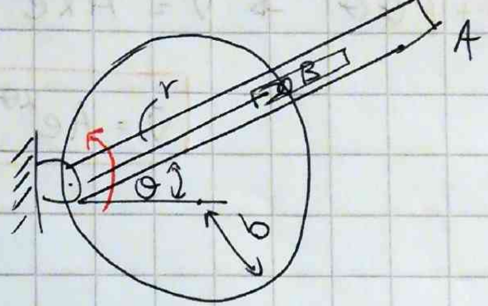
Taller 2 - Secciones 7 y 8

Francisco Zamorano

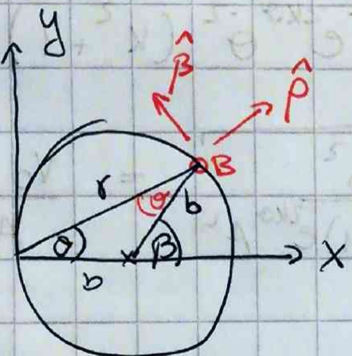
Pregunta 1



Por enunciado,

$$\hat{r} = \hat{\rho}$$

Reescribimos el sistema coordenado



* $\hat{\beta}$ es otra forma de decir $\hat{\theta}$ pero cambiamos la notación para no confundir con el ángulo θ.

Por otra parte, en ese instante

$$\vec{r} = b\hat{\imath} + b\hat{\rho}$$

Por materia, $\vec{v} = b\frac{d\rho}{dt} = b\dot{\beta}\hat{\beta}$ No hay en $\hat{\rho}$ porque $\dot{\rho} = 0$ ($\rho = cte$)

Además, $2\theta + (180 - \beta) = 180$

$$\beta = 2\theta$$

$$\boxed{\dot{\beta} = 2\dot{\theta}} \longrightarrow \boxed{V = 2b\dot{\theta}}$$

Además, $\vec{a} = \frac{d\vec{v}}{dt} = b\ddot{\beta}\hat{\beta} - b\dot{\beta}^2\hat{\rho} = b2\ddot{\theta}\hat{\beta} - b4\dot{\theta}^2\hat{\rho}$

$$= 2b[\ddot{\theta}\hat{\beta} - 2\dot{\theta}^2\hat{\rho}]$$

$$\boxed{|\vec{a}| = 2b\sqrt{\ddot{\theta}^2 + 4\dot{\theta}^4}}$$

Es lo mismo que decir

$$\vec{a} = (\ddot{\rho} + \rho\dot{\theta}^2)\hat{\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\hat{\theta}$$

Pregunta 2

$$\rho = Ae^{k\theta} \quad , \quad \overbrace{v = cte = V_0}^{\text{Rapidez, o sea } |\vec{v}|}$$

a) $\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} \rightarrow \vec{v} = Ake^{k\theta}\dot{\theta}\hat{\rho} + Ae^{k\theta}\dot{\theta}\hat{\theta}$

$$\boxed{\vec{v} = Ae^{k\theta}\dot{\theta}[k\hat{\rho} + \hat{\theta}]}$$

Notar que:

$\rho = Ae^{k\theta}$

$\dot{\rho} = Ake^{k\theta}\dot{\theta}$

$\ddot{\rho} = Ak^2e^{k\theta}\dot{\theta}^2 + Ake^{k\theta}\ddot{\theta} = Ake^{k\theta}[k\dot{\theta}^2 + \ddot{\theta}]$

$\dot{\theta}$ es desconocido $\Rightarrow |\vec{v}|^2 = A^2e^{2k\theta}\dot{\theta}^2(k^2+1) = V_0^2$

$$\dot{\theta} = \sqrt{\frac{V_0^2}{(k^2+1)e^{2k\theta}A^2}} = \frac{V_0 e^{-k\theta}(k^2+1)^{-1/2}}{A}$$

$$\Rightarrow \boxed{\vec{v} = V_0(k^2+1)^{-1/2}[k\hat{\rho} + \hat{\theta}]}$$

b) $\vec{a} = (\ddot{\rho} - \rho\dot{\theta}^2)\hat{\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\hat{\theta}$

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = -\frac{kV_0}{A}(k^2+1)^{-1/2}e^{-k\theta}\cdot\dot{\theta} = \frac{-kV_0^2}{A^2}\cdot\frac{e^{-2k\theta}}{(k^2+1)}$$

Reemplazando $\rho, \dot{\rho}, \ddot{\rho}, \dot{\theta}$ y $\ddot{\theta}$ en $\vec{a}$ se obtiene

$$\boxed{\vec{a} = \frac{V_0^2}{A}\cdot\frac{e^{-k\theta}}{(k^2+1)}[k\hat{\theta} - \hat{\rho}]}$$

c) Para demostrar que dos vectores son perpendiculares se verifica que su producto punto sea 0.

$$\vec{a}\cdot\vec{v} = \frac{V_0^2 e^{-k\theta}}{A(k^2+1)}(-1, k) \cdot \frac{1}{(k^2+1)}(k, 1)$$

$$\vec{a}\cdot\vec{v} = \left(\begin{matrix}\text{constante}\\ \text{Horrible :)}\end{matrix}\right)(-k+k) = 0 //$$

d) Sabemos que $\dot{\theta} = \frac{d\theta}{dt}$ $\longrightarrow$ $\Rightarrow \frac{d}{dt}\left(e^{k\theta}\right) = \frac{kV_0}{A}(k^2+1)^{-1/2}$

$$\Rightarrow \dot{\theta}\, e^{k\theta} = \frac{V_0}{A}(k^2+1)^{-1/2}$$ $\rightarrow$ Calculado en a)

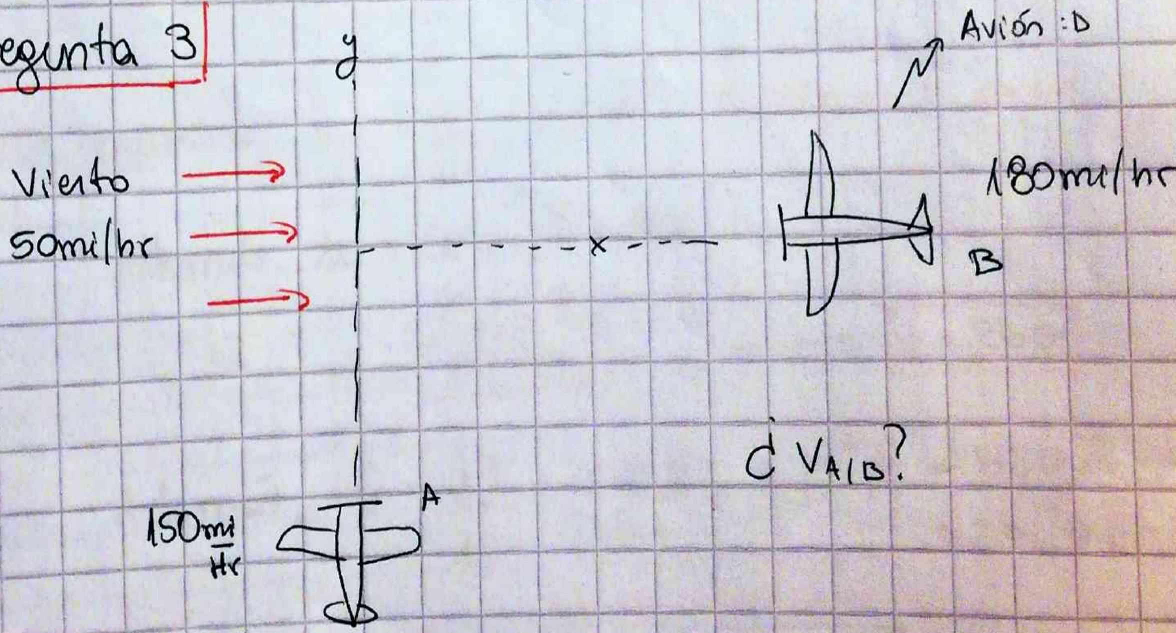
$$\Rightarrow \int_{\theta_0}^{\theta(t)} \frac{d}{dt}\left(e^{k\theta}\right) dt = \int_0^t \frac{kV_0}{A}(1+k^2)^{-1/2}\, dt$$

$$e^{k\theta} - e^{k\theta_0} = \frac{kV_0}{A} \cdot \frac{t}{(1+k)^{1/2}} - 0$$

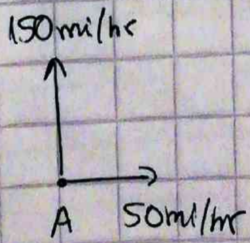
$$\boxed{\theta(t) = \frac{1}{k}\ln\left[e^{k\theta_0} + \frac{kV_0}{A}\,\frac{t}{(1+k^2)^{3/2}}\right]}$$

Nota: $\theta(0) = \theta_0$ (Depende de las condiciones iniciales)

Pregunta 3



Para A



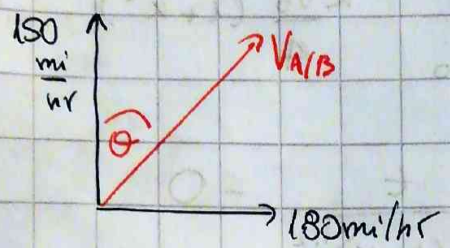
$$\vec{V}_A = 50\,\hat{i} + 150\,\hat{j}\ \frac{mi}{hr}$$

Para B

$$\vec{V}_B = (180 - 50)\,\hat{\imath} = -130\,\hat{\imath} \text{ mi/hr}$$

$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$

$$= 180\hat{\imath} + 150\hat{\jmath} \text{ mi/hr} \rightarrow \boxed{|\vec{V}_{A/B}| = 234 \text{ mi/hr}}$$



$$\boxed{\theta = tg^{-1}\left(\frac{180}{150}\right) = 50.2^\circ}$$