Table 2 represent the value of the common outcome, z. According to branch independence, preferences should not change as a function of the common outcome. Instead, the percentages choosing (x, y, z) over (x', y', z) decrease from left to right in each row as z is increased. This decrease indicates that preferences changed, and that more changed in one direction than the other. When z changes from smallest to largest, preference switches from the gamble with the small range (x, y) to the gamble with the large range (x', y').

Table 3 shows crosstabulations for Row 3 of Table 2, with data for Experiment 1 shown below the diagonal and data for Experiment 2 above the diagonal. For each of the 15 combinations of z and z', we can examine the two by two crosstabulation of preferring the smaller ranged gamble (z, $40, $44), designated "S," or preferring (z, $10, $98), designated "R," combined with preferences between ($40, $44, z') and ($10, $98, z'), labeled again with "R" indicating preference for the wider range ($10, $98) and "S" for the smaller range. The binomial sign test for correlated proportions is used to

TABLE 3

Preference between ($10, $98, z) vs ($40, $44, z) Crosstabulated by Preference between ($10, $98, z') vs ($40, $44, z')

	z = 2		z = 5		z = 33		z = 42		z = 108		z = 136	
Common	S	R	S	R	S	R	S	R	S	R	S	R
z = 2												
S			32	4	16	20*	22	14*	16	20*	15	21*
R			2	10	0	12	2	10	3	9	1	11
z = 5												
S	60	9			15	19*	21	13*	16	18*	15	19*
R	11	26			1	13	3	11	3	11	1	13
z = 33												
S	45	24*	45	26*			12	4	9	7	10	6
R	8	29	8	27			12	20	10	22	6	26
z = 42												
S	49	20	48	23	45	8			13	11	11	13
R	10	27	11	24	14	39			6	18	5	19
z = 108												
S	33	36*	35	36*	28	25*	32	27*			12	7
R	8	29	6	29	13	40	9	38			4	25
z = 136												
S	44	25*	46	25*	35	18	42	17	32	9		
R	8	29	6	29	17	36	10	37	20	45		

Note. S indicates preference for gamble containing small range ($40, $44) pair; R indicates preference for gamble containing higher range pair ($10, $98). Entries show the number of subjects who exhibit each conjunction of preferences. Data for Experiment 1 are shown below diagonal; data for Experiment 2 are shown above diagonal.

* $p < .05$, by two-tailed sign test of symmetry of violations of branch independence.

test the significance of the changes in proportion due to changes in z (violations of branch independence). This test does not use instances in which choices conform to branch independence; instead, it compares the numbers in the off-diagonal cells, where branch independence is violated. For example, the two by two crosstabulation in the upper, right corner of Table 3 shows that 21 subjects out of 48 in Experiment 2 preferred ($2, $40, $44) over ($2, $10, $98) *and* preferred ($10, $98, $136) over ($40, $44, $136), and only 1 violated branch independence in the opposite direction. If the violations were due to random error, then the 22 violations should be equally likely to split in either direction. Instead, the binomial is "significant," since the two-tailed probability of obtaining 1 or fewer or 21 or more (out of 22 binomial trials with $p = .5$) is .00001, which is less than .05.

Crosstabulations corresponding to Table 3 were examined for all rows of Table 2, with similar results. Out of 90 two by two crosstabulations (6 rows by 15 comparisons) in Experiment 1, 77 showed the same pattern: as z was increased, the proportion of preference for the wider range pair increased, 44 of these were significant by the two-tailed sign test, including significant changes in every row. Of the 13 remaining crosstabs, 3 showed equal splits of violations of branch independence and 10 showed the opposite pattern; none of these were statistically significant. For Experiment 2, 67 crosstabs showed the same pattern as the majority in Experiment 1, 23 of these were significant including at least one in each row, 17 showed the opposite pattern (none of which were significant), and six showed equal splits.

Twelve of the crosstabulations represent tests of comonotonic independence. None of these crosstabs in either experiment showed a significant asymmetry in violations. Five of these twelve crosstabs in Experiment 1 and seven in Experiment 2 showed the opposite pattern from that shown by the majority. These results are consistent with rank-dependent utility theory, which requires comonotonic independence to be satisfied. However, the comonotonic changes in z in this experiment are smaller than the noncomonotonic changes, so the manipulation is not really comparable to the larger changes in z that produced significantly asymmetric violations of noncomonotonic independence.

To examine individual differences, we examined two comparisons in each row for each subject, counting whether there was a violation between $z = 2$ and $z = 136$ and also between $z = 5$ and $z = 108$. There are thus 12 comparisons per subject. Out of 106 subjects in Experiment 1, only 13 subjects had no violations of branch independence among these 12 comparisons; 66 showed more violations in the direction of switching from preferring the low range pair when z was low to choosing the high range pair when z was highest (the pattern in Table 2); 18 showed more violations of the opposite type; and 9 subjects made an equal number of violations in either direction. Out of 48 subjects in Experiment 2, only 2 had no violations for these 12 comparisons; 33 showed more violations switching from preference for (x, y) to (x', y') as z was changed from lowest to highest; 10 showed more violations of the opposite type; and 3 had an even split.

These individual violations were also examined to investigate Median Theory. A person who always selects the gamble with the highest median would show 12 violations of branch independence, and would show no tendency to increase preference for the wide range (x', y') pair as x and y are decreased in successive rows. Combining both experiments, there were 29 subjects who had 7 or more violations among these 12 tests; 26 of these had more violations in the direction of the medians, but only one did not show increasing preference for (x', y') as $x + y$ decreased. Therefore, we did not find evidence that a subgroup of subjects chose consistently according to medians.

In summary, the data of both experiments show the same pattern of violations of branch independence. The pattern is consistent with rank-dependent utility theory under the assumption of Expression 6, as depicted in Fig. 1.

Modeling of Strength of Preference

To approximate strength of preference judgments according to the rank-dependent model, mean judgments of strength of preference were fit by the following model:

$$\mathrm{Pref}(x, y, z; x', y', z') = \mathrm{w_L}(x - x') + \mathrm{w_M}(y - y') + \mathrm{w_H}(z - z') \quad (8)$$

where $x < y < z$ and $x' < y' < z'$; $\mathrm{Pref}(x, y, z; x', y', z')$ is the judged strength of preference between gamble (x, y, z) and gamble (x', y', z'); $\mathrm{w_L}$, $\mathrm{w_M}$, and $\mathrm{w_H}$ are the weights of the lowest, middle, and highest outcomes within each gamble, respectively. This model assumes (a) that the preference judgment is proportional to the difference in utilities between the gambles; (b) that the utility of gambles can be represented by a rank-dependent, configural weight average of the utilities of the outcomes; and (c) that the utility of money can be approximated by $u(x) = x$ for this range of outcomes. Assumptions (b) and (c) are consistent with the results of Birnbaum *et al.* (1992) for this range of outcomes.

This model was fit to all 84 cells in Design 1 and Design 2 of Experiment 1. The correlation between predicted and obtained mean judgments was only .95. This model did most poorly in predicting judgments in Design 2 for cases in which one gamble dominated the other on all three values by small amounts. In those cases, the judgments were more extreme than predicted by the model, as if preference judgments are a mixture of strength of preference and certainty of choice. This pattern of residuals appears to be similar to the type of higher order configural effect discussed by Birnbaum, Thompson, and Bean (in press); Mellers, Chang, Birnbaum, and Ordóñez (1992); and Stuhlmacher and Stevenson (1994), among others, in which a small difference that is easy to discriminate can produce a large judgment of strength of preference.

A Premium for Dominance

Equation (8) was modified to allow an additive constant to represent a premium for dominance and also allowed the weights to be different for the cases of comparison with or without strict dominance on all three ranked outcomes. The weights for the nondominated case were .28, .17, and .09 for lowest, middle, and highest outcomes, respectively. These weights satisfy Expression 6 because $.28/.17 < 1.8 < .17/.09$, predicting a violation in Row 3 of Tables 1 and 2. For the dominant choices, the weights were .26, .04, and .03, respectively, with a premium for dominance of \$8.17. This model correlated .98 with the judgments of strength of preference in Experiment 1 and did a better job of fitting both the violations of branch independence in Design 1 and the strength of dominated choices in Design 2. Subjects appeared to offer too much to get dominant choices (often more than the expected value difference); furthermore, the weights indicate that they attend mostly to the improvement in the worst outcome in these dominating cases. Similar results were obtained for the ratings in Experiment 2, with even more extreme ratings in the case of dominant choices (the modal and median judgments in these cases were all 9 or −9).

For the nondominated comparisons of Experiment 1, the ratios of weights for the group fit are $\mathrm{w_L}/\mathrm{w_M} = 1.62$ and $\mathrm{w_M}/\mathrm{w_H} = 1.92$, which "straddle" 1.8 and therefore predict a violation of branch independence in Row 3 of Table 1, assuming $u(x) = x$. This row indeed showed a violation in the mean judgments of strength of preference, which changed signs in that row in both experiments. Analysis of variance of the strengths of preference showed that all main effects and interactions in both designs were statistically significant, except for the five way interaction between the outcomes in Design 2. The main effect of the common consequence was

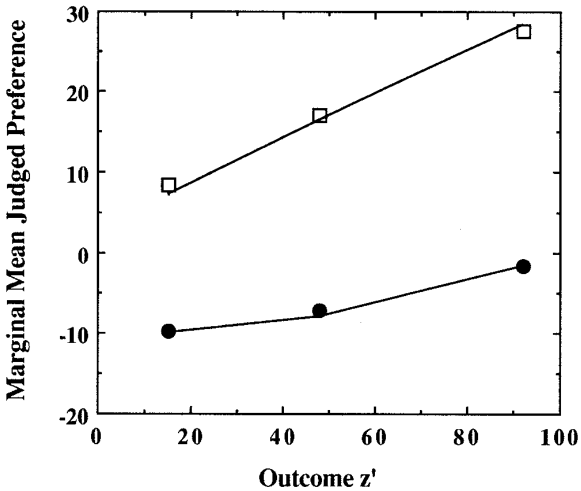

FIG. 5. Fit of configural weight model. Filled circles and open squares show empirical marginal mean judgments for ($13, $14, *z*) and ($84, $85, *z*), respectively. Lines show predictions.

statistically significant in Design 1 of both experiments, $F(5, 525) = 4.69$, and $F(5, 235) = 8.58$, for Experiments 1 and 2 respectively, contrary to branch independence. The main effect of the total of $x + y$ in the first design was also significant, $F(5, 525) = 59.27$, and $F(5, 235) = 52.90$, for Experiments 1 and 2, respectively, contrary to Median Theory.

To further examine the fit of the model and examine the interactions among outcomes, marginal means of Design 2 of Experiment 1 were calculated, along with the marginal means of the predictions. These are plotted in Fig. 5. Symbols show the marginal mean strength of preference judgments; curves show predictions of the model. Note that the curves diverge to the right, as predicted by the model, which assigns greater weighting to lower outcomes. The model appears to approximate the interaction fairly well. Similar results were also obtained for the row marginal means in Design 2 of Experiment 1, and similar divergent interactions were also obtained in Experiment 2 for both rows and columns.

The revised model was also fit to the data of each individual subject in Experiment 1, with a median correlation of .83 with the individual judgments. The median value of the premium for dominance was $7.56; 88 of the 106 subjects had their greatest weight on the lowest outcome for dominated comparisons; median weights for the dominated case were .20, .02, and .003. For the nondominated case, 70 of the subjects placed their greatest weight on the lowest outcome; 26 placed greatest weight on the middle outcome, and 10 subjects placed most weight on the highest outcome; median weights for the nondominated case were .264, .144, and .071 for lowest, middle, and highest outcomes respectively. These median weights satisfy the inequality of Expression 6, $w_L/w_M < w_M/w_H$; the majority of individuals had weights that also satisfied this inequality, consistent with the systematic pattern of violations of branch independence in Table 2.

Power Function for Utility Produces Minimal Improvement

The model was further generalized to allow a power function approximation for the utility function, $u(x) = x^b$. This model assumes that subjects compare two gambles taking the difference in their certainty equivalents. For the nondominated cases, the model is as follows:

$$\text{Pref}(x, y, z; x', y', z') = a[\text{CE}(x, y, z) - \text{CE}(x', y', z')] \quad (9a)$$

where

$$\text{CE}(x, y, z) = (w_L u(x) + w_M u(y) + w_H u(z))^{(1/b)} \quad (9b)$$

$$\text{CE}(x', y', z') = (w_L u(x') + w_M u(y') + w_H u(z'))^{(1/b)}, \quad (9c)$$

where CE(x, y, z) represents the certainty equivalent of the gamble; the weights are defined as in Equation (2); a and b are constants. In addition to the exponent for the utility function, this model includes a conversion factor, a, between the certainty equivalent difference and the judged value; therefore, weights can be restricted to sum to one. When $b = 1$, this model is equivalent to Eq. (8). The least-squares estimates of a and b were .535 and 1.10; the weights were .54, .31, and .15 for lowest, middle, and highest outcomes, respectively. This version of the model reduced the sum of squares in the residual from 415.4 to 414.2, a trivial improvement.

Expressions 9 were also fit to the data for nondominated choices of individual subjects, with similar results. The median value of b was .99. Only 10 of 106 subjects had improvements of more than 1% of the variance with values of b significantly different from 1; 7 of these had values of $b \geqslant 5$; but none produced a substantial improvement of fit. We concluded that the assumption of a linear function for $u(x)$ need not be rejected in favor of a power function to fit the individual subject data, once rank-dependent configural weighting is allowed.

DISCUSSION

These results show systematic violations of branch independence. The pattern is predicted by rank-dependent configural weighting, according to Expression 6, illustrated in Fig. 1. The pattern is also consistent with the predictions in Fig. 4. Because branch independence is a weaker form of Savage's axiom, the present results rule out EU and SEU theories and other weighted utility theories that would require branch independence.

No Support for Editing of Common Components

The present results provide no support for the theory that subjects consistently edit and cancel common components when making choices (Kahneman & Tversky, 1979). If subjects had eliminated common outcomes from consideration, then there would have been no systematic violations of branch independence. It may be that subjects place less weight on a common outcome and perhaps more weight on differing outcomes in making their comparisons, so the general idea of placing more emphasis on differing outcomes is still viable. Nevertheless, the strong form of the principle of editing and cancellation can be rejected in this study by the systematic violations of branch independence.

Editing and cancellation in the strong form may indeed be in the repertoire of subjects, if the experiment facilitates the use of such a strategy. In the present experiment, fewer than half of the experimental trials would have allowed a cancellation, and no two successive trials would allow a cancellation. Therefore, in this experiment, the strategy of cancellation would not suffice to handle most of the trials. Wakker *et al.* (1994) did not employ these design features and did not detect systematic violations of independence. Weber and Kirsner (in press) concluded that the previous failure to find violations of independence may have been due to the use of a cancellation strategy in the Wakker *et al.* study. Thus, it may be that subjects can use cancellation if the experimental design promotes such a strategy.

Another difference between the present study and that of Wakker *et al.* is that they tested for violations of independence using comparisons in which the expected values (EVs) of the gambles were always equal. In contrast, the present study compared lower EV, smaller range pairs [e.g., (z, \$40, \$44)] against higher range, higher EV pairs [e.g., (z, \$10, \$98)]. The median weights of the present study would not predict violations of branch independence in Table 1 for equal EV pairs, and extrapolation from the first row of Table 2 suggests that most subjects would have chosen the lower range pair combined with any value of z, if the EVs had been equal. Assuming that $u(x) = x$, the subject would violate branch independence with equal EVs if the weights straddled 1, i.e., if the subject placed the most or least weight on the middle outcome. If few subjects have such a pattern of weights (as suggested by the present results), violations would be infrequent when EVs are restricted to be equal.

Median Theory Refuted

The data of Experiments 1 and 2 are not consistent with Median Theory. Although Median Theory correctly predicts the direction of the violation of branch independence within rows of Table 2 (the effect of columns), it does not account for the systematic decrease in choice proportions in Table 2 as $x + y$ is decreased down the rows. Close inspection of individual data did not find any subgroup of subjects whose data appeared consistent with Median Theory.

Generic Rank-Dependent Utility Theory Satisfied

The present results are quite compatible with generic rank-dependent utility theory (Luce, 1992). The violations of branch independence observed were systematic for changes in z that were not comonotonic. The pattern of violations of branch independence are consistent with greater weight on the lowest outcome, followed by the middle outcome, then the highest outcome; the pattern of violations also indicates that ratios stand in the order, $w_L/w_M < w_M/w_H$, as in Expression 6.

Tests of comonotonic independence did not find systematic violations, consistent with rank-dependent utility theory, although the present experiment does not provide comparable manipulations. The tests of comonotonic independence involved smaller changes in z. Therefore, the property of comonotonic independence has not been put to a strenuous test in this experiment. As shown in Appendix A, minimum loss theory can yield violations of comonotonic independence when the largest outcome is taken to very high levels, and this implication has not yet been tested.

Related Research on Configural Weighting

The results of this study appear consistent with the pattern of weighting observed by Birnbaum & Stegner (1979), Birnbaum and Sutton (1992), Birnbaum *et al.* (1992), Birnbaum and Zimmermann (1995), Birnbaum and Beeghley (in press) and Weber and Kirsner (in press) for the buyer's point of view: lowest outcomes receive the greatest weight, followed by middle, followed by highest. One might think that the task of Experiment 1 may induce a buyer's viewpoint in the subjects, since they are asked to judge how much they would pay to receive their preferred gamble rather than the other. Although very similar choice proportions (Table 2) were also obtained in Experiment 2, which used a rating task that seems more neutral, it would be interesting to study whether weights would change if the task were to judge the price required to give up the preferred gamble and receive the less preferred one.

The present data are also consistent with results of recent studies of strength of preference between two-outcome gambles (Birnbaum, Thompson, & Bean, in press). Those experiments asked subjects to express preferences between gambles consisting of two equally likely outcomes by stating how much they would pay to receive one gamble rather than the other or by rating the strength of preference. The purpose of those studies was to assess interval independence: the effect of a common consequence on the judgment of a strength of preference due to a particular contrast. Subjects judged the strength of preference between ($6, $8) and ($6, $74) to be less than the strength of preference between ($8, $100) and ($74, $100). If strengths of preference are monotonically related to differences in utility between gambles, then their results indicate that lower outcomes receive higher weights.

Birnbaum *et al.* (in press) estimated the relative weights of the lower and higher outcomes to be .63 and .37 for strength of preference judgments. These values agree with those estimated from judgments of the "fair" prices of gambles between two positive outcomes (Birnbaum *et al.*, 1992), and (as shown below) they also predict choice-based certainty equivalents for binary gambles (Tversky & Kahneman, 1992). Results for binary gambles refute the rank-dependent theory of Quiggen (1982), which assumes that the weights of two equally likely outcomes should both be $\frac{1}{2}$.

The result of Birnbaum *et al.* (in press) is analogous to that of Birnbaum (1974), who asked subjects to judge "differences" in likeableness between persons described by pairs of adjectives. Birnbaum found that the difference due to variation in one trait was greater if the common trait was high in likeableness than when the common trait was low. For example, subjects rated the difference in likeableness between "LOYAL & UNDERSTANDING" and "LOYAL & OBNOXIOUS" to be greater than the difference in likeableness between a person who is "MALICIOUS & UNDERSTANDING" and one who is "MALICIOUS & OBNOXIOUS." Birnbaum concluded that subjects use a rank-dependent weighted average, placing the greatest weight on the lowest-valued trait in forming their integrated impressions. Birnbaum and Jou (1990) found that the comparative response times were also compatible with the "difference" ratings: greater "differences" in likeableness take less time, also indicating that the less favorable traits carry greater weight.

Birnbaum *et al.* (in press) found that strength of preference judgments conformed to weak transitivity, but they also found deviations from scalability, especially when one gamble dominated another by small amounts. Similar to the findings in the present study, judgments seemed too high when one gamble dominates another by small differences on each value.

It is important to emphasize that the present experiment does not rest its test of branch independence on the subtractive model, as does the test of interval independence in Birnbaum *et al.* (in press). All that is required to test branch independence is the preference relation. The analyses in Tables 1, 2, and 3 make use only of the direction of preference, not the magnitudes of strength of preference.

Cumulative Prospect Model Makes Wrong Prediction

The violations of branch independence observed here are not consistent with predictions (Figure 2) based on the weighting function of cumulative prospect theory (Tversky & Kahneman, 1992). This weighting function implies that the middle outcome should have less weight than either extreme outcome. Figure 2, which shows the pattern predicted by this weighting function, was not descriptive of the present data, since the present results (e.g., Table 2) show systematic violations in the opposite direction from that predicted by the model of Tversky and Kahneman (1992).

Tversky and Kahneman (1992) estimated their weighting function from certainty equivalents of gambles to win x with probability p and otherwise receive $0 ($x, p$; 0) as follows: $\mathrm{CE}(x, p; 0) = u^{-1}[\mathrm{W}(p)u(x)]$. They fit the $\mathrm{W}(p)$ function as follows:

$$\mathrm{W}(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1 - p)^{\gamma}]^{1/\gamma}}, \tag{10a}$$

where γ is the parameter of the weighting function, estimated to be .61; and $u(x) = x^{.88}$. This model implies an inverse-S relationship between $\mathrm{W}(p)$ and p, as shown by the dashed curve in Fig. 6. According to the theory, when there are more than two outcomes, weights of the outcomes are given by the expression,

$$\mathrm{w}(i) = \mathrm{W}(p_i) - \mathrm{W}(q_i), \tag{10b}$$

where $\mathrm{w}(i)$ is the weight of outcome, x_i; p_i is the (decumulative) probability that an outcome is greater than or equal to x_i given the gamble and q_i is the probability that an outcome is greater than x_i. For the highest outcome, the weight is given by $\mathrm{W}(p)$; for a middle out-

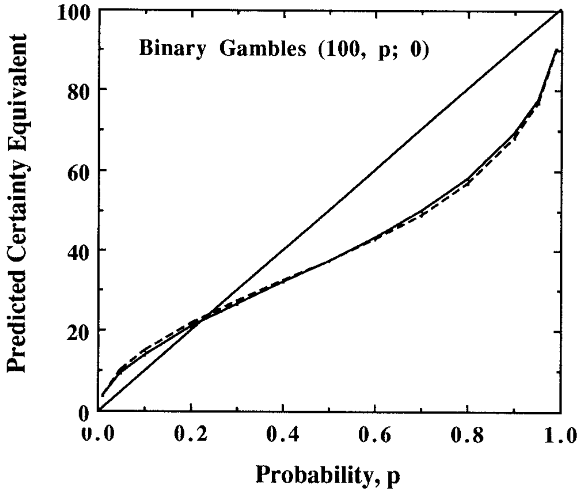

FIG. 6. Cumulative prospect model (dashed curve) vs relative weighting model (solid curve shows Eq. (11), with $u(x) = x$). These models are virtually identical for binary gambles.

come in a set of three, the weight depends not only on the outcome's probability, but also the probability of higher outcomes in the same gamble. With binary gambles, Eq. (10a) can be fit, but Eq. (10b) remains untested.

The relationship between judged proportions and objective proportions has also been found to resemble an inverse-S. Varey, Mellers, and Birnbaum (1990) noted that the inverse-S relationship can be explained by a relative ratio model in which subjective frequencies are negatively accelerated functions of objective frequencies.

In the configural-weight averaging models of Birnbaum and Stegner (1979) and Birnbaum *et al.* (1992), each weight is divided by the sum of the weights, producing a relative ratio. According to the configural weight model of Birnbaum *et al.* (1992), the absolute weight of an outcome is the product of the configural weight parameter, which depends on the point of view and rank of the outcome (apart from the probability distribution), and a function of the outcome's probability. The relative weight (dividing each weight by the sum of the weights) is given as in the following rewriting of Birnbaum *et al.* (1992, Eq. (4)):

$$\frac{w_H f(p)}{w_H f(p) + w_L f(1-p)}. \tag{11}$$

This expression, with $w_L = .63$, $w_H = .37$, and $f(p) = p^{.6}$, produces an inverse-S that is virtually identical to Tversky and Kahneman's weighting function, as shown by the solid curve in Fig. 6. The same weights (.63 and .37) that can account for Tversky and Kahneman's (1992) data also predict judgments of strength of preference (Birnbaum *et al.*, in press), and judgments from the neutral, or "fair price" point of view (Birnbaum *et al.*, 1992). Thus, in configural-weight theory, the inverse-S prediction is merely a consequence of relative weighting, and it does not imply anything about the weight of a middle outcome. See Appendix B for further implications.

However, in cumulative prospect theory, the inverse-S for binary gambles is taken as a direct measure of the cumulative weighting function, so Eq. (10b) makes the incorrect prediction that the middle of three equally likely outcomes should have the least weight.

The present failure of the weighting function derived from binary gambles to predict choices among three outcome gambles seems in accord with Tversky and Kahneman's (1992, p. 317) pessimistic assessment of the generality of their model. However, the present results do not test the structural assumptions of the theory, so the basic idea of cumulative weighting has not been directly refuted here. It should be possible to get direct tests of the theory by combining the approach of Wu and Gonzalez (in press) with the present approach within the same experiment. Systematic violations of monotonicity or stochastic dominance would also refute the theory directly.

Branch Independence vs Monotonicity Violations

The property of branch independence seems similar to the property of outcome monotonicity, but they are distinct. Monotonicity can be defined as follows:

If gamble A and A′ differ in one outcome on one branch:

$$A = (x, p(x); a_2, p(a_2); \ldots; a_i, p(a_i))$$
$$A' = (y, p(x); a_2, p(a_2); \ldots; a_i, p(a_i)),$$

where $p(x) = p(y)$ is the probability to receive outcomes x (or y) given choice A (or A′), respectively. Monotonicity requires that subjects prefer gamble A to A′ if and only if they prefer gamble B to B′, where

$$B = (x, p'(x); b_2, p(b_2); \ldots; b_i, p(b_i))$$
$$B' = (y, p'(x); b_2, p(b_2); \ldots; b_i, p(b_i))$$

for all $p'(x) = p'(y)$, b_i, $p(b_i)$. Monotonicity requires that if a subject prefers x to y in one gamble, then the preference should be in the same direction in the context of any other gamble. In contrast, branch independence requires the trade-off of two or more outcomes to be independent of the value of the common outcome.

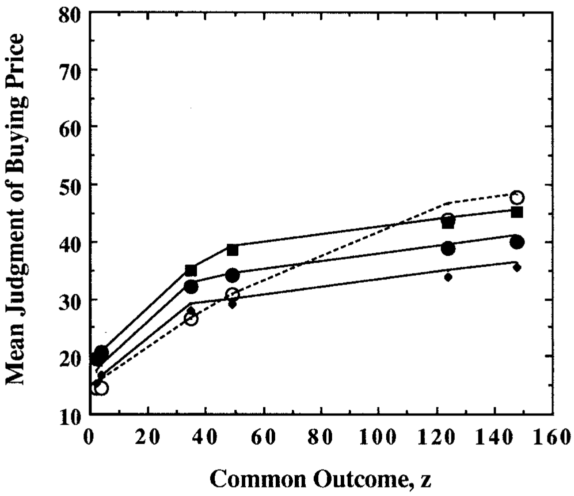

FIG. 7. Buyer's point of view. Open circles show mean judgments of (z, \$12, \$96). Filled diamonds, circles, and squares show means for (z, \$27, \$33), (z, \$33, \$39), and (z, \$39, \$45), respectively. Lines show predictions of configural weight model.

Even though outcome monotonicity seems more compelling than the sure thing principle, outcome monotonicity has been violated in studies of judgment. For example, Birnbaum *et al.* (1992) found that subjects judge the gamble (\$96, p, \$0) to be worth more than (\$96, p, \$24) when $p \geqslant .8$, but the order of judgments is reversed when $p < .8$. Similar results have been obtained by Birnbaum and Sutton (1992); Birnbaum (1992); Mellers, Weiss, and Birnbaum (1992); Birnbaum and Thompson (in press); and Mellers, Berretty, and Birnbaum (1995).

Birnbaum and Sutton (1992) found that although judgments show consistent violations of monotonicity, subjects rarely violate the principle in direct choices between the gambles. Von Winterfeldt, Chung, Luce, and Cho (in press) found different rates of violations with different procedures. They found few violations when gambles were ordered according to certainty equivalents determined using PEST, a sequential, staircase method for determining certainty equivalents. However, Birnbaum (1992) and Birnbaum and Thompson (in press) found violations of monotonicity when certainty equivalents based on choices between gambles and a fixed set of amounts of money were compared. Because monotonicity is a fundamental assumption of many utility theories, it is important to pin down the situations in which it is satisfied or not.

Judgment vs Choice

Violations of branch independence have also been investigated for judgments of the value of gambles by Birnbaum and Beeghley (in press). They asked subjects to judge the "highest price that a buyer should pay" to buy each of 168 three-outcome gambles and also asked them to judge the "least that a seller should accept to sell" each gamble, rather than to play it. Selected results from Birnbaum and Beeghley are shown for buyer's and seller's viewpoints in Figs. 7 and 8, respectively, plotted in a fashion similar to Figs. 1, 2, and 4. Note that the curves for wide range gambles have steeper slopes as a function of the common outcome than narrower range gambles. The pattern of violations of branch independence is similar to that shown in the present study (as in Figs. 1 and 4 but not 2), yet different small range gambles cross over in different points of view.

Birnbaum and Beeghley found that for the buyer's viewpoint, increasing the range of (x, y) pair (holding $x + y$ constant) always decreased the judgment, for any value of z. However, from the seller's point of view, increasing the range increased the judgment when z was the highest outcome, but it decreased the judgment when z was the lowest, suggesting that the middle outcome has the greatest weight in the seller's point of view. This change in violations is consistent with Birnbaum and Stegner's (1979) theory that point of view affects the configural weights of the outcomes. Birnbaum and Beeghley (in press) estimated the weights of lowest, middle, and highest outcomes to be .47, .30, and .07 for the buyer's point of view and .23, .44, and .18 for the seller's point of view, respectively, with $u(x) = x$.

For the present study, the relative weights for non-

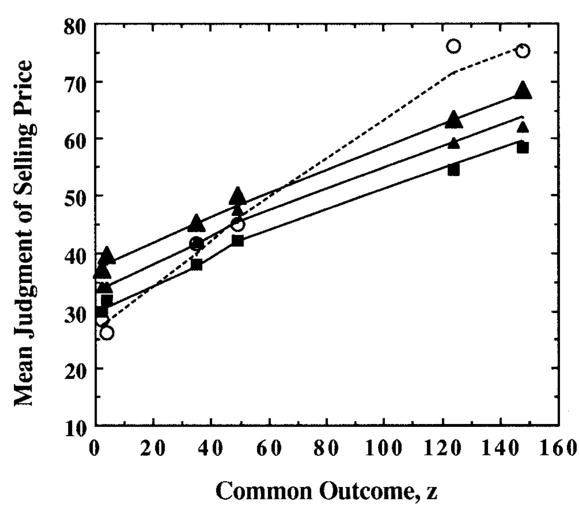

FIG. 8. Seller's point of view. Open circles show mean judgments of (z, \$12, \$96). Filled squares, small triangles, and large triangles show means for (z, \$39, \$45), (z, \$45, \$51), and (z, \$51, \$57), respectively. Lines show predictions of configural weight model.

TABLE 4

Estimated Relative Weights of Three Equally Likely Outcomes as a Function of Rank

Experiment	Rank of outcome: Lowest	Middle	Highest
Buyer's prices	.56	.36	.08
Seller's prices	.27	.52	.21
Preferences	.51	.33	.16

Note. Relative weights are normalized to sum to one by dividing by the sum of weights in each case. Values for Preferences are based on the model of strength of preference judgments for the nondominated choices in this experiment; values for Buyer's and Seller's Prices are from Birnbaum and Beeghley (in press). All three studies were fit with the same utility function.

dominated comparisons are .51, .33, and .16 for low, medium, and high value outcomes. Relative weights from Birnbaum and Beeghley (in press) are shown in Table 4. All three situations are compatible with the same utility function, $u(x) = x$; the different rank orders produced by the different tasks are explained entirely by changing configural weights. Although the preference orders differ in each case, the ratios of weights conform to the same inequality (Expression 6), which implies a similar pattern of violations of branch independence in all three studies.

In this case, despite differences in weighting, judgments and choices seem to agree, because patterns of judgments in both viewpoints agree with the pattern of violations observed in the present choice experiments. These findings suggest that the pattern of violations is not due to some process of comparison, such as editing, that would be unique to choice experiments, but rather that the pattern is produced by a combination process that is common to all three experiments.

Preference Reversals and Scale Convergence

The configural weights in Table 4 can be used with the same utility function to explain different preference orders for different points of view in the judgment task and for choice. Changing preference orders between different situations are sometimes termed "preference reversals." Contingent weighting theory (Tversky, Sattath, & Slovic, 1988) is a theory of preference reversals that should not be confused with configural weight theory. In contingent weight theory, the relative weights of probabilities vs outcomes depends on the task; whereas in configural weight theory, the weights of higher or lower outcomes depend on the configuration of outcomes and the subject's point of view. Contingent weight theory was fit to the relationship between ratings of attractiveness and prices assigned to binary gambles (Tversky *et al.*, 1988). However, tests of scale convergence (the assumption that the utility function in this case is invariant) required rejection of contingent weighting theory (Mellers, Ordóñez, and Birnbaum, 1992) for this situation in favor of the theory that the operation combining probability and outcome changed between those two tasks.

CONCLUSIONS

In summary, the present experiments show clear violations of branch independence. Therefore, these results deal a severe blow to descriptive theories that represent the value of a gamble as the sum of weighted products of a function of probability and a function of the outcome.

The data also show that subjects do not necessarily cancel common outcomes when comparing gambles. The particular pattern of violations of branch independence are inconsistent with the weighting function of probability used in cumulative prospect theory. Instead, the violations show the opposite pattern: when the common outcome is the lowest in each gamble, people tend to prefer the narrower range pair; when the common outcome is highest, they tend to prefer the wider range pair.

These results are compatible with rank-dependent utility theory, with the assumption that the lowest outcome receives the greatest weight, followed by the middle value, and least weight is given to the highest outcome; furthermore, the ratio of weight of the middle outcome to the highest is greater than the ratio of the lowest to the middle outcome. That comparison of ratios appears to hold for choice experiments, and for judgment experiments involving buying and selling prices, despite changing configural weights in the three tasks.

The present experiment found no systematic evidence of violation of comonotonic independence, but the experiment did not provide a strenuous test of this property. The theory that configural weighting is due to minimization of a loss function is also consistent with the present results, with the assumption that it is more costly to overestimate the value of a chosen gamble than to underestimate its value.

APPENDIX A: MINIMIZING ASYMMETRIC LOSS FUNCTIONS CAN VIOLATE COMONOTONIC INDEPENDENCE

Suppose we choose t to minimize the following loss function:

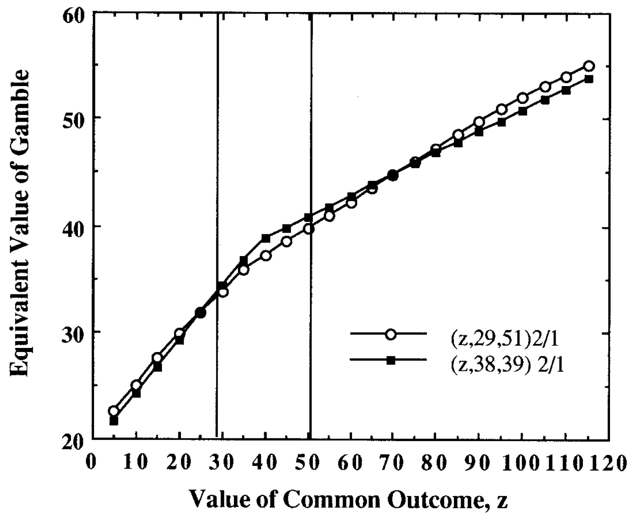

FIG. 9. Violation of comonotonic independence predicted by asymmetric loss function with $r = 2$.

$$L(t) = w_L |u(x_i) - t|^r \text{ for } x_i > t \quad (12a)$$

$$L(t) = w_H |u(x_i) - t|^r \text{ for } x_i \leq t \quad (12b)$$

Suppose that $r = 2$. If $w_L = w_H = w$, then this expression leads to expected utility theory. The solution for t in this case is $t = \Sigma w u(x_i)/\Sigma w$. For equally likely outcomes, $p(x) = w/\Sigma w$ for all outcomes. Because t represents expected utility, one would need to apply the u-inverse function to convert t to a cash equivalent.

When the weights of over- and underestimation are not equal ($w_L \neq w_H$), the loss function is said to be asymmetric. From the premise of an asymmetric loss function, we can derive an interesting family of configurally weighted models.

The case of $r = 1$ leads to the median and its generalizations. This situation is sometimes illustrated with the example of the "newsboy" problem. The newsboy must buy his papers to sell, faced with uncertainty concerning the number of customers that will buy papers any given day. If he buys too many papers, he loses the cost of each unsold paper. If he buys too few, he loses the chance to make profits on those sales. The cost of each paper and the cost of each lost profit are in general not equal, so this loss function is asymmetric. This case leads to the conclusion that the best solution is a percentile that depends on the asymmetry of costs. With symmetric costs, the solution is the median.

The case of $r = 2$ is convenient to study because calculus gives us simple, unique solutions. For three outcomes, however, there two cases to consider, which give different equations for the solutions. Consider three outcomes, $x < y < z$. Suppose $u(x) = x^b$. If the solution is between x^b and y^b, then we can minimize the loss function by taking the derivative of the following equation with respect to t, setting it to zero, and solving for t as follows:

$$L(t) = w_L(x^b - t)^2 + w_H(y^b - t)^2 + w_H(z^b - t)^2$$

$$L'(t) = 0 = -2w_L(x^b - t) - 2w_H(y^b - t) - 2w_H(z^b - t)$$

$$0 = w_L x^b - w_L t + w_H y^b - w_H t + w_H z^b - w_H t$$

$$(w_L + w_H + w_H)t = w_L x^b + w_H y^b + w_H z^b$$

therefore,

$$t = \frac{w_L x^b + w_H y^b + w_H z^b}{w_L + w_H + w_H}. \quad (13)$$

The above expression is a rank-dependent, configural weighted average of the utilities of the gambles. By a similar derivation, the case in which the solution is between y^b and z^b leads to the following solution:

$$t = \frac{w_L x^b + w_L y^b + w_H z^b}{w_L + w_L + w_H}. \quad (14)$$

This equation is also a rank-dependent, configural weight average. However, in second case, the absolute weight of y^b has changed from the weight of a high in Eq. (13) to the weight of a low in Eq. (14). Because the denominator also changes, the relative weights of the other two outcomes change as well. To convert to a cash value, one would apply the inverse function to the value of t at the solution [i.e., $t^{(1/b)}$].

Although these equations are equivalent to rank-dependent utility theory on a given subdomain (outcomes

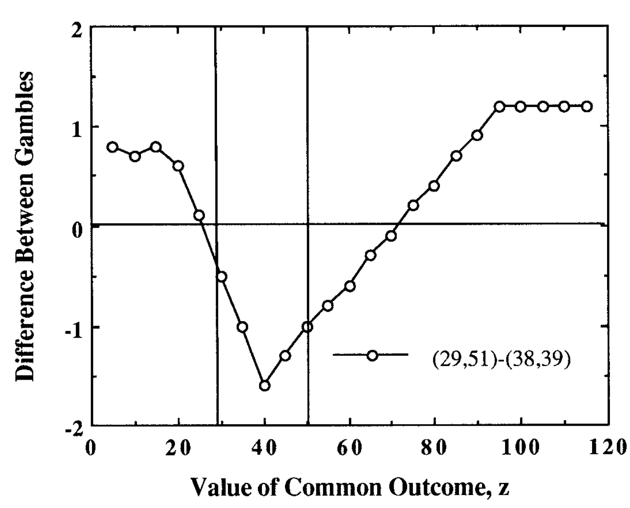

FIG. 10. Difference between values of gambles in Fig. 9.

are chosen so that the solution is always in the same interval), this model can violate comonotonic independence in a large enough experiment, when comonotonic changes in the outcomes move the solution from one subdomain to another. An example of violation of comonotonic independence is shown in Figs. 9 and 10, in which $b = 1$, $r = 2$, $w_L = 2$, $w_H = 1$. The values of t that minimize the loss function are plotted as a function of the common outcome, z, with separate curves for the gamble (z, \$38, \$39) and the gamble (z, \$29, \$51).

Note that the curves in Fig. 9 cross when z is increased from \$55 to \$105. Any crossover is a violation of branch independence; because this crossover occurs when z is increased without changing the rank order of the outcomes, this crossover represents a violation of comonotonic independence.

It is instructive to plot the difference between the curves, as in Fig. 10. Note that for values of z above \$100, the difference between the curves is constant. In this region, comonotonicity will be satisfied. This loss function theory has an interesting psychological interpretation. For the case of three outcomes, the subject treats them as either one low outcome and two high ones, or as two low outcomes and one high one. Thus, (\$29, \$51, \$55) would be treated as one low outcome (29) and two high ones (51 and 55). However, in the case of (\$29, \$51, \$105), the subject interprets the array as two low values and one high one. Thus, the spacing of the outcomes (as well as their ranks) determines their weights. Changing the value of the highest outcome caused the medium outcome to change weight from that of a high to that of a low; furthermore, since the total weight increased, the relative weight of the lowest and highest outcomes also decreased.

If there are many outcomes, this loss function approach is simpler than the full rank-dependent model, because there are only two weights. However, the theory adds the metric parameter, r, and the additional aspect of spatial configuration: the spacing among the values determines the weights assigned to the outcomes as well as their ranks.

APPENDIX B: GENERALIZATION OF ANALYSIS OF BRANCH INDEPENDENCE

The findings of the present experiment, that people violate branch independence and do not edit and eliminate common components in comparison, suggests that the approach of Eqs. (1–7) can be applied to more general situations to learn more about patterns of configural weighting. In this section, we sketch out a generalization to situations involving outcomes of unequal probability and to situations with different numbers of outcomes. Violations of branch independence provide powerful constraints on the configural weighting functions.

For three outcomes with unequal probabilities, we can rewrite branch independence as follows:

$$\begin{gathered}(x, p; y, q; z, r) \text{ preferred to } (x', p'; y', q'; z, r)\\ \text{if and only if}\\ (x, p; y, q; z', r) \text{ preferred to } (x', p'; y', q'; z', r).\end{gathered} \tag{15}$$

Where $(x, p; y, q; z, r)$ is the gamble to win x with probability p, y with probability q, and z with probability r; $p + q + r = p' + q' + r = 1$.

It is useful to analyze a choice experiment with respect to rank-dependent configural weight theory in which the relative weights of equally likely outcomes depend entirely on the ranks of the outcomes. Outcomes are selected so that $0 < z < x' < x < y < y' < z'$. We restrict attention to the case in which $p = p'$ and $q = q'$; $r = 1 - p - q$ (With equal distributions the denominator of the relative weights is constant, allowing common components to be subtracted off in configural weight theory). Consider the preference relation, $\succ$, between two such gambles, with a common value of z or z' that is lowest or highest outcome, respectively.

Suppose

$$(z, r; x, p; y, q) \succ (z, r; x', p; y', q).$$

and

$$(x, p; y, q; z', r) \prec (x', p; y', q; z', r).$$

In order to observe this violation of branch independence, the ratios of successive weights must "straddle" the ratio of differences in utility as follows:

$$\frac{w_L(p)}{w_M(q)} < \frac{u(y') - u(y)}{u(x) - u(x')} < \frac{w_M(p)}{w_H(q)}. \tag{16}$$

The opposite pattern of violations of branch independence is also possible, when the following holds:

$$\frac{w_L(p)}{w_M(q)} > \frac{u(y') - u(y)}{u(x) - u(x')} > \frac{w_M(p)}{w_H(q)}. \tag{17}$$

By conducting a series of experiments with different probabilities (p and q), each one like that in Table 1, in which different values of the utility interval are varied to determine a violation of branch independence, it is possible to obtain an ordering on the ratios of weights for any pair of probabilities. One can then construct weighting functions for outcomes of different probabilities in different rank positions. Such an experiment could test the distinction between cumulative prospect theory, for example, and the prediction of the configural weight model of Birnbaum *et al.* (1992, Eq. 4). According to that theory, weights are the product of a function of probability and a configural weighting parameter that depends on rank and point of view. In that theory, the same preferences should hold whenever $p = q > 0$, since the ratios in Eqs. (16–17) would depend only on the ratios of the configural weight parameters. Furthermore, if the function of probability ($f(p)$ in Eq. (11)) is a power function, then the same violations of branch independence should hold whenever p/q is constant, in this experiment. Cumulative prospect theory makes very different predictions.

Extension of the present approach to a greater number of outcomes is straightforward. For example, with four equally likely outcomes, there can be one or two common branches. With two common branches, the analysis will resemble Eqs. (1–7), except that there are six cases to consider: the common branches could be the two lowest, the two highest, the two in the middle, the two extremes, or they could alternate in two different ways. Each of these six experiments would then be designed to find violations of branch independence, which would then provide orderings on six comparisons of ratios of the four weights, two at a time. By determining weights for 2, 3, 4, etc. equally likely outcomes, it will be possible to test theories of how configural weighting depends on the number of outcomes.

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