## Formulario Examen

**Igualdades** 

$$(a+b)^{n} = \sum_{k=0}^{n} \binom{n}{k} a^{k} b^{n-k}; \qquad \sum_{k=x}^{\infty} \phi^{k} = \frac{\phi^{x}}{1-\phi} \quad \text{si } |\phi| < 1;$$
$$\sum_{k=0}^{\infty} \frac{\lambda^{k}}{k!} = \exp(\lambda); \qquad \sum_{x=0}^{\infty} \binom{x+k-1}{k-1} \phi^{x} = \frac{1}{(1-\phi)^{k}} \quad \text{si } 0 < \phi < 1 \text{ y } k \in \mathbb{N}$$

Propiedades función  $\Gamma(\cdot)$  y  $B(\cdot, \cdot)$ 

(1) 
$$\Gamma(k) = \int_0^\infty u^{k-1} e^{-u} du;$$
 (2)  $\Gamma(a+1) = a \Gamma(a);$  (3)  $\Gamma(n+1) = n!,$  si  $n \in \mathbb{N}_0;$ 

(4) 
$$\Gamma(1/2) = \sqrt{\pi};$$
 (5)  $B(q, r) = \int_0^1 x^{q-1} (1-x)^{r-1} dx;$  (6)  $B(q, r) = \frac{\Gamma(q) \Gamma(r)}{\Gamma(q+r)}$ 

#### Distribución Gamma

(1) Si 
$$T \sim \text{Gamma}(k, \nu)$$
, con  $k \in \mathbb{N} \longrightarrow F_T(t) = 1 - \sum_{r=0}^{k-1} \frac{(\nu t)^x e^{-\nu t}}{x!}$ 

(2) 
$$Gamma(1, \nu) = Exp(\nu)$$
 (3)  $Gamma(\eta/2, 1/2) = \chi^2(\eta)$ 

## Medidas descriptivas

$$\mu_X = \mathcal{E}(X), \quad \sigma_X^2 = \mathcal{E}\left[(X - \mu_X)^2\right], \quad \delta_X = \frac{\sigma_X}{\mu_X}, \quad \theta_X = \frac{\mathcal{E}\left[(X - \mu_X)^3\right]}{\sigma_X^3}, \quad K_X = \frac{\mathcal{E}\left[(X - \mu_X)^4\right]}{\sigma_X^4} - 3$$

$$M_X(t) = \mathcal{E}\left(e^{tX}\right), \quad \mathcal{E}[g(X)] = \begin{cases} \sum_{x \in \Theta_X} g(x) \cdot p_X(x) \\ \int_{-\infty}^{\infty} g(x) \cdot f_X(x) \, dx \end{cases}, \quad \text{Rango} = \text{máx} - \text{mín}, \quad \text{IQR} = x_{75\%} - x_{25\%}$$

$$x_p$$
: Percentil  $p \times 100 \% \to F_X(x_p) = p$ ,  $Cov(X, Y) = E[(X - \mu_X) \cdot (Y - \mu_Y)]$ ,  $\rho = \frac{Cov(X, Y)}{\sigma_X \cdot \sigma_Y}$ 

#### Teorema de Probabilidades Totales

$$p_{Y}(y) = \sum_{x \in \Theta_{X}} p_{X,Y}(x,y); \qquad f_{X}(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y) \, dy$$
$$p_{X}(x) = \int_{-\infty}^{+\infty} p_{X|Y=y}(x) \cdot f_{Y}(y) \, dy; \qquad f_{Y}(y) = \sum_{x \in \Theta_{X}} f_{Y|X=x}(y) \cdot p_{X}(x)$$

## Transformación

Sea Y = g(X) una función cualquiera, con k raíces:

$$f_Y(y) = \sum_{i=1}^k f_X(g_i^{-1}(y)) \cdot \left| \frac{d}{dy} g_i^{-1}(y) \right| \quad \text{o} \quad p_Y(y) = \sum_{i=1}^k p_X(g_i^{-1}(y))$$

Sea Z = g(X, Y) una función cualquiera:

$$p_Z(z) = \sum_{g(x,y)=z} p_{X,Y}(x,y)$$

Sea Z = g(X, Y) una función invertible para X o Y fijo:

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}(g^{-1}, y) \left| \frac{\partial}{\partial z} g^{-1} \right| dy = \int_{-\infty}^{\infty} f_{X,Y}(x, g^{-1}) \left| \frac{\partial}{\partial z} g^{-1} \right| dx$$

#### Esperanza y Varianza Condicional

$$\mathrm{E}(Y) = \mathrm{E}[\mathrm{E}(Y \,|\, X)] \quad \mathrm{y} \quad \mathrm{Var}(Y) = \mathrm{Var}[\mathrm{E}(Y \,|\, X)] + \mathrm{E}[\mathrm{Var}(Y \,|\, X)]$$

#### Distribución Normal Bivariada

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \times \exp\left\{-\frac{1}{2(1-\rho^2)} \left[ \left(\frac{x-\mu_X}{\sigma_X}\right)^2 + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right) \left(\frac{y-\mu_Y}{\sigma_Y}\right) \right] \right\}$$

$$Y \mid X = x \sim \text{Normal}\left(\mu_Y + \frac{\rho\sigma_Y}{\sigma_X}\left(x-\mu_X\right), \, \sigma_Y\sqrt{(1-\rho^2)}\right)$$

#### Teorema del Límite Central

Sean  $X_1, \ldots, X_n$  variables aleatorias independientes e idénticamente distribuidas, entonces

$$Z_n = \frac{\sum_{i=1}^n X_i - n \cdot \mu}{\sqrt{n} \, \sigma} = \frac{\overline{X}_n - \mu}{\sigma / \sqrt{n}} \longrightarrow Z \sim \text{Normal}(0, 1),$$

cuando  $n \to \infty$ ,  $E(X_i) = \mu$  y  $Var(X_i) = \sigma^2$ .

## Mínimo y Máximo

Sean  $X_1, \ldots, X_n$  variables aleatorias continuas independientes con idéntica distribución  $(f_X y F_X)$ , entonces para:

$$Y_1 = \min\{X_1, \dots, X_n\} \longrightarrow F_{Y_1}(y) = 1 - [1 - F_X(y)]^n \to f_{Y_1}(y) = n [1 - F_X(y)]^{n-1} f_X(y)$$

$$Y_n = \max\{X_1, \dots, X_n\} \longrightarrow F_{Y_n}(y) = [F_X(y)]^n \to f_{Y_n}(y) = n [F_X(y)]^{n-1} f_X(y)$$

Mientras que la distribución conjunta entre  $Y_1$  e  $Y_n$  está dada por:

$$f_{Y_1,Y_n}(u,v) = n(n-1) [F_X(v) - F_X(u)]^{n-2} f_X(v) f_X(u), \qquad u \le v$$

## Función Generadora de Momentos

En el caso que  $X_1, \ldots, X_n$  sean variables aleatorias independientes con funciones generadoras de momentos  $M_{X_1}, \ldots, M_{X_n}$  respectivamente, se tiene si  $Z = \sum_{i=1}^n X_i \to M_Z(t) = M_{X_1}(t) \times \cdots \times M_{X_n}(t)$ .

## Propiedades Esperanza, Varianza y Covarianza

Sean  $X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_m$  variables aleatorias y  $a_0, a_1, \ldots, a_n, b_0, b_1, \ldots, b_m$  constantes conocidas.

• 
$$E\left(a_0 + \sum_{i=1}^n a_i \cdot X_i\right) = a_0 + \sum_{i=1}^n a_i \cdot E(X_i).$$

• 
$$\operatorname{Cov}\left(a_0 + \sum_{i=1}^n a_i \cdot X_i, \ b_0 + \sum_{j=1}^m b_j \cdot Y_j\right) = \sum_{i=1}^n \sum_{j=1}^m a_i \cdot b_j \cdot \operatorname{Cov}\left(X_i, Y_j\right).$$

• Si 
$$X_1, \ldots, X_n$$
 son variables aleatorias independientes, entonces  $\operatorname{Var}\left(a_0 + \sum_{i=1}^n a_i \cdot X_i\right) = \sum_{i=1}^n a_i^2 \cdot \operatorname{Var}\left(X_i\right)$ 

#### Aproximación de Momentos

Sea X una variable aleatoria e Y = g(X), la aproximación de 4to orden está dada por

$$Y = g(X) \approx g(\mu_X) + \frac{(X - \mu_X)g'(\mu_X)}{1!} + \frac{(X - \mu_X)^2g''(\mu_X)}{2!} + \frac{(X - \mu_X)^3g'''(\mu_X)}{3!} + \frac{(X - \mu_X)^4g''''(\mu_X)}{4!}$$

Sean  $X_1, \ldots, X_n$  variables aleatorias con valores esperados  $\mu_{X_1}, \ldots, \mu_{X_n}$  y varianzas  $\sigma^2_{X_1}, \ldots, \sigma^2_{X_n}$  e  $Y = g(X_1, \ldots, X_n)$ , la aproximación de primer orden está dada por

$$Y \approx g(\mu_{X_1}, \dots, \mu_{X_n}) + \sum_{i=1}^n (X_i - \mu_{X_i}) \frac{\partial}{\partial X_i} g(\mu_{X_1}, \dots, \mu_{X_n})$$

$$E(Y) \approx g(\mu_{X_1}, \dots, \mu_{X_n})$$

$$Var(Y) \approx \sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \, \sigma_{X_i} \, \sigma_{X_j} \, \left[ \frac{\partial}{\partial X_i} g(\mu_{X_1}, \dots, \mu_{X_n}) \cdot \frac{\partial}{\partial X_j} g(\mu_{X_1}, \dots, \mu_{X_n}) \right], \quad \text{con } \rho_{ij} = \text{Corr}(X_i, X_j)$$

#### Estimador Máximo Verosímil

Sea  $X_1, \ldots, X_n$  una muestra aleatoria independiente e idénticamente distribuida con función de probabilidad  $p_X$  o de densidad  $f_X$ , determinada por un parámetro  $\theta$ . Si  $\hat{\theta}$  es el estimador máximo verosímil del parámetro  $\theta$ , entonces:

- $E(\hat{\theta}) \to \theta$ , cuando  $n \to \infty$ .
- $\sqrt{I_n(\theta)}(\hat{\theta} \theta) \stackrel{\cdot}{\sim} \text{Normal}(0, 1)$ , cuando  $n \to \infty$ .
- El estimador máximo verosímil de  $g(\theta)$  es  $g(\hat{\theta})$ , cuya varianza está dada por:  $Var[g(\hat{\theta})] = \frac{[g'(\theta)]^2}{I_n(\theta)}$

#### Error Cuadrático Medio

El error cuadrático medio de un estimador  $\hat{\theta}$  de  $\theta$  se define como:

$$\mathrm{ECM}(\hat{\theta}) = \mathrm{E}\left[\left(\hat{\theta} - \theta\right)^2\right] = \mathrm{Var}(\hat{\theta}) + \mathrm{Sesgo}^2$$

#### Distribuciones Muestrales

Sean  $X_1, \ldots, X_n$  variables aleatorias independientes e idénticamente distribuidas Normal $(\mu, \sigma)$ , entonces

$$\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \sim \text{Normal}(0, 1), \quad \frac{\overline{X}_n - \mu}{s/\sqrt{n}} \sim \text{t-student}(n - 1), \quad \frac{s^2 \left(n - 1\right)}{\sigma^2} \sim \chi^2(n - 1)$$

con 
$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2$$
.

### Potencia

Sean  $X_1, \dots, X_n$  variables aleatorias independientes e idénticamente distribuidas Normal $(\mu, \sigma)$ , entonces para  $H_0$ :  $\mu = \mu_0$  y  $\sigma$  conocido:

$$1 - \Phi\left(k_{1-\alpha/2} - \Delta\frac{\sqrt{n}}{\sigma}\right) + \Phi\left(k_{\alpha/2} - \Delta\frac{\sqrt{n}}{\sigma}\right), \qquad 1 - \Phi\left(k_{1-\alpha} - \Delta\frac{\sqrt{n}}{\sigma}\right), \qquad \Phi\left(k_{\alpha} - \Delta\frac{\sqrt{n}}{\sigma}\right)$$

#### Comparación de Poblaciones

Sean  $X_1, \ldots, X_n$  e  $Y_1, \ldots, Y_m$  dos muestras aleatorias independientes con distribución Normal $(\mu_X, \sigma_X)$  y Normal $(\mu_Y, \sigma_Y)$  respectivamente. Con medias y varianzas muestrales dadas por:

$$\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \qquad \overline{Y}_m = \frac{1}{m} \sum_{i=1}^m Y_i \qquad S_X^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X}_n)^2 \qquad S_Y^2 = \frac{1}{m-1} \sum_{i=1}^m (Y_i - \overline{Y}_m)^2$$

Entonces

■ Si  $\sigma_X$  y  $\sigma_Y$  son conocidos:

$$\frac{(\overline{X}_n - \overline{Y}_m) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim \text{Normal}(0, 1)$$

• Si  $\sigma_X$  y  $\sigma_Y$  son desconocidos pero iguales:

$$\frac{(\overline{X}_n - \overline{Y}_m) - (\mu_X - \mu_Y)}{S_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \sim t - \text{Student}(n + m - 2)$$

con 
$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}$$

• Si  $\sigma_X$  y  $\sigma_Y$  son desconocidos:

$$\frac{(\overline{X}_n - \overline{Y}_m) - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}} \sim t - \text{Student}(\nu) \quad \text{con} \quad \nu = \left[ \frac{\left(S_X^2 / n + S_Y^2 / m\right)^2}{\frac{\left(S_X^2 / n\right)^2}{n - 1} + \frac{\left(S_Y^2 / m\right)^2}{m - 1}} \right]$$

• Si  $\mu_X$  y  $\mu_Y$  son desconocidos:

$$\frac{\left[ (n-1) S_X^2 / \sigma_X^2 \right] / (n-1)}{\left[ (m-1) S_Y^2 / \sigma_Y^2 \right] / (m-1)} = \frac{S_X^2}{S_Y^2} \cdot \frac{\sigma_Y^2}{\sigma_X^2} \sim F(n-1, m-1)$$

Sean  $X_1, \ldots, X_n$  e  $Y_1, \ldots, Y_m$  dos muestras aleatorias independientes con distribución Bernoulli $(p_X)$  y Bernoulli $(p_Y)$  respectivamente, entonces

$$\frac{(\overline{X}_n - \overline{Y}_m) - (p_X - p_Y)}{\sqrt{\frac{p_X(1 - p_X)}{n} + \frac{p_Y(1 - p_Y)}{m}}} \overset{\text{aprox}}{\sim} \text{Normal}(0, 1) \qquad \text{y} \qquad \frac{(\overline{X}_n - \overline{Y}_m) - (p_X - p_Y)}{\sqrt{\frac{\overline{X}_n(1 - \overline{X}_n)}{n} + \frac{\overline{Y}_m(1 - \overline{Y}_m)}{m}}} \overset{\text{aprox}}{\sim} \text{Normal}(0, 1)$$

Sean  $X_1, \ldots, X_n$  e  $Y_1, \ldots, Y_m$  dos muestras aleatorias independientes con distribución  $Poisson(\lambda_X)$  y  $Poisson(\lambda_Y)$  respectivamente, entonces

$$\frac{(\overline{X}_n - \overline{Y}_m) - (\lambda_X - \lambda_Y)}{\sqrt{\frac{\lambda_X}{n} + \frac{\lambda_Y}{m}}} \stackrel{\text{aprox}}{\sim} \text{Normal}(0, 1) \qquad \text{y} \qquad \frac{(\overline{X}_n - \overline{Y}_m) - (\lambda_X - \lambda_Y)}{\sqrt{\frac{\overline{X}_n}{n} + \frac{\overline{Y}_m}{m}}} \stackrel{\text{aprox}}{\sim} \text{Normal}(0, 1)$$

Sean  $X_1, \ldots, X_n$  e  $Y_1, \ldots, Y_m$  dos muestras aleatorias independientes con distribución Exponencial $(\nu_X)$  y Exponencial $(\nu_Y)$  respectivamente, entonces

$$\frac{(\overline{X}_n - \overline{Y}_m) - \left(\frac{1}{\nu_X} - \frac{1}{\nu_Y}\right)}{\sqrt{\frac{1}{n\nu_X^2} + \frac{1}{m\nu_Y^2}}} \overset{\text{aprox}}{\sim} \text{Normal}(0, 1) \qquad \text{y} \qquad \frac{(\overline{X}_n - \overline{Y}_m) - \left(\frac{1}{\nu_X} - \frac{1}{\nu_Y}\right)}{\sqrt{\frac{\overline{X}_n^2}{n} + \frac{\overline{Y}_m^2}{m}}} \overset{\text{aprox}}{\sim} \text{Normal}(0, 1)$$

## Bondad de Ajuste

$$X^{2} = \sum_{i=1}^{k} \frac{(O_{i} - E_{i})^{2}}{E_{i}} \sim \chi^{2}(k - 1 - \nu)$$

 $con \ \nu$  igual al número de estadísticos muestrales utilizados para estimar los parámetros del modelo ajustado.

#### Regresión Lineal Simple

Para el modelo de regresión lineal simple  $y' = \hat{y} = \alpha + \beta x$ , se tiene que

$$\hat{\alpha} = \overline{y} - \hat{\beta} \, \overline{x}, \quad \hat{\beta} = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n} (x_i - \overline{x})^2}, \quad r^2 = 1 - \frac{s_{Y|x}^2}{s_Y^2}, \qquad s_{Y|x}^2 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - y_i')^2$$

$$\hat{\rho} = \hat{\beta} \, \frac{s_X}{s_Y}, \qquad \hat{\rho}^2 = 1 - \frac{(n-2)}{(n-1)} \, \frac{s_{Y|x}^2}{s_Y^2} \qquad , \langle \mu_{Y|x_i} \rangle_{1-\alpha} = \overline{y}_i \pm t_{(1-\alpha/2), \, n-2} \cdot s_{Y|x} \sqrt{\frac{1}{n} + \frac{(x_i - \overline{x})^2}{\sum_{j=1}^{n} (x_j - \overline{x})^2}}$$

#### Regresión Lineal Múltiple

$$SCT = SCR + SCE$$

$$\sum_{i=1}^{n} (y_i - \overline{y})^2 = \sum_{i=1}^{n} (\hat{y}_i - \overline{y})^2 + \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$R^2 = \frac{SCR}{SCT} = 1 - \frac{SCE}{SCT} = 1 - \frac{(n-k-1)}{(n-1)} \frac{s_{Y|X}^2}{s_Y^2}, \qquad r^2 = 1 - \frac{(n-1)}{(n-k-1)} \frac{SCE}{SCT} = 1 - \frac{s_{Y|X}^2}{s_Y^2}$$

$$T_{b_j} = \frac{b_j - \beta_j}{s_{b_j}} \sim \text{t-Student}(n-k-1), \qquad F = \frac{SCR/k}{SCE/(n-k-1)} \sim F(k, n-k-1)$$

con k regresores en el modelo,  $b_j$  estimador de  $\beta_j$  y  $s_{b_j} = \sqrt{\widehat{\mathrm{Var}(b_j)}}$ .

$$F = \frac{\left(SCE_{(r)} - SCE\right)/r}{SCE/(n - (k + r) - 1)} \sim F(r, n - (k + r) - 1)$$

con  $SCE_{(r)}$  y SCE son suma de errores al cuadrado de dos modelos anidados en k regresores comunes.

# Tablas de Percentiles p

				Distrib	ución N	Normal I	Estánda	$k_p$					Distribu	ción t-stı	$_{ m l}$	$_{0}( u)$
$k_p$	0,00	0,01	0,02			0,04	0,05	0,06	0,07	0,08	0,09	ν	$t_{0,90}$	$t_{0,95}$	$t_{0,975}$	$t_{0,99}$
0,0	0,5000	0,5040	,			*	0,5199	0,5239	0,5279	0,5319	0,5359	1	3,078	6,314	12,706	31,821
0,1	0,5398	0,5438	,			*	0,5596	0,5636	0,5675	0,5714	0,5753	2	1,886	2,920	4,303	6,965
0,2	0,5793	0,5832	,	,		*	0,5987	0,6026	0,6064	0,6103	0,6141	3	1,638	2,353	3,182	4,541
0,3	0,6179	0,6217	,	,		*	0,6368	0,6406	0,6443	0,6480	0,6517	4	1,533	2,132	2,776	3,747
0,4	0,6554	0,6591	0,662	,		*	0,6736	0,6772	0,6808	0,6844	0,6879	5	1,476	2,015	2,571	3,365
0,5	0,6915	0,6950	,	,		*	0,7088	0,7123	0,7157	0,7190	0,7224	6	1,440	1,943	2,447	3,143
0,6	0,7257	0,7291		,			0,7422	0,7454	0,7486	0,7517	0,7549	7	1,415	1,895	2,365	2,998
0,7	0,7580 0,7881	0,7611 $0,7910$	,	,		*	0,7734 $0,8023$	0,7764 $0,8051$	0,7794 $0,8078$	0,7823 $0,8106$	0,7852	8 9	1,397	1,860 $1,833$	2,306 2,262	2,896 $2,821$
$0.8 \\ 0.9$	0,7881	0,7910	,	,		*	0,8023 0,8289	0,8031 $0,8315$	0,8340	0.8365	0,8133 $0,8389$	10	$\begin{vmatrix} 1,383 \\ 1,372 \end{vmatrix}$	1,833 $1,812$	2,202 $2,228$	2,821 $2,764$
$^{0,9}$	0,8139	0,8138	,	,		*	0,8531	0,8513 $0,8554$	0,8540 $0,8577$	0,8599	0,8621	11	1,363	1,796	2,220 2,201	2,704 $2,718$
1,0 $1,1$	0,8413	0,8665					0,8749	0,8334	0,8790	0,8333	0,8830	12	1,356	1,780 $1,782$	2,201 $2,179$	2,681
$^{1,1}_{1,2}$	0,8849	0,8869	,	,		*	0.8944	0,8962	0,8790	0,8997	0,9015	13	1,350	1,771	2,179 $2,160$	2,650
1,2 $1,3$	0,9032	0,9049	,	,		*	0,9115	0,8302 $0,9131$	0,8380 $0,9147$	0,8337	0,9013 $0,9177$	14	1,345	1,761	2,100 $2,145$	2,624
$^{1,0}_{1,4}$	0,9192	0,9207	,	,		*	0,9265	0,9279	0,9292	0,9306	0,9319	15	1,341	1,753	2,140 $2,131$	2,602
$^{1,4}_{1,5}$	0,9332	0,9345	,			*	0,9394	0,9406	0,9418	0,9429	0.9441	16	1,337	1,746	2,120	2,583
1,6	0,9452	0,9463					0,9505	0,9515	0,9410 $0,9525$	0,9425 $0,9535$	0,9545	17	1,333	1,740 $1,740$	2,120 $2,110$	2,565 $2,567$
$^{1,0}_{1,7}$	0,9554	0,9564	,	,		*	0,9599	0,9608	0,9616	0,9625	0,9633	18	1,330	1,734	2,110 $2,101$	2,552
1,8	0,9641	0,9649	,	,		*	0,9678	0,9686	0,9693	0,9699	0,9706	19	1,328	1,729	2,093	2,539
1,9	0,9713	0,9719	,			*	0,9744	0,9750	0,9756	0,9761	0,9767	20	1,325	1,725	2,086	2,528
2,0	0,9772	0,9778	,	,		*	0,9798	0,9803	0,9808	0,9812	0,9817	21	1,323	1,721	2,080	2,518
$^{2,0}_{2,1}$	0,9821	0,9826	,	,		*	0,9842	0,9846	0,9850	0,9854	0,9857	22	1,321	1,717	2,074	2,508
$^{2,1}_{2,2}$	0,9861	0,9864	,	,		*	0,9878	0,9881	0,9884	0,9887	0,9890	23	1,319	1,714	2,069	2,500
2,3	0,9893	0,9896	,			*	0,9906	0,9909	0,9911	0,9913	0,9916	$\frac{1}{24}$	1,318	1,711	2,064	2,492
$^{2,3}_{2,4}$	0,9918	0,9920	,	,		*	0,9929	0,9931	0.9932	0,9934	0,9936	25	1,316	1,708	2,060	2,485
2,5	0,9938	0,9940	,	,		*	0,9946	0,9948	0,9949	0,9951	0,9952	26	1,315	1,706	2,056	2,479
2,6	0,9953	0,9955	,	,		*	0,9960	0,9961	0,9962	0,9963	0,9964	27	1,314	1,703	2,052	2,473
$^{-,\circ}_{2,7}$	0,9965	0,9966	,			*	0,9970	0,9971	0,9972	0,9973	0,9974	28	1,313	1,701	2,048	2,467
2,8	0.9974	0,9975	,			*	0,9978	0,9979	0,9979	0.9980	0,9981	29	1,311	1,699	2,045	2,462
2,9	0,9981	0,9982		,		*	0,9984	0,9985	0,9985	0,9986	0,9986	30	1,310	1,697	2,042	2,457
3,0	0,9987	0,9987	0,998	87 0,99	988 0	,9988	0,9989	0,9989	0,9989	0,9990	0,9990	$\infty$	1,282	1,645	1,960	2,326
							Distrib	ución Chi-	Cuadrado	$c_p(\nu)$						
ν	c <sub>0,005</sub>	c <sub>0,001</sub>	c <sub>0,025</sub>	c <sub>0,05</sub>	c <sub>0,1</sub>	c <sub>0,2</sub>	c <sub>0,3</sub>	$c_{0,4}$	c <sub>0,6</sub>	c <sub>0,7</sub>	c <sub>0,8</sub>	c <sub>0,9</sub>	c <sub>0,95</sub>	c <sub>0,975</sub>	c <sub>0,99</sub>	c <sub>0,995</sub>
1	0,000	0,000	0,001	0,004	0,016	0,064	0,148	0,275	0,708	1,074	1,642	2,706	3,841	5,024	6,635	7,879
2 3	0,010 0,072	$0,002 \\ 0,024$	$0,051 \\ 0,216$	$0,103 \\ 0,352$	0,211 $0,584$	$0,446 \\ 1,005$	0,713 $1,424$	1,022 1,869	$^{1,833}_{2,946}$	$2,408 \\ 3,665$	3,219 $4,642$	4,605 $6,251$	5,991 7,815	7,378 9,348	9,210 11,345	
4 5	$0,207 \\ 0,412$	$0,091 \\ 0,210$	0,484	0,711 $1,145$	1,064 1,610	1,649 2,343	2,195 3,000	2,753 3,655	4,045 $5,132$	4,878	5,989	7,779	9,488	11,143 12,833	13,277	14,860
6	0,676	0,381	0,831 $1,237$	1,635	2,204	3,070	3,828	4,570	6,211	$6,064 \\ 7,231$	$7,289 \\ 8,558$	9,236 $10,645$	$11,070 \\ 12,592$	14,449	15,086 $16,812$	18,548
7 8	0,989 1,344	$0,598 \\ 0,857$	1,690 2,180	2,167 $2,733$	2,833 3,490	3,822 $4,594$	4,671 $5,527$	5,493 $6,423$	7,283 8,351	8,383 $9,524$	9,803 11,030	12,017 $13,362$	14,067 15,507	16,013 17,535	18,475 20,090	
9	1,735	1,152	2,700	3,325	4,168	5,380	6,393	7,357	9,414	10,656	12,242	14,684	16,919	19,023	21,666	23,589
10 11	2,156 2,603	1,479 $1,834$	$3,247 \\ 3,816$	$3,940 \\ 4,575$	4,865 $5,578$	6,179 $6,989$	$7,267 \\ 8,148$	$8,295 \\ 9,237$	10,473 $11,530$	11,781 $12,899$	13,442 $14,631$	15,987 $17,275$	18,307 19,675	20,483 $21,920$	23,209 $24,725$	25,188 $26,757$
12 13		2,214	4,404	5,226	6,304	$7,807 \\ 8,634$	9,034 9,926	10,182 $11,129$	12,584 $13,636$	14,011 15,119	15,812 16,985	18,549 $19,812$	21,026 $22,362$	23,337 24,736	26,217 27,688	28,300
	3,074 3,565			5.892					14,685	16,222			23,685	26,119	29,141	31,319
14	3,565 4,075	$^{2,617}_{3,041}$	5,009 $5,629$	5,892 6,571	7,042 7,790	9,467	10,821	12,078			18,151	21,064				
14 15 16	3,565	2,617 $3,041$ $3,483$	5,009		$7,790 \\ 8,547$		11,721	13,030	15,733	16,222 17,322 18,418	18,151 $19,311$ $20,465$	22,307	24,996	27,488 28,845	30,578	32,801
15 16 17	3,565 4,075 4,601 5,142 5,697	2,617 3,041 3,483 3,942 4,416	5,009 5,629 6,262 6,908 7,564	6,571 7,261 7,962 8,672	7,790 8,547 9,312 10,085	9,467 10,307 11,152 12,002	11,721 $12,624$ $13,531$	13,030 13,983 14,937	15,733 16,780 17,824	17,322 $18,418$ $19,511$	19,311 20,465 21,615	22,307 $23,542$ $24,769$	24,996 26,296 27,587	27,488 28,845 30,191	30,578 32,000 33,409	32,801 $34,267$ $35,718$
15 16	3,565 4,075 4,601 5,142	2,617 3,041 3,483 3,942 4,416 4,905 5,407	5,009 5,629 6,262 6,908	6,571 7,261 7,962	7,790 8,547 9,312	9,467 $10,307$ $11,152$	$11,721 \\ 12,624$	13,030 13,983 14,937 15,893 16,850	15,733 16,780	17,322 $18,418$ $19,511$ $20,601$ $21,689$	19,311 $20,465$	22,307 $23,542$	24,996 26,296	27,488 $28,845$	30,578 32,000	32,801 34,267 35,718 37,156 38,582
15 16 17 18 19 20	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591	6,571 7,261 7,962 8,672 9,390 10,117 10,851	7,790 8,547 9,312 10,085 10,865 11,651 12,443	9,467 10,307 11,152 12,002 12,857 13,716 14,578	11,721 $12,624$ $13,531$ $14,440$ $15,352$ $16,266$	13,030 13,983 14,937 15,893 16,850 17,809	15,733 16,780 17,824 18,868 19,910 20,951	17,322 18,418 19,511 20,601 21,689 22,775	19,311 20,465 21,615 22,760 23,900 25,038	22,307 23,542 24,769 25,989 27,204 28,412	24,996 26,296 27,587 28,869 30,144 31,410	27,488 28,845 30,191 31,526 32,852 34,170	30,578 32,000 33,409 34,805 36,191 37,566	32,801 34,267 35,718 37,156 38,582 39,997
15 16 17 18 19 20 21 22	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041	$\begin{array}{c} 9,467 \\ 10,307 \\ 11,152 \\ 12,002 \\ 12,857 \\ 13,716 \\ 14,578 \\ 15,445 \\ 16,314 \end{array}$	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796
15 16 17 18 19 20 21 22 23	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848	9,467 $10,307$ $11,152$ $12,002$ $12,857$ $13,716$ $14,578$ $15,445$ $16,314$ $17,187$	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181
15 16 17 18 19 20 21 22 23 24 25	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085 8,649	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 15,659 16,473	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187 18,062 18,940	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690 21,652 22,616	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 39,364 40,646	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928
15 16 17 18 19 20 21 22 23 24	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 15,659	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690 21,652 22,616 23,579 24,544	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 39,364	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645
15 16 17 18 19 20 21 22 23 24 25 26 27 28	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,643 9,260 9,886 10,520 11,160 11,808 12,461	2,617 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085 8,649 9,222 9,803 10,391	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,401 13,120 13,844 14,573 15,308	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,928	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 15,659 16,473 17,292 18,114 18,939	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187 18,062 18,940 19,820 20,703 21,588	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867 21,792 22,719 23,647	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690 21,652 22,616 23,579 24,544 25,509	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113 41,337	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 40,646 41,923 43,195 44,461	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520 11,160 11,808 12,461 13,121 13,787	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085 8,649 9,222 9,803 10,391 10,986 11,588	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120 13,844 14,573 15,308 16,047 16,791	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,928 17,708 18,493	7,790 8,547 9,312 10,085 11,651 12,443 13,240 14,041 14,848 15,659 16,473 17,292 18,114 18,939 19,768 20,599	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187 18,940 19,820 20,703 21,588 22,475 23,364	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,943 20,867 21,792 22,719 23,647 24,577 25,508	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690 21,652 22,616 23,579 24,544 25,509 26,475 27,442	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249 30,283 31,316	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391 32,461 33,530	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027 35,139 36,250	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916 39,087 40,256	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113 41,337 42,557 43,773	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 40,646 41,923 43,195 44,461 45,722 46,979	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278 49,588 50,892	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993 52,336 53,672
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 40	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520 11,160 11,808 12,461 13,121 13,787 20,707	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085 8,649 9,222 9,803 10,391 10,986 11,588 17,916	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120 13,844 14,5308 16,047 16,791 24,433	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,928 17,708 18,493 26,509	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 16,659 16,473 17,292 18,114 18,939 19,768 20,599 29,051	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187 18,062 18,940 19,820 20,703 21,588 22,475 23,364 32,345	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867 21,792 22,719 23,647 24,577 25,508 34,872	13,030 13,983 14,937 15,893 16,850 17,809 18,768 19,729 20,690 21,652 22,616 23,579 24,544 25,509 26,475 27,442 37,134	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249 30,283 31,316 41,622	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391 32,461 33,530 44,165	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027 35,139 36,250 47,269	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916 39,087 40,256 51,805	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113 41,337 42,557 43,773 55,758	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 39,364 40,646 41,923 43,195 44,461 45,722 46,979 59,342	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278 49,588 50,892 63,691	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993 52,336 53,672 66,766
15 16 17 18 19 20 21 22 23 24 25 26 27 28 30 40 50 60	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520 11,160 11,808 12,461 13,121 13,787 20,707 27,991 35,534	2,617 3,041 3,942 4,416 4,905 5,407 6,921 6,447 6,983 8,085 9,223 10,391 10,986 17,516 9,203 10,391 11,588 17,916 24,674	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120 13,844 14,573 15,308 16,047 16,791 24,433 32,357	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,28 17,708 18,493 26,509 34,764 43,188	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 15,659 16,473 17,292 18,114 18,939 19,768 20,599 29,051 37,689 46,459	9,467 10,307 11,152 12,052 13,716 14,578 15,445 16,314 17,187 18,062 18,940 19,820 20,703 21,588 22,475 23,364 41,449	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867 21,792 22,719 23,647 24,577 25,508 34,872 44,313 53,809	13,030 13,983 14,937 15,893 16,850 17,809 18,768 20,690 21,652 22,616 23,579 24,544 25,509 26,475 27,442 37,134 46,864 456,620	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249 30,283 31,316 41,622 51,892 62,135	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391 32,461 33,530 44,165 54,723 65,227	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027 35,139 36,250 47,269 58,164 68,972	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916 39,087 40,256 51,805 63,167 74,397	24,996 26,296 27,587 28,869 30,144 31,410 32,671 35,172 36,415 37,652 38,885 40,113 41,337 42,557 55,758 67,505	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 39,364 40,646 41,923 43,195 44,461 45,722 46,979 59,342 71,420 83,298	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278 49,588 50,892 63,691 76,154 88,379	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993 52,336 53,672 66,766 79,490 91,952
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 40 50	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520 11,160 11,808 12,461 13,121 13,787 20,707 27,991 35,534 43,275 51,172	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,447 6,983 7,529 8,085 8,649 9,222 9,803 10,391 10,986 11,588 17,916	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120 13,844 14,573 15,308 16,047 16,791 24,433 32,357	6,571 7,261 7,962 8,672 9,390 10,117 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,928 17,708 18,493 26,509 34,764	7,790 8,547 9,312 10,085 10,865 11,651 12,443 13,240 14,041 14,848 15,659 16,473 17,292 18,114 18,939 19,768 20,599 29,051 37,689	9,467 10,307 11,152 12,002 12,857 13,716 14,578 15,445 16,314 17,187 18,062 18,940 20,703 21,588 22,475 23,364 32,345 41,449	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867 21,792 22,719 23,647 24,577 25,508 34,872 44,313	13,030 13,983 14,937 15,893 16,850 17,809 18,769 20,690 21,652 22,616 23,579 24,544 25,509 26,475 27,442 37,134 46,864	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249 30,283 31,316 41,622 51,892	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391 32,461 33,530 44,165 54,723	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027 35,139 36,250 47,269 58,164	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916 39,087 40,256 51,805 63,167	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113 41,337 42,557 43,773 55,758 67,505	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 40,646 41,923 43,195 44,461 45,722 46,979 59,342 71,420	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278 49,588 50,892 63,691 76,154	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993 52,336 67,490 91,952 104,215
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 40 50 60 70	3,565 4,075 4,601 5,142 5,697 6,265 6,844 7,434 8,034 8,643 9,260 9,886 10,520 11,160 11,808 12,461 13,121 13,787 20,707 27,991 35,534 43,275	2,617 3,041 3,483 3,942 4,416 4,905 5,407 5,921 6,983 7,522 8,085 8,649 9,222 9,803 10,391 10,986 11,588 11,588 11,588 11,588	5,009 5,629 6,262 6,908 7,564 8,231 8,907 9,591 10,283 10,982 11,689 12,401 13,120 13,844 14,573 15,308 16,047 16,791 24,433 32,357 40,485	6,571 7,261 7,962 8,672 9,390 10,117 10,851 11,591 12,338 13,091 13,848 14,611 15,379 16,151 16,928 17,708 18,493 26,509 34,764 43,188	7,790 8,547 9,312 10,865 11,651 12,443 13,240 14,041 14,848 15,659 16,473 17,292 18,114 18,939 19,768 20,599 29,051 37,689 46,459 55,329	9,467 10,307 11,152 12,052 13,716 14,578 15,445 16,314 17,187 18,062 18,940 19,820 20,703 21,588 22,475 23,364 41,449 50,641 59,898	11,721 12,624 13,531 14,440 15,352 16,266 17,182 18,101 19,021 19,943 20,867 21,792 22,719 22,719 23,647 24,577 24,373 44,313 53,809	13,030 13,983 14,937 15,893 16,850 17,809 20,690 21,652 22,616 23,579 24,544 25,509 26,475 27,442 37,134 46,864 56,620 66,396	15,733 16,780 17,824 18,868 19,910 20,951 21,991 23,031 24,069 25,106 26,143 27,179 28,214 29,249 30,283 31,316 41,622 51,892 62,135 72,358	17,322 18,418 19,511 20,601 21,689 22,775 23,858 24,939 26,018 27,096 28,172 29,246 30,319 31,391 32,461 33,530 44,165 54,723 65,227 75,689	19,311 20,465 21,615 22,760 23,900 25,038 26,171 27,301 28,429 29,553 30,675 31,795 32,912 34,027 35,139 36,250 47,269 58,164 68,972 79,715	22,307 23,542 24,769 25,989 27,204 28,412 29,615 30,813 32,007 33,196 34,382 35,563 36,741 37,916 51,805 63,167 74,397 74,397 74,397 785,527	24,996 26,296 27,587 28,869 30,144 31,410 32,671 33,924 35,172 36,415 37,652 38,885 40,113 41,337 42,557 43,773 55,758 67,505 79,082 90,531	27,488 28,845 30,191 31,526 32,852 34,170 35,479 36,781 38,076 40,646 41,923 43,195 44,461 45,722 46,979 59,342 71,420 83,298 95,023	30,578 32,000 33,409 34,805 36,191 37,566 38,932 40,289 41,638 42,980 44,314 45,642 46,963 48,278 49,588 50,892 63,691 76,154 88,379 100,425	32,801 34,267 35,718 37,156 38,582 39,997 41,401 42,796 44,181 45,559 46,928 48,290 49,645 50,993 52,336 53,672 66,766 79,490 91,952 104,215 116,321

## Percentiles p Distribución Fisher: $F_p(df_1, df_2)$

qf(p = 0.950, df1, df2): df2=2 df2=3 df2=4df2=5 df2=6 df2=7df2=8 df2=9 df2=10 df2=11 df2=12 df2=13 df2=14 df2=15 df1=1 161.45 18.51 10.13 7.71 6.61 5.99 5.59 5.32 5.12 4.96 4.84 4.75 4.67 4.60 199.50 3.74 19.00 9.55 6.94 5.79 5.14 4.74 4.46 4.26 4.10 3.98 3.89 3.81 5.41 df1=3215.71 19.16 9.28 6.59 4.76 4.35 4.07 3.86 3.71 3.59 3.49 3.41 3.34 3.29 df1=4 224.58 19.25 9.12 6.39 5.19 4.53 4.12 3.84 3.63 3.48 3.36 3.26 3.18 3.11 3.06 df1=5 230.16 19.30 9.01 6.26 5.05 4.39 3.97 3.69 3.48 3.33 3.20 3.11 3.03 2.96 2.90 233.99 8.94 4.28 3.37 3.09 2.92 2.85 df1=6 19.33 6.16 4.95 3.87 3.58 3.22 3.00 2.79 236.77 19.35 8.89 6.09 4.88 4.21 3.79 3.50 3.29 3.14 3.01 2.91 2.83 2.76 df1=7 2.71 df1=8 238.88 19.37 8.85 6.04 4.82 4.15 3.73 3.44 3.23 3.07 2.95 2.85 2.77 2.70 2.64 df1=9 240.54 19.38 8.81 6.00 4.77 4.10 3.68 3.39 3.18 3.02 2.90 2.80 2.71 2.65 df1=10 241.88 19.40 8.79 5.96 4.74 4.06 3.64 3.35 3.14 2.98 2.85 2.75 2.67 2.60 2.54 df1=11 242.98 19.40 8.76 5.94 4.70 4.03 3.60 3.31 3.10 2.94 2.82 2.72 2.63 2.57 2.51 df1=12 243.91 19.41 8.74 5.91 4.68 4.00 3.57 3.28 3.07 2.91 2.79 2.69 2.60 2.53 df2=16 df2=17 df2=18 df2=19 df2=20 df2=21 df2=22 df2=23 df2=24 df2=25 df2=26 df2=27 df2=28 df2=29 df2=30 df1=1 4.41 4.38 4.35 4.30 4.28 4.23 4.21 4.20 4.17 4.49 4.45 4.32 4.26 4.24 4.18 df1=2 3.63 3.59 3.55 3.52 3.49 3.47 3.44 3.42 3.40 3.39 3.37 3.35 3.34 3.33 df1=3 3.24 3.20 3.16 3.13 3.10 3.07 3.05 3.03 3.01 2.99 2.98 2.96 2.95 2.93 2.92 df1=4 3.01 2.96 2.93 2.90 2.87 2.84 2.82 2.80 2.78 2.76 2.74 2.73 2.71 2.70 2.69 df1=5 2.85 2.81 2.77 2.74 2.71 2.68 2.66 2.64 2.62 2.60 2.59 2.57 2.55 2.56 2.53 2.45 df1=6 2.74 2.70 2.66 2.63 2.60 2.57 2.55 2.53 2.51 2.49 2.47 2.46 2.43 2.42 df1=7 2.66 2.61 2.58 2.54 2.51 2.49 2.46 2.44 2.42 2.40 2.39 2.37 2.36 2.35 2.33 df1=8 2.59 2.55 2.51 2.48 2.45 2.42 2.40 2.37 2.36 2.34 2.32 2.31 2.29 2.28 2.27 df1=9 2.54 2.49 2.46 2.42 2.39 2.37 2.34 2.32 2.30 2.28 2.27 2.25 2.24 2.22 2.21 df1=10 2.49 2.45 2.41 2.38 2.35 2.32 2.30 2.27 2.25 2.24 2.22 2.20 2.19 2.18 2.16 df1=11 2.37 2.28 2.26 2.24 2.20 2.18 2.46 2.41 2.34 2.31 2.22 2.17 2.15 2.14 2.13 df1=12 2.38 2.34 2.31 2.28 2.25 2.23 2.20 2.18 2.16 2.15 df2=31 df2=32 df2=33 df2=34 df2=35 df2=36 df2=37 df2=38 df2=40 df2=41 df2=42 df2=43 df2=44 df2=45 df1=1 4.15 4.14 4.13 4.12 4.11 4.11 4.10 4.09 4.08 4.08 4.07 4.07 4.06 4.06 4.16 df1=2 3.30 3.29 3.28 3.28 3.27 3.26 3.25 3.24 3.24 3.23 3.23 3.22 3.21 3.21 3.20 2.89 2.87 2.86 2.85 2.85 2.83 2.83 2.82 2.82 df1 = 32.91 2.90 2.88 2.87 2.84 2.81 df1=4 2.68 2.67 2.66 2.65 2.64 2.63 2.63 2.62 2.61 2.61 2.60 2.59 2.59 2.58 2.58 df1=5 2.52 2.51 2.50 2.49 2.49 2.48 2.47 2.46 2.46 2.45 2.44 2.44 2.43 2.43 2.42 df1=6 2.41 2.40 2.39 2.38 2.37 2.36 2.36 2.35 2.34 2.34 2.33 2.32 2.32 2.31 2.31 df1=7 2.32 2.31 2.30 2.29 2.29 2.28 2.27 2.26 2.26 2.25 2.24 2.24 2.23 2.23 2.22 df1=8 2.25 2.24 2.23 2.23 2.22 2.21 2.20 2.19 2.19 2.18 2.17 2.17 2.16 2.16 2.15 df1=9 2.20 2.19 2.18 2.17 2.16 2.15 2.14 2.14 2.13 2.12 2.12 2.11 2.11 2.10 2.10 df1=10 2.15 2.14 2.13 2.12 2.11 2.11 2.10 2.09 2.08 2.08 2.07 2.06 2.06 2.05 2.05 df1=11 2.11 2.10 2.09 2.08 2.07 2.07 2.06 2.05 2.04 2.04 2.03 2.03 2.02 2.01 2.01 df1=12 2.08 2.07 2.06 2.05 2.04 2.03 2.02 2.02 2.01 2.00 2.00 1.99 1.99 1.98 df2=46 df2=47 df2=48 df2=49 df2=50 df2=51 df2=52 df2=53 df2=54 df2=55 df2=56 df2=57 df2=58 df2=59 df2=60 df1=1 4.05 4.04 4.04 4.03 4.03 4.03 4.02 4.02 4.02 4.01 4.01 4.01 df1=2 3.20 3.20 3.19 3.19 3.18 3.18 3.18 3.17 3.17 3.16 3.16 3.16 3.16 3.15 3.15 df1=3 2.81 2.80 2.80 2.79 2.79 2.79 2.78 2.78 2.78 2.77 2.77 2.77 2.76 2.76 2.76 df1=4 2.57 2.57 2.57 2.56 2.56 2.55 2.55 2.55 2.54 2.54 2.54 2.53 2.53 2.53 2.41 2.41 2.39 2.38 df1=5 2.42 2.40 2.40 2.40 2.39 2.39 2.38 2.38 2.37 2.37 2.37 df1=6 2.30 2.30 2.29 2.29 2.29 2.28 2.28 2.28 2.27 2.27 2.27 2.26 2.26 2.26 2.25 df1=7 2.22 2.21 2.21 2.20 2.20 2.20 2.19 2.19 2.18 2.18 2.18 2.18 2.17 2.17 2.17 df1=8 2.15 2.14 2.14 2.13 2.13 2.13 2.12 2.12 2.12 2.11 2.11 2.11 2.10 2.10 2.10 df1=9 2.09 2.09 2.08 2.08 2.07 2.07 2.07 2.06 2.06 2.06 2.05 2.05 2.05 2.04 2.04 df1=10 2.04 2.04 2.03 2.03 2.03 2.02 2.02 2.01 2.01 2.01 2.00 2.00 2.00 2.00 1.99 df1=11 2.00 1.99 1.99 1.99 1.98 1.98 1.97 1.97 1.97 1.96 1.96 1.96 1.96 1.95 df1=12 1.96 1.96 1.95 1.94 1.94 1.93 1.93 1.97 1.96 1.95 1.94 1.93 1.92 1.92 1.92

## Propiedad:

Si 
$$F \sim F(df_1, df_2)$$
, entonces  $F_p(df_1, df_2) = \frac{1}{F_{1-p}(df_2, df_1)}$ .

		0	1	7
Distribucion	Densidad de Frobabilidad	×	Farametros	Esperanza y varianza
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, \dots, n$	u, p	$\mu X = n p$ $\sigma_X^2 = n p (1 - p)$ $M(t) = [p e^t + (1 - p)]^n,  t \in \mathbb{R}$
Geométrica	$p (1-p)^{x-1}$	$x=1,2,\dots$	d	$M(t) = p e^{t} / [1 - (1 - p)/p^{2}]$ $M(t) = p e^{t} / [1 - (1 - p) e^{t}], t < -\ln(1 - p)$
Binomial-Negativa	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$	$x = r, r + 1, \dots$	r, p	$\mu X = r/p$ $\frac{\sigma_X^2 = r (1 - p)/p^2}{r (1 - p) (1 - p)} M(t) = \left\{ p e^t / [1 - (1 - p) e^t] \right\}^T,  t < -\ln(1 - p)$
Poisson	$\frac{(\nu t)^x e^{-\nu t}}{x!}$	$x = 0, 1, \dots$	7	$\mu X = \nu t$ $\sigma_X^2 = \nu t$ $M(t) = \exp \left[ \lambda \left( e^t - 1 \right) \right],  t \in \mathbb{R}$
Exponencial	V e - V B	0 ∧I 8	7	$\mu_X = 1/\nu$ $\sigma_X = 1/\nu^2$ $\sigma_X = 1/\nu^2$ $M(t) = \nu/(\nu - t),  t < \nu$
Gamma	$\frac{\nu^k}{\Gamma(k)}  x^{k-1}  e^{-\nu}  x$	О ЛІ в	k, v	$\mu_X = k/\nu$ $\sigma_X^2 = k/\nu^2$ $\sigma_X^2 = k/\nu^2$ $M(t) = [\nu/(\nu - t)]^k,  t < \nu$
Normal	$\frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$	8 V e V 8	μ, σ	$\mu_X = \mu$ $\sigma_X^2 = \sigma^2$ $M(t) = \exp(\mu t + \sigma^2 t^2/2),  t \in \mathbb{R}$
Log-Normal	$\frac{1}{\sqrt{2\pi}\left(\zetax\right)}\exp\left[-\frac{1}{2}\left(\frac{\lnx-\lambda}{\zeta}\right)^2\right]$	s VI O	У, С	$\mu_X = \exp\left(\lambda + \frac{1}{2}\zeta^2\right)$ $\sigma_X^2 = \mu_X^2 \left(e^{\zeta^2} - 1\right)$ $E(X^r) = e^{r\lambda} M_Z(r\zeta), \text{ con } Z \sim \text{Normal}(0,1)$
Uniforme	$\frac{1}{(b-a)}$	a	a, $b$	$\begin{split} \mu  X &= (a+b)/2 \\ \sigma_X^2 &= (b-a)^2/12 \\ M(t) &= [e^t b^* - e^t a]/[t  (b-a)],  t \in \mathcal{R} \end{split}$
Beta	$\frac{1}{B(q,r)} \frac{(x-a)^{q-1} (b-x)^{r-1}}{(b-a)^{q+r-1}}$	a   A   A   A   A   A   A   A   A   A	ę.	$\mu_X = a + \frac{q}{q+r} (b-a)$ $\sigma_X^2 = \frac{q r (b-a)^2}{(q+r)^2 (q+r+1)}$
Hipergeométrica	$\frac{\binom{m}{x}\binom{N-m}{n}}{\binom{n}{n}}$	$\max\{0,n+m-N\}\leq x\leq \min\{n,m\}$	$N,\ m,\ n$	$\mu_X = n \stackrel{\mathcal{R}}{X}$ $\sigma_X^2 = \left(\frac{N-n}{N-1}\right) n \stackrel{\mathcal{R}}{Y} \left(1 - \frac{m}{Y}\right)$

#### Otras distribuciones

• Si  $T \sim \text{Weibull}(\eta, \beta)$ , se tiene que

$$F_T(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \quad f_T(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta - 1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], \quad t > 0$$

Con  $\beta>0$ , es un parámetro de forma y  $\eta>0$ , es un parámetro de escala. Si  $t_p$  es el percentil  $p\times 100\,\%$ , entonces

$$\ln(t_p) = \ln(\eta) + \frac{1}{\beta} \cdot \Phi_{\text{Weibull}}^{-1}(p), \quad \Phi_{\text{Weibull}}^{-1}(p) = \ln[-\ln(1-p)]$$

Mientras que su m-ésimo momento está dado por

$$\begin{split} E(\boldsymbol{T}^m) &= \boldsymbol{\eta}^m \, \Gamma(1+m/\beta) \\ \mu_T &= \boldsymbol{\eta} \, \Gamma\left(1+\frac{1}{\beta}\right), \quad \boldsymbol{\sigma}_T^2 = \boldsymbol{\eta}^2 \, \left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right] \end{split}$$

• Si  $Y \sim \text{Log}(\text{stica}(\mu, \sigma))$ , se tiene que

$$F_Y(y) = \Phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right); \qquad f_Y(y) = \frac{1}{\sigma}\,\phi_{\text{Logística}}\left(\frac{y-\mu}{\sigma}\right), \quad -\infty < y < \infty$$

donde

$$\Phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1 + \exp(z)]} \quad \text{y} \quad \phi_{\text{Logística}}(z) = \frac{\exp(z)}{[1 + \exp(z)]^2}$$

son la función de probabilidad y de densidad de una Logística Estándar.  $\mu \in \mathbb{R}$ , es un parámetro de localización y  $\sigma > 0$ , es un parámetro de escala. Si  $y_p$  es el percentil  $p \times 100\%$ , entonces

$$y_p = \mu + \sigma \, \Phi_{\text{Logística}}^{-1}(p) \quad \text{con} \quad \Phi_{\text{Logística}}^{-1}(p) = \log \left(\frac{p}{1-p}\right)$$

Su esperanza y varianza están dadas por:  $\mu_Y = \mu$  y  $\sigma_Y^2 = \frac{\sigma^2 \pi^2}{3}$ .

• Si  $T \sim \text{Log-Log}(\text{stica}(\mu, \sigma))$ , se tiene que

$$F_T(t) = \Phi_{\text{Logística}}\left(\frac{\ln(t) - \mu}{\sigma}\right); \quad f_T(t) = \frac{1}{\sigma\,t}\,\phi_{\text{Logística}}\left(\frac{\ln(t) - \mu}{\sigma}\right) \quad t > 0$$

Donde  $\exp(\mu)$ , es un parámetro de escala y  $\sigma>0$ , es un parámetro de forma. Si  $t_p$  es el percentil  $p\times 100\,\%$ , entonces

$$\ln(t_p) = \mu + \sigma \, \Phi_{\text{Logística}}^{-1}(p)$$

Para un entero m > 0 se tiene que

$$E(T^{m}) = \exp(m \mu) \Gamma(1 + m \sigma) \Gamma(1 - m \sigma)$$

El *m*-ésimo momento no es finito si  $m \sigma \geq 1$ .

Para  $\sigma < 1$ :  $\mu_T = \exp(\mu) \Gamma(1 + \sigma) \Gamma(1 - \sigma)$ 

y para 
$$\sigma < 1/2$$
:  $\sigma_T^2 = \exp(2\mu) \left[ \Gamma(1+2\sigma) \Gamma(1-2\sigma) - \Gamma^2(1+\sigma) \Gamma^2(1-\sigma) \right]$ 

 $\blacksquare$  Un variable aleatoria T tiene distribución t-student si su función de densidad está dada por:

$$f_T(t) = \frac{\Gamma[(\nu+1)/2]}{\sqrt{\pi \nu} \Gamma(\nu/2)} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad -\infty < t < \infty$$

- $\mu_T = 0$ , para  $\nu > 1$ .
- $\sigma_T^2 = \frac{\nu}{\nu 2}$ , para  $\mu > 2$ .
- Si  $T \sim \text{Fisher}(\eta, \nu)$ , se tiene que

$$f_T(t) = \frac{\Gamma(\frac{\eta+\nu}{2})}{\Gamma(\eta/2)\Gamma(\nu/2)} \left(\frac{\eta}{\nu}\right)^{\frac{\eta}{2}} \frac{t^{\frac{\eta}{2}-1}}{\left(\frac{\eta}{2}, t+1\right)^{\frac{\eta+\nu}{2}}}, \quad t > 0$$

- $\mu_T = \frac{\nu}{\nu 2}$ , para  $\nu > 2$ .
- $\sigma_T^2 = \frac{2\nu^2 (\eta + \nu 2)}{\eta (\nu 2)^2 (\nu 4)}$ , para  $\nu > 4$ .