Formulario I3

Igualdades

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k\, b^{n-k}; \qquad \sum_{k=x}^{\infty} \phi^k = \frac{\phi^x}{1-\phi} \quad \text{si } |\phi| < 1;$$

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = \exp(\lambda); \qquad \sum_{x=0}^{\infty} \binom{x+k-1}{k-1} \phi^x = \frac{1}{(1-\phi)^k} \quad \text{si } 0 < \phi < 1 \text{ y } k \in \mathbb{N}$$

Propiedades función $\Gamma(\cdot)$ y $B(\cdot,\cdot)$

$$(1) \quad \Gamma(k) = \int_0^{\infty} u^{k-1}\, e^{-u}\, du; \quad (2) \quad \Gamma(a+1) = a\,\Gamma(a); \quad (3) \quad \Gamma(n+1) = n!, \quad \text{si } n \in \mathbb{N}_0;$$

$$(4) \quad \Gamma(1/2) = \sqrt{\pi}; \quad (5) \qquad B(q,\,r) = \int_0^1 x^{q-1}\,(1-x)^{r-1}\, dx; \quad (6) \quad B(q,\,r) = \frac{\Gamma(q)\,\Gamma(r)}{\Gamma(q+r)}$$

Distribución Gamma

$$(1) \quad \text{Si } T \sim \text{Gamma}(k,\,\nu), \text{ con } k \in \mathbb{N} \longrightarrow F_T(t) = 1 - \sum_{x=0}^{k-1} \frac{(\nu\,t)^x\, e^{-\nu\,t}}{x!}$$

$$(2) \quad \text{Gamma}(1,\,\nu) = \text{Exp}(\nu) \qquad (3) \quad \text{Gamma}(\eta/2,\,1/2) = \chi^2(\eta)$$

Medidas descriptivas

$$\mu_X = \text{E}(X), \quad \sigma_X^2 = \text{E}\left[(X-\mu_X)^2\right], \quad \delta_X = \frac{\sigma_X}{\mu_X}, \quad \theta_X = \frac{\text{E}\left[(X-\mu_X)^3\right]}{\sigma_X^3}, \quad K_X = \frac{\text{E}\left[(X-\mu_X)^4\right]}{\sigma_X^4} - 3$$

$$M_X(t) = \text{E}\left(e^{t\,X}\right), \quad \text{E}[g(X)] = \begin{cases} \displaystyle\sum_{x \in \Theta_X} g(x) \cdot p_X(x) \\ \displaystyle\int_{-\infty}^{\infty} g(x) \cdot f_X(x)\, dx \end{cases}, \quad \text{Rango} = \text{máx} - \text{mín}, \quad \text{IQR} = x_{75\,\%} - x_{25\,\%}$$

$$x_p: \text{ Percentil } p \times 100\,\% \to F_X(x_p) = p, \qquad \text{Cov}(X,\,Y) = \text{E}[(X-\mu_X)\cdot(Y-\mu_Y)] \qquad , \qquad \rho = \frac{\text{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

Teorema de Probabilidades Totales

$$p_Y(y) = \sum_{x \in \Theta_X} p_{X,Y}(x,y); \qquad f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\, dy$$

$$p_X(x) = \int_{-\infty}^{+\infty} p_{X\,|\,Y=y}(x) \cdot f_Y(y)\, dy; \qquad f_Y(y) = \sum_{x \in \Theta_X} f_{Y\,|\,X=x}(y) \cdot p_X(x)$$

Transformación

Sea $Y = g(X)$ una función cualquiera, con k raíces:

$$f_Y(y) = \sum_{i=1}^{k} f_X\left(g_i^{-1}(y)\right) \cdot \left|\frac{d}{dy}\, g_i^{-1}(y)\right| \quad \text{o} \quad p_Y(y) = \sum_{i=1}^{k} p_X\left(g_i^{-1}(y)\right)$$

Sea $Z = g(X,Y)$ una función cualquiera:

$$p_Z(z) = \sum_{g(x,y)=z} p_{X,Y}(x,y)$$

Sea $Z = g(X,Y)$ una función invertible para X o Y fijo:

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(g^{-1}, y)\, \left|\frac{\partial}{\partial z} g^{-1}\right|\, dy = \int_{-\infty}^{\infty} f_{X,Y}(x, g^{-1})\, \left|\frac{\partial}{\partial z} g^{-1}\right|\, dx$$

Esperanza y Varianza Condicional

$$\mathrm{E}(Y) = \mathrm{E}[\mathrm{E}(Y \mid X)] \quad \text{y} \quad \mathrm{Var}(Y) = \mathrm{Var}[\mathrm{E}(Y \mid X)] + \mathrm{E}[\mathrm{Var}(Y \mid X)]$$

Distribución Normal Bivariada

$$f_{X,Y}(x, y) = \frac{1}{2\,\pi\,\sigma_X\,\sigma_Y\,\sqrt{1-\rho^2}} \times \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2 - 2\,\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right)\right]\right\}$$

$$Y \mid X = x \sim \text{Normal}\left(\mu_Y + \frac{\rho\,\sigma_Y}{\sigma_X}\,(x-\mu_X),\, \sigma_Y\,\sqrt{(1-\rho^2)}\right)$$

Teorema del Límite Central

Sean $X_1, \ldots, X_n$ variables aleatorias independientes e idénticamente distribuidas, entonces

$$Z_n = \frac{\displaystyle\sum_{i=1}^{n} X_i - n \cdot \mu}{\sqrt{n}\,\sigma} = \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \longrightarrow Z \sim \text{Normal}(0, 1),$$

cuando $n \to \infty$, $\mathrm{E}(X_i) = \mu$ y $\mathrm{Var}(X_i) = \sigma^2$.

Mínimo y Máximo

Sean $X_1, \ldots, X_n$ variables aleatorias continuas independientes con idéntica distribución (f_X y F_X), entonces para:

$Y_1 = \text{mín}\{X_1, \ldots, X_n\} \longrightarrow f_{Y_1} = n\,[1 - F_X(y)]^{n-1}\, f_X(y)$; $Y_n = \text{máx}\{X_1, \ldots, X_n\} \longrightarrow f_{Y_n} = n\,[F_X(y)]^{n-1}\, f_X(y)$

Mientras que la distribución conjunta entre Y_1 e Y_n está dada por:

$$f_{Y_1,Y_n}(u, v) = n\,(n-1)\,[F_X(v) - F_X(u)]^{n-2}\, f_X(v)\, f_X(u), \qquad u \leq v$$

Función Generadora de Momentos

En el caso que $X_1, \ldots, X_n$ sean variables aleatorias independientes con funciones generadoras de momentos $M_{X_1}, \ldots, M_{X_n}$ respectivamente, se tiene si $Z = \displaystyle\sum_{i=1}^{n} X_i \to M_Z(t) = M_{X_1}(t) \times \cdots \times M_{X_n}(t)$.

Propiedades Esperanza, Varianza y Covarianza

Sean $X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m$ variables aleatorias y $a_0, a_1, \ldots, a_n, b_0, b_1, \ldots, b_m$ constantes conocidas.

- $\mathrm{E}\left(a_0 + \displaystyle\sum_{i=1}^{n} a_i \cdot X_i\right) = a_0 + \displaystyle\sum_{i=1}^{n} a_i \cdot \mathrm{E}(X_i)$.
- $\mathrm{Cov}\left(a_0 + \displaystyle\sum_{i=1}^{n} a_i \cdot X_i,\, b_0 + \displaystyle\sum_{j=1}^{m} b_j \cdot Y_j\right) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} a_i \cdot b_j \cdot \mathrm{Cov}\,(X_i, Y_j)$.
- $\mathrm{Var}\left(a_0 + \displaystyle\sum_{i=1}^{n} a_i \cdot X_i\right) = \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} a_i \cdot a_j \cdot \mathrm{Cov}\,(X_i, X_j)$.
- Si $X_1, \ldots, X_n$ son variables aleatorias independientes, entonces $\mathrm{Var}\left(a_0 + \displaystyle\sum_{i=1}^{n} a_i \cdot X_i\right) = \displaystyle\sum_{i=1}^{n} a_i^2 \cdot \mathrm{Var}\,(X_i)$

Aproximación de Momentos

Sea X una variable aleatoria e $Y = g(X)$, la aproximación de 4to orden está dada por

$$Y = g(X) \approx g(\mu_X) + \frac{(X-\mu_X)\, g'(\mu_X)}{1!} + \frac{(X-\mu_X)^2\, g''(\mu_X)}{2!} + \frac{(X-\mu_X)^3\, g'''(\mu_X)}{3!} + \frac{(X-\mu_X)^4\, g''''(\mu_X)}{4!}$$

Sean $X_1, \ldots, X_n$ variables aleatorias con valores esperados $\mu_{X_1}, \ldots, \mu_{X_n}$ y varianzas $\sigma^2_{X_1}, \ldots, \sigma^2_{X_n}$ e $Y = g(X_1, \ldots, X_n)$, la aproximación de primer orden está dada por

$$\begin{aligned}
Y &\approx g(\mu_{X_1}, \ldots, \mu_{X_n}) + \sum_{i=1}^{n} (X_i - \mu_{X_i})\, \frac{\partial}{\partial X_i} g(\mu_{X_1}, \ldots, \mu_{X_n}) \\
\mathrm{E}(Y) &\approx g(\mu_{X_1}, \ldots, \mu_{X_n}) \\
\mathrm{Var}(Y) &\approx \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\, \sigma_{X_i}\, \sigma_{X_j} \left[\frac{\partial}{\partial X_i} g(\mu_{X_1}, \ldots, \mu_{X_n}) \cdot \frac{\partial}{\partial X_j} g(\mu_{X_1}, \ldots, \mu_{X_n})\right], \qquad \text{con } \rho_{ij} = \mathrm{Corr}(X_i,\, Xj)
\end{aligned}$$

Estimador Máximo Verosímil

Sea $X_1, \ldots, X_n$ una muestra aleatoria independiente e idénticamente distribuida con función de probabilidad p_X o de densidad f_X, determinada por un parámetro θ. Si $\hat{\theta}$ es el estimador máximo verosímil del parámetro θ, entonces:

- $\mathrm{E}(\hat{\theta}) \to \theta$, cuando $n \to \infty$.
- $\mathrm{Var}(\hat{\theta}) = \dfrac{1}{I_n(\theta)}$, con $I_n(\theta) = -\mathrm{E}\left[\dfrac{\partial^2}{\partial \theta^2} \ln L(\theta)\right]$.
- $\sqrt{I_n(\theta)}(\hat{\theta} - \theta) \,\dot{\sim}\, \text{Normal}\,(0,\, 1)$, cuando $n \to \infty$.
- El estimador máximo verosímil de $g(\theta)$ es $g(\hat{\theta})$, cuya varianza está dada por: $\mathrm{Var}[g(\hat{\theta})] = \dfrac{[g'(\theta)]^2}{I_n(\theta)}$.

Error Cuadrático Medio

El error cuadrático medio de un estimador $\hat{\theta}$ de θ se define como:

$$\mathrm{ECM}(\hat{\theta}) = \mathrm{E}\left[\left(\hat{\theta} - \theta\right)^2\right] = \mathrm{Var}(\hat{\theta}) + \text{Sesgo}^2$$

Distribuciones Muestrales

Sean $X_1, \ldots, X_n$ variables aleatorias independientes e idénticamente distribuidas Normal$(\mu,\, \sigma)$, entonces

$$\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1), \qquad \frac{\overline{X}_n - \mu}{s/\sqrt{n}} \sim \text{t-student}(n-1), \qquad \frac{s^2\,(n-1)}{\sigma^2} \sim \chi^2(n-1)$$

con $s^2 = \dfrac{1}{n-1} \displaystyle\sum_{i=1}^{n} (X_i - \overline{X}_n)^2$.

Tablas de Percentiles p

Distribución Normal Estándar k_p

k_p	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990

Distribución t-student $t_p(\nu)$

ν	$t_{0,90}$	$t_{0,95}$	$t_{0,975}$	$t_{0,99}$
1	3,078	6,314	12,706	31,821
2	1,886	2,920	4,303	6,965
3	1,638	2,353	3,182	4,541
4	1,533	2,132	2,776	3,747
5	1,476	2,015	2,571	3,365
6	1,440	1,943	2,447	3,143
7	1,415	1,895	2,365	2,998
8	1,397	1,860	2,306	2,896
9	1,383	1,833	2,262	2,821
10	1,372	1,812	2,228	2,764
11	1,363	1,796	2,201	2,718
12	1,356	1,782	2,179	2,681
13	1,350	1,771	2,160	2,650
14	1,345	1,761	2,145	2,624
15	1,341	1,753	2,131	2,602
16	1,337	1,746	2,120	2,583
17	1,333	1,740	2,110	2,567
18	1,330	1,734	2,101	2,552
19	1,328	1,729	2,093	2,539
20	1,325	1,725	2,086	2,528
21	1,323	1,721	2,080	2,518
22	1,321	1,717	2,074	2,508
23	1,319	1,714	2,069	2,500
24	1,318	1,711	2,064	2,492
25	1,316	1,708	2,060	2,485
26	1,315	1,706	2,056	2,479
27	1,314	1,703	2,052	2,473
28	1,313	1,701	2,048	2,467
29	1,311	1,699	2,045	2,462
30	1,310	1,697	2,042	2,457
∞	1,282	1,645	1,960	2,326

Distribución Chi-Cuadrado $c_p(\nu)$

ν	$c_{0,005}$	$c_{0,001}$	$c_{0,025}$	$c_{0,05}$	$c_{0,1}$	$c_{0,2}$	$c_{0,3}$	$c_{0,4}$	$c_{0,6}$	$c_{0,7}$	$c_{0,8}$	$c_{0,9}$	$c_{0,95}$	$c_{0,975}$	$c_{0,99}$	$c_{0,995}$
1	0,000	0,000	0,001	0,004	0,016	0,064	0,148	0,275	0,708	1,074	1,642	2,706	3,841	5,024	6,635	7,879
2	0,010	0,002	0,051	0,103	0,211	0,446	0,713	1,022	1,833	2,408	3,219	4,605	5,991	7,378	9,210	10,597
3	0,072	0,024	0,216	0,352	0,584	1,005	1,424	1,869	2,946	3,665	4,642	6,251	7,815	9,348	11,345	12,838
4	0,207	0,091	0,484	0,711	1,064	1,649	2,195	2,753	4,045	4,878	5,989	7,779	9,488	11,143	13,277	14,860
5	0,412	0,210	0,831	1,145	1,610	2,343	3,000	3,655	5,132	6,064	7,289	9,236	11,070	12,833	15,086	16,750
6	0,676	0,381	1,237	1,635	2,204	3,070	3,828	4,570	6,211	7,231	8,558	10,645	12,592	14,449	16,812	18,548
7	0,989	0,598	1,690	2,167	2,833	3,822	4,671	5,493	7,283	8,383	9,803	12,017	14,067	16,013	18,475	20,278
8	1,344	0,857	2,180	2,733	3,490	4,594	5,527	6,423	8,351	9,524	11,030	13,362	15,507	17,535	20,090	21,955
9	1,735	1,152	2,700	3,325	4,168	5,380	6,393	7,357	9,414	10,656	12,242	14,684	16,919	19,023	21,666	23,589
10	2,156	1,479	3,247	3,940	4,865	6,179	7,267	8,295	10,473	11,781	13,442	15,987	18,307	20,483	23,209	25,188
11	2,603	1,834	3,816	4,575	5,578	6,989	8,148	9,237	11,530	12,899	14,631	17,275	19,675	21,920	24,725	26,757
12	3,074	2,214	4,404	5,226	6,304	7,807	9,034	10,182	12,584	14,011	15,812	18,549	21,026	23,337	26,217	28,300
13	3,565	2,617	5,009	5,892	7,042	8,634	9,926	11,129	13,636	15,119	16,985	19,812	22,362	24,736	27,688	29,819
14	4,075	3,041	5,629	6,571	7,790	9,467	10,821	12,078	14,685	16,222	18,151	21,064	23,685	26,119	29,141	31,319
15	4,601	3,483	6,262	7,261	8,547	10,307	11,721	13,030	15,733	17,322	19,311	22,307	24,996	27,488	30,578	32,801
16	5,142	3,942	6,908	7,962	9,312	11,152	12,624	13,983	16,780	18,418	20,465	23,542	26,296	28,845	32,000	34,267
17	5,697	4,416	7,564	8,672	10,085	12,002	13,531	14,937	17,824	19,511	21,615	24,769	27,587	30,191	33,409	35,718
18	6,265	4,905	8,231	9,390	10,865	12,857	14,440	15,893	18,868	20,601	22,760	25,989	28,869	31,526	34,805	37,156
19	6,844	5,407	8,907	10,117	11,651	13,716	15,352	16,850	19,910	21,689	23,900	27,204	30,144	32,852	36,191	38,582
20	7,434	5,921	9,591	10,851	12,443	14,578	16,266	17,809	20,951	22,775	25,038	28,412	31,410	34,170	37,566	39,997
21	8,034	6,447	10,283	11,591	13,240	15,445	17,182	18,768	21,991	23,858	26,171	29,615	32,671	35,479	38,932	41,401
22	8,643	6,983	10,982	12,338	14,041	16,314	18,101	19,729	23,031	24,939	27,301	30,813	33,924	36,781	40,289	42,796
23	9,260	7,529	11,689	13,091	14,848	17,187	19,021	20,690	24,069	26,018	28,429	32,007	35,172	38,076	41,638	44,181
24	9,886	8,085	12,401	13,848	15,659	18,062	19,943	21,652	25,106	27,096	29,553	33,196	36,415	39,364	42,980	45,559
25	10,520	8,649	13,120	14,611	16,473	18,940	20,867	22,616	26,143	28,172	30,675	34,382	37,652	40,646	44,314	46,928
26	11,160	9,222	13,844	15,379	17,292	19,820	21,792	23,579	27,179	29,246	31,795	35,563	38,885	41,923	45,642	48,290
27	11,808	9,803	14,573	16,151	18,114	20,703	22,719	24,544	28,214	30,319	32,912	36,741	40,113	43,195	46,963	49,645
28	12,461	10,391	15,308	16,928	18,939	21,588	23,647	25,509	29,249	31,391	34,027	37,916	41,337	44,461	48,278	50,993
29	13,121	10,986	16,047	17,708	19,768	22,475	24,577	26,475	30,283	32,461	35,139	39,087	42,557	45,722	49,588	52,336
30	13,787	11,588	16,791	18,493	20,599	23,364	25,508	27,442	31,316	33,530	36,250	40,256	43,773	46,979	50,892	53,672
40	20,707	17,916	24,433	26,509	29,051	32,345	34,872	37,134	41,622	44,165	47,269	51,805	55,758	59,342	63,691	66,766
50	27,991	24,674	32,357	34,764	37,689	41,449	44,313	46,864	51,892	54,723	58,164	63,167	67,505	71,420	76,154	79,490
60	35,534	31,738	40,482	43,188	46,459	50,641	53,809	56,620	62,135	65,227	68,972	74,397	79,082	83,298	88,379	91,952
70	43,275	39,036	48,758	51,739	55,329	59,898	63,346	66,396	72,358	75,689	79,715	85,527	90,531	95,023	100,425	104,215
80	51,172	46,520	57,153	60,391	64,278	69,207	72,915	76,188	82,566	86,120	90,405	96,578	101,879	106,629	112,329	116,321
90	59,196	54,155	65,647	69,126	73,291	78,558	82,511	85,993	92,761	96,524	101,054	107,565	113,145	118,136	124,116	128,299
100	67,328	61,918	74,222	77,929	82,358	87,945	92,129	95,808	102,946	106,906	111,667	118,498	124,342	129,561	135,807	140,169

Distribución	Densidad de Probabilidad	Θ_X	Parámetros	Esperanza y Varianza
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, \ldots, n$	$n,\ p$	$\mu_X = n\,p$ $\sigma^2_X = n\,p\,(1-p)$ $M(t) = [p\,e^t + (1-p)]^n, \quad t \in \mathbb{R}$
Geométrica	$p\,(1-p)^{x-1}$	$x = 1, 2, \ldots$	p	$\mu_X = 1/p$ $\sigma^2_X = (1-p)/p^2$ $M(t) = p\,e^t/[1-(1-p)\,e^t], \quad t < -\ln(1-p)$
Binomial-Negativa	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$	$x = r, r+1, \ldots$	$r,\ p$	$\mu_X = r/p$ $\sigma^2_X = r\,(1-p)/p^2$ $M(t) = \left\{p\,e^t/[1-(1-p)\,e^t]\right\}^r, \quad t < -\ln(1-p)$
Poisson	$\dfrac{(\nu\,t)^x\,e^{-\nu\,t}}{x!}$	$x = 0, 1, \ldots$	ν	$\mu_X = \nu\,t$ $\sigma^2_X = \nu\,t$ $M(t) = \exp\left[\lambda\left(e^t - 1\right)\right], \quad t \in \mathbb{R}$
Exponencial	$\nu\,e^{-\nu\,x}$	$x \geq 0$	ν	$\mu_X = 1/\nu$ $\sigma^2_X = 1/\nu^2$ $M(t) = \nu/(\nu - t), \quad t < \nu$
Gamma	$\dfrac{\nu^k}{\Gamma(k)}\,x^{k-1}\,e^{-\nu\,x}$	$x \geq 0$	$k,\ \nu$	$\mu_X = k/\nu$ $\sigma^2_X = k/\nu^2$ $M(t) = [\nu/(\nu - t)]^k, \quad t < \nu$
Normal	$\dfrac{1}{\sqrt{2\,\pi}\,\sigma}\ \exp\left[-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right]$	$-\infty < x < \infty$	$\mu,\ \sigma$	$\mu_X = \mu$ $\sigma^2_X = \sigma^2$ $M(t) = \exp(\mu\,t + \sigma^2\,t^2/2), \quad t \in \mathbb{R}$
Log-Normal	$\dfrac{1}{\sqrt{2\,\pi}\,(\zeta\,x)}\ \exp\left[-\dfrac{1}{2}\left(\dfrac{\ln x - \lambda}{\zeta}\right)^2\right]$	$x \geq 0$	$\lambda,\ \zeta$	$\mu_X = \exp\left(\lambda + \dfrac{1}{2}\,\zeta^2\right)$ $\sigma^2_X = \mu^2_X\left(e^{\zeta^2} - 1\right)$ $E(X^r) = e^{r\,\lambda}\,M_Z(r\,\zeta)$, con $Z \sim$Normal(0,1)
Uniforme	$\dfrac{1}{(b-a)}$	$a \leq x \leq b$	$a,\ b$	$\mu_X = (a+b)/2$ $\sigma^2_X = (b-a)^2/12$ $M(t) = [e^{t\,b} - e^{t\,a}]/[t\,(b-a)], \quad t \in \mathbb{R}$
Beta	$\dfrac{1}{B(q,\ r)}\ \dfrac{(x-a)^{q-1}\,(b-x)^{r-1}}{(b-a)^{q+r-1}}$	$a \leq x \leq b$	$q,\ r$	$\mu_X = a + \frac{q}{q+r}\,(b-a)$ $\sigma^2_X = \frac{q\,r\,(b-a)^2}{(q+r)^2\,(q+r+1)}$
Hipergeométrica	$\dfrac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}$	$\text{máx}\{0, n+m-N\} \leq x \leq \text{mín}\{n, m\}$	$N,\ m,\ n$	$\mu_X = n\,\frac{m}{N}$ $\sigma^2_X = \left(\frac{N-n}{N-1}\right)\,n\,\frac{m}{N}\,\left(1 - \frac{m}{N}\right)$

Otras distribuciones

- Si $T \sim \text{Weibull}(\eta,\, \beta)$, se tiene que

$$F_T(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \qquad f_T(t) = \frac{\beta}{\eta}\,\left(\frac{t}{\eta}\right)^{\beta-1}\,\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], \quad t > 0$$

  Con $\beta > 0$, es un parámetro de forma y $\eta > 0$, es un parámetro de escala. Si t_p es el percentil $p \times 100\,\%$, entonces

$$\ln(t_p) = \ln(\eta) + \frac{1}{\beta}\cdot\Phi^{-1}_{\text{Weibull}}(p), \quad \Phi^{-1}_{\text{Weibull}}(p) = \ln[-\ln(1-p)]$$

  Mientras que su m-ésimo momento está dado por

$$E(T^m) = \eta^m\,\Gamma(1 + m/\beta)$$

$$\mu_T = \eta\,\Gamma\left(1 + \frac{1}{\beta}\right), \quad \sigma^2_T = \eta^2\,\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

- Si $Y \sim \text{Logística}(\mu,\, \sigma)$, se tiene que

$$F_Y(y) = \Phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right); \qquad f_Y(y) = \frac{1}{\sigma}\,\phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right), \quad -\infty < y < \infty$$

  donde

$$\Phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1+\exp(z)]} \quad \text{y} \quad \phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1+\exp(z)]^2}$$

  son la función de probabilidad y de densidad de una Logística Estándar. $\mu \in \mathbb{R}$, es un parámetro de localización y $\sigma > 0$, es un parámetro de escala. Si y_p es el percentil $p \times 100\,\%$, entonces

$$y_p = \mu + \sigma\,\Phi^{-1}_{\text{Logística}}(p) \quad \text{con} \quad \Phi^{-1}_{\text{Logística}}(p) = \log\left(\frac{p}{1-p}\right)$$

  Su esperanza y varianza están dadas por: $\mu_Y = \mu$ y $\sigma^2_Y = \frac{\sigma^2\,\pi^2}{3}$.

- Si $T \sim \text{Log-Logística}(\mu,\, \sigma)$, se tiene que

$$F_T(t) = \Phi_{\text{Logística}}\left(\frac{\ln(t)-\mu}{\sigma}\right); \quad f_T(t) = \frac{1}{\sigma\,t}\,\phi_{\text{Logística}}\left(\frac{\ln(t)-\mu}{\sigma}\right) \quad t > 0$$

  Donde $\exp(\mu)$, es un parámetro de escala y $\sigma > 0$, es un parámetro de forma. Si t_p es el percentil $p \times 100\,\%$, entonces

$$\ln(t_p) = \mu + \sigma\,\Phi^{-1}_{\text{Logística}}(p)$$

  Para un entero $m > 0$ se tiene que

$$E(T^m) = \exp(m\,\mu)\,\Gamma(1 + m\,\sigma)\,\Gamma(1 - m\,\sigma)$$

  El m-ésimo momento no es finito si $m\,\sigma \geq 1$.

  Para $\sigma < 1$: $\mu_T = \exp(\mu)\,\Gamma(1+\sigma)\,\Gamma(1-\sigma)$

  y para $\sigma < 1/2$: $\sigma^2_T = \exp(2\,\mu)\,\left[\Gamma(1 + 2\,\sigma)\,\Gamma(1 - 2\,\sigma) - \Gamma^2(1+\sigma)\,\Gamma^2(1-\sigma)\right]$

- Un variable aleatoria T tiene distribución t-student si su función de densidad está dada por:

$$f_T(t) = \frac{\Gamma[(\nu+1)/2]}{\sqrt{\pi\,\nu}\,\Gamma(\nu/2)}\,\left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad -\infty < t < \infty$$

  - $\mu_T = 0$, para $\nu > 1$.
  - $\sigma^2_T = \frac{\nu}{\nu-2}$, para $\mu > 2$.

- Si $T \sim \text{Fisher}(\eta,\, \nu)$, se tiene que

$$f_T(t) = \frac{\Gamma(\frac{\eta+\nu}{2})}{\Gamma(\eta/2)\Gamma(\nu/2)}\,\left(\frac{\eta}{\nu}\right)^{\frac{\eta}{2}}\,\frac{t^{\frac{\eta}{2}-1}}{\left(\frac{\eta}{\nu}\,t + 1\right)^{\frac{\eta+\nu}{2}}}, \quad t > 0$$

  - $\mu_T = \frac{\nu}{\nu-2}$, para $\nu > 2$.
  - $\sigma^2_T = \frac{2\,\nu^2\,(\eta+\nu-2)}{\eta\,(\nu-2)^2\,(\nu-4)}$, para $\nu > 4$.