Francisco Zamorano

Taller 1 - Secciones 7 y 8

Problema 1 $a = g(1-cv^2)$, ¿$v(t_1) = ??$ ¿$v_{ter}, t\to\infty$?

$t_1 = 5 seg$, $g = 9{,}81 m/s$, $c = 10^{-4} s^2/m^2$

$$a = \frac{dv}{dt} = g(1-cv^2)$$

$$\int_0^{V_1} \frac{dv}{1-cv^2} = \int_0^{t_1} g\,dt \to \int_0^{V_1} \frac{dv}{1-cv^2} = \int \frac{dv}{(1+\sqrt{c}v)(1-\sqrt{c}v)}$$

$$\frac{A}{1+\sqrt{c}v} + \frac{B}{1-\sqrt{c}\cdot v}$$

$$A(1-\sqrt{c}\cdot v) + B(1+\sqrt{c}v)$$
$$A+B+(\sqrt{c}v)(B-A)$$

$\Rightarrow A+B=1$
$B-A=0$

$B = \frac{1}{2}$

$A = \frac{1}{2}$

$$\Rightarrow \int_0^{V_1}\left[\frac{1}{2(1+\sqrt{c}\cdot v)} + \frac{1}{2(1-\sqrt{c}v)}\right]dv = gt_1$$

$$\frac{1}{2}\int_0^{V_1}\left[\frac{1}{1+\sqrt{c}v} + \frac{1}{1-\sqrt{c}\cdot v}\right]dv = gt_1$$

$u = 1+\sqrt{c}\cdot v / d/d$ $\quad$ $x = 1-\sqrt{c}\cdot v$

$du = \sqrt{c}\cdot dv$ $\quad$ $dx = -\sqrt{c}\,dv$

$$\frac{1}{2}\int_0^{V_1} \frac{du}{\sqrt{c}\cdot u} + \frac{1}{2}\int_0^{V_1} \frac{dx}{-\sqrt{c}\,x}$$

$$\frac{1}{2\sqrt{c}} \ln(u) - \frac{1}{2\sqrt{c}} \ln(x)$$

$$\frac{1}{2\sqrt{c}}\ln\left(1+\sqrt{c}\,v_1\right)-\frac{1}{2\sqrt{c}}\ln\left(1-\sqrt{c}\,v_1\right)=gt_1$$

$$\frac{1}{2\sqrt{c}}\ln\left(\frac{1+\sqrt{c}\,v_1}{1-\sqrt{c}\,v_1}\right)=gt_1$$

$$\frac{1+\sqrt{c}\,v_1}{1-\sqrt{c}\,v_1}=e^{2\sqrt{c}gt_1}$$

$$1+\sqrt{c}\,v_1=e^{2\sqrt{c}gt_1}-e^{2\sqrt{c}gt_1}\sqrt{c}\,v_1$$

$$1+\sqrt{c}\,v_1+e^{2\sqrt{c}gt_1}\sqrt{c}\,v_1=e^{2\sqrt{c}gt_1}$$

$$v_1\left(\sqrt{c}+\sqrt{c}\,e^{2\sqrt{c}gt_1}\right)=e^{2\sqrt{c}gt_1}-1$$

$c=10^{-4}$
$g=9{,}81$
$t_1=5$

$$v_1=\frac{e^{2\sqrt{c}gt_1}-1}{\sqrt{c}+\sqrt{c}\,e^{2\sqrt{c}gt_1}}$$

$$\boxed{v_1=\frac{1{,}667}{0{,}037}=45{,}46\ m/s}$$

Velocidad terminal $\Rightarrow a=0$

$$0=g\left(1-cV_{TERM}^2\right)$$

$$V_{Term}=\sqrt{\frac{1}{c}}\qquad \boxed{V_{term}=100\ m/s}$$

Problema 2 $v = 24t - t^2 + 5\sqrt{t}$

$$\frac{ds}{dt} = 24t - t^2 + 5\sqrt{t}$$

$$\int ds = \int \left[24t - t^2 + 5\sqrt{t}\right] dt$$

$$\boxed{S(t) = 12t^2 - \frac{t^3}{3} + \frac{10}{3} t^{3/2} + C}$$

→ según el gráfico, $V = 0$ en $t = 0$

$$\Rightarrow S(t=10) = 12 \cdot 10^2 - \frac{10^3}{3} + \frac{10}{3} 10^{3/2}$$

$$\boxed{S(t=10) = 972{,}07 \text{ ft/seg}}$$

Problema 3 $a = a_0 - kv^2$ ¿s? → $v = 250 \frac{km}{h}$

$a_0 = 2 m/s^2$

$k = 0{,}00004 m^{-1}$

a) si $a = a_0$

Fórmula de Cinemática → $V_f^2 = V_0^2 - 2ad$

$V_f^2 = -2 \cdot 2 \cdot s$

$$-\frac{(250^{km}/3{,}6)^2}{4} = s$$

$$\boxed{s = 1205{,}6 m}$$

b) $\cancel{\text{si } a = a_0 - kv^2}$

$$\cancel{\frac{dv}{dt} = a_0 - kv^2}$$

$$\cancel{\int^{v} \frac{dv}{-kv^2} = \int_0^t a_0 \, dt} \rightarrow \cancel{-\frac{1}{k}\left(-\frac{1}{v}\right) = a_0 t}$$

$\cancel{\frac{1}{kv} = a_0 t}$ $\qquad$ $\cancel{v = \frac{1}{a_0 t k}}$

$\cancel{\int_0^s ds = \int_0 \frac{dt}{a_0 t k}}$

$$a = a_0 - kv^2$$

$$v\frac{dv}{ds} = a_0 - kv^2$$

$$\int_0^v \frac{v\,dv}{a_0 - kv^2} = \int_0^s ds$$

$k = 0{,}00004$

$a_0 = 2$

$v = 250\,km/H = 69{,}4\,m/s$

$$x = a_0 - kv^2$$

$$dx = -kv2dv$$

$$\int_0^v \frac{\cancel{v}}{x} \cdot \frac{dx}{-2k\cancel{v}} = s$$

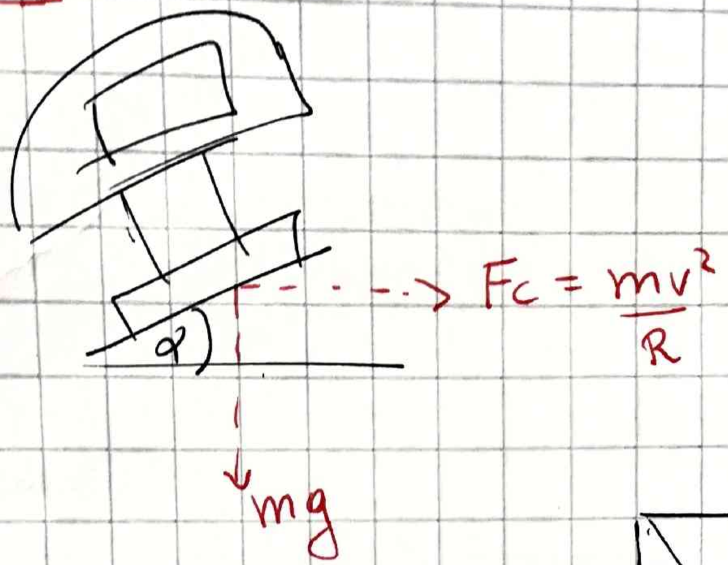
$$-\frac{1}{2k}\int_0^v \frac{dx}{x} = s$$

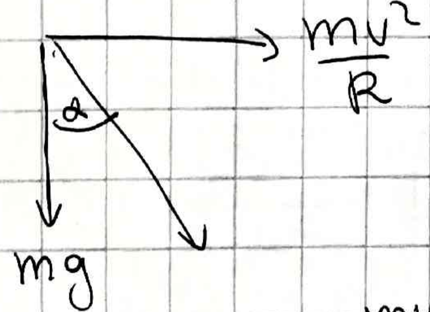
$$-\frac{1}{2k}\ln\left(\frac{a_0 - kv^2}{a_0}\right) = s$$

$\cancel{a_0 - k \cdot v^2 = e^{-ks}}$

$$\boxed{s = 1268m}$$

Problema 4





$$tg(\alpha) = \frac{\frac{mv^2}{R}}{mg}$$

$$\boxed{tg(\alpha) = \frac{v^2}{gR}}$$