Answers to Selected Problems

Following are answers to those odd-numbered problems for which a short answer can be given. No proofs, graphs, or extensive data analysis are given.

Chapter 1

1. a. $\Omega = \{hhh, hht, htt, hth, ttt, tth, thh, tht\}$
b. $A = \{hhh, hht, hth, thh\}$
$B = \{hht, hhh\}$
$C = \{hht, htt, ttt, tht\}$
c. $A^c = \{htt, ttt, tth, tht\}$
$A \cap B = \{hht, hhh\}$
$A \cup C = \{hhh, hht, hth, thh, htt, ttt, tht\}$

3. $\Omega = \{rrr, rrg, rrw, rwg, rgw, rgr, rwr, rgg, ggr, ggw, grr, grw, gwr, grg, gwg, wrr, wgg, wrg, wgr\}$

5. $\Omega = (A \cap B)^c \cap (A \cup B)$ **9.** Not 50% **11.** $7 \times 6 \times 5 \times 4/10^4$

13. a. $10(4^5 - 4)/\binom{52}{5}$ **b.** $13 \times 48/\binom{52}{5}$ **c.** $13 \times 12 \times 4 \times 6/\binom{52}{5}$

15. 72 **19. a.** $5 \times 3 \times 2 \times 2/\binom{12}{4}$ **b.** $240/\binom{12}{5}$

21. $\frac{8}{32}$ **23.** $n(n-1)$ **25.** 6

27. $26 \times 25 \times 24 \times 23 \times 22/26^5$ **29.** $\binom{10}{2}/\binom{47}{2}$ **31.** $6^2 \times 5^2 \times 4^2 \times 3^2 \times 2^2$

33. $7 \times 6 \times 5 \times 4 \times 3/7^5$ **37.** 210

39. a. $21!/26!$ **b.** 1.818×10^7

41. a. $\left[\binom{7}{2} + \binom{8}{2} + \binom{9}{2}\right]/\binom{24}{2}$ **b.** $\binom{7}{2}/\binom{24}{2}$

43. $\binom{10}{3\ 3\ 4}$ **47. a.** 11/45 **b.** 6/11

49. a. 4/7 **b.** 3/7 **51.** 2/5

53. 0.35 **55. a.** .48, .70 **b.** .064, .614, .322

57. 2/3 **59. a.** 2/3 **b.** 5/6

61. .86 **63.** 1/3 **69.** Yes

73. $\sum_{j=k}^{n}\binom{n}{j}p^j(1-p)^{n-j}$ **75.** p^3-2p^2+1; .597 **77.** 14

79. a. $P(aa)=1/4,\ P(Aa)=1/2,\ P(AA)=1/4$
b. 2/3
c. $P(aa)=p/6,\ P(Aa)=1/3+p/6,\ P(AA)=2/3-p/3$
d. $p_c=[(1-p/4)(2/3)]/(1-p/6)$

Chapter 2

3. $p(1)=.1,\ p(2)=.2,\ p(3)=.4,\ p(4)=.1,\ p(5)=.2$

7. $F(x)=\begin{cases}0 & x<0\\ 1-p & 0\le x<1\\ 1 & x\ge 1\end{cases}$ **9.** $p<.5$ **11.** $[(n+1)p]$

13. a. .0130 **b.** .2517 **15.** 3 of 5

17. $P(X=k)=p(1-p)^k, k=0,\ 1,\ldots$ **19.** $F(n)=1-(1-p)^n$

23. $\binom{k+r-1}{r}p^r(1-p)^k$ **25. a.** .9987 **b.** 9×10^{-7}

27. $p(k)=100^k e^{-100}/k!$, approximately **29.** $P(X\le 4)=.532104$

31. a. .28 **b.** 20.79 min **33.** $f(x)=\alpha\beta x^{\beta-1}\exp(-\alpha x^{\beta})$ **37.** 2/3

39. b. $f(x)=[\pi(1+x^2)]^{-1},\ -\infty<x<\infty$ **c.** 3.08

41. $-\log(1/4)/\lambda,\ -\log(3/4)/\lambda$ **43.** $f(x)=4\lambda\pi x^2\exp(-4\lambda\pi x^3/3)$

45. a. $1-e^{-1}$ **b.** $e^{-.5}-e^{-1.5}$ **c.** 46.1

53. a. 0.3085 **b.** 0.8351 **c.** 21.5

55. $c=1.96\sigma$ **59.** $f(x)=x^{-1/2}/2$

61. $(\lambda/c)^{\alpha}t^{\alpha-1}\exp(-\lambda t/c)/\Gamma(\alpha)$ **63.** $[\pi(1+x^2)]^{-1}$

65. $X=[-1+2\sqrt{1/4-\alpha(1/2-\alpha/4-U)}]/\alpha$, where U is uniform

67. a. $f(x)=(\beta/\alpha^{\beta})x^{\beta-1}\exp(-(x/\alpha)^{\beta})$

69. $f(x)=(\lambda/3)(3/4\pi)^{1/3}x^{-2/3}\exp(-\lambda(3x/4\pi)^{1/3})$

Chapter 3

1. a. $p_1=.19,\ p_2=.32,\ p_3=.31,\ p_4=.18$, for both X and Y
b. $p(1\,|1)=.526,\ p(2\,|1)=.263,\ p(3\,|1)=.105,\ p(4\,|1)=.105$, for both X and Y

3. Multinomial, $n = 10,\ p_1 = p_2 = p_3 = 1/3$

7. $f_{XY}(x, y) = \alpha\beta \exp[-\alpha x - \beta y]$; $f_x(x) = \alpha \exp[-\alpha x]$, $f_Y(y) = \beta \exp[-\beta y]$

9. **a.** $f_X(x) = 3(1 - x^2)/4,\ -1 \le x \le 1,\ f_Y(y) = 3\sqrt{1-y}/2,\ 0 \le y \le 1$
b. $f_{X|Y}(x|y) = 1/(2\sqrt{1-y})$, $f_{Y|X}(y|x) = 1/(1 - x^2)$

11. $1/9$

13. $p(0) = 1/2,\ p(1) = p(2) = 1/4$

15. **a.** $c = 3/2\pi$ **c.** $\dfrac{2\sqrt{2}-1}{2\sqrt{2}}$

d. $f_Y(y) = \frac{3}{2}(1 - y^2),\quad -1 \le y \le 1$
$f_X(x) = \frac{3}{2}(1 - x^2),\quad -1 \le x \le 1$
X and Y are not independent.

e. $f_{Y|X}(y|x) = \dfrac{\sqrt{1 - x^2 - y^2}}{\pi(1 - x^2)}$

$f_{X|Y}(x|y) = \dfrac{\sqrt{1 - x^2 - y^2}}{\pi(1 - y^2)}$

17. **b.** $f_X(x) = 1 - |x|,\ -1 \le x \le 1$; $f_Y(y) = 1 - |y|, -1 \le y \le 1$
c. $f_{X|Y}(x|y) = 1/(2 - 2|y|),\ 1 - |y| \le x \le 1 + |y|$
$f_{Y|X}(y|x) = 1/(2 - 2|x|),\ 1 - |x| \le y \le 1 + |x|$

19. **a.** $\beta/(\alpha + \beta)$ **b.** $\beta/(2\alpha + \beta)$

23. Binomial (m, pr)

29. $h(x, y) = \lambda\mu e^{-\lambda x} e^{-\mu y}[1 + \alpha(1 - 2e^{-\lambda x})(1 - 2e^{-\mu y})]$

33. **a.** $f_{\Theta|N}(\theta|n) = n(n+1)\theta(1-\theta)^{n-1}$

43. $f_S(s) = s$ for $0 \le s \le 1$ and $= 2 - s$ for $1 \le s \le 2$

49. $\lambda e^{-\lambda S/2} - \lambda e^{-\lambda S}$ **53.** $5/9$

55. $f_{XY}(x, y) = (x^2 + y^2)^{-1/2},\ x^2 + y^2 \le 1$

57. $x_1 = y_1$; $x_2 = -y_1 + y_2$

61. $f_{UV}(u, v) = \dfrac{1}{bd} f_{XY}\left(\dfrac{u-a}{b}, \dfrac{v-c}{d}\right)$

63. **a.** $f_{UV}(u, v) = \dfrac{1}{2} f_{XY}\left(\dfrac{u+v}{2}, \dfrac{u-v}{2}\right)$ where $U = X + Y,\ V = X - Y$

b. $f_{UV}(u, v) = \dfrac{1}{2|v|} f_{XY}((uv)^{1/2},\ (u/v)^{1/2})$ where $U = XY,\ V = X/Y$

67. $f(t) = n(n-1)\lambda[\exp(-(n-1)\lambda t) - \exp(-n\lambda t)]$

69. $n\beta v^{\beta-1}\alpha^{-\beta}\exp(-n(v/\alpha)^{\beta})$

71. $1-\gamma^n$

75. Let $U = X_{(i)},\ V = X_{(j)}$

$$f_{UV}(u,v) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!} \times [F(u)]^{i-1}f(u)[F(v)-F(u)]^{j-i-1}f(v)[1-F(v)]^{n-j}$$

77. $n(1-x)^{n-1}$

79. Exponential (λ)

81. **a.** $n/(n+1)$ **b.** $(n-1)/(n+1)$

Chapter 4

3. $E(X) = 3.1;\ \text{Var}(X) = 1.49$

5. $E(X) = \alpha/3;\ \text{Var}(X) = 1/3 - \alpha^2/9$

7. **a.** $E(X) = 5/8$
b. $p_Y(0) = 1/2,\ p_Y(1) = 3/8,\ p_Y(4) = 1/8,\ E(Y) = 7/8$
c. $E(X^2) = 7/8$
d. $\text{Var}(X) = 31/64$

9. That value of n such that $s\sum_{k=n}^{\infty} p(k) > c\sum_{k=1}^{n-1} p(k)$ and $s\sum_{k=n+1}^{\infty} p(k) < c\sum_{k=1}^{n} p(k)$

15. It makes no difference.

17. **a.** $E(X_{(k)}) = k/(n+1)$
b. $\text{Var}(X_{(k)}) = k(n-k+1)/[(n+1)^2(n+2)]$

19. $1/(n+1)$

21. $1/3$

23. $2/\lambda^2$ (square), $1/\lambda^2$ (rectangle)

25. $2\alpha(\alpha+1)/\lambda^2$

27. 1

31. no

35. r/p

37. $p > (1/k)^{1/k}$

39. **a.** 4606 **b.** 10,000

41. The expected number of occurrences is 4.62. Using Markov's inequality, the chance of 100 or more occurrences is less than 0.0462, so you should be surprised.

45. $\text{Cov}(N_i, N_j) = -np_ip_j$

47. $\text{Cov}(X, Z) = -\sigma_X^2;\ \text{Corr}(X, Z) = -\dfrac{\sigma_X}{(\sigma_X^2+\sigma_Y^2)^{1/2}}$

49. **b.** $\alpha = \sigma_Y^2/(\sigma_Y^2+\sigma_X^2)$
c. $(X+Y)/2$ is better when $1/3 < \sigma_X^2/\sigma_Y^2 < 3$.

51. $\pi_i = n^{-1}$ for the optimal portfolio. If each individual return has standard deviation σ, the standard deviation of the return from this portfolio is $\sigma/\sqrt{n}$. If the entire investment is in one security, the standard deviation of the return is σ.

55. $E(T) = n(n+1)\mu/2;\ \text{Var}(T) = n(n+1)(2n+1)\sigma^2/6$

57. $\sigma_X^2\sigma_Y^2 + \mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2$

61. **a.** $\text{Cov}(x, Y) = 1/36$; $\text{Corr}(X, Y) = 1/2$
b. $E(X|Y) = Y/2$, $E(Y|X) = (X + 1)/2$
c. If $Z = E(X|Y)$, the density of Z is $f_Z(z) = 8z$, $0 \le z \le 1/2$
If $Z = E(Y|X)$, the density of Z is $f_Z(z) = 8(1 - z)$, $1/2 \le z \le 1$
d. $\hat{Y} = \frac{1}{2} + \frac{1}{2}X$; the mean squared prediction error is 1/24
e. $\hat{Y} = \frac{1}{2} + \frac{1}{2}X$; the mean squared prediction error is 1/24

63. **a.** $\text{Cov}(X, Y) = -.0085$; $\rho_{XY} = -.1256$
b. $E(Y|X) = (6X^2 + 8X + 3)/[4(3X^2 + 3X + 1)]$

65. In the claim that $E(T\,|N = n) = nE(X)$

67. 3/2, 1/6

71. $p_{Y|X}(y\,|x)$ is hypergeometric. $E(Y|X = x) = mx/n$

73. $np(1 + p)$

75. **a.** $1/2\lambda$; **b.** $5/12\lambda^2$

77. $E(X|Y) = Y/2$, $E(Y|X) = X + 1$

79. $M(t) = \frac{1}{2} + \frac{3}{8}e^t + \frac{1}{8}e^{2t}$

81. $M(t) = 1 - p + pe^t$

85. $M(t) = e^t p/[1 - (1 - p)e^t]$; $E(X) = 1/p$; $\text{Var}(X) = (1 - p)/p^2$

87. Same p

93. Exponential

99. **b.** $E[g(X)] \approx \log\mu - \sigma^2/2\mu^2$; $\text{Var}[g(X)] \approx \sigma^2/\mu^2$

101. $E(Y) \approx \sqrt{\lambda} - 1/(8\sqrt{\lambda})$; $\text{Var}(Y) \approx 1/4$

103. .0628 mm

Chapter 5

3. .0228

13. $N(0,\ 150{,}000)$; most likely to be where he started

15. $p = .017$

17. $n = 96$

21. **b.** $$\text{Var}(\hat{I}(f)) = \frac{1}{n}\left[\int_a^b \frac{f^2(x)}{g(x)}dx - I^2(f)\right]$$

29. Let $Z_n = n(U_{(n)} - 1)$. Then $P(Z_n \le z) \to e^z, -1 \le z \le 0$

Chapter 6

3. $c = .17$

9. $E(S^2) = \sigma^2$; $\text{Var}(S^2) = 2\sigma^4/(n - 1)$

Chapter 7

1. $p(1.5) = 1/5$, $p(2) = 1/10$, $p(2.5) = 1/10$, $p(3) = 1/5$, $p(4.5) = 1/10$, $p(5) = 1/5$, $p(6) = 1/10$; $E(\overline{X}) = 17/5$; $\text{Var}(\overline{X}) = 2.34$

3. d, f, h

7. $n = 319$, ignoring the fpc

9. $\text{SE} = .026$. CI: $(.05, .15)$

11. **a.** 6 samples. **b.** Yes

15. b.

n	Δ_1	Δ_2
20	211.6	86.8
40	145.6	59.7
80	96.9	39.8

17. no

19. 1.28, 1.645

21. The sample size should be multiplied by 4.

29. a. $\hat{Q} = \dfrac{R - t(1-p)}{p}$, where t = probability of answering yes to unrelated question

c. $\mathrm{Var}(\hat{Q}) = r(1-r)/(np^2)$, where $r = P(yes) = qp + t(1-p)$

31. $n = 395$

33. The sample size for each survey should be 1250.

35. a. $\overline{X} = 98.04$

b. $s^2 \dfrac{N-1}{N} = 133.64$, $\dfrac{s^2}{n}\left(1 - \dfrac{n}{N}\right) = 5.28$

c. 98.04 ± 4.50 and $196{,}080 \pm 9008$

37. a. $\alpha + \beta = 1$

b. $\alpha = \dfrac{\sigma^2_{\overline{X}_2}}{\sigma^2_{\overline{X}_1} + \sigma^2_{\overline{X}_2}}$ $\quad \beta = \dfrac{\sigma^2_{\overline{X}_1}}{\sigma^2_{\overline{X}_1} + \sigma^2_{\overline{X}_2}}$

39. Choose n such that $p = 1 - \dfrac{(N-k)(N-k-1)\cdots(N-n+k-1)}{N(N-1)\cdots(N-k+1)}$, which can be done by a recursive computation; $n = 581$

41. b. $\dfrac{N^2}{n}\left(\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B\right)$

c. The proposed method has smaller variance if $\rho > \dfrac{\sigma_B^2}{2\sigma_A\sigma_B}$.

d. The ratio estimate is biased. The approximate variance of the ratio estimate is greater if $\dfrac{\mu_A}{\mu_B} > 1$.

43. $R = \dfrac{\overline{V}}{\overline{O}} = .73$, $s_R = .02$, $.73 \pm .04$

47. The bias is .96 for $n = 64$ and .39 for $n = 128$.

49. Ignoring the fpc,

a. $R = 31.25$;

b. $s_R = .835$; 31.25 ± 1.637;

c. $T = 10^7$; $10^7 \pm 5, 228, 153$;

d. $s_{T_R} = 266{,}400$, which is much better.

53. a. For optimal allocation, the sample sizes are 10, 18, 17, 19, 12, 9, 15. For proportional allocation they are 20, 23, 19, 17, 8, 6, 7.

b. $\mathrm{Var}(\overline{X}_{SO}) = 2.90$, $\mathrm{Var}(\overline{X}_{sp}) = 3.4$, $\mathrm{Var}(\overline{X}_{srs}) = 6.2$

55. a. $\frac{1}{6}\overline{X}_H + \frac{5}{6}\overline{X}_L$
b. 0.68
c. No, the standard error would be 0.87.
d. No, the standard error would be 0.71.

57. $p(2.2) = 1/6$, $p(2.8) = 1/3$, $p(3.8) = 1/6$, $p(4.4) = 1/3$; $E(\overline{X}_s) = 3.4$; $\text{Var}(\overline{X}_s) = .72$

61. a. $w_1 + w_2 + w_3 = 0$ and $w_1 + 2w_2 + 3w_3 = 1$
b. $w_1 = -1/2$, $w_2 = 0$, $w_3 = 1/2$

Chapter 8

3. For concentration (1),

a. $\hat{\lambda} = .6825$; **b.** $.6825 \pm .081$;
c. There are not gross differences between observed and expected counts.

5. a. $\hat{\theta} = 1/3$ **b.** $\text{Lik}(\theta) = \theta(1-\theta)^2$
c. $\hat{\theta} = 1/3$ **d.** $\beta(2, 3)$

7. a. $\hat{p} = 1/\overline{X}$ **b.** $\tilde{p} = 1/\overline{X}$
c. $\text{Var}(\tilde{p}) \approx p^2(1-p)/n$
d. The posterior distribution is $\beta(2, k)$; the posterior mean is $2/(k+2)$.

13. $P(|\hat{\alpha}| > .5) \approx .1489$

17. b. $\hat{\alpha} = n(8\Sigma_{i=1}^n X_i^2 - 2n)^{-1} - 1/2$

c. $$\frac{\Gamma'(2\alpha)}{\Gamma(2\alpha)} - \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} + \frac{1}{2n}\sum_{i=1}^{n}\log[X_i(1-X_i)] = 0$$

d. $$\left(2n\left[\frac{\Gamma''(\alpha)\Gamma(\alpha) - \Gamma'(\alpha)^2}{\Gamma(\alpha)^2} - \frac{2\Gamma''(2\alpha)\Gamma(2\alpha) - \Gamma'(2\alpha)^2}{\Gamma(2\alpha)^2}\right]\right)^{-1}$$

19. a. $\hat{\sigma} = \sqrt{n^{-1}\Sigma_{i=1}^n(X_i - \mu)^2}$ **b.** $\hat{\mu} = \overline{X}$ **c.** no

21. a. $\overline{X} - 1$ **b.** $\min(X_1, X_2, \ldots, X_n)$ **c.** $\min(X_1, X_2, \ldots, X_n)$

23. Method of moments estimate is 1775. MLE is 888.

27. Let T be the time of the first failure.

a. $\frac{5}{\tau}\exp\left(-\frac{5t}{\tau}\right)$ **b.** $\hat{\tau} = 5T$

c. $\hat{\tau} \sim \exp\left(\frac{1}{\tau}\right)$ **d.** $\sigma_{\hat{\tau}} = \tau$

31. a. $3p(1-p)^6$ **b.** $\hat{p} = 1/7$

33. Let q be the .95 quantile of the t distribution with $n-1$ df; $c = qs_{\overline{X}}$.

41. For α the relative efficiency is approximately .444; for λ it is approximately .823.

47. a. $\hat{\theta} = \overline{X}/(\overline{X} - x_0)$
b. $\tilde{\theta} = n/(\Sigma \log X_i - n \log x_0)$
c. $\mathrm{Var}(\tilde{\theta}) \approx \theta^2/n$

49. a. Let $\hat{p}$ be the proportion of the n events that go forward. Then $\hat{\alpha} = 4\hat{p} - 2$.
b. $\mathrm{Var}(\hat{\alpha}) = (2 - \alpha)(2 + \alpha)/n$

53. a. $\hat{\theta} = 2\overline{X}$; $E(\hat{\theta}) = \theta$; $\mathrm{Var}(\hat{\theta}) = \theta^2/3n$
b. $\tilde{\theta} = \max(X_1, X_2, \ldots, X_n)$
c. $E(\tilde{\theta}) = n\theta/(n+1)$; bias $= -\theta/(n+1)$; $\mathrm{Var}(\hat{\theta}) = n\theta^2/(n+2)(n+1)^2$; $\mathrm{MSE} = 2\theta^2/(n+1)(n+2)$
d. $\theta^* = (n+1)\tilde{\theta}/n$

55. a. Let n_1, n_2, n_3, n_4 denote the counts. The mle of θ is the positive root of the equation

$$(n_1 + n_2 + n_3 + n_4)\theta^2 - (n_1 - 2n_2 - 2n_3 - n_4)\theta - 2n_4 = 0$$

The asymptotic variance is $\mathrm{Var}(\hat{\theta}) = 2(2+\theta)(1-\theta)\theta/(n_1 + n_2 + n_3 + n_4)(1+\theta)$. For these data, $\hat{\theta} = .0357$ and $s_{\hat{\theta}} = .0057$.
b. An approximate 95% confidence interval is $.0357 \pm .0112$.

57. a. s^2 is unbiased. **b.** $\hat{\sigma}^2$ has smaller MSE. **c.** $\rho = 1/(n+1)$

59. b. $\hat{\alpha} = (n_1 + n_2 - n_3)/(n_1 + n_2 + n_3)$ if this quantity is positive and 0 otherwise.

63. In case (1) the posterior is $\beta(4, 98)$ and the posterior mean is 0.039. In case (2) the posterior is $\beta(3.5, 102)$ and the posterior mean is 0.033. The posterior for case (2) rises more steeply and falls off more rapidly than that of case (1).

65. $\mu_0 = 16.25$, $\xi_0 = 80$

71. $\prod_{i=1}^{n} (1 + X_i)$

73. $\sum_{i=1}^{n} X_i^2$

Chapter 9

1. a. $\alpha = .002$ **b.** power $= .349$

3. a. $\alpha = .046$

5. F, F, F, F, F, F, F, T

7. Reject when $\sum X_i > c$. Since under H_0, $\sum X_i$ follows a Poisson distribution with parameter $n\lambda$, c can be chosen so that $P(\sum X_i > c | H_0) = \alpha$.

9. For $\alpha = .10$, the test rejects for $\overline{X} > 2.56$, and the power is .2981. For $\alpha = .01$, the test rejects for $\overline{X} > 4.66$, and the power is .0571.

17. a. $LR = \frac{\sigma_1}{\sigma_0} \exp\left[\frac{1}{2}x^2\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\right)\right]$. A level α test rejects for $X^2 > \sigma_0^2 \chi_1^2(\alpha)$.
b. Reject for $\sum_{i=1}^{n} X_i^2 > \sigma_0^2 \chi_n^2(\alpha)$ **c.** Yes

19. a. $X < 2/3$ **b.** Reject for large values of X
c. Reject for $X > \sqrt{1 - \alpha}$ **d.** $1 - (1 - \alpha)^3/2$

21. **a.** Reject for $X > 1$; power $= 1/2$
b. Significance level $= \alpha$, power $= 1 - \alpha/2$
c. Significance level $= \alpha$, power $= 1 - \alpha/2$
d. Reject when $(1-\alpha)/2 \le X \le (1+\alpha)/2$
e. For $\alpha > 0$, the rejection region is not uniquely determined.
f. The rejection region is not uniquely determined.

23. yes **25.** $-2 \log \Lambda = 54.6$. Strongly rejects **27.** ≥ 12.02

29. yes **31.** 2.6×10^{-1}, 9.8×10^{-3}, 3×10^{-4}, 7×10^{-7}

33. $-2 \log \Lambda$ and X^2 are both approximately 2.93. $.05 < p < .10$; not significant for Chinese and Japanese; both $\approx .3$.

35. $X^2 = .0067$ with 1 df and $p \approx .90$. The model fits well.

37. $X^2 = 79$ with 11 df and $p \approx 0$. The accidents are not uniformly distributed, apparently varying seasonally with the greatest number in November–January and the fewest in March–June. There is also an increased incidence in the summer months, July–August.

39. $\chi^2 = 85.5$ with 9 df, and thus provides overwhelming evidence against the null hypothesis of constant rate.

41. Let $\hat{p}_i = X_i/n_i$ and $\hat{p} = \sum X_i / \sum n_i$. Then

$$\Lambda = \frac{\hat{p}^{\Sigma n_i \hat{p}_i}(1-\hat{p})^{\Sigma n_i(1-\hat{p}_i)}}{\prod \hat{p}_i^{n_i \hat{p}_i}(1-\hat{p}_i)^{n_i(1-\hat{p}_i)}}$$

and

$$-2 \log \Lambda \approx \sum \frac{(X_i - n_i \hat{p})^2}{n_i \hat{p}(1-\hat{p})}$$

is approximately distributed as χ^2_{m-1} under H_0.

43. **a.** 9207 heads out of 17950 tosses is not consistent with the null hypothesis of 17950 independent Bernoulli trials with probability .5 of heads. ($X^2 = 11.99$ with 1 df).
b. The data are not consistent with the model ($X^2 = 21.57$ with 5 df, $p \approx .001$).
c. A chi-square test gives $X^2 = 8.74$ with 4 df and $p \approx .07$. Again, the model looks doubtful.

45. The binomial model does not fit the data ($X^2 = 110.5$ with 11 df). Relative to the binomial model, there are too many families with very small and very large numbers of boys. The model might fail because the probability of a male child differs from family to family.

51. The horizontal bands are due to identical data values.

57. The tails decrease less rapidly than do those of a normal probability distribution, causing the normal probability plot to deviate from a straight line at the ends by curving below the line on the left and above the line on the right.

59. The rootogram shows no systematic deviation.

Chapter 10

3. $q_{.25} \approx 63.4$; $q_{.5} \approx 63.6$; $q_{.75} \approx 63.8$

7. Differences are about 50 days for the weakest, 150 days for the median. Can't tell for the strongest.

9. Bias $\approx -\dfrac{1}{2n}\dfrac{F(x)}{1-F(x)}$, which is large for large x.

11. $h(t) = \alpha\beta t^{\beta-1}$

13. The uniform distribution on [0, 1] is an example.

15. $h(t) = (24-t)^{-1}$. It increases from 0 to 24. It is more likely after 5 hours.

23. $(n+1)\left(\dfrac{k+1}{n+1} - p\right) X_{(k)} + (n+1)\left(p - \dfrac{k}{n+1}\right) X_{(k+1)}$

29. **b.** $\approx .018$ **c.** ≈ 18 **d.** $\approx 2.4 \times 10^{-19}$

31. **a.** n^n

b.

x	1/3	5/3	2	7/3	8/3	3	10/3	11/3
$p(x)$	1/27	3/27	3/27	3/27	8/27	3/27	3/27	3/27

33. The mean and standard deviation

37. Median = 14.57, $\bar{x} = 14.58$, $\bar{x}_{.10} = 14.59$, $\bar{x}_{.20} = 14.59$; $s = .78$, IQR/1.35 = .74, MAD/.65 = .82

41. The interval $(X_{(r)}, X_{(s)})$ covers x_p with probability $\sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i}$.

Chapter 11

7. Throughout. For example, all are used in the assertion that $\mathrm{Var}(\overline{X} - \overline{Y}) = \sigma^2(n^{-1} + m^{-1})$. All are used in Theorem A and Corollary A. Independence is used in the expression for the likelihood.

11. Use the test statistic

$$t = \frac{(\overline{X} - \overline{Y}) - \Delta}{s_p\sqrt{\dfrac{1}{n} + \dfrac{1}{m}}}$$

13. The power of the sign test is .35, and the power of the normal theory test is .46.

15. $n = 768$

19. **a.** $\sqrt{2}$ **b.** $\overline{Y} - \overline{X} > 2.33$ **c.** 0.17

d. Yes **e.** $\overline{Y} - \overline{X} > 2.78$; power = 0.11

21. **a.** A pooled t test gives a p-value of .053.

b. The p-value from the Mann-Whitney test is .064.

c. The sample sizes are small and normal probability plots suggest skewness, so the Mann-Whitney test is more appropriate.

25. **a.** No **b.** No **c.** Yes, yes

27.

w	0	1	2	3	4	5	6	7	8	9	10
$p(w)$	.0625	.0625	.0625	.125	.125	.125	.125	.125	.0625	.0625	.0625

31. $E\hat{\pi} = 1/2$; $\text{Var}\,(\hat{\pi}) = \dfrac{1}{12}\dfrac{m+n+1}{mn}$, which is smallest when $m = n$.

33. Let $\theta = \sigma_X^2/\sigma_Y^2$ and $\hat{\theta} = s_X^2/s_Y^2$.

a. For $H_1\colon\ \theta > 1$, reject if $\hat{\theta} > F_{n-1,m-1}(\alpha)$. For $H_2\colon\ \theta \neq 1$, reject if $\hat{\theta} > F_{n-1,m-1}\,(\alpha/2)$ or $\hat{\theta} < F_{n-1,m-1}(1-\alpha/2)$.

b. A $100(1-\alpha)\%$ confidence interval for θ is

$$\left[\frac{\hat{\theta}}{F_{n-1,m-1}(\alpha/2)}, \frac{\hat{\theta}}{F_{n-1,m-1}(1-\alpha/2)}\right].$$

c. $\hat{\theta} = .60$. The p-value for a two-sided test is .42. A 95% confidence interval for θ is (.13, 2.16).

37. **a.** For each patient, compute a difference score (after – before), and compare the difference scores of the treatment and control by a signed rank test or a paired t test. A signed rank test gives for Ward A $W_+ = 36$, $p = .124$ and for Ward B $W_+ = 22$, $p = .205$.

b. To compare the two wards, use a two-sample t test or a Mann-Whitney test on difference scores. Using a Mann-Whitney test, there is strong evidence that the stelazine group in Ward A improved more than the stelazine group in Ward B $(p = .02)$ and weaker evidence that the placebo group improved more in Ward A than in Ward B $(p = .09)$.

45. **a.** For example, for 1957 by a Wilcoxon signed rank test there is no evidence that seeding is effective $(p = .73)$. For this and other years, it appears that the gain in seeding over not seeding may be greatest when rainfall in the unseeded area is low.

b. Randomization guards against possibly confounding the effect of seeding with cyclical weather patterns. Pairing is effective if rainfall on successive days is positively correlated; in these data, the correlation is weak.

47. **a.** To test for an effect of seeding, compare the differences (target – control) to each other by a two-sample t test or a Mann-Whitney test. A Mann-Whitney test gives a p-value of .73.

b. The square root transformation makes the distribution of the data less skewed.

c. Using a control area is effective if the correlation between the target and control areas is large enough that the standard deviation of the difference (target – control) is smaller than the standard deviation of the target rainfalls. This was indeed the case.

49. 95% CI: (8.9, 13.1). Null hypothesis is overwhelmingly rejected.

51. The durations of the bottle-fed are typically much longer. Because the distribution is very skewed with some large outliers, a nonparametric test is preferable. The p-value from a signed-rank test is 0.012.

53. The lettuce leaf cigarettes were controls to ensure that the effects of the experiment were due to tobacco specifically, not just due to smoking a lit cigarette. The unlit cigarettes were controls to ensure that the effects were due to lit tobacco, not just unlit tobacco.

Chapter 12

11.

	A	B	C	D
I	2	3	4	5
II	3	4	5	6
III	4	5	6	7

17. $\hat{\alpha}_i = \overline{Y}_{i..} - \overline{Y}_{...}$
$\hat{\beta}_j = \overline{Y}_{.j.} - \overline{Y}_{...}$
$\hat{\delta}_{ij} = \overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{Y}_{...}$
$\hat{\mu} = \overline{Y}_{...}$

19. $Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \upsilon_{jk} + \rho_{ik} + \phi_{ijk} + \epsilon_{ijkl}$
The main effects $\alpha_i,\ \beta_j,\ \gamma_k$ satisfy constraints of the form $\sum \alpha_i = 0$. The two-factor interactions, δ, υ, and ρ, satisfy constraints of the form $\sum_i \delta_{ij} = \sum_j \delta_{ij} = 0$. The three-factor interactions, ϕ_{ijk}, sum to zero over each subscript.

21. A graphical display suggests that Group IV may have a higher infestation rate than the other groups, but the F test only gives a p-value of .12 ($F_{3,16} = 2.27$). The Kruskal–Wallis test results in $K = 6.2$ with a p-value of .10 (3 df).

23. For the male rats, both dose and light are significant (LH increases with dose and is higher in normal light), and there is an indication of interaction ($p = .07$) (the difference in LH production between normal and constant light increases with dose), summarized in the following anova table:

Source	*df*	*SS*	*MS*
Dose	4	545549	136387
Light	1	242189	242189
Interaction	4	55099	13775
Error	50	301055	6021

The variability is not constant from cell to cell but is proportional to the mean. When the data are analyzed on a log scale, the cell variability is stabilized and the interaction disappears. The effects of light and dose are still clear.

25. The following anova table shows that none of the main effects or interactions are significant:

Source	df	SS	MS
Position	9	83.84	9.32
Bar	4	46.04	11.51
Interaction	36	334.36	9.29
Error	50	448.00	8.96

There are some odd things about the data. The first reading is almost always larger than the second, suggesting that the measurement procedure changed somehow between the first and second measurements. One notable exception to this is position 7 on bar 50, which looks anomalous.

27.

Source	df	SS	MS
Species	2	836131	418066
Error	131	446758	3410
Total	133	1282889	

The variance increases with the mean and is stabilized by a square root transformation. The Bonferroni method shows that there are significant differences between all the species.

29.

Source	Df	Sum Sq	Mean Sq	F value	p-value
Furnace	2	4.1089	2.0544	1.4460	0.26159
Wafer.Type	2	5.8756	2.9378	2.0678	0.15547
Furnace x Wafer.Type	4	21.3489	5.3372	3.7566	0.02162
Residuals	18	25.5733	1.4207		

Only interactions are significant. Lines are not parallel in the interaction plot, in which the relationship of thickness of external wafers to furnaces appears quite different than that of the other two wafer types.

33. **a.** N/R50 and R/R50 **b.** N/R50 and lopro **c.** N/R50 and N/R40

Chapter 13

1. $X^2 = 5.10$ with 1 df; $p < .025$

3. For the ABO group there is a significant association ($X^2 = 15.37$ with 6 df, $p = .02$), due largely to the higher than average incidence of moderate-advanced TB in B. For the MN group there is no significant association ($X^2 = 4.73$ with 4 df, $p = .32$).

5. $X^2 = 6.03$ with 7 df and $p = .54$, so there is no convincing evidence of a relationship.

7. $X^2 = 12.18$ with 6 df and $p = .06$. It appears that psychology majors do a bit worse and biology majors a bit better than average.

9. In this aspect of her style, Jane Austen was not consistent. *Sense and Sensibility* and *Emma* do not differ significantly from each other ($X^2 = 6.17$ with 5 df and $p = .30$), but *Sanditon* I differs from them, *and* not being followed by *I* less frequently and *the* not being preceded by *on* more frequently ($X^2 = 23.29$ with 10 df and $p = .01$). *Sanditon* I and II were not consistent ($X^2 = 17.77$, df $= 5$, $p < .01$), largely due to the different incidences of *and* followed by *I*.

11. **a.** In both cases the statistic is

$$-2 \log \Lambda = 2 \sum_i \sum_j O_{ij} \log(O_{ij}/E_{ij})$$

b. $-2 \log \Lambda = 12.59$
c. $-2 \log \Lambda = 16.52$

13. Arrange a table with the number of children of an older sister as rows and the number of children of her younger sister as columns.

a. $H_o: \pi_{ij} = \pi_{i.}\pi_{j.}$. This is the usual test for independence, with

$$X^2 = \sum_{ij} (n_{ij} - n_{i.}n_{.j}/n_{..})^2/(n_{i.}n_{.j}/n_{..})$$

b. H_0: $\sum_{i \neq j} \pi_{ij} = \sum_{j \neq i} \pi_{ji}$ is equivalent to $H_0: \pi_{ij} = \pi_{ji}$. The test statistic is

$$X^2 = \sum_{i \neq j} (n_{ij} - (n_{ij} + n_{ji})/2)^2/((n_{ij} + n_{ji})/2)$$

which follows a χ_2^2 distribution under H_0. The null hypotheses of (a) and (b) are not equivalent. For example, if the younger sister had exactly the same number of children as the older, (a) would be false and (b) would be true.

15. For males, $X^2 = 13.39$, df $= 4$, $p = .01$. For females, $X^2 = 4.47$, df $= 4$, $p = .35$. We would conclude that for males the incidence was especially high in Control I and especially low in Medium Dose and that there was no evidence of a difference in incidence rates among females.

17. There is clear evidence of different rates of ulcers for A and O in both London and Manchester ($X^2 = 43.4$ and 5.52 with 1 df respectively). Comparing London A to Manchester A, we see that the incidence rate is higher in Manchester ($X^2 = 91$, df $= 1$), whereas the incidence rate is higher for London O than for Manchester O ($X^2 = 204$, df $= 1$).

19. $p = .01$

21. $\hat{\Delta} = 3.77$

23. McNemar's test gives a chi-square statistic equal to 28.5. Comparing this to the chi-square distribution with 1 df, the result is highly significant: heavy exertion is associated with myocardial infarction. This design is similar to the cell phone study in that each subject acts as his own control.

25. a. The total incidence of myocardial infarction (MCI) is reduced by aspirin ($X^2 = 26.4$ with 1 df). The odds ratio is 0.58, which is a considerable reduction in risk due to aspirin. The incidences of fatal and nonfatal are both significantly reduced as well ($X^2 = 6.2, 20.43$, df $= 1$). There is no indication that among those having MCI, the fatality rate was reduced (p-value $= 0.32$). The difference in the incidence of strokes was not statistically significant, $X^2 = 1.67$, df $= 1$.

b. There is no evidence that total cardiovascular mortality is decreased by aspirin, but the reduction in mortality due to myocardial infarction is significant.

27. The death penalty was given in 13% of the cases in which the victim was white and the defendant was not. In all other cases the death penalty was given only 5–6% of the time. A chi-square test of independence yields a statistic equal to 15.9 with 3 df, so the p-value is 0.001. Whether such a test is valid is debatable. The use of the test could be criticized on the grounds that these are all the data there are for the years 1993–97, the numbers speak for themeselves, and there is no plausible probability model on which to base probability calculations, like p-values. The use of the test could be defended by arguing that for a table with these row and column marginal totals, it would be very unlikely that there would be such variation of the proportions between rows if only chance were at work.

29. It depends on how the sampling is done. If the number of males and females are determined prior to the sample being drawn, a test of homogeneity would be appropriate. If only the total sample size are fixed, a test of independence would be appropriate. Management won't care, because the qualitative nature of the conclusion would be the same in either case.

Chapter 14

1. b. $\log y = \log a - bx$. Let $u = \log y$ and $v = \log x$.

d. $y^{-1} = ax^{-1} + b$. Let $u = y^{-1}$ and $v = x^{-1}$.

5. This can be set up as a least squares problem with the parameter vector $\beta = (p_1, p_2, p_3)^T$ and the design matrix

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

The least squares estimate is $\hat{\beta} = (X^T X)^{-1} X^T Y$. This gives, for example,

$$\hat{p}_1 = \frac{1}{2}Y_1 + \frac{1}{4}Y_2 + \frac{1}{4}Y_3 + \frac{1}{4}Y_4 + \frac{1}{4}Y_5$$

13. a. $\mathrm{Var}(\hat{\mu}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]$

c. $\hat{\mu}_0 \pm s_{\hat{\mu}_0} t_{n-2}(\alpha/2)$, where

$$s_{\hat{\mu}_0} = s\left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum(x_i - \bar{x})^2}\right]^{1/2}$$

15. $\hat{\beta} = \left(\sum x_i y_i\right)/\left(\sum x_i^2\right)$

21. Place half the x_i at –1 and half at +1.

23. a. 85 **b.** 80 **25.** true

31. $\text{Cov}(U, V) = 0$

37. $\hat{A} = 18.18$, $s_{\hat{A}} = .14$; $18.18 \pm .29$
$\hat{B} = -2.126 \times 10^4$, $s_{\hat{B}} = 1.33 \times 10^2$; $-2.126 \times 10^4 \pm 2.72 \times 10^2$

39. Neither a linear nor a quadratic function fits the data.

41. One possibility is DBH versus the square root of age.

43. A physical argument suggests that settling times should be inversely proportional to the squared diameter; empirically, such a fit looks reasonable. Using the model $T = \beta_0 + \beta_1/D^2$ and weighted least squares, we find (standard errors listed in parentheses)

	10	**25**	**28**
$\hat{\beta}_0$	–.403(1.59)	1.48 (2.50)	2.25 (2.08)
$\hat{\beta}_1$	28672 (371)	18152 (573)	16919 (474)

From the table we see that the intercept can be taken to be 0.

51. For 1998, RSS = .016. For the 1999 predictions, RSS = .055, which is much larger. The predicted values for 1999 appear unrelated to the observed values. The poor performance in 1999 of the predictions formed from the 1998 data is due to over-fitting—4 parameters were estimated from 5 data points.

53. a. There appear to be two regimes corresponding to durations less than or greater than 3 min, and it is best to fit separate linear regressions to each regime.

b. For a duration of 2 min the prediction would be 54.3 min. The standard error of this fitted value is 1.04 min. But there are two parts to the prediction error: the error of the fitted value and the variability of a new observation around its expected value. This latter is measured by the standard deviation of the residuals, 5.9 min. For a duration of 4.5 min, the prediction is 80.3 min. The standard error of this prediction is 1.09 min and the residual standard deviation is 6.7 min. A 95% prediction interval is (67 min, 94 min). See problems 13 and 14.